Università degli Studi di Napoli Federico II

Corsi di
Dinamica e Simulazione del Volo (prof. D. Coiro)
Aeroelasticità (prof. F. Marulo)

DYNAMICS OF FLEXIBLE AIRCRAFT

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Outline

Part 1: Introduction to the dynamics of flexible aircraft

- Motivations for the study of flight dynamics of flexible aircraft
  - faster, lighter, more deformable
  - active control of deformations for improved riding qualities
- Qualitative analysis of flexibility static effects on A/C dynamics
  - fuselage deformation
  - aileron reversal

Part 2: Writing the equations of motion of flexible aircraft

- An hystorical perspective (with some technical considerations):
  - transport variables (ODE's) and deformation variables (PDE's)
  - the problem of the "body frame"
- A mixed Newtonian-Lagrangian approach
  - generalized Euler equation
  - assumed modes method

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– faster, lighter, more deformable
– active control

Analysis of static & dyn. effects
– fuselage def.
– aileron reversal

Part 2:
EoM 4 flex. A/C

Hystorical perspective
– transport vs deformation variables
– the "body frame"

Mixed Newt.-Lagr. Approach
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Part 1

Introduction to the dynamics of flexible aircraft

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Why a flexible aircraft model?

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Trend in aircraft technologies:

- faster aircraft → higher dynamic pressure → higher loads;
- lighter aircraft → less structural weight; } → more deformation.
- more efficient aircraft → slender structures; }

Result:

- wider deformations significantly alter aircraft shape, depending on flight and manoeuvre condition;
- lower structural frequencies close to flight dynamic frequencies and/or control system bandwidth.

Consequence:

- coupling between structural dynamics and piloting tasks needs to be taken into account at all levels;
- rigid body dynamics no longer sufficient for a satisfactory description of aircraft behaviour within its operational flight envelope

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What happens?

Static effects

- variation of stability margins, stability derivatives and control power (always in a worsening direction!).

Dynamic effects:

- aeroelastic response: coupling between aerodynamics and structural dynamics (includes significant unsteady aerodynamic effects) at relatively high frequency → usually not relevant for flight dynamics and control within pilot tasks bandwidth;
- **flexible aircraft response: coupling between pilot/SCAS input to control surfaces, aircraft response and deformation**

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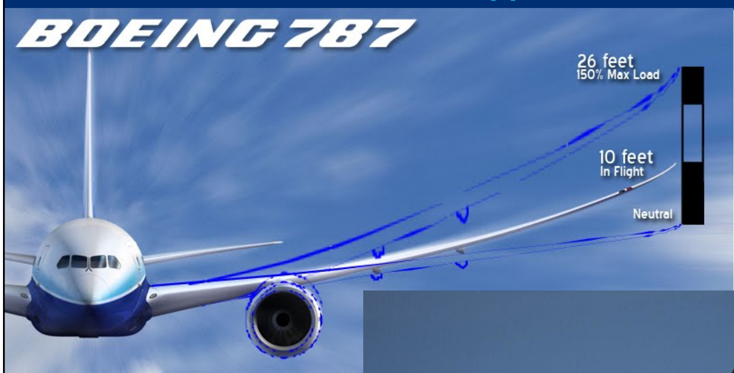
What happens?



Need for active structural control

Performance degradation in terms of

- reduced stability margins and control power
- lighter damping

requires active control systems to compensate for the effects of structural deformations on aircraft response to controls.

First application of active structural control on flying vehicles:

- rocket launchers (tested; nowadays current technology);
- transatmospheric vehicles (only theory!).

First application to fixed wing aircraft: Rockwell B1-B (Structural Mode Control System).

Many modern transport jet aircraft now features…

- active flutter suppression system;
- active structure control for improved ride qualities in turbulent air;
- and more…

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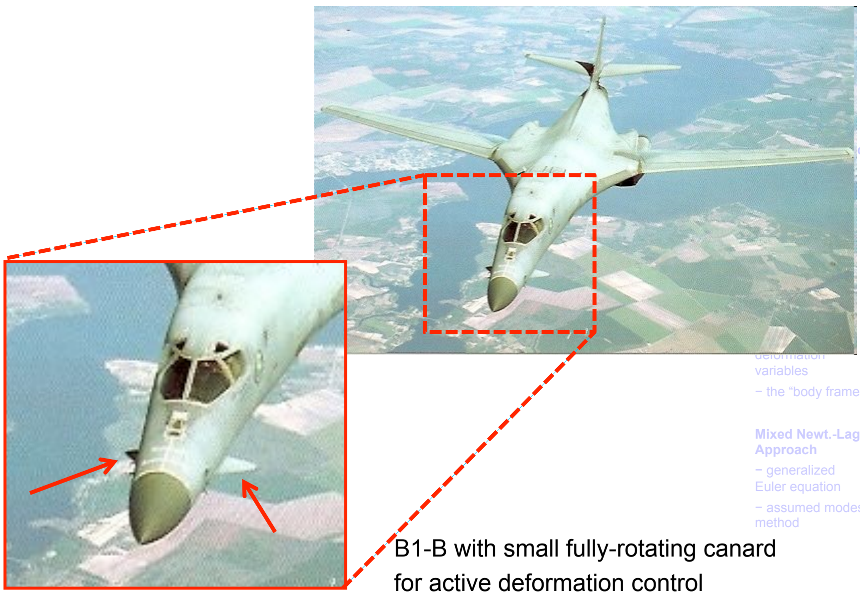
Need for active structural control



B1-B with small fully-rotating canard for active deformation control

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Modelling issues

- Flexible aircraft dyamics described by means of a **hybrid system of differential equations** (mixed and coupled ODE's and PDE's);
- Some form of discretisation for elastic DoF's is needed for direct simulation;
- Need for a simple yet reliable aerodynamic model, that allows for direct numerical simulation within a reasonable CPU time;
- Elastic DoF's frequencies may induce (usually unwanted) couplings with
  - pilot commands and response to aircraft acceleration (degradation of handling qualities);
  - actuator dynamics (aero-servo-elastic problem);
  - external disturbances (e.g. turbulence).

