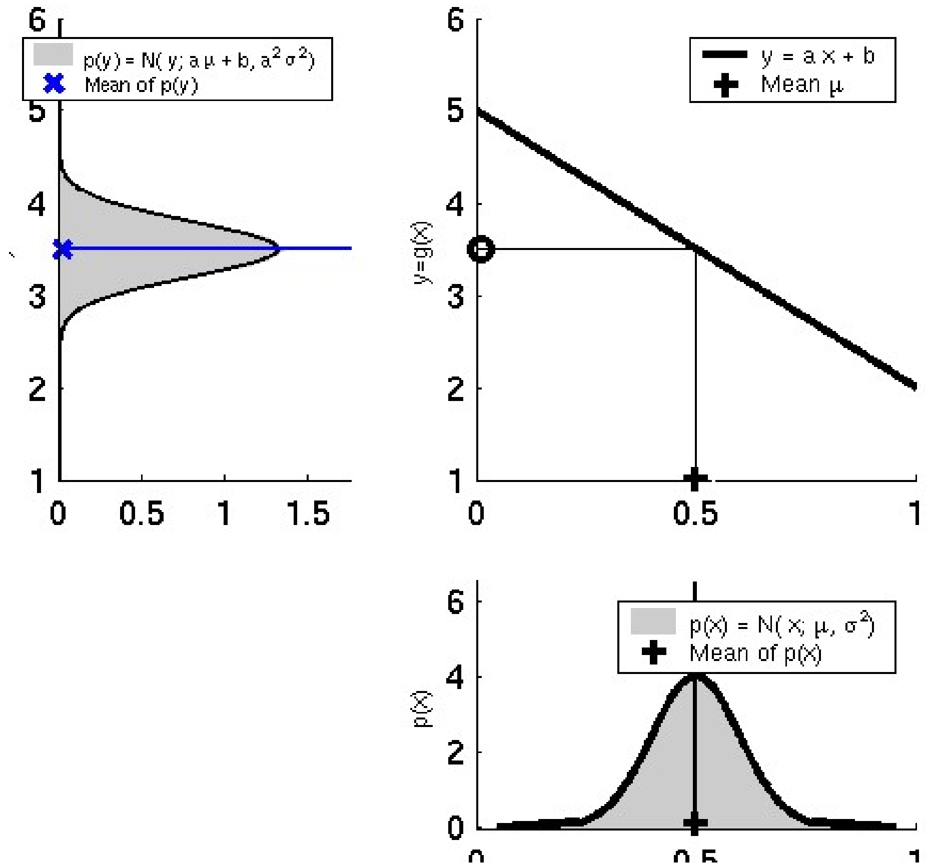
Sistemi Dinamici Nonlineari

- Molti problemi robotici realistici richiedono funzioni nonlineari

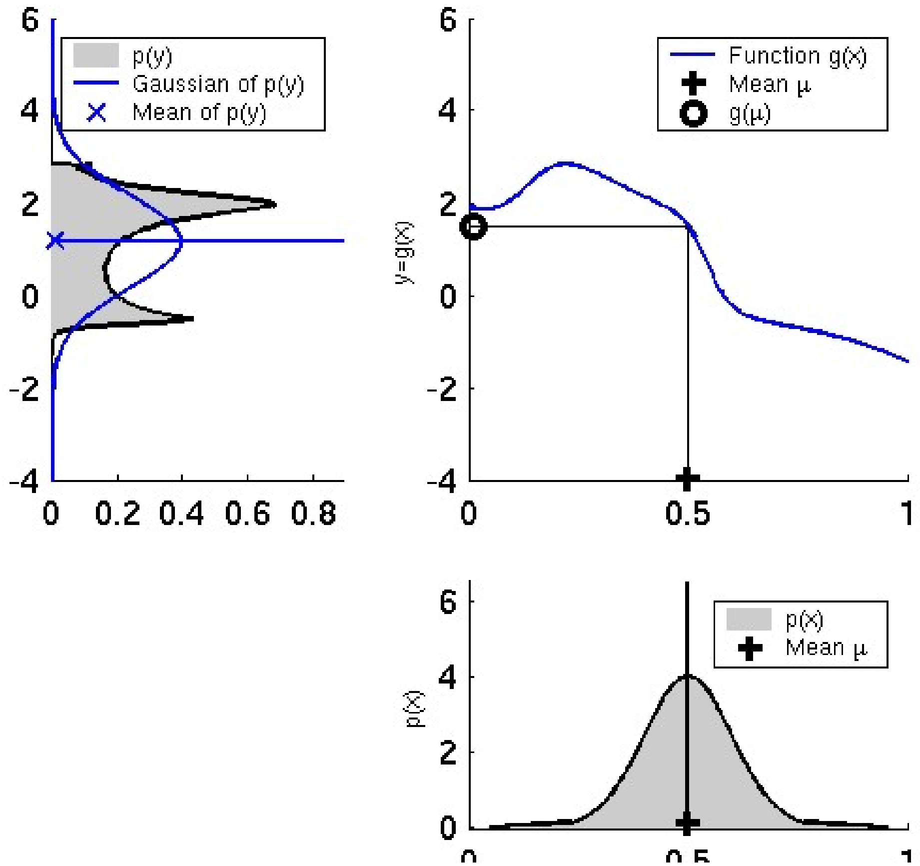
$$x_t = g(u_t, x_{t-1})$$

$$z_t = h(x_t)$$

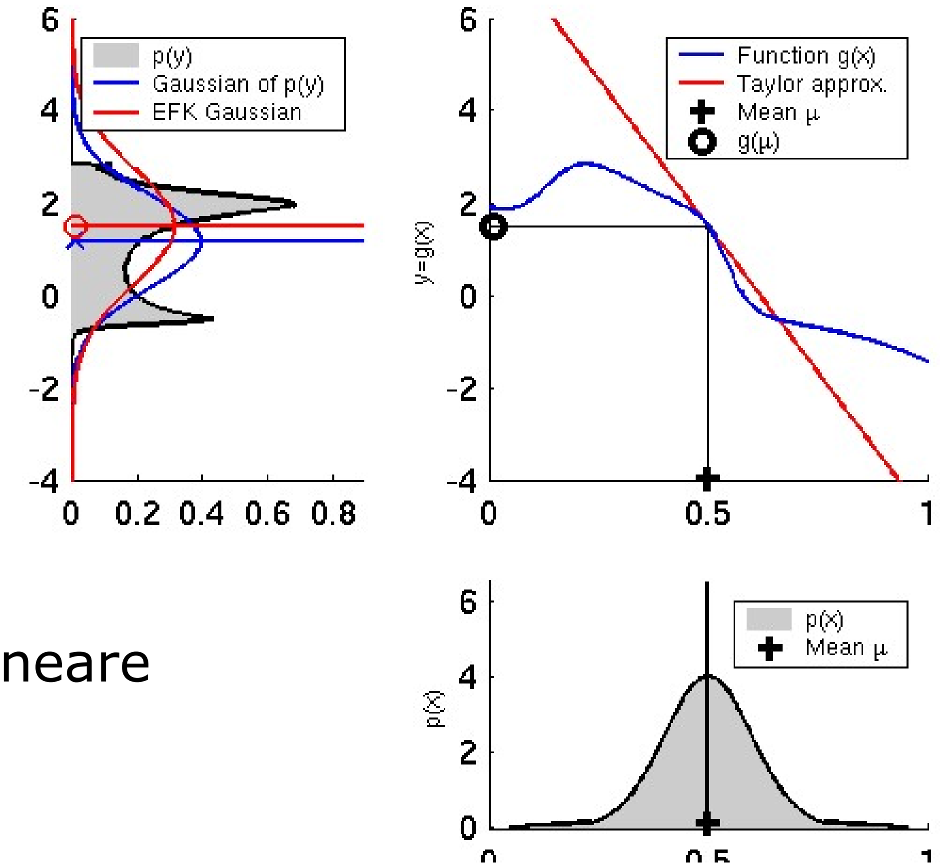
Assunzione lineare rivisitata



Funzione Non-lineare

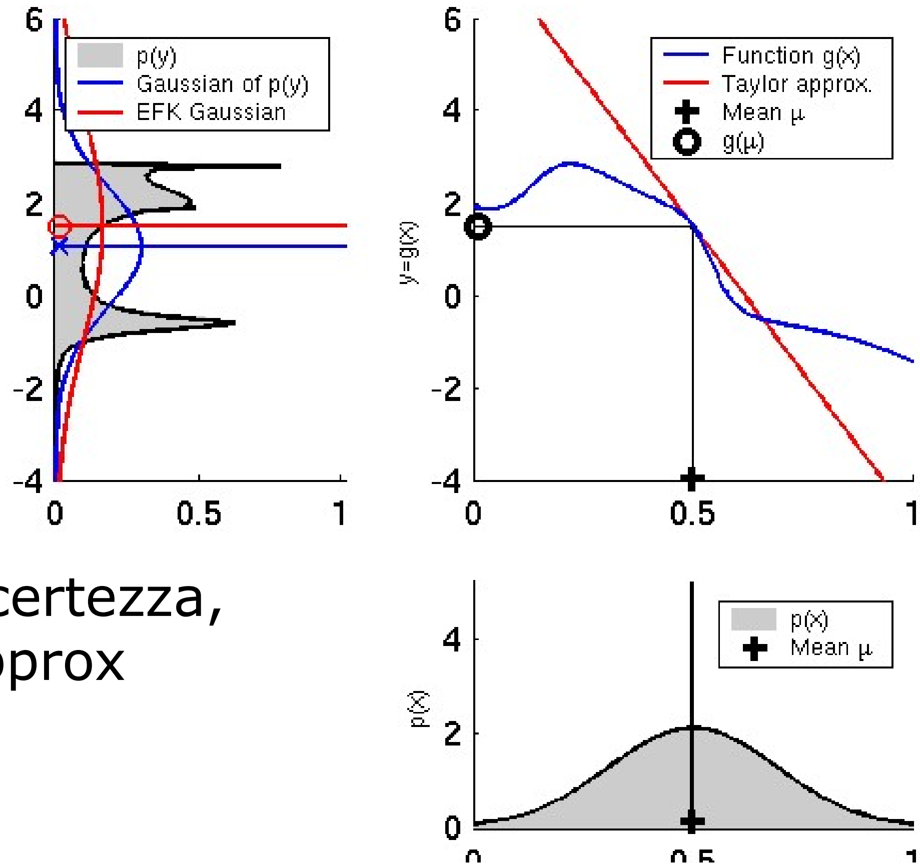


Linearizazione EKF (1)



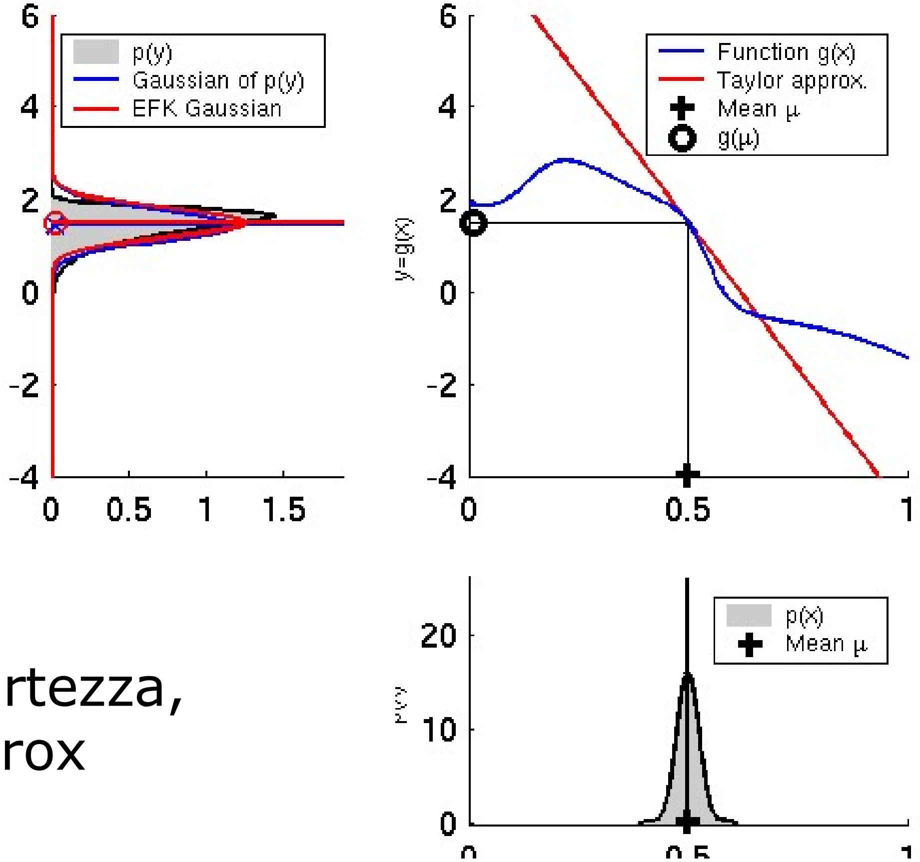
Approx lineare

Linearizazione EKF (2)



Maggiore incertezza,
maggiore approx

Linearizazione EKF (3)



Minore incertezza,
minore approx

Linearizazione EKF: Espansione del Primo Ordine della serie di Taylor

- Predizione:

$$g(u_t, x_{t-1}) \approx g(u_t, \mu_{t-1}) + \frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}} (x_{t-1} - \mu_{t-1})$$

$$g(u_t, x_{t-1}) \approx g(u_t, \mu_{t-1}) + G_t \, (x_{t-1} - \mu_{t-1})$$

G_t corrisponde a:

$A_t \quad B_t$

- Correzione:

$$h(x_t) \approx h(\overline{\mu}_t) + \frac{\partial h(\overline{\mu}_t)}{\partial x_t} (x_t - \overline{\mu}_t)$$

$$h(x_t) \approx h(\overline{\mu}_t) + H_t \, (x_t - \overline{\mu}_t)$$

H_t corrisponde a:

C_t

Jacobiano

- Matrice m x n
- Data la funzione:

$$g(x) = \begin{pmatrix} g_1(x) \\ \dots \\ g_m(x) \end{pmatrix}$$

- Lo Jacobiano è:

