Variable Stiffness Actuation: Modeling and Control

Gianluca Palli

DEIS - University of Bologna
LAR - Laboratory of Automation and Robotics

Viale Risorgimento 2, 40136, Bologna
TEL: +39 051 2093903
E-mail: *gianluca.palli@unibo.it*

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Why Variable Stiffness Actuation?

Improves the safety of the robotic device with respect to:

- interaction with unknown environment
- unexpected collisions
- limited controller and sensors bandwidth
- actuator failures

A. Bicchi and G. Tonietti. "Fast and soft arm tactics: Dealing with the safety-performance trade-off in robot arms design and control". IEEE Robotics and Automation Magazine, 2004.

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Drawbacks of the Variable Stiffness Actuation:

- A more complex mechanical design
- The number of actuators increases
- Non-linear transmission elements must be used
- High non-linear and cross coupled dynamic model

Dynamic Model of Robots with Variable Joint Stiffness

- Robot dynamic equations

$$\begin{aligned} M(q)\,\ddot{q} + N(q,\dot{q}) + K\,(q-\theta) &= 0 \\ B\,\ddot{\theta} + K\,(\theta - q) &= \tau \end{aligned}$$

- The diagonal joint stiffness matrix is considered time-variant

$$K = \mathrm{diag}\{k_1,\, \ldots,\, k_n\}, \quad K = K(t) > 0$$

- Alternative notation

$$K\,(q-\theta) = \Phi k, \quad \Phi = \mathrm{diag}\{(q_1-\theta_1), \ldots, (q_n-\theta_n)\}, \quad k = [k_1,\, \ldots,\, k_n]^T$$

1. The joint stiffness k can be directly changed by means of a (suitably scaled) additional command τ_k

$$k = \tau_k$$

2. The variation of joint stiffness may be modeled as a second-order dynamic system

$$\ddot{k} = \phi(x, k, \dot{k}, \tau_k)$$

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Dynamic Model of Robots with Variable Joint Stiffness

- The input u and the robot state x are:

$$u = \begin{bmatrix} \tau \\ \tau_k \end{bmatrix} \in \mathbb{R}^{2n}, \quad x = \begin{bmatrix} q^T & \dot{q}^T & \theta^T & \dot{\theta}^T \end{bmatrix}^T \in \mathbb{R}^{4n}$$

- In the case of second-order stiffness variation model, the state vector of the robot becomes:

$$x_e = \begin{bmatrix} q^T & \dot{q}^T & \theta^T & \dot{\theta}^T & k^T & \dot{k}^T \end{bmatrix}^T \in \mathbb{R}^{6n}$$

- In all cases, the objective will be to simultaneously control the following set of outputs

$$y = \begin{bmatrix} q \\ k \end{bmatrix} \in \mathbb{R}^{2n}$$

namely the link positions (and thus, through the robot direct kinematics, the end-effector pose) and the joint stiffness

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Dynamic Inversion

- The motion is specified in terms of a desired smooth position trajectory $q = q_d(t)$ and joint stiffness matrix $K = K_d(t)$ (or, equivalently, of the vector $k = k_d(t)$)
- Assuming $k = \tau_k$, we have simply $\tau_{k,d} = k_d(t)$ and only the computation of the nominal motor torque τ_d is of actual interest
- The robot dynamic equation is differentiated twice with respect to time

$$M(q)\, q^{[3]} + \dot{M}(q)\, \ddot{q} + \dot{N}(q, \dot{q}) + \dot{K}\,(q - \theta) + K\,(\dot{q} - \dot{\theta}) = 0$$

and

$$M(q)\, q^{[4]} + 2\,\dot{M}(q)\, q^{[3]} + \ddot{M}(q)\, \ddot{q} + \ddot{N}(q, \dot{q}) + \\ + K\,(\ddot{q} - \ddot{\theta}) + 2\,\dot{K}\,(\dot{q} - \dot{\theta}) + \ddot{K}\,(q - \theta) = 0$$

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Dynamic Inversion

- Reference motor position along the desired robot trajectory

$$\theta_d = q_d + K_d^{-1} \left(M(q_d)\ddot{q}_d + N(q_d, \dot{q}_d) \right).$$

- Reference motor velocity

$$\begin{aligned} \dot{\theta}_d &= \dot{q}_d + K_d^{-1} \Big(M(q_d) q_d^{[3]} + \dot{M}(q_d)\ddot{q}_d + \dot{N}(q_d, \dot{q}_d) \\ &\quad - \dot{K}_d K_d^{-1} (M(q_d)\ddot{q}_d + N(q_d, \dot{q}_d)) \Big). \end{aligned}$$