Engineering issues

- When an automatic control system is required, a state observer (e.g. a Kalman filter) is needed in order to provide reasonable estimate of "elastic states" from available measurements;
- The problem of estimation is made more demanding by the high level of "noise" (e.g. vibrations, turbulence, etc.);
- High performance actuators are necessary in order to control the system up to relatively high bandwidths;
- An engineering choice needs to be made between
  - **all purpose configurations** (a single set of aerodynamic surfaces used for full aircraft control, including flexible DoF's);
  - **dedicated surfaces** for deformation control (conventional aerodynamic surfaces used for aircraft control).

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Static vs dynamic flexible A/C models

- Method of **quasi-static deformations**:
  - deformations are sufficiently large to affect aircraft stability derivatives and/or control power;
  - structural mode frequencies are sufficiently high, such that only steady-state deformations affect aircraft dynamics.
- Lagrangian or Hamiltonian approaches to derive **fully dynamic models** of flexible aircraft that couples structural and transport dynamic variables:
  - based on FEM (high order, highly accurate, complex, may require unsteady aerodynamics);
  - based on low-order approximations of deformation (e.g. method of assumed modes);

  Both techniques require choice of a suitable set of "body" axes and the definition of transport variables.
- **Note:** Dynamic models discussed in Part 2.

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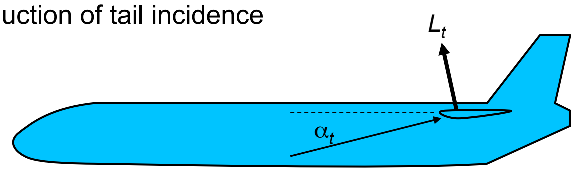
Effects of fuselage deformation (static case)

Primary effect of fuselage deformation in the longitudinal plane is a reduction of tail incidence



Tail angle of attack:

rigid aircraft $\alpha_t^* = \alpha_{wb} + i_t - \varepsilon$

with

α_{wb}	aircraft angle of attack
i_t	tail incidence setting
ε	downwash angle

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Effects of fuselage deformation

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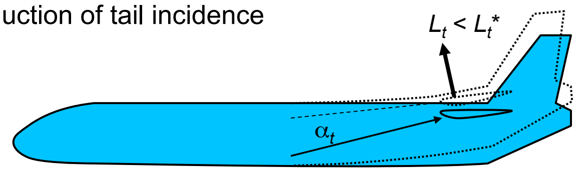

Tail angle of attack:

rigid aircraft $\alpha_t^* = \alpha_{wb} + i_t - \varepsilon$

elastic aircraft $\alpha_t = \alpha_{wb} + i_t - \varepsilon - k\, L_t$

Tail incidence is reduced by fuselage flexibility!

Tail lift coefficient: $C_{Lt} = C_{L\alpha t}\,[\alpha_{wb} + i_t - \varepsilon - k\,(½\,\rho V_t^2 S_t C_{Lt})]$

Solving for C_{Lt}: $C_{Lt} = \dfrac{C_{L\alpha t}\,(\alpha_{wb} + i_t - \varepsilon\,)}{1 + k\,½\,\rho V_t^2 S_t C_{L\alpha t}}$

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Effects of fuselage deformation

It is possible to define an effective tail lift gradient:

$$C_{L\alpha t}^{eff} = \frac{C_{L\alpha t}}{1 + k\,½\,\rho V_t^2 S_t C_{L\alpha t}} < C_{L\alpha t}$$

Effects of fuselage deformation on longitudinal static stability

Position of neutral point: $\dfrac{x_N}{\bar{c}} = \dfrac{x_{AC_{wb}}}{\bar{c}} + \dfrac{C_{L\alpha_t}}{C_{L\alpha_{wb}}}\bar{V}_H\left(1 - \dfrac{\partial\varepsilon}{\partial\alpha}\right)$

Static stability derivative: $C_{m\alpha} = C_{L\alpha}\left(\dfrac{x_{CG}}{\bar{c}} - \dfrac{x_N}{\bar{c}}\right) < 0$ if $x_{CG} < x_N$

Variation of x_N: $\dfrac{\Delta x_N}{\bar{c}} = \dfrac{\Delta C_{L\alpha_t}}{C_{L\alpha_{wb}}}\bar{V}_H\left(1 - \dfrac{\partial\varepsilon}{\partial\alpha}\right)$

where $\Delta C_{L\alpha_t} = C_{L\alpha_t}^{eff} - C_{L\alpha_t} = -\dfrac{\frac{1}{2}k\rho V_t^2 S_t C_{L\alpha_t}^2}{1 + \frac{1}{2}k\rho V_t^2 S_t C_{L\alpha_t}} < 0$

that is, **static margin is reduced**!