$$G_x = \begin{pmatrix} \frac{\partial g_1}{\partial x_1} & \cdots & \frac{\partial g_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_m}{\partial x_1} & \cdots & \frac{\partial g_m}{\partial x_n} \end{pmatrix}$$

Algoritmo EKF

1. **Extended_Kalman_filter**($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$):

2. Prediction:

3. $\bar{\mu}_t = g(u_t, \mu_{t-1})$ ⟵ $\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$

4. $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$ ⟵ $\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$

5. Correction:

6. $K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$ ⟵ $K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$

7. $\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$ ⟵ $\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$

8. $\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$ ⟵ $\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$

9. Return μ_t, Σ_t

$$H_t = \frac{\partial h(\bar{\mu}_t)}{\partial x_t} \qquad G_t = \frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}}$$

$$R = V_t M_t V_t^T$$

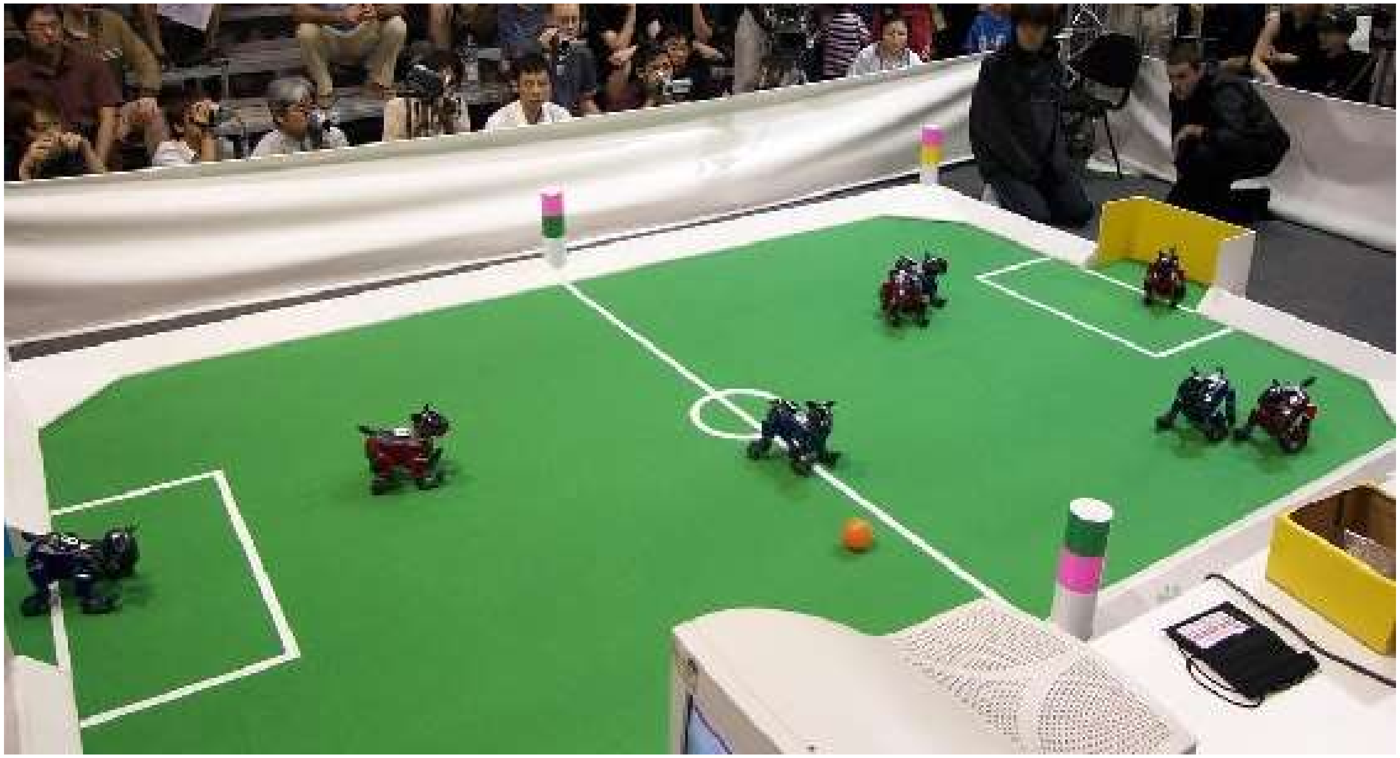
Localizazione

> "Using sensory information to locate the robot in its environment is the most fundamental problem to providing a mobile robot with autonomous capabilities." [Cox '91]