- Actuators dynamic model inversion

$$\ddot{\theta} = B^{-1} \left[\tau - K(\theta - q) \right],$$

Actuator Torques Computation

- Reference motor torque along the desired trajectory

$$\tau_d = M(q_d)\ddot{q}_d + N(q_d, \dot{q}_d) + BK_d^{-1}\alpha_d\left(q_d, \dot{q}_d, \ddot{q}_d, q_d^{[3]}, q_d^{[4]}, k_d, \dot{k}_d, \ddot{k}_d\right)$$

- Some minimal smoothness requirements are imposed

$$q_d(t) \in \mathbb{C}^4 \quad \text{and} \quad k_d(t) \in \mathbb{C}^2$$

- Discontinuous models of friction or actuator dead-zones on the motor side can be considered without problems
- Discontinuous phenomena acting on the link side should be approximated by a smooth model
- The command torques τ_d can be kept within the saturation limits by a suitable time scaling of the manipulator trajectory

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Second-Order Stiffness Model

- The dynamics of the joint stiffness k is written as a generic nonlinear function of the system configuration

$$\ddot{k} = \beta(q, \theta) + \gamma(q, \theta)\, \tau_k$$

- Double differentiation wrt time of the robot dynamics

$$\begin{aligned} M\, q^{[4]} + 2\, \dot{M}\, q^{[3]} + \ddot{M}\, \ddot{q} + \ddot{N} \\ + K\, \left(\ddot{q} - B^{-1}\left[\tau - K(\theta - q)\right]\right) \\ + 2\, \dot{K}\, (\dot{q} - \dot{\theta}) + \Phi\, (\beta + \gamma\, \tau_k) = 0 \end{aligned}$$

  where both the inputs τ and τ_k appear

- Important notes

$$\ddot{q} = \ddot{q}(\dot{q}, q),\;\; q^{[3]} = q^{[3]}(\dot{q}, q),\;\; q^{[4]} = q^{[4]}(\dot{q}, q)$$

Feedback Linearized Model

- The overall system can be written as

$$\begin{bmatrix} q^{[4]} \\ \ddot{k} \end{bmatrix} = \begin{bmatrix} \alpha(x_e) \\ \beta(q,\theta) \end{bmatrix} + Q(x_e) \begin{bmatrix} \tau \\ \tau_k \end{bmatrix}$$

where $Q(x_e)$ is the decoupling matrix:

$$Q(x_e) = \begin{bmatrix} M^{-1}KB^{-1} & M^{-1}\Phi\,\gamma(q,\theta) \\ 0_{n\times n} & \gamma(q,\theta) \end{bmatrix}$$

- Non-Singularity Conditions

$$\left.\begin{array}{c} k_i > 0 \\ \gamma_i(q_i,\theta_i) \neq 0 \end{array}\right\} \forall\ i = 1, \ldots, n$$

- By applying the static state feedback

$$\begin{bmatrix} \tau \\ \tau_k \end{bmatrix} = Q^{-1}(x_e)\left(-\begin{bmatrix} \alpha(x_e) \\ \beta(q,\theta) \end{bmatrix} + \begin{bmatrix} v_q \\ v_k \end{bmatrix} \right)$$

the full feedback linearized model is obtained

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Dynamic Feedback Linearization

- Considering the very simple stiffness variation model

$$k_i = \tau_{k_i}$$

the dynamics of the system becomes:

$$\begin{bmatrix} \ddot{q} \\ k \end{bmatrix} = \begin{bmatrix} -M^{-1}N \\ 0_{n\times n} \end{bmatrix} + \begin{bmatrix} 0_{n\times n} & -M^{-1}\Phi \\ 0_{n\times n} & I_{n\times n} \end{bmatrix} \begin{bmatrix} \tau \\ \tau_k \end{bmatrix}$$

Problem

The decoupling matrix of the system is <u>structurally</u> singular

Solution

Dynamic extension on the input τ_k is needed

$$\ddot{\tau}_k = u_k$$

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Control Strategy

- A static state feedback in the state space of the feedback linearized system is used:

$$v_c = \begin{bmatrix} v_q \\ v_k \end{bmatrix}, \quad v_f = \begin{bmatrix} q_d^{[4]} \\ \ddot{k}_d \end{bmatrix}$$