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Effects of fuselage deformation

Effects of fuselage deformation on longitudinal control power

Tail lift coefficient: $C_{Lt} = C_{L\alpha t}\,[\alpha_{wb} + i_t - \varepsilon - k\,L_t] + C_{Lt\delta E}\,\delta_E$

Solving for C_{Lt}:

$$C_{L_t} = \frac{C_{L\alpha_t}(\alpha_{wb}+i_t-\varepsilon)}{1+\frac{1}{2}k\rho V_t^2 S_t C_{L\alpha_t}} + \left[\frac{C_{Lt\delta_E}}{1+\frac{1}{2}k\rho V_t^2 S_t C_{L\alpha_t}}\right]\delta_E$$

Effective control power:

$$C^{eff}_{Lt\delta_E} = \frac{C_{Lt\delta_E}}{1+\frac{1}{2}k\rho V_t^2 S_t C_{L\alpha_t}} < C_{Lt\delta_E}$$

Again, **control power is reduced**!!

Effects of fuselage deformation on pitch damping

Tail incidence induced by pitch motion: $\Delta\alpha_t = q\,l_t / V_t$

Tail contribution to pitch damping coefficient: $C_{m_q} = -2C_{L\alpha_t}\bar{V}_H\frac{l_t}{\bar{c}}$

The reduction of $C_{L\alpha t}$ **decreases pitch damping coefficient too**!!!

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Effects of fuselage deformation

Effects of fuselage deformation on longitudinal modes

- Static margin reduction lowers short-period frequency
- Reduced horizontal tail lift gradient lowers damping

Consequence: worse HQ expected

- A contribution to pitch moment vs speed derivative shows up

In general $|M_u| \ll 1$ is neglected. It becomes non-negligible at high speed because of
- aerodynamic effects due to transonic aerodynamics;
- fuselage flexibility

Remembering that $C_{m_t} = -\bar{V}_H C_{L_t} = -\bar{V}_H \frac{C_{L\alpha_t}(\alpha_{wb}+i_t-\varepsilon)}{1+\frac{1}{2}k\rho V_t^2 S_t C_{L\alpha_t}}$

one gets $C_{m_u} = \frac{\partial C_{m_t}}{\partial(\Delta u / V)} = -\bar{V}_H V\frac{\partial C_{L_t}}{\partial u}$

$$\approx \bar{V}_H\frac{C_{L\alpha_t}(\alpha_{wb}+i_t-\varepsilon)}{\left(1+\frac{1}{2}k\rho V_t^2 S_t C_{L\alpha_t}\right)^2}k\rho V_t^2 S_t C_{L\alpha_t} = -\frac{2kQ_t S_t C_{L\alpha_t}}{1+kQ_t S_t C_{L\alpha_t}}C_{m_t}$$

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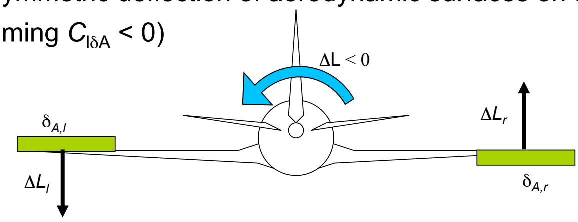
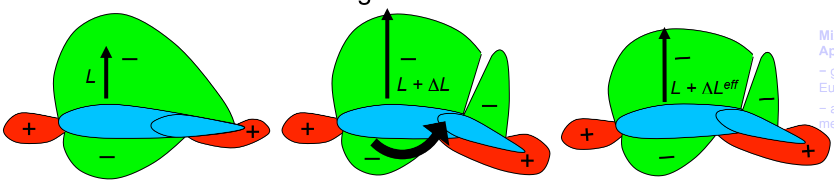
Wing torsion induced by aileron deflection

- Aileron deflection generates a rolling moment by means of an anti-symmetric deflection of aerodynamic surfaces on the wing (assuming $C_{l\delta A} < 0$)



- A significant torsion moment is also developed, that reduces wing section incidence and, as a consequence, the lift increment and the resulting control moment



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Wing torsion induced by aileron deflection

- Torsion moment is roughly proportional to dynamic pressure and aileron deflection: $M_T \propto \frac{1}{2}\rho V^2 \delta_A$
- Torsional deformation ϑ is proportional to torsion moment M_T, and it corresponds to a variation of airfoil incidence in the opposite direction, that is: $\vartheta(y) = -\Delta\alpha(y) \propto M_T$
- As a consequence, the variation of roll control moment coefficient ΔC_ℓ due to torsion effects can be written in the form

$$\Delta C_\ell = k_T \tfrac{1}{2}\rho V^2 \delta_A$$

- The resulting total roll control moment is thus given by

$$\Delta C_\ell^{tot} = \Delta C_\ell^{rig} + \Delta C_\ell^{flex} = C_{\ell_{\delta_A}} \delta_A + \tfrac{1}{2} k_T \rho V^2 \delta_A$$

and the effective control moment gradient becomes

$$C_{\ell_{\delta_A}}^{eff} = C_{\ell_{\delta_A}} + \tfrac{1}{2} k_T \rho V^2$$

- Aileron reversal speed V_R is defined as the velocity such that the control gradient vanishes $C_{\ell_{\delta_A}} + \frac{1}{2} k_T \rho V_R^2 = 0$
- The effective roll control gradient becomes $C_{\ell_{\delta_A}}^{eff} = C_{\ell_{\delta_A}} \left[1 - \frac{V^2}{V_R^2}\right]$

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Part 2

Equations of motion of flexible aircraft

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Why is it so difficult?