- **Data**
  - Mappa dell'ambiente
  - Sequenza di misure di sensori.
- **Ricerca**
  - Stima della posa del robot.
- **Problemi**
  - Position tracking
  - Global localization
  - Kidnapped robot problem (recovery)

Localizazione Landmark-based

Si assume una mappa definita da un insieme di landmark, l'osservazione genera un vettore $\{z_1, \ldots, z_n\}$ di misure di feature univocamente identificabili



6 marker colorati diversamente

Localizzazione Markoviana

Algoritmo generale

Algorithm Markov_localization($bel(x_{t-1}), u_t, z_t, m$)**:**

  for all x_t do

    $\overline{bel}(x_t) = \int p(x_t \mid u_t, x_{t-1}, m)\ bel(x_{t-1})\ dx$

    $bel(x_t) = \eta\ p(z_t \mid x_t, m)\ \overline{bel}(x_t)$

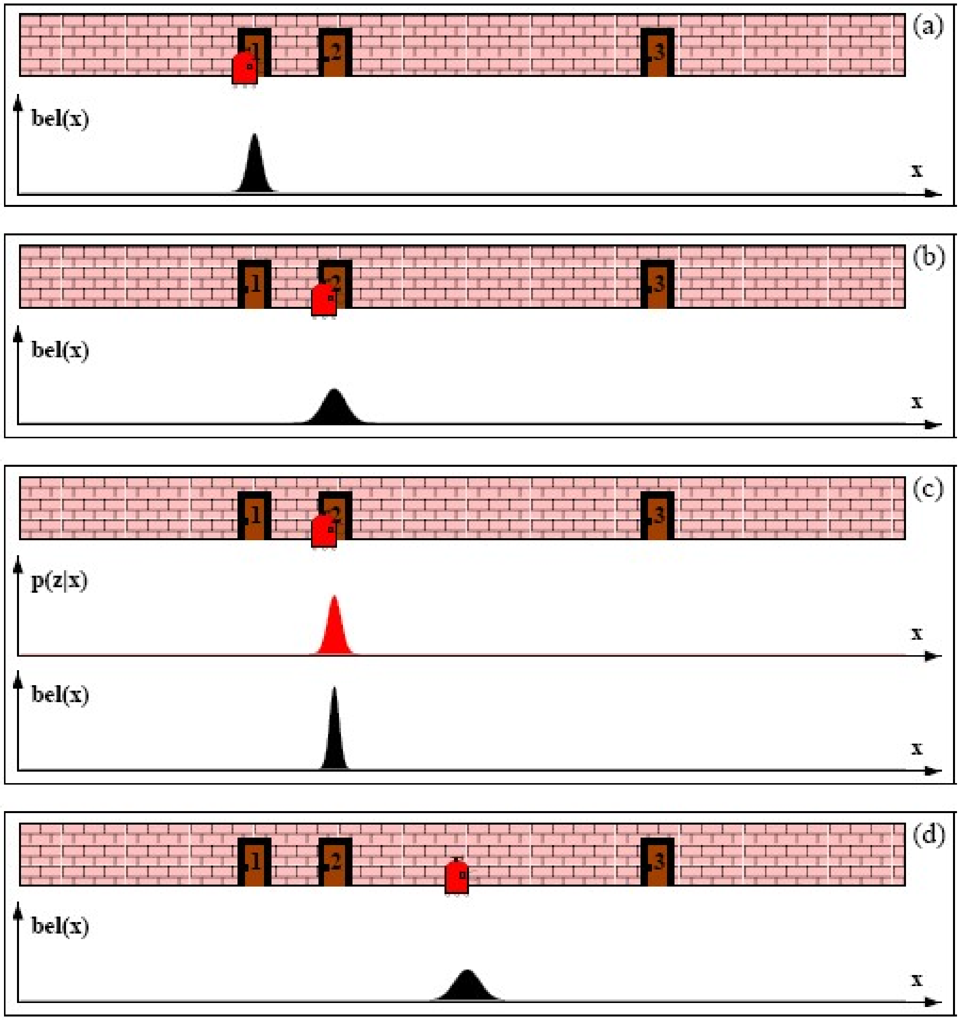
  endfor

  return $bel(x_t)$

Assunzione Gaussiana

La distribuzione si assume gaussiana

1. Mappa features con corrispondenze note
2. Posizione iniziale quasi nota (approx da una gaussiana)



Caso Concreto

- Consideriamo il modello in velocità
- La mappa feature-based con corrispondenze note
- Dato belief (gaussiano) al tempo t-1
  - valore medio e covarianza al tempo t-1
- Genera stima di belief al tempo t
  - valore medio e covarianza al tempo t

Modello in Velocità:

$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x_c + \frac{v}{\omega} \sin(\theta + \omega\Delta t) \\ y_c - \frac{v}{\omega} \cos(\theta + \omega\Delta t) \\ \theta + \omega\Delta t \end{pmatrix}$$

$$= \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} -\frac{v}{\omega}\sin\theta + \frac{v}{\omega} \sin(\theta + \omega\Delta t) \\ \frac{v}{\omega}\cos\theta - \frac{v}{\omega} \cos(\theta + \omega\Delta t) \\ \omega\Delta t \end{pmatrix}$$

Ma c'e' il rumore, quindi le coordinate reali sono:

$$\begin{pmatrix} \hat{v} \\ \hat{\omega} \end{pmatrix} = \begin{pmatrix} v \\ \omega \end{pmatrix} + \begin{pmatrix} \varepsilon_{\alpha_1|v|+\alpha_2|\omega|} \\ \varepsilon_{\alpha_3|v|+\alpha_4|\omega|} \end{pmatrix}$$

Quindi:

$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} -\frac{\hat{v}}{\hat{\omega}}\sin\theta + \frac{\hat{v}}{\hat{\omega}} \sin(\theta + \hat{\omega}\Delta t) \\ \frac{\hat{v}}{\hat{\omega}}\cos\theta - \frac{\hat{v}}{\hat{\omega}} \cos(\theta + \hat{\omega}\Delta t) \\ \hat{\omega}\Delta t \end{pmatrix}$$

Mappa feature-based

Feature-based map: mappa come insieme di features localizzate $\{m_1, \ldots, m_n\}$

Con $m_{i,x}$ $m_{i,y}$ si indica la locazione x,y per la feature i-esima

Legame probabilistico tra feature j nella mappa globale e feature estratta nella mappa locale

$$\begin{pmatrix} r_t^i \\ \phi_t^i \\ s_t^i \end{pmatrix} = \begin{pmatrix} \sqrt{(m_{j,x}-x)^2+(m_{j,y}-y)^2} \\ \operatorname{atan2}(m_{j,y}-y, m_{j,x}-x)-\theta \\ s_j \end{pmatrix} + \begin{pmatrix} \varepsilon_{\sigma_r^2} \\ \varepsilon_{\sigma_\phi^2} \\ \varepsilon_{\sigma_s^2} \end{pmatrix}$$

Derivazione EKF

Modello di moto:

$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} -\frac{v_t}{\omega_t}\sin\theta + \frac{v_t}{\omega_t}\sin(\theta+\omega_t\Delta t) \\ \frac{v_t}{\omega_t}\cos\theta - \frac{v_t}{\omega_t}\cos(\theta+\omega_t\Delta t) \\ \omega_t\Delta t + \gamma_t\Delta t \end{pmatrix} \qquad u_t = \begin{pmatrix} v_t \\ \omega_t \end{pmatrix}$$

Introducendo il rumore:

$$\underbrace{\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix}}_{x_t} = \underbrace{\begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} -\frac{v_t}{\omega_t}\sin\theta + \frac{v_t}{\omega_t}\sin(\theta+\omega_t\Delta t) \\ \frac{v_t}{\omega_t}\cos\theta - \frac{v_t}{\omega_t}\cos(\theta+\omega_t\Delta t) \\ \omega\Delta t \end{pmatrix}}_{g(u_t, x_{t-1})} + \mathcal{N}(0, R_t)$$

