Variable Stiffness Actuation: Modeling and Control

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***Abstract*— Physical human-robot interaction requires the development of safe and dependable robots. This involves the mechanical design of lightweight and compliant manipulators and the definition of motion control laws that allow to combine compliant behavior in reaction to possible collisions, while preserving accuracy and performance of rigid robots in free space. In this talk, a basic control study for a general class of multi-dof manipulators with variable joint stiffness is presented. It is shown that nonlinear control laws, based either on static or dynamic state feedback, are able to exactly linearize the closed loop equations and allow to simultaneously impose a desired behavior to the robot motion and to the joint stiffness in a decoupled way.**

***Index Terms*— Robot Manipulators, Elastic Joints, Variable Stiffness, Feedback Linearization, Nonlinear Systems.**

I. Introduction

It has been shown in [1] how the intrinsic safety of robotic arms can be improved, besides maintaining a low level of inertia, introducing also an high compliance at the mechanical level both in the joints of the robot and in the interface between the robot and the environment. In order to obtain also an adequate level of both static and dynamic performances, the use of variable stiffness devices allows to satisfy all the requirements for a safe and accurate interaction with humans and unknown environments.

This talk aims to show how the full state linearization and simultaneous control of both the position and the stiffness of a robotic manipulator with variable joint stiffness can be achieved via static or dynamic feedback. We suppose that the mechanical stiffness of the joint can be modulated by means of external control inputs.

II. Dynamic Model of the System

The model of a n-DOF robotic manipulator with elastic joints is composed by the dynamics of $2n$ rigid bodies (n links and n actuators), coupled through the elastic joints. Let $q \in \mathbb{R}^n$ and $\theta \in \mathbb{R}^n$ be, respectively, the generalized coordinates of the driven links and of the driving actuators. Under the simplifying modeling assumption used in [2] , the dynamic model can be written as:

$$M(q)\,\ddot{q} + N(q,\dot{q}) + K\,(q-\theta) = 0 \qquad (1)$$

$$B\,\ddot{\theta} + K\,(\theta - q) = \tau, \qquad (2)$$

where $M(q)$ is the inertia matrix of the robot links, vector $N(q,\dot{q})$ contains the centrifugal, Coriolis, and gravity forces, $K = \mathrm{diag}\{k_1,\,\dots,\,k_n\} > 0$ is the joint stiffness matrix, $B = \mathrm{diag}\{b_1,\,\dots,\,b_n\}$ is the inertia matrix of the actuators, and $\tau \in \mathbb{R}^n$ are the motor torques. Damping at the joints can be also included —see, e.g., [3]. In the following, we will also use the equivalent notation

$$K\,(q-\theta) = \Phi k, \qquad (3)$$

with matrix

$$\Phi = \mathrm{diag}\{(q_1-\theta_1),\,(q_2-\theta_2),\,\dots,(q_n-\theta_n)\} \qquad (4)$$

and vector $k = \begin{bmatrix} k_1 & \dots & k_n \end{bmatrix}^T \in \mathbb{R}^n$.

Here, the joint stiffness matrix K in eqs. (1–2) will not be considered constant but, in general, a function of time:

$$K = K(t) > 0\ \forall t. \qquad (5)$$

The simplest situation is when the joint stiffness k_i can be directly changed by means of a (suitably scaled) additional command τ_{k_i}, for $i = 1,\dots,n$. In vector form,

$$k = \tau_k. \qquad (6)$$

Therefore, the overall available input u and the robot state x are:

$$u = \begin{bmatrix} \tau & \tau_k \end{bmatrix}^T \in \mathbb{R}^{2n}, \quad x = \begin{bmatrix} q & \dot{q} & \theta & \dot{\theta} \end{bmatrix}^T \in \mathbb{R}^{4n}.$$

Indeed, the dynamics of change of the stiffness of the joints may not be neglected. In this case, we can model the variation of joint stiffness as a second-order dynamic system

$$\ddot{k} = \phi(x,k,\dot{k},\tau_k), \qquad (7)$$

in which the dependence include also the stiffness and their time derivatives. In this case, equations (1–2) should be complemented by (7) in order to represent the complete dynamic model of a robot with variable joint stiffness.