Transport vs deformation variables

- Transport variables (position vector, speed, Euler angles and angular velocity)
  - depend on time *t* only;
  - describe "global" properties of aircraft motion;
  - their evolution can be described in terms of a set of ODE's.
- Deformation variables
  - depend on time *t* and position in an aircraft-"fixed" frame;
  - describe "local" properties of structure motion;
  - their evolution is described by means of a set of PDE's;
- A hybrid set of highly coupled highly nonlinear ordinary and partial differential equations is obtained.
- Some form of discretization of the latter is required.
- Coupling extends to inertial terms (highest differential order), so that the system of O+PDE's is in non-normal form.

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Why is it so difficult?

Is there something like an aircraft-fixed frame?
Unfortunately the answer is NO!!!

- Body-fixed frame centred in aircraft CoG defined for the rigid aircraft case only.
- Position of CoG and other inertial properties (e.g. moments of inertia) depend on deformation state.
- At the same time a frame representative of aircraft position and attitude is necessary!
- Two possible choices:
    1. Mean axes (Milne, 1964)
    2. Pseudo-body axes (Tuzcu and Meirovitch, 2003)

Linear and angular momentum balance

Momentum balance $\frac{d}{dt}(mv_C) = \Sigma F$

Absolute angular momentum balance (pole O fixed in the inertial frame) $\frac{d}{dt}(h_O) = \Sigma M_O$

Relative angular momentum balance (only when pole C is the centre of mass) $\frac{d}{dt}(h_C) = \Sigma M_C$

For a deformable body it is

$$h_C = \int_{\mathcal{B}} (r \times v)\delta m = \int_{\mathcal{B}} \left[r \times (v_C + \omega \times r + \dot{r})\right]\delta m =$$

$$= \underbrace{\left(\int_{\mathcal{B}} r\delta m\right)}_{0} \times v_C + \underbrace{\int_{\mathcal{B}} \left[r \times (\omega \times r)\right]\delta m}_{I\omega} + \underbrace{\int_{\mathcal{B}} (r \times \dot{r})\delta m}_{}$$

0 by definition of CoM; $I\omega$ "rigid-body" term; distortional component of angular momentum

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Definition of the “mean axes” frame

Statement: whichever the deformation state, there always exists a frame centred in the centre of mass such that

$$\int_{\mathcal{B}} \left(r \times \dot{r} \right) \delta m = 0$$

This frame is known as the **mean axes frame**.

Proof: assume two different body-frames, F_{B1} and F_{B2}. It is

$$h_{C1} = I_{B1}\omega_{B1} + \int_{\mathcal{B}1} \left(r_{B1} \times \dot{r}_{B1} \right) \delta m$$

$$h_{C2} = I_{B2}(\omega + \Delta\omega)_{B2} + \int_{\mathcal{B}2} \left(r_{B2} \times \dot{r}_{B2} \right) \delta m$$

$$\begin{aligned} \int_{\mathcal{B}2} \left(r_{B2} \times \dot{r}_{B2} \right) \delta m &= \int_{\mathcal{B}2} \left(T_{B2B1} \tilde{R}_{B1} T_{B1B2} \right) \left[T_{B2B1} \left(\dot{r}_{B1} + \Delta\omega_{B1} \times r_{B1} \right) \right] \delta m \\ &= T_{B2B1} \int_{\mathcal{B}2} \left(\tilde{R}_{B1} \dot{r}_{B1} \right) \delta m + T_{B2B1} \int_{\mathcal{B}2} \left[\tilde{R}_{B1} \left(\Delta\omega_{B1} \times r_{B1} \right) \right] \delta m \\ &= T_{B2B1} \left[\int_{\mathcal{B}2} \left(r_{B1} \times \dot{r}_{B1} \right) \delta m + I_{B1} \Delta\omega_{B1} \right] \end{aligned}$$

The distortional term thus disappears if $\Delta\omega_{B1} = -I_{B1}^{-1} \int_{\mathcal{B}2} \left(r_{B1} \times \dot{r}_{B1} \right) \delta m$

that is, the coordinate transformation matrix evolves according to the equation $T_{B1B2} = \Delta\tilde{\Omega}_{B1} T_{B1B2}$.

In such a case the angular momentum has the form $h_C = I\omega$

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Pros & cons of the “mean axes” frame

Statement: whichever the deformation state, there always exists a frame centred in the centre of mass such that

$$\int_{\mathcal{B}} \left(r \times \dot{r} \right) \delta m = 0$$

This frame is known as the **mean axes frame**.

Pros:
- frame centred in the vehicle centre of mass → relative angular momentum balance equation holds;
- major inertial coupling term between transport and deformation DoF’s is removed from E.o.M. → if variations of inertia tensor are negligible, no coupling on the LHS of E.o.M.

Cons:
- significant coupling between transport and deformation DoF’s is always present in the aerodynamic terms;
- centre of mass depends on deformation state, so that its velocity depends on deformation rate too;
- most important: mean axes always exist, but the identification of their actual position is far from trivial (i.e. truly difficult!).

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Pros & cons of the "mean axes" frame

Definition: A set of **pseudo-body axes** is given by a reference frame attached to the centre of mass of the undeformed aircraft structure and fixed with respect to it.

Pros:
- simple definition;
- deformation state naturally described with respect to the undeformed condition.

Cons:
- the actual centre of mass moves with respect to the origin of the frame → usual angular momentum balance equations (w.r.t. the CoM) can no longer be used;
- fully coupled equations are obtained (local inertial acceleration depends on transport acceleration, which in turn depends on time derivative of deformation rates).