Linearizzazione di Taylor:

$$g(u_t, x_{t-1}) \approx g(u_t, \mu_{t-1}) + G_t\,(x_{t-1} - \mu_{t-1})$$

Rispetto ad $x_{t-1} = \mu_{t-1}$

Jacobiano:

$$G_t = g'(u_t, \mu_{t-1}) = \begin{pmatrix} 1 & 0 & -\frac{v_t}{\omega_t}\cos\mu_{t-1,\theta} + \frac{v_t}{\omega_t}\cos(\mu_{t-1,\theta}+\omega_t\Delta t) \\ 0 & 1 & \frac{v_t}{\omega_t}\sin\mu_{t-1,\theta} - \frac{v_t}{\omega_t}\sin(\mu_{t-1,\theta}+\omega_t\Delta t) \\ 0 & 0 & 1 \end{pmatrix}$$

1. **EKF_localization** ($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, m$):

Predizione:

3. $$G_t = \frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}} = \begin{pmatrix} \frac{\partial x'}{\partial \mu_{t-1,x}} & \frac{\partial x'}{\partial \mu_{t-1,y}} & \frac{\partial x'}{\partial \mu_{t-1,\theta}} \\ \frac{\partial y'}{\partial \mu_{t-1,x}} & \frac{\partial y'}{\partial \mu_{t-1,y}} & \frac{\partial y'}{\partial \mu_{t-1,\theta}} \\ \frac{\partial \theta'}{\partial \mu_{t-1,x}} & \frac{\partial \theta'}{\partial \mu_{t-1,y}} & \frac{\partial \theta'}{\partial \mu_{t-1,\theta}} \end{pmatrix}$$ Jacobiano di *g* rispetto alla locazione stimata

5. $$M_t = \begin{pmatrix} (\alpha_1 |v_t| + \alpha_2 |\omega_t|)^2 & 0 \\ 0 & (\alpha_3 |v_t| + \alpha_4 |\omega_t|)^2 \end{pmatrix}$$ Rumore di movimento

6. $$V_t = \frac{\partial g(u_t, \mu_{t-1})}{\partial u_t} = \begin{pmatrix} \frac{\partial x'}{\partial v_t} & \frac{\partial x'}{\partial \omega_t} \\ \frac{\partial y'}{\partial v_t} & \frac{\partial y'}{\partial \omega_t} \\ \frac{\partial \theta'}{\partial v_t} & \frac{\partial \theta'}{\partial \omega_t} \end{pmatrix}$$ Jacobiano di *g* rispetto al controllo (v,w) per mappare il rumore di movimento

7. $\bar{\mu}_t = g(u_t, \mu_{t-1})$ Posa predetta

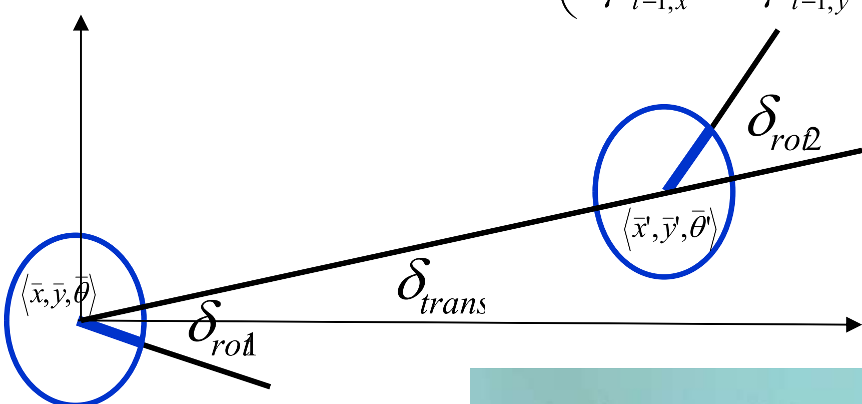
8. $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + V_t M_t V_t^T$ Covarianza predetta

Derivazione EKF

Modello odometrico:

$$G_t = \frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}} = \begin{pmatrix} \frac{\partial x'}{\partial \mu_{t-1,x}} & \frac{\partial x'}{\partial \mu_{t-1,y}} & \frac{\partial x'}{\partial \mu_{t-1,\theta}} \\ \frac{\partial y'}{\partial \mu_{t-1,x}} & \frac{\partial y'}{\partial \mu_{t-1,y}} & \frac{\partial y'}{\partial \mu_{t-1,\theta}} \\ \frac{\partial \theta'}{\partial \mu_{t-1,x}} & \frac{\partial \theta'}{\partial \mu_{t-1,y}} & \frac{\partial \theta'}{\partial \mu_{t-1,\theta}} \end{pmatrix}$$

Jacobiano di *g* rispetto alla locazione stimata

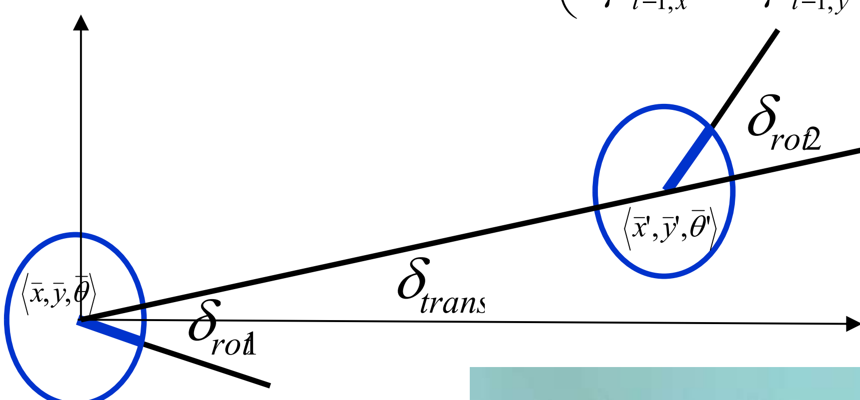


$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \underbrace{\begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} \delta_{\text{trans}} \cos(\theta + \delta_{rot_1}) \\ \delta_{\text{trans}} \sin(\theta + \delta_{rot_1}) \\ \delta_{rot_1} + \delta_{rot_2} \end{pmatrix}}_{g(u_t, x_{t-1})} + \mathcal{N}(0, R_t)$$

Derivazione EKF

Modello odometrico:

$$G_t = \frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}} = \begin{pmatrix} \frac{\partial x'}{\partial \mu_{t-1,x}} & \frac{\partial x'}{\partial \mu_{t-1,y}} & \frac{\partial x'}{\partial \mu_{t-1,\theta}} \\ \frac{\partial y'}{\partial \mu_{t-1,x}} & \frac{\partial y'}{\partial \mu_{t-1,y}} & \frac{\partial y'}{\partial \mu_{t-1,\theta}} \\ \frac{\partial \theta'}{\partial \mu_{t-1,x}} & \frac{\partial \theta'}{\partial \mu_{t-1,y}} & \frac{\partial \theta'}{\partial \mu_{t-1,\theta}} \end{pmatrix} \quad \begin{bmatrix} 1 & 0 & -\delta_{trans}\sin(\theta + \delta_{rot_1}) \\ 0 & 1 & \delta_{trans}\cos(\theta + \delta_{rot_1}) \\ 0 & 0 & \end{bmatrix}$$



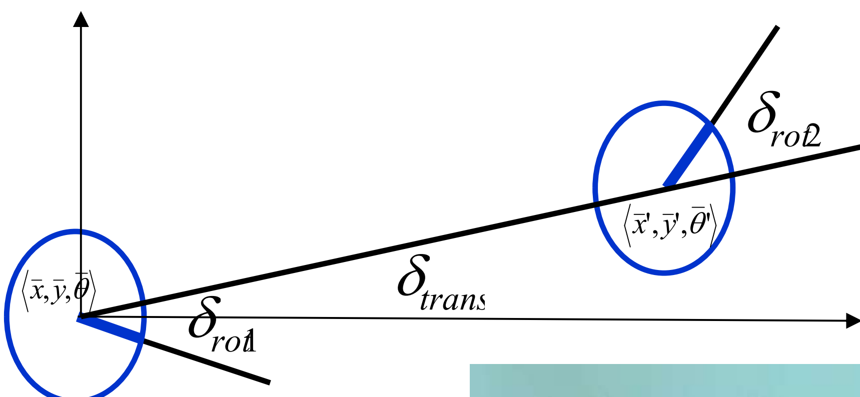
$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \underbrace{\begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} \delta_{\text{trans}}\cos(\theta + \delta_{rot_1}) \\ \delta_{\text{trans}}\sin(\theta + \delta_{rot_1}) \\ \delta_{rot_1} + \delta_{rot_2} \end{pmatrix}}_{g(u_t, x_{t-1})} + \mathcal{N}(0, R_t)$$