As a result, the state vector of the robot is extended and becomes:

$$x_e = \begin{bmatrix} q^T & \dot{q}^T & \theta^T & \dot{\theta}^T & k^T & \dot{k}^T \end{bmatrix}^T \in \mathbb{R}^{6n}, \qquad (8)$$

so that eq. (7) can be rewritten as:

$$\ddot{k} = \phi(x_e, \tau_k). \qquad (9)$$

In all cases, the objective will be to simultaneously control the following set of outputs

$$y = \begin{bmatrix} q & k \end{bmatrix}^T \in \mathbb{R}^{2n},$$

namely the link positions and the joint stiffness.

III. Feedback Linearization

It is possible to write the joint stiffness k as generic nonlinear functions of the system state variables q and θ:

$$\ddot{k} = \beta(q,\theta) + \gamma(q,\theta)\,\tau_k. \qquad (10)$$

Then, the overall system can be written in a more compact form:

$$\begin{bmatrix} q^{[4]} \\ \ddot{k} \end{bmatrix} = \begin{bmatrix} \alpha(x_e) \\ \beta(q,\theta) \end{bmatrix} + Q(x_e) \begin{bmatrix} \tau \\ \tau_k \end{bmatrix} \qquad (11)$$

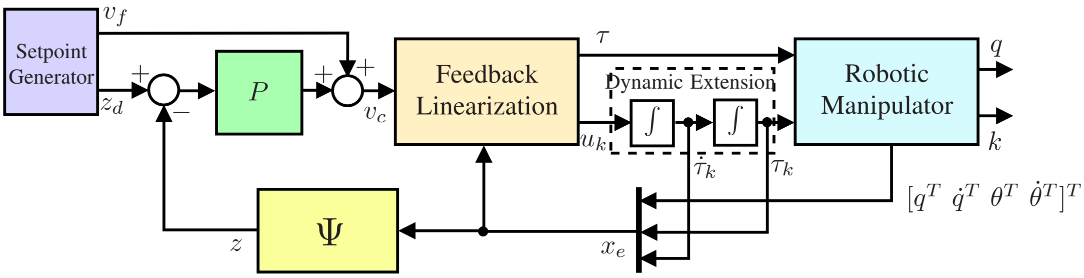

Fig. 1. Scheme of the feedback linearization controller with dynamic extension.

where

$$\alpha(x_e) = -M^{-1}\Big[2\,\dot{M}\,q^{[3]} + (\ddot{M} + K)\,\ddot{q} + \ddot{N} + KB^{-1}K\,(\theta - q) + 2\,\dot{K}\,(\dot{q} - \dot{\theta}) + \Phi\,\beta\Big] \quad (12)$$

and $Q(x_e)$ is the so called decoupling matrix:

$$Q(x_e) = \begin{bmatrix} M^{-1}KB^{-1} & M^{-1}\Phi\,\gamma(q,\theta) \\ 0_{n\times n} & \gamma(q,\theta) \end{bmatrix} \quad (13)$$

By defining the static control law:

$$\begin{bmatrix} \tau \\ \tau_k \end{bmatrix} = Q^{-1}(x_e)\left(-\begin{bmatrix} \alpha(x_e) \\ \beta(q,\theta) \end{bmatrix} + \begin{bmatrix} v_q \\ v_k \end{bmatrix}\right) \quad (14)$$

we obtain the full linearized form of the overall system:

$$\begin{bmatrix} q^{[4]} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} v_q \\ v_k \end{bmatrix}$$

where v_q and v_k are the new inputs of the linearized system used to control, respectively, the positions and the stiffness of the joints of the manipulator. By taking into account the following very simple stiffness variation model:

$$k_i = \tau_{k_i} \quad (15)$$

the full state linearization problem cannot be solved by means of a static state feedback.