Why is it so difficult? Last issue

Derivation of a finite order model (ODE system) may follow different paths:

- Direct derivation from a global Hamiltonian discretized by means of FEM in a set of mean axes (Cavin III & Dusto, 1977);
- Inertially decoupled models with aerodynamic coupling only in a set of approximate mean axes (Waszak et al., 1987);
- Direct derivation from a global Lagrangian function in terms of quasi-velocity variables expressed in a set of pseudo-body axes; discretization performed on the full set of hybrid O+PDE's (Tuzcu and Meirovitch, 2003);
- Mixed Newtonian-Lagrangian approach in a set of pseudo-body axes (Avanzini, Capello, Piacenza, 2014; from approach by Junkins et al. for space structures); discretization of deformation variables performed on the Lagrangian → **amplitudes of assumed modes** used as **generalized variables**; transport dynamics derived by **generalized Euler equation**.

Generalized Euler equation

Momentum balance (for constant mass): $\frac{d}{dt}(mv_C) = F$

Relative angular momentum balance (referred to body centre of mass) $\frac{d}{dt}(h_C) = M_C$

For a different pole A ≠ C it is $M_A = M_C + r_{AC} \times F$ and $a_C = a_A + \frac{d^2 r_{AC}}{dt^2}$

$$h_A = \int_{\mathcal{B}} \left(r_{AP} \times \frac{dr_{AP}}{dt} \right) \delta m = \int_{\mathcal{B}} \left[(r_{AC} + r_{CP}) \times \frac{d}{dt}(r_{AC} + r_{CP}) \right] \delta m$$

$$= \int_{\mathcal{B}} \left(r_{AC} \times \frac{dr_{AC}}{dt} \right) \delta m + \cancel{\int_{\mathcal{B}} \left(r_{AC} \times \frac{dr_{CP}}{dt} \right) \delta m} + \cancel{\int_{\mathcal{B}} \left(r_{CP} \times \frac{dr_{AC}}{dt} \right) \delta m} + \int_{\mathcal{B}} \left(r_{CP} \times \frac{dr_{CP}}{dt} \right) \delta m$$

0 — from the definition of CoM

$$= m \left(r_{AC} \times \frac{dr_{AC}}{dt} \right) + h_C$$

$$\frac{d}{dt} \left(h_A - m r_{AC} \times \frac{dr_{AC}}{dt} \right) = M_A - m r_{AC} \times \left(a_A + \frac{d^2 r_{AC}}{dt^2} \right)$$

$$\frac{dh_A}{dt} - \cancel{m r_{AC} \times \frac{d^2 r_{AC}}{dt^2}} = M_A - m r_{AC} \times a_A - \cancel{m r_{AC} \times \frac{d^2 r_{AC}}{dt^2}}$$

Generalized Euler equation: $\boxed{\frac{dh_A}{dt} + s_A \times a_A = M_A}$

where $s_A = m r_{AC}$ is the static moment

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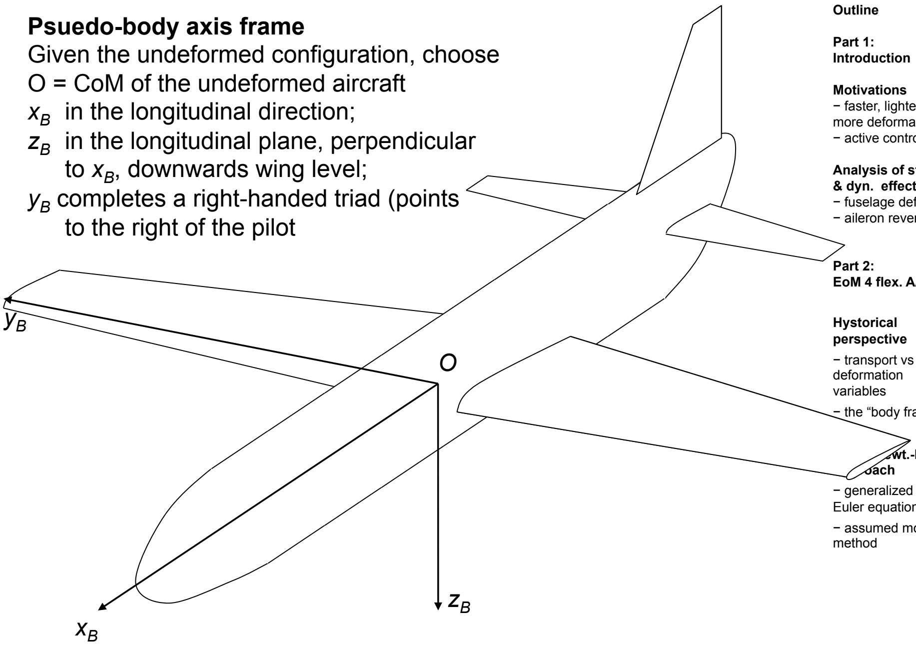
Description of flexible aircraft

Psuedo-body axis frame
Given the undeformed configuration, choose
O = CoM of the undeformed aircraft
x_B in the longitudinal direction;
z_B in the longitudinal plane, perpendicular to x_B, downwards wing level;
y_B completes a right-handed triad (points to the right of the pilot