Derivazione EKF

Modello odometrico:

$$V_t = \frac{\partial g(u_t, \mu_{t-1})}{\partial u_t} =$$

$$u_t = (\delta_{rot_1} \; \delta_{trans} \; \delta_{rot_2})$$

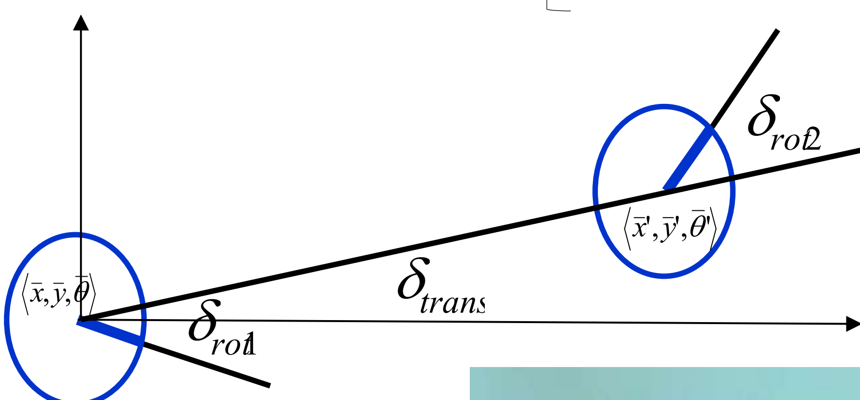


$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \underbrace{\begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} \delta_{\text{trans}} \cos(\theta + \delta_{rot_1}) \\ \delta_{\text{trans}} \sin(\theta + \delta_{rot_1}) \\ \delta_{rot_1} + \delta_{rot_2} \end{pmatrix}}_{g(u_t, x_{t-1})} + \mathcal{N}(0, R_t)$$

Derivazione EKF

Modello odometrico:

$$V_t = \frac{\partial g(u_t, \mu_{t-1})}{\partial u_t} = \begin{bmatrix} -\delta_{trans}\sin(\theta + \delta_{rot_1}) & \cos(\theta + \delta_{rot_1}) & 0 \\ \delta_{trans}\cos(\theta + \delta_{rot_1}) & \sin(\theta + \delta_{rot_1}) & 0 \\ 1 & & \end{bmatrix}$$



$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \underbrace{\begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} \delta_{\text{trans}}\cos(\theta + \delta_{rot_1}) \\ \delta_{\text{trans}}\sin(\theta + \delta_{rot_1}) \\ \delta_{rot_1} + \delta_{rot_2} \end{pmatrix}}_{g(u_t, x_{t-1})} + \mathcal{N}(0, R_t)$$

Passo di Correzione

Corrispondenza features

$$z_t^i = \begin{pmatrix} r_t^i \\ \phi_t^i \\ s_t^i \end{pmatrix} = \begin{pmatrix} \sqrt{(m_{j,x}-x)^2+(m_{j,y}-y)^2} \\ \text{atan2}(m_{j,y}-y, m_{j,x}-x)-\theta \\ m_{j,s} \end{pmatrix} + \begin{pmatrix} \mathcal{N}(0,\sigma_r) \\ \mathcal{N}(0,\sigma_\phi) \\ \mathcal{N}(0,\sigma_s) \end{pmatrix}$$

$$z_t^i = h(x_t, j, m) + \mathcal{N}(0, Q_t) \qquad h(x_t, j, m) = \begin{pmatrix} \sqrt{(m_{j,x}-x)^2+(m_{j,y}-y)^2} \\ \text{atan2}(m_{j,y}-y, m_{j,x}-x)-\theta \\ m_{j,s} \end{pmatrix}$$

$$Q_t = \begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_\phi & 0 \\ 0 & 0 & \sigma_s \end{pmatrix}$$

Covarianza dell'errore di misura addizionale

segnatura

Si procede con la linearizzazione di Taylor:

$$h(x_t, j, m) \approx h(\bar{\mu}_t, j, m) + H_t^i \,(x_t - \bar{\mu}_t)$$

Rispetto alla posa stimata: $\bar{\mu}_t = (\bar{\mu}_{t,x} \;\; \bar{\mu}_{t,y} \;\; \bar{\mu}_{t,\theta})^T$

Non ha effetto

H jacobiano (posa stimata):

$$H_t^t = h'(\bar{\mu}_t, j, m) = \begin{pmatrix} \frac{\partial r_t^i}{\partial \bar{\mu}_{t,x}} & \frac{\partial r_t^i}{\partial \bar{\mu}_{t,y}} & \frac{\partial r_t^i}{\partial \bar{\mu}_{t,\theta}} \\ \frac{\partial \phi_t^i}{\partial \bar{\mu}_{t,x}} & \frac{\partial \phi_t^i}{\partial \bar{\mu}_{t,y}} & \frac{\partial \phi_t^i}{\partial \bar{\mu}_{t,\theta}} \\ \frac{\partial s_t^i}{\partial \bar{\mu}_{t,x}} & \frac{\partial s_t^i}{\partial \bar{\mu}_{t,y}} & \frac{\partial s_t^i}{\partial \bar{\mu}_{t,\theta}} \end{pmatrix} \qquad H_t^i = \begin{pmatrix} \frac{m_{j,x}-\bar{\mu}_{t,x}}{\sqrt{q_t}} & \frac{y_t-\bar{\mu}_{t,y}}{\sqrt{q_t}} & 0 \\ \frac{\bar{\mu}_{t,y}-y_t}{q_t} & \frac{m_{j,x}-\bar{\mu}_{t,x}}{q_t} & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

with $q_t = (m_{j,x}-\bar{\mu}_{t,x})^2 + (m_{j,y}-\bar{\mu}_{t,y})^2$, and $j = c_t^i$

1. **EKF_localization** (μ_{t-1}, Σ_{t-1}, u_t, z_t, m):

Correzione:

3. $$\hat{z}_t = \begin{pmatrix} \sqrt{(m_x - \bar{\mu}_{t,x})^2 + (m_y - \bar{\mu}_{t,y})^2} \\ \operatorname{atan2}(m_y - \bar{\mu}_{t,y}, m_x - \bar{\mu}_{t,x}) - \bar{\mu}_{t,\theta} \end{pmatrix}$$ Pred. media di misura

5. $$H_t = \frac{\partial h(\bar{\mu}_t, m)}{\partial x_t} = \begin{pmatrix} \frac{\partial r_t}{\partial \bar{\mu}_{t,x}} & \frac{\partial r_t}{\partial \bar{\mu}_{t,y}} & \frac{\partial r_t}{\partial \bar{\mu}_{t,\theta}} \\ \frac{\partial \varphi_t}{\partial \bar{\mu}_{t,x}} & \frac{\partial \varphi_t}{\partial \bar{\mu}_{t,y}} & \frac{\partial \varphi_t}{\partial \bar{\mu}_{t,\theta}} \end{pmatrix}$$ Jacobiano di *h* rispetto alla locazione

6. $$Q_t = \begin{pmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_r^2 \end{pmatrix}$$

7. $$S_t = H_t \bar{\Sigma}_t H_t^T + Q_t$$ Pred. misura di covariance

8. $$K_t = \bar{\Sigma}_t H_t^T S_t^{-1}$$ Guadagno di Kalman

9. $$\mu_t = \bar{\mu}_t + K_t (z_t - \hat{z}_t)$$ Posa aggiornata

10. $$\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$$ Covarianza aggiornata

Algorithm EKF_localization_known_correspondences$(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, c_t, m)$**:**

$$\bar{\mu}_t = \mu_{t-1} + \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \mu_{t-1,\theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1,\theta} + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \mu_{t-1,\theta} - \frac{v_t}{\omega_t} \cos(\mu_{t-1,\theta} + \omega_t \Delta t) \\ \omega_t \Delta t \end{pmatrix}$$

Nuova posa stimata

$$G_t = \begin{pmatrix} 1 & 0 & -\frac{v_t}{\omega_t} \cos \mu_{t-1,\theta} + \frac{v_t}{\omega_t} \cos(\mu_{t-1,\theta} + \omega_t \Delta t) \\ 0 & 1 & \frac{v_t}{\omega_t} \sin \mu_{t-1,\theta} - \frac{v_t}{\omega_t} \sin(\mu_{t-1,\theta} + \omega_t \Delta t) \\ 0 & 0 & 1 \end{pmatrix}$$

Jacobiano

$$\bar{\Sigma}_t = G_t \, \Sigma_{t-1} \, G_t^T + R_t$$

Covarianza predizione:
aggiornamento + rumore di movimento:

$$R = V_t M_t V_t^T$$

$$Q_t = \begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_\phi & 0 \\ 0 & 0 & \sigma_s \end{pmatrix}$$