Then, we define the auxiliary control input u_k by adding a chain of two integrators on the input τ_k (see also Fig. 1):

$$\ddot{\tau}_k = u_k$$

By substituting τ_k and $\dot{\tau}_k$ with k and $\dot{k}$ respectively, the system can be then rewritten as:

$$\begin{bmatrix} q^{[4]} \\ \ddot{k} \end{bmatrix} = \begin{bmatrix} \alpha(x_e) \\ 0_{n\times n} \end{bmatrix} + Q(x_e)\begin{bmatrix} \tau \\ u_k \end{bmatrix} \quad (16)$$

where

$$Q(x_e) = \begin{bmatrix} M^{-1}KB^{-1} & -M^{-1}\Phi \\ 0_{n\times n} & I_{n\times n} \end{bmatrix} \quad (17)$$

$$\alpha(x_e) = -M^{-1}\Big[2\,\dot{M}\,q^{[3]} + (\ddot{M} + K)\,\ddot{q} + \ddot{N} + 2\,\dot{\Phi}\,\dot{\tau}_k + KB^{-1}K\,(\theta - q)\Big], \quad (18)$$

from which it follows that the decoupling matrix is non-singular if K is non-singular, or, in other words, if the joint stiffness are strictly positive. By defining the control law:

$$\begin{bmatrix} \tau \\ u_k \end{bmatrix} = Q^{-1}(x_e)\left(-\begin{bmatrix} \alpha(x_e) \\ 0_{n\times n} \end{bmatrix} + \begin{bmatrix} v_q \\ v_k \end{bmatrix}\right) \quad (19)$$

we obtain the linearized form of the (16):

$$\begin{bmatrix} q^{[4]} \\ \ddot{k} \end{bmatrix} = \begin{bmatrix} v_q \\ v_k \end{bmatrix}$$

Both the positions and the stiffness of the joint of the robot can be now controlled by means of a static state feedback plus feedforward action:

$$v_c = v_f + P[z_d - z] = v_f + P[z_d - \Psi(x_e)] \quad (20)$$

where

$$P = \begin{bmatrix} P_{q_0} & P_{q_1} & P_{q_2} & P_{q_3} & 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & P_{k_0} & P_{k_1} \end{bmatrix}$$

A scheme of the proposed controller is depicted in Fig. 1, while in Fig.2 the simulative results of a two-link planar manipulator are reported.

IV. Conclusions

In this talk, the feedforward control action needed to perform a desired motion profile on a robotic manipulator with joint variable stiffness has been computed and the problem of feedback linearization of these devices has been analyzed.

The simultaneous non-interactive stiffness-position control can be implemented by means of an outer linear control loop, that can be seen as a static state feedback in the state space of the linearized system. The asymptotic trajectory tracking problem can then be solved with arbitrary dynamics if the position and the stiffness trajectories are continuous together with their time derivatives up to the 4th and 2nd order respectively.

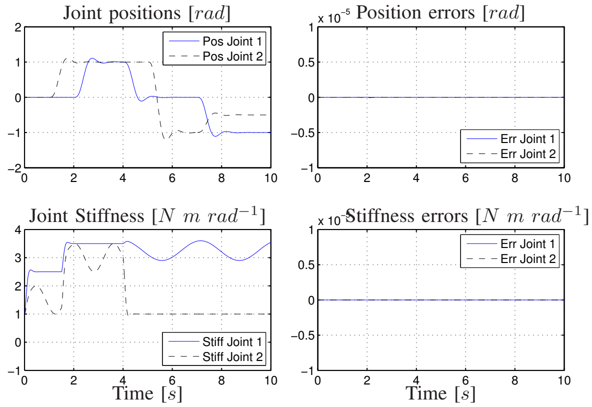

Fig. 2. Full state linearization via dynamic feedback: (a) Joint positions, (b) position errors, (c) joint stiffness and (d) stiffness errors.

References

[1] A. Bicchi and G. Tonietti, “Fast and soft arm tactics: Dealing with the safety-performance trade-off in robot arms design and control,” *IEEE Robotics and Automation Magazine*, vol. 11, no. 2, pp. 22–33, 2004.

[2] M. W. Spong, “Modeling and control of elastic joint robots,” *Journal of Dynamic Systems, Measurement, and Control*, vol. 109, no. 4, pp. 310–319, 1987.

[3] A. D. Luca, R. Farina, and P. Lucibello, “On the control of robots with visco-elastic joints,” in *Proc. IEEE Int. Conf. on Robotics and Automation*, 2005.