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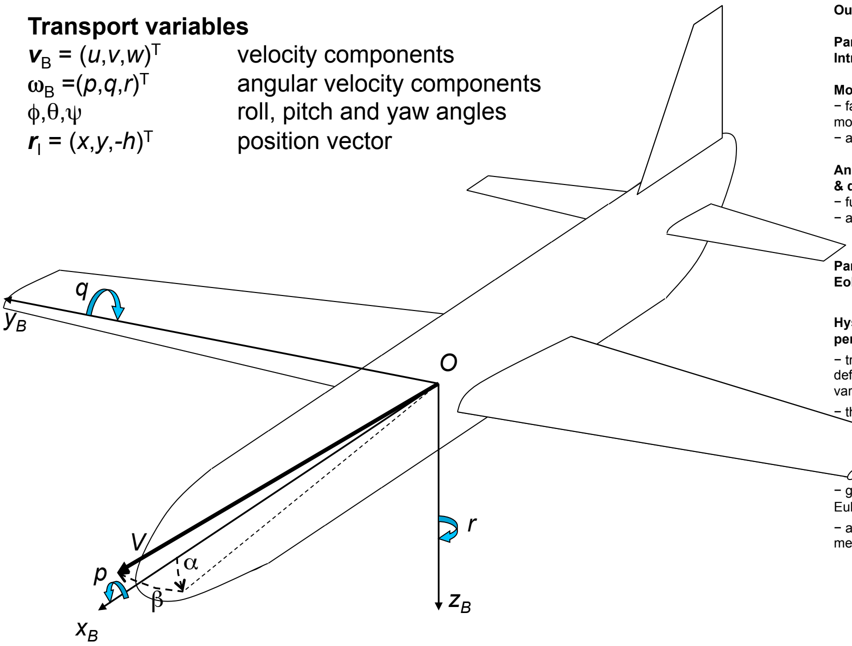
Description of flexible aircraft

Transport variables

$\boldsymbol{v}_B = (u,v,w)^T$ velocity components
$\omega_B = (p,q,r)^T$ angular velocity components
ϕ,θ,ψ roll, pitch and yaw angles
$\boldsymbol{r}_I = (x,y,-h)^T$ position vector



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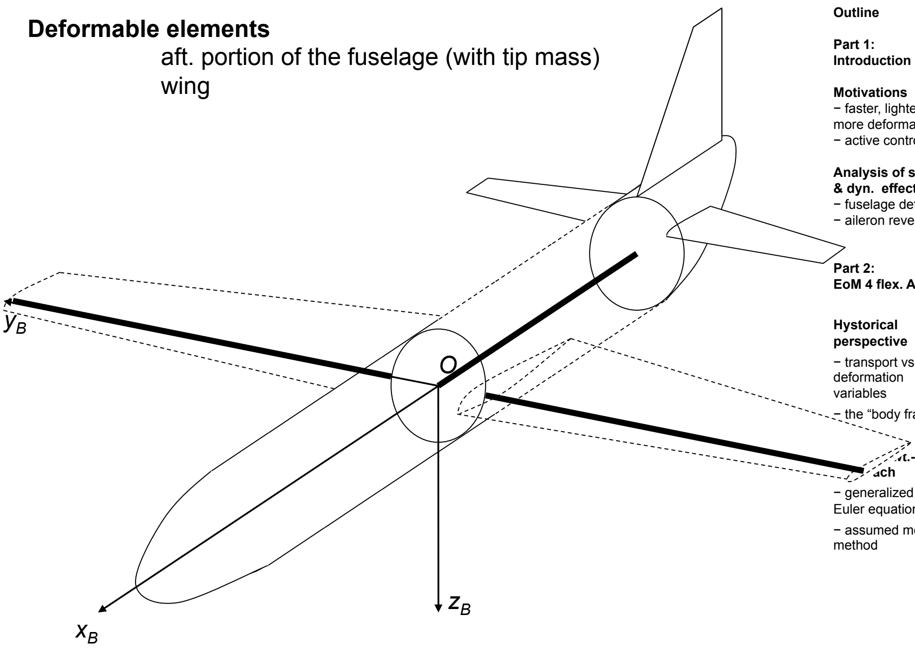
Description of flexible aircraft

Deformable elements

aft. portion of the fuselage (with tip mass)
wing



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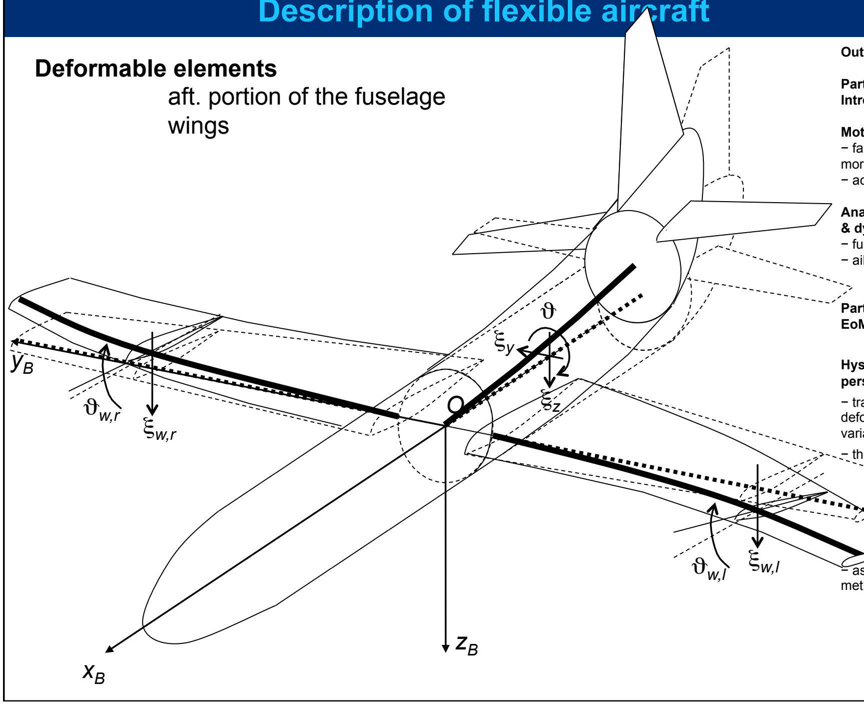