Covarianza rumore misura addizionale

for all observed features $z_t^i = (r_t^i \ \ \phi_t^i \ \ s_t^i)^T$ *do*

$$j = c_t^i$$

corrispondenza

$$\delta = \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} m_{j,x} - \bar{\mu}_{t,x} \\ m_{j,y} - \bar{\mu}_{t,y} \end{pmatrix}$$

posizione

$$q = \delta^T \delta$$

$$\hat{z}_t^i = \begin{pmatrix} \sqrt{q} \\ \operatorname{atan2}(\delta_y, \delta_x) - \bar{\mu}_{t,\theta} \\ m_{j,s} \end{pmatrix}$$

Distanza rel. e angolo

$$H_t^i = \frac{1}{q} \begin{pmatrix} \sqrt{q}\delta_x & -\sqrt{q}\delta_y & 0 \\ \delta_y & \delta_x & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

Jacobiano del modello di misura

$$K_t^i = \bar{\Sigma}_t \, H_t^{i,T} (H_t^i \, \bar{\Sigma}_t \, H_t^{i,T} + Q_t)^{-1}$$

Guadagno di Kalman per ogni osservazione

endfor

$$\mu_t = \bar{\mu}_t + \sum_i K_t^i (z_t^i - \hat{z}_t^i)$$

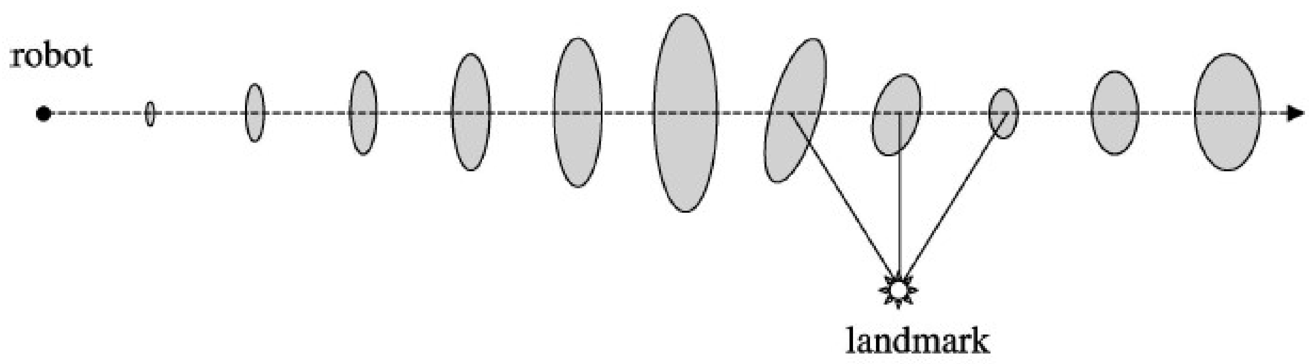
$$\Sigma_t = (I - \sum_i K_t^i \, H_t^i) \, \bar{\Sigma}_t$$

Nuova Posa e covarianza

return μ_t, Σ_t

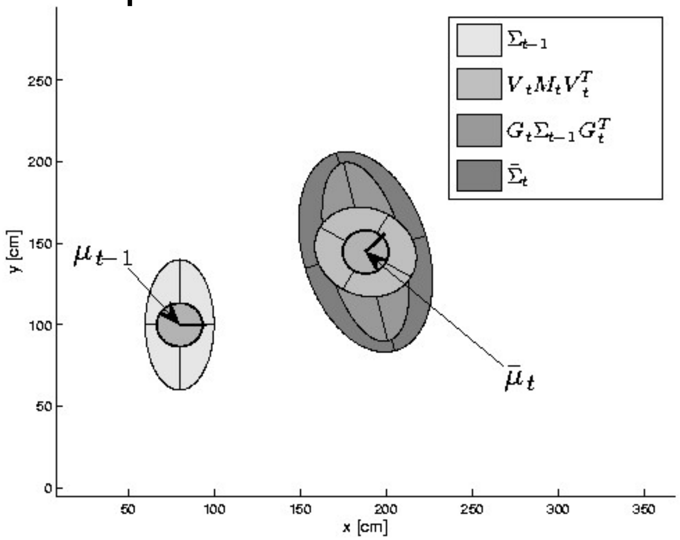
Localizzazione EKF

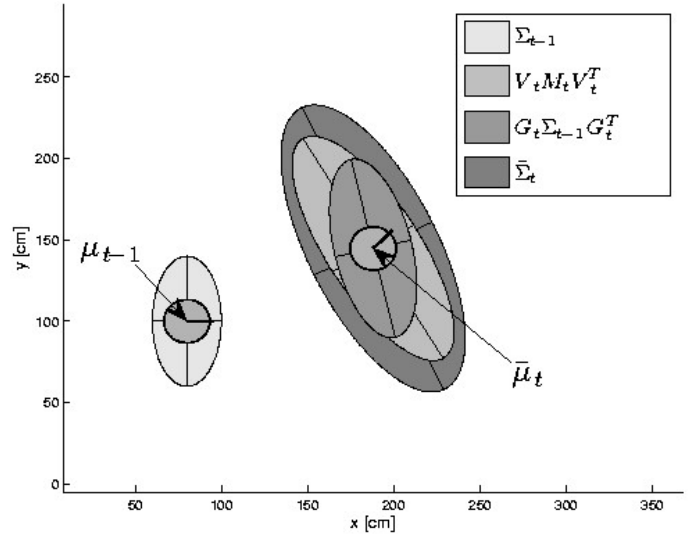
Incertezza diminuisce con percezione landmark, poi aumenta di nuovo