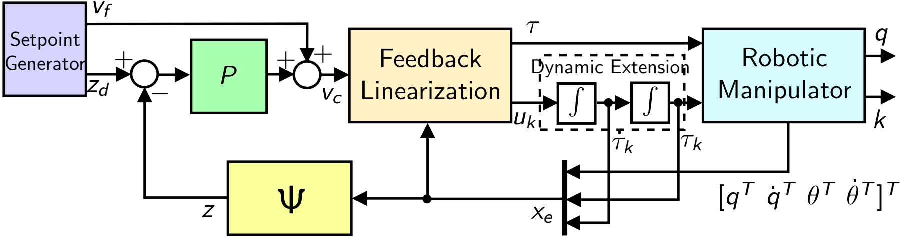
$$z_d = \begin{bmatrix} q_d^T & \dot{q}_d^T & \ddot{q}_d^T & q_d^{[3]^T} & k_d^T & \dot{k}_d^T \end{bmatrix}^T$$

- The state vector z of the feedback linearized system and a suitable nonlinear coordinate transformation are defined:

$$z = \begin{bmatrix} q^T & \dot{q}^T & \ddot{q}^T & q^{[3]^T} & k^T & \dot{k}^T \end{bmatrix}^T = \Psi(x_e) =$$

$$\begin{bmatrix} q \\ \dot{q} \\ -M^{-1}\left[N + \Phi\, k\right] \\ -M^{-1}\left[-\dot{M}\, M^{-1}\left[N + \Phi\, k\right] + \dot{N} + \Phi\, \dot{k} + \dot{\Phi}\, k\right] \\ k \\ \dot{k} \end{bmatrix}$$

Control System Architecture



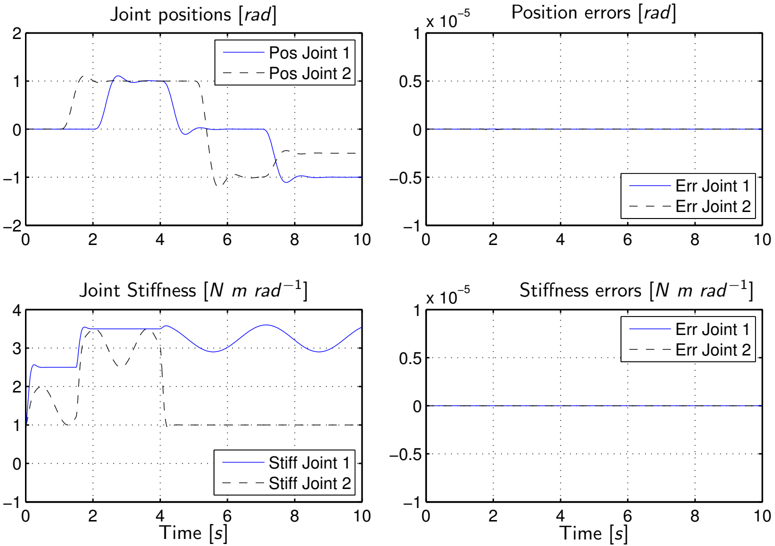
- The controller can be then rewritten as:

$$v_c = v_f + P[z_d - z] = v_f + P[z_d - \Psi(x_e)]$$

where

$$P = \begin{bmatrix} P_{q_0} & P_{q_1} & P_{q_2} & P_{q_3} & 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & P_{k_0} & P_{k_1} \end{bmatrix}$$

Simulation of a two-link Planar Manipulator



Application to Antagonistic Variable Stiffness Devices

- Dynamic model of an antagonistic variable stiffness robot

$$
\begin{aligned}
M(q)\,\ddot{q} + N(q,\dot{q}) + \eta_\alpha - \eta_\beta &= 0 \\
B\,\ddot{\theta}_\alpha + \eta_\alpha &= \tau_\alpha \\
B\,\ddot{\theta}_\beta + \eta_\beta &= \tau_\beta
\end{aligned}
$$

- By introducing the auxiliary variables

$p = \frac{\theta_\alpha - \theta_\beta}{2}$	positions of the generalized joint actuators
$s = \theta_\alpha + \theta_\beta$	state of the virtual stiffness actuators
$F(s)$	generalized joint stiffness matrix (diagonal)
$g(q-p)$	strictly monotonically increasing functions (generalized joint displacements)
$h(q-p,s)$	such that $h_i(0,0) = 0$

$\tau = \tau_\alpha - \tau_\beta$, $\tau_k = \tau_\alpha + \tau_\beta$

it is possible to write

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\begin{aligned}
M(q)\,\ddot{q} + N(q,\dot{q}) + F(s)g(q-p) &= 0 \\
2B\ddot{p} + F(s)g(p-q) &= \tau \\
B\ddot{s} + h(q-p,s) &= \tau_k
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M(q)\,\ddot{q} + N(q,\dot{q}) + F(s)g(q-p) &= 0 \\
2B\ddot{p} + F(s)g(p-q) &= \tau \\
B\ddot{s} + h(q-p,s) &= \tau_k
\end{aligned}
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Some Considerations on the Antagonistic Model