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State variables and discretization

State vector $\boldsymbol{x} = (\boldsymbol{x}_T^T, \boldsymbol{x}_D^T, \dot{\boldsymbol{x}}_D^T)^T$

Transport variables: $\boldsymbol{x}_T = (u,v,w;p,q,r;\phi,\theta,\psi;\Delta x,\Delta y,-h)^T$

Deformation variables:

1. Discretization

Fuselage (i=z,y) flexural deformation (z=longitudinal; y=lateral)

$$\xi_{f,i}(x_f,t)=\sum_{k=1}^{N}\eta_k^{(f,i)}(t)\Phi_k(x_f)$$

Fuselage torsional deformation

$$\vartheta_f(x_f,t)=\sum_{k=1}^{N}\sigma_k^{(f)}(t)\Theta_k(x_f)$$

Wing (i=r,l) flexural deformations (r=right; l=left)

$$\xi_{w,i}(x_{w,i},t)=\sum_{k=1}^{N}\eta_k^{(w,i)}(t)\Phi_k(x_f)$$

Wing (i=r,l) torsional deformation (r=right; l=left)

$$\vartheta_{w,i}(x_{w,i},t)=\sum_{k=1}^{N}\sigma_k^{(w,i)}(t)\Theta_k(x_f)$$

2. Resulting state vector for deformation variables:

$$\boldsymbol{x}_D=(\eta_1^{(f,y)},\ldots,\eta_N^{(f,y)};\ \eta_1^{(f,z)},\ldots,\eta_N^{(f,z)};\ \sigma_1^{(f)},\ldots,\sigma_N^{(f)};\ \eta_1^{(w,l)},\ldots,\eta_N^{(w,l)};\ \eta_1^{(w,r)},\ldots,\eta_N^{(w,r)};\ \sigma_1^{(w,l)},\ldots,\sigma_N^{(w,l)};\ \sigma_1^{(w,r)},\ldots,\sigma_N^{(w,r)})^T$$

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Lagrangian derivation of flexible dynamics

Kinetic energy:

Definition $T = \frac{1}{2}\int_{\mathcal{B}} \left(v_P \cdot v_P\right)\delta m$

$$= \int_{\mathcal{B}} \left[v_O + \omega\times\left(r_P+\rho_P\right)+\rho_P\right]\cdot\left[v_O + \omega\times\left(r_P+\rho_P\right)+\rho_P\right]\delta m$$

Terms $\mathbf{T} = \mathbf{T}\,(\boldsymbol{v}_B, \omega_B, \dot{\boldsymbol{x}}_D)$

$$= \mathbf{T}_{\text{RIG}}(\boldsymbol{v}_B, \omega_B) + \mathbf{T}_{\text{FLEX}}(\dot{\boldsymbol{x}}_D) + \mathbf{T}_{\text{COUP}}(\boldsymbol{v}_B, \omega_B, \dot{\boldsymbol{x}}_D)$$

Potential energy:

$$V = \frac{1}{2}\int_0^{\ell_f}\left[EI_{f,y}(x_f)\left(\frac{\partial^2 \xi_{f,z}}{\partial x^2}\right)^2 + EI_{f,z}(x_f)\left(\frac{\partial^2 \xi_{f,y}}{\partial x^2}\right)^2 + GJ_{f,x}(x_f)\left(\frac{\partial \vartheta_f}{\partial x}\right)^2\right]dx_f$$

$$+\frac{1}{2}\int_0^{\ell_w}\left[EI_{w,l}(x_w)\left(\frac{\partial^2 \xi_{w,l}}{\partial x^2}\right)^2 + GJ_{w,l}(x_w)\left(\frac{\partial \vartheta_{w,l}}{\partial x}\right)^2\right]dx_w$$

$$+\frac{1}{2}\int_0^{\ell_w}\left[EI_{w,r}(x_w)\left(\frac{\partial^2 \xi_{w,r}}{\partial x^2}\right)^2 + GJ_{w,r}(x_w)\left(\frac{\partial \vartheta_{w,r}}{\partial x}\right)^2\right]dx_w$$

Terms $\mathbf{V} = \mathbf{V}\,(\boldsymbol{x}_D) = ½\, \boldsymbol{x}_D^{\mathrm{T}}\, \boldsymbol{K}\, \boldsymbol{x}_D$ with $\boldsymbol{K}$ block diagonal

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Lagrangian derivation of flexible dynamics

Lagrange equations

$$\frac{d}{dt}\left(\frac{d\mathrm{T}}{dx_D}\right) - \left(\frac{d\mathrm{T}}{dx_D}\right) + \left(\frac{d\mathrm{V}}{dx_D}\right) = Q$$

with Q a vector of generalized non-conservative forces (including aerodynamic ones)

$$\delta W = \sum_{k=1}^{N_{tot}} Q_k \delta x_{D_k} = \int_B f_P^{(NC)} \cdot \delta\xi$$

Comments:

- Aerodynamics forces can be expressed in the form of a Raileigh function
  $$\mathbf{F} = \mathbf{F}\,(\dot{\boldsymbol{x}}_D) = ½\, \dot{\boldsymbol{x}}_D^{\mathrm{T}}\, \boldsymbol{F}\, \dot{\boldsymbol{x}}_D$$
  such that $Q_A = \boldsymbol{F}\dot{\boldsymbol{x}}_D = \partial\mathbf{F}/\,\partial\dot{x}_D$ (omitted for sake of brevity).
- The coupling term $\mathbf{T}_{\text{COUP}}(\boldsymbol{v}_B, \omega_B, \dot{\boldsymbol{x}}_D)$ in the kinetic energy causes inertial coupling between flexible variable dynamics and transport acceleration.
- The additional term in the generalized Euler equation ($m r_{AC} x a_A$) and the variation of mass properties with deformation couple transport variable dynamics with deformation rate derivatives.

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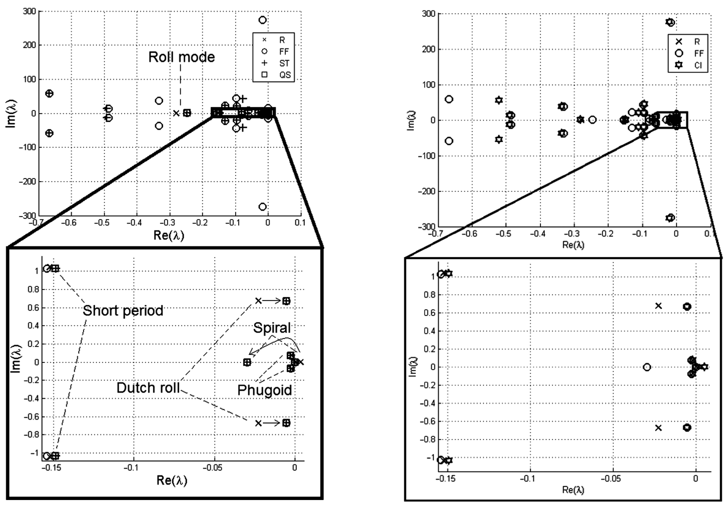
Root loci



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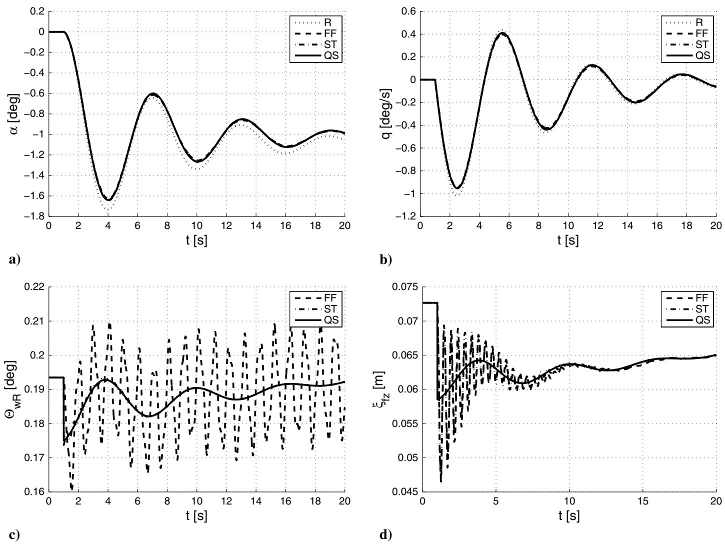
Time-domain response



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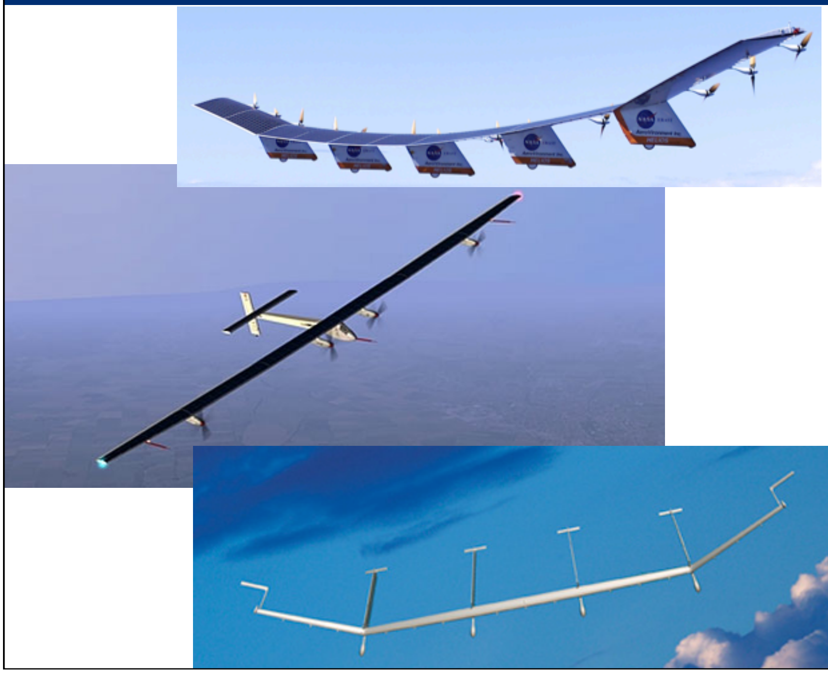
Open issues…



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… that someone should solve!

– the "body frame"

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Questions?

If not too bored...

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