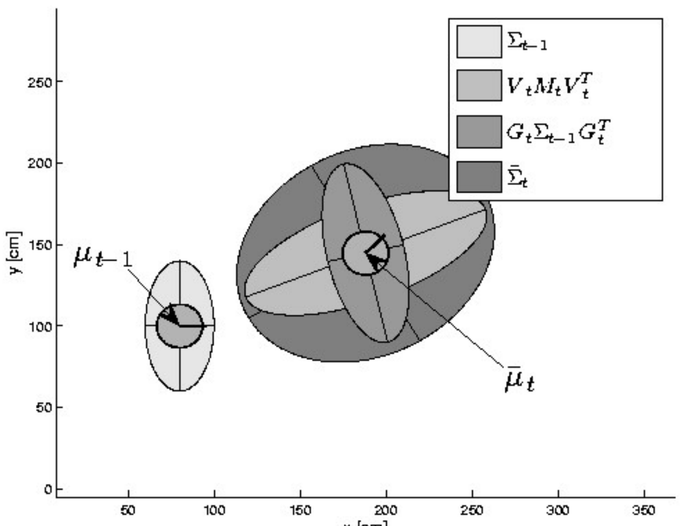


Passo di predizione EKF

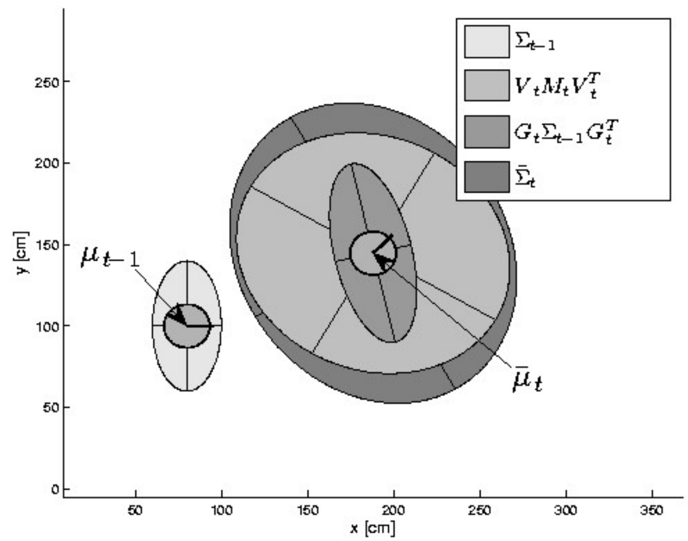
Spostamento di 90cm e 45 g





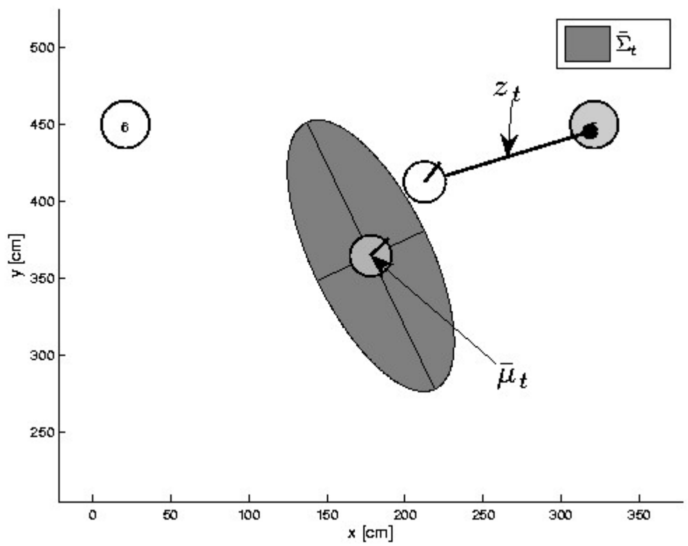


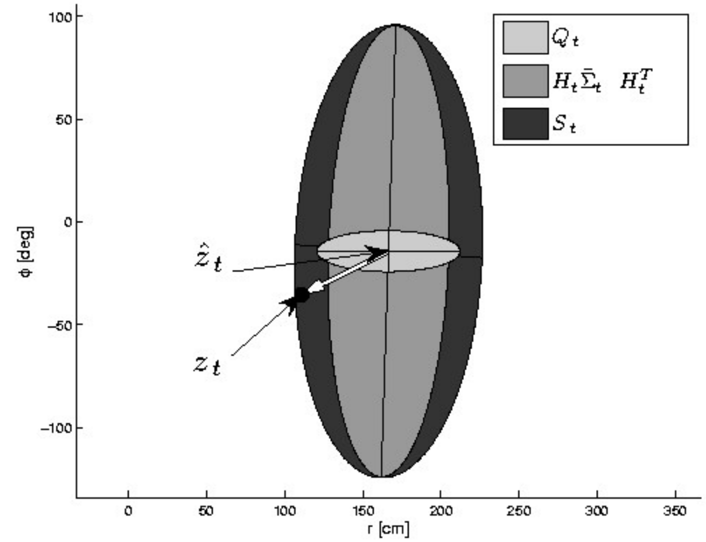
Rumore traslazione



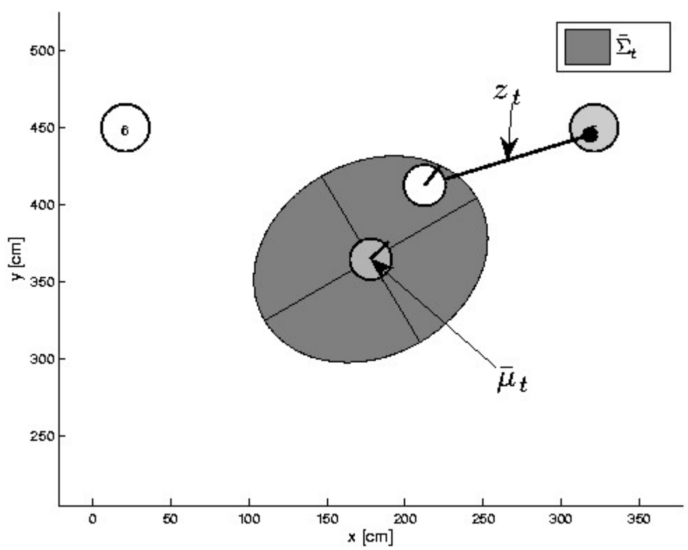
Rumore rotazione

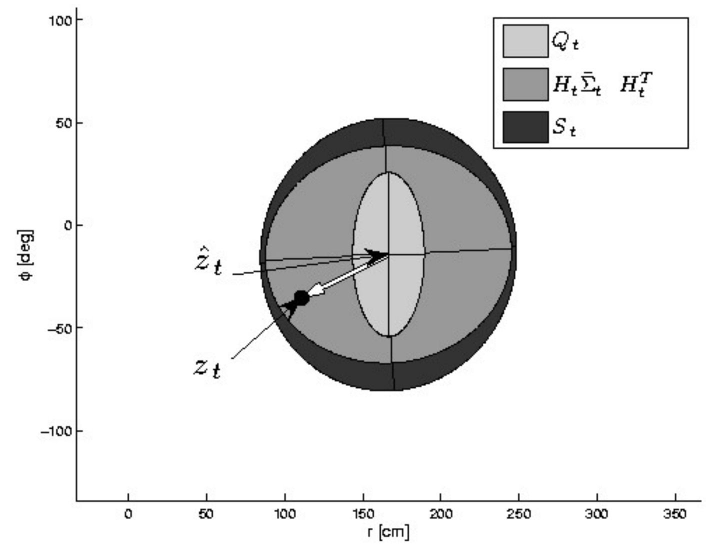
EKF Observation Prediction Step



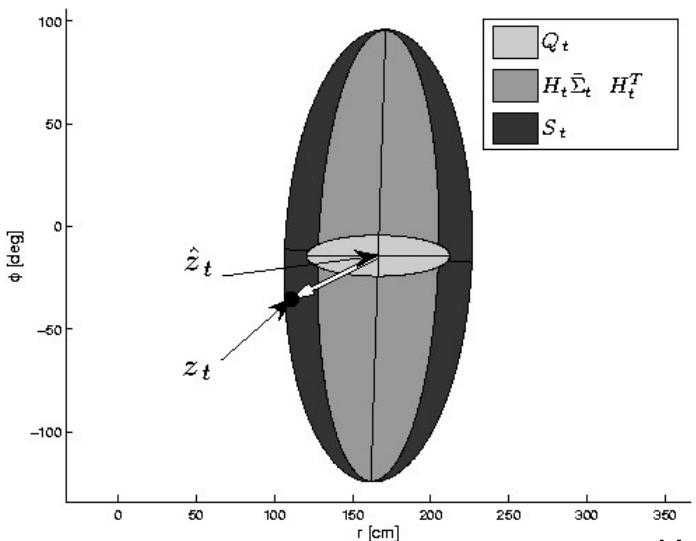


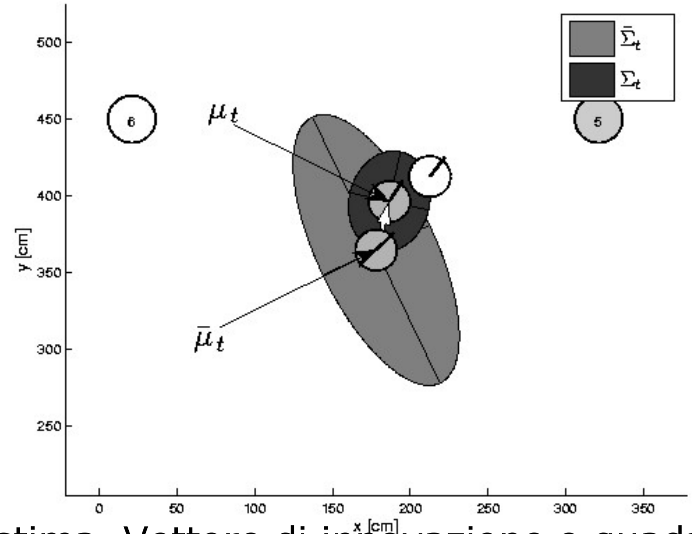
Vettore di innovazione



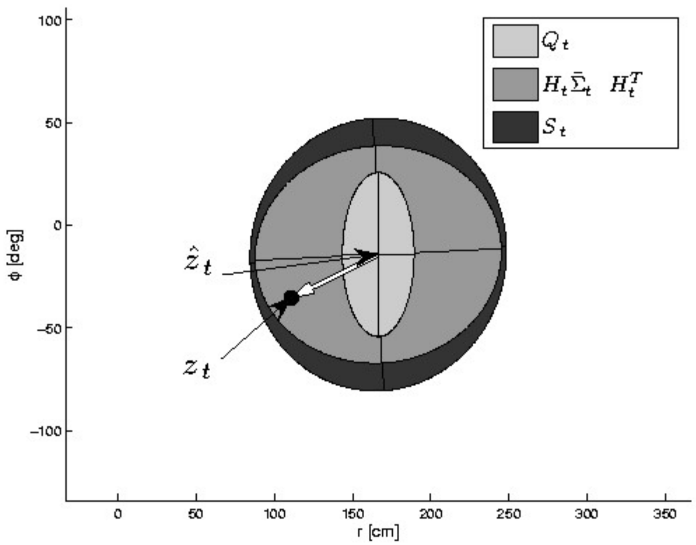