- The system is composed by 3N rigid bodies (N links and 2N actuators)
- The state space dimension is 6N (position and velocity of each rigid body)
- The input dimension is 2N (actuator torques)
- The output dimension is 3N (joint and actuator positions)
- y has dimension 2N (position and stiffness of each joint)
- The system has 2N DOF (N positioning DOF and N joint stiffnesses DOF)

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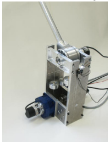
Actual Variable Stiffness Joint Implementations

- For antagonistic actuated robot with exponential force/compression characteristic (Palli et al. 2007)



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f_i(s_i) &= e^{a\,s_i} \\
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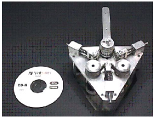

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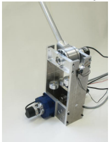

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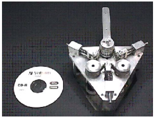

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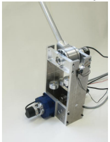

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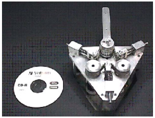

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State reconstruction

- All the state variables are known (full state feedback)

State Reconstruction

The whole state of the system can be reconstructed by means of:

- State Observers
  - Increase the complexity of the system
  - Parameters adaptation is needed
  - Require a measure (or a estimation) of the external forces
- Filtering of position information
  - Generates noisy velocity signals
  - High-speed acquition and computation system
- Tachometers
  - Increase costs
  - Difficulties due to the integration into the system

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Disturbance decoupling problem

- The vector relative degrees of the outputs with respect to the input w is:

$$\begin{array}{lclclcl} L_d q & = & 0_{N\times N} & , & L_d F(s) & = & 0_{N\times N} \\ L_d L_f q & = & M(q)^{-1} & , & L_d L_f F(s) & = & M(q)^{-1}\frac{\partial g(q-p)}{\partial q} \end{array}$$

- The disturbance decoupling problem can't be solved
- The joint positions can't be decoupled from the disturbance
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External load estimation

- The dynamic equation of the robot manipulator can be rewritten to take into account for external load

$$M(q)\ddot{q} + N(q, \dot{q}) + \eta_\alpha - \eta_\beta = \tau_{ext}$$

- The generalized momenta of the robotic arm is:

$$p = M(q)\dot{q}$$
$$\dot{p} = \dot{M}(q)\dot{q} + M(q)\ddot{q} = \dot{M}(q)\dot{q} - N(q, \dot{q}) - \eta_\alpha + \eta_\beta + \tau_{ext}$$

- Recalling the general property

$$q^T[\dot{M}(q) - 2C(q, \dot{q})]q = 0 \quad \Rightarrow \quad \dot{M}(q) = C(q, \dot{q}) + C^T(q, \dot{q})$$

we obtain

$$\dot{p} = -C^T(q, \dot{q})\dot{q} - D\dot{q} - g(q) - \eta_\alpha + \eta_\beta + \tau_{ext} = \bar{N}(q, \dot{q}) - \tau_q + \tau_{ext}$$

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$$\hat{\tau}_{ext} = L \left[\int \left(\tau_q - \bar{N}(q, \dot{q}) - \hat{\tau}_{ext} \right) dt + p \right]$$

whit positive defined (diagonal) L, the torque extimation dynamic is:

$$\dot{\hat{\tau}}_{ext} = -L\hat{\tau}_{ext} + L\tau_{ext}$$

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Feedback linearization of the complete model

- Complete dynamic model of the antagonistic actuated arm:

$$\begin{aligned} M(q)\ddot{q} + H_\alpha\ \ddot{\theta}_\alpha + H_\beta\ \ddot{\theta}_\beta + N(q,\dot{q}) + \eta_\alpha - \eta_\beta &= \tau_{ext} \\ B\ddot{\theta}_\alpha + H_\alpha^T\ \ddot{q} + \eta_\alpha &= \tau_{\theta_\alpha} \\ B\ddot{\theta}_\beta + H_\beta^T\ \ddot{q} + \eta_\beta &= \tau_{\theta_\beta} \end{aligned}$$

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- The feedback linearization problem with decoupled control has been solved taking into account different stiffness variation models
- The simultaneous asymptotic trajectory tracking of both the position and the stiffness has been achieved by means of an outer linear control loop
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