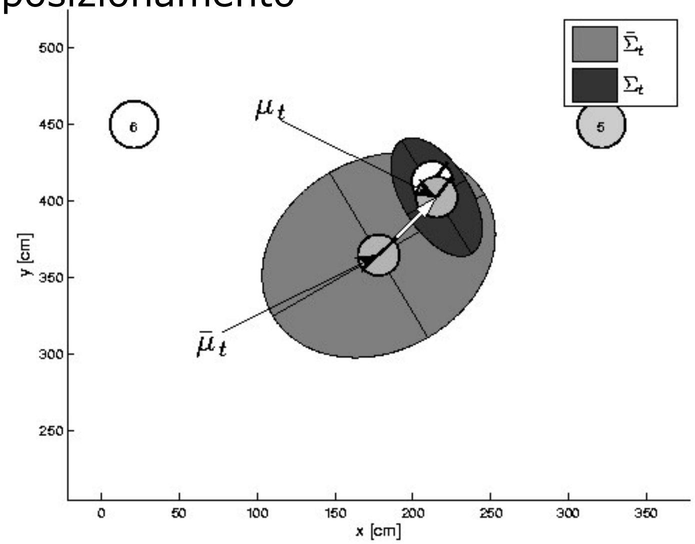
Passo di correzione EKF



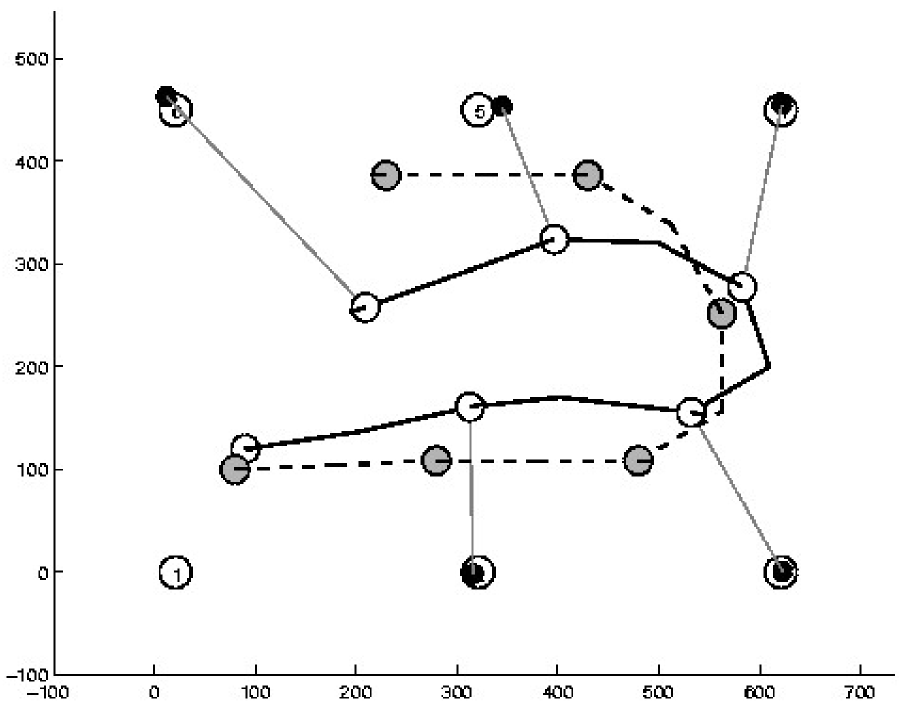


Update della stima. Vettore di innovazione e guadagno di Kalman per riposizionamento

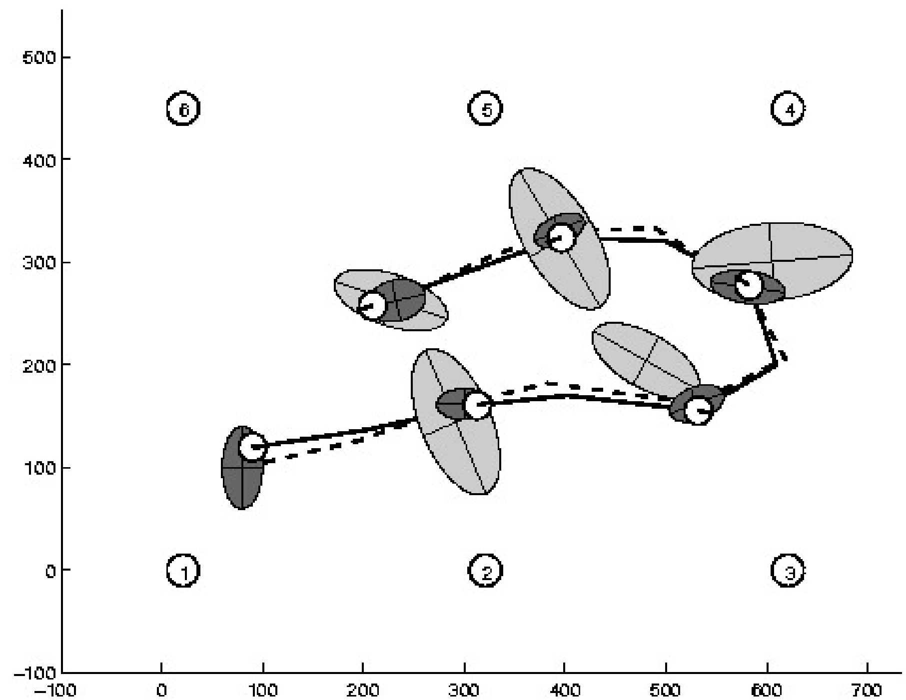




Estimation Sequence (1)

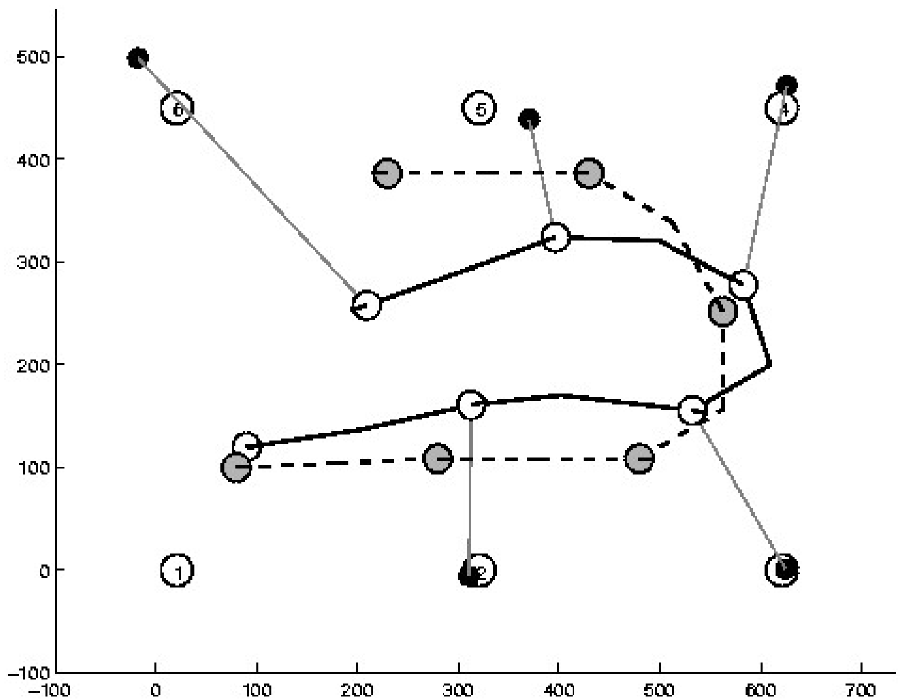


Traiettorie reali (solide),
traiettorie stimate (a tratti)

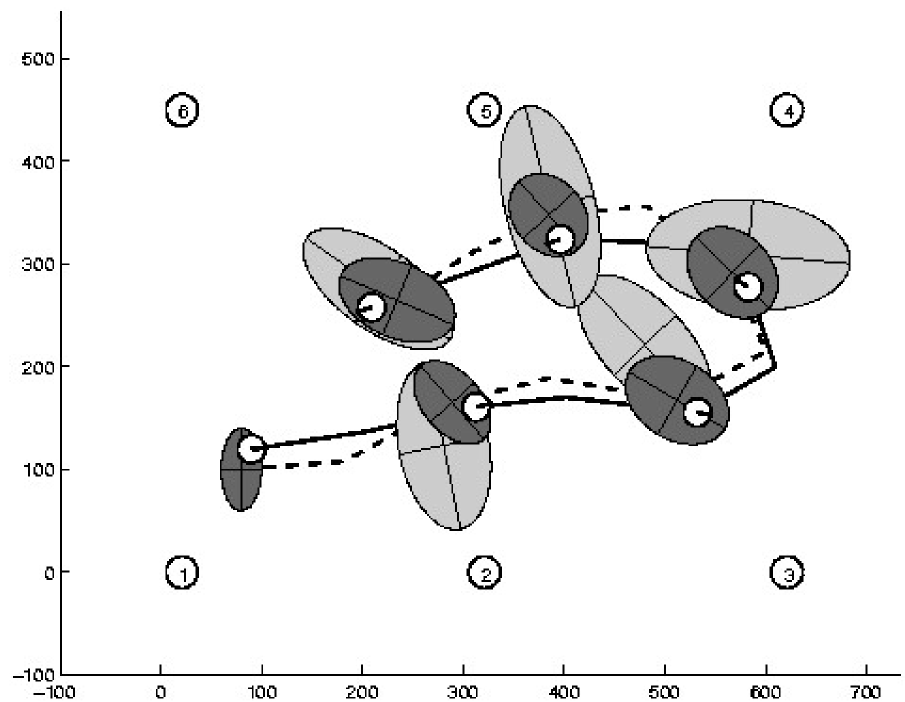


Basso errore

Estimation Sequence (2)



Traiettorie reali (solide),
traiettorie controllate (a tratti)



Errore di misura

Confronto con la GroundTruth