

SLAM Grid-based

- Possiamo risolvere il problema SLAM senza landmark predefiniti?
- Possiamo usare le idee di FastSLAM per costruire grid map?
- Come con i landmark, la mappa dipende dalle pose del robot durante l'acquisizione dati
- Se le pose sono note, grid-based mapping è facile

Occupancy Grid Maps

- Introdotte da Moravec and Elfes nel 1985
- Rappresentano l'ambiente mediante griglia
- Stima la probabilità che una cella sia occupata da un ostacolo.
- **Assunzioni**
 - Occupazione delle celle ($m[xy]$) indipendente

$$\begin{aligned} Bel(m_t) &= P(m_t \mid u_1, z_2 \dots, u_{t-1}, z_t) \\ &= \prod_{x,y} Bel(m_t^{[xy]}) \end{aligned}$$

- Ogni cella è una variabile aleatoria binaria:
 - $P(c) = 1$ occupata, $P(c) = 0$ libera, $P(c) = 0.5$

Occupancy Grid Maps Update

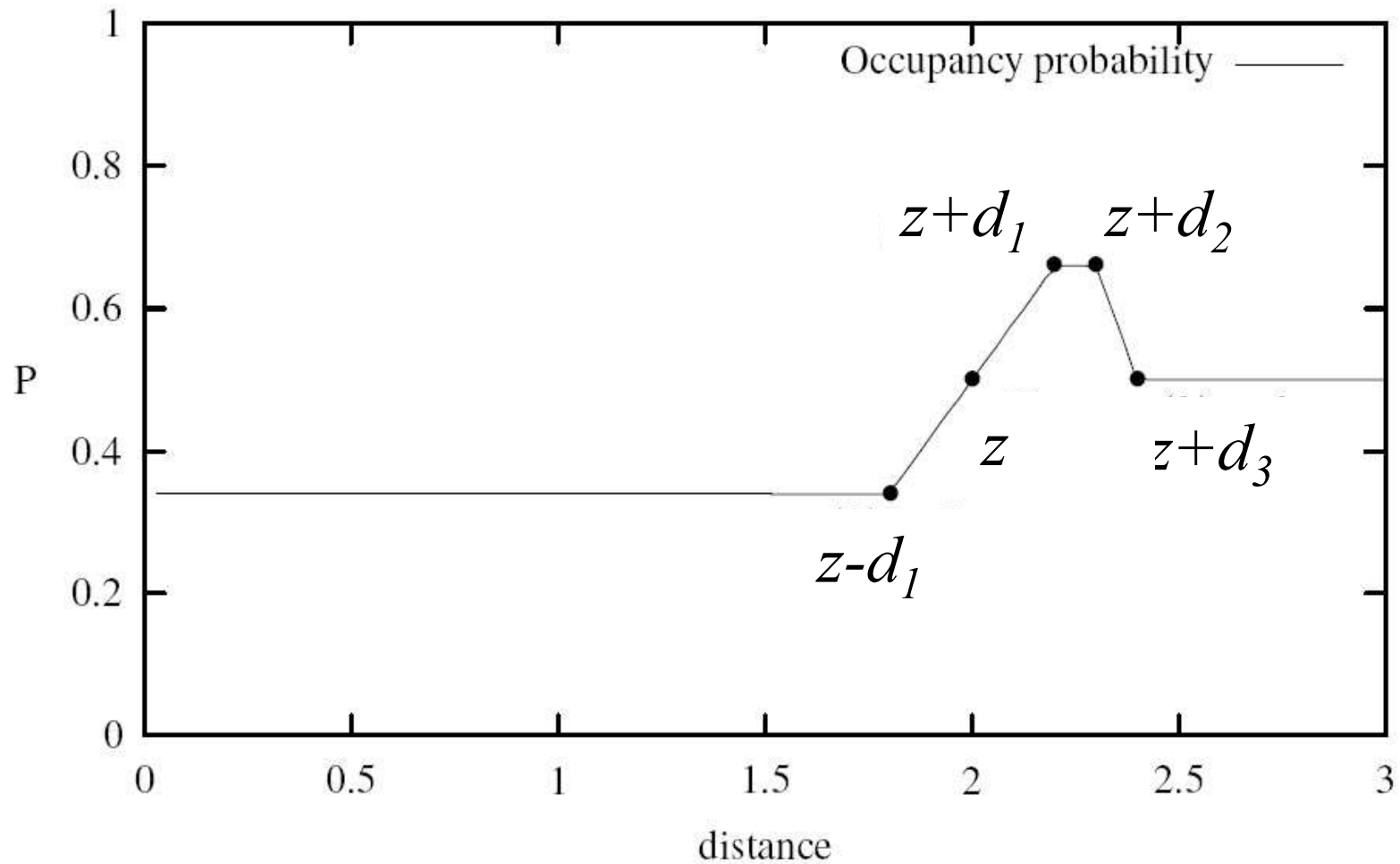
- Aggiornare ogni cella con un **binary Bayes filter**.

$$Bel(m_t^{[xy]}) = \eta p(z_t | m_t^{[xy]}) \int p(m_t^{[xy]} | m_{t-1}^{[xy]}, u_{t-1}) Bel(m_{t-1}^{[xy]}) dm_{t-1}^{[xy]}$$

- **Assunzione:** Mappa statica

$$Bel(m_t^{[xy]}) = \eta p(z_t | m_t^{[xy]}) Bel(m_{t-1}^{[xy]})$$

Occupancy Value Depending on the Measured Distance



Updating Occupancy Grid Maps

- Update the map cells using the **inverse sensor model**

$$Bel(m_t^{[xy]}) = 1 - \left(1 + \frac{P(m_t^{[xy]} | z_t, u_{t-1})}{1 - P(m_t^{[xy]} | z_t, u_{t-1})} \cdot \frac{1 - P(m_t^{[xy]})}{P(m_t^{[xy]})} \cdot \frac{Bel(m_{t-1}^{[xy]})}{1 - Bel(m_{t-1}^{[xy]})} \right)^{-1}$$

- Or use the **log-odds representation**

$$\begin{aligned} \bar{B}(m_t^{[xy]}) &= \log odds(m_t^{[xy]} | z_t, u_{t-1}) \\ &\quad - \log odds(m_t^{[xy]}) \\ &\quad + \bar{B}(m_{t-1}^{[xy]}) \end{aligned}$$

$$\begin{aligned} \bar{B}(m_t^{[xy]}) &:= \log odds(m_t^{[xy]}) \\ odds(x) &:= \left(\frac{P(x)}{1 - P(x)} \right) \end{aligned}$$

Occupancy Grid Maps Update

Algorithm occupancy_grid_mapping($\{l_{t-1,i}\}, x_t, z_t$):

for all cells m_i do

if m_i in perceptual field of z_t then

$$l_{t,i} = l_{t-1,i} + \text{inverse_sensor_model}(m_i, x_t, z_t) - l_0$$

else

$$l_{t,i} = l_{t-1,i}$$

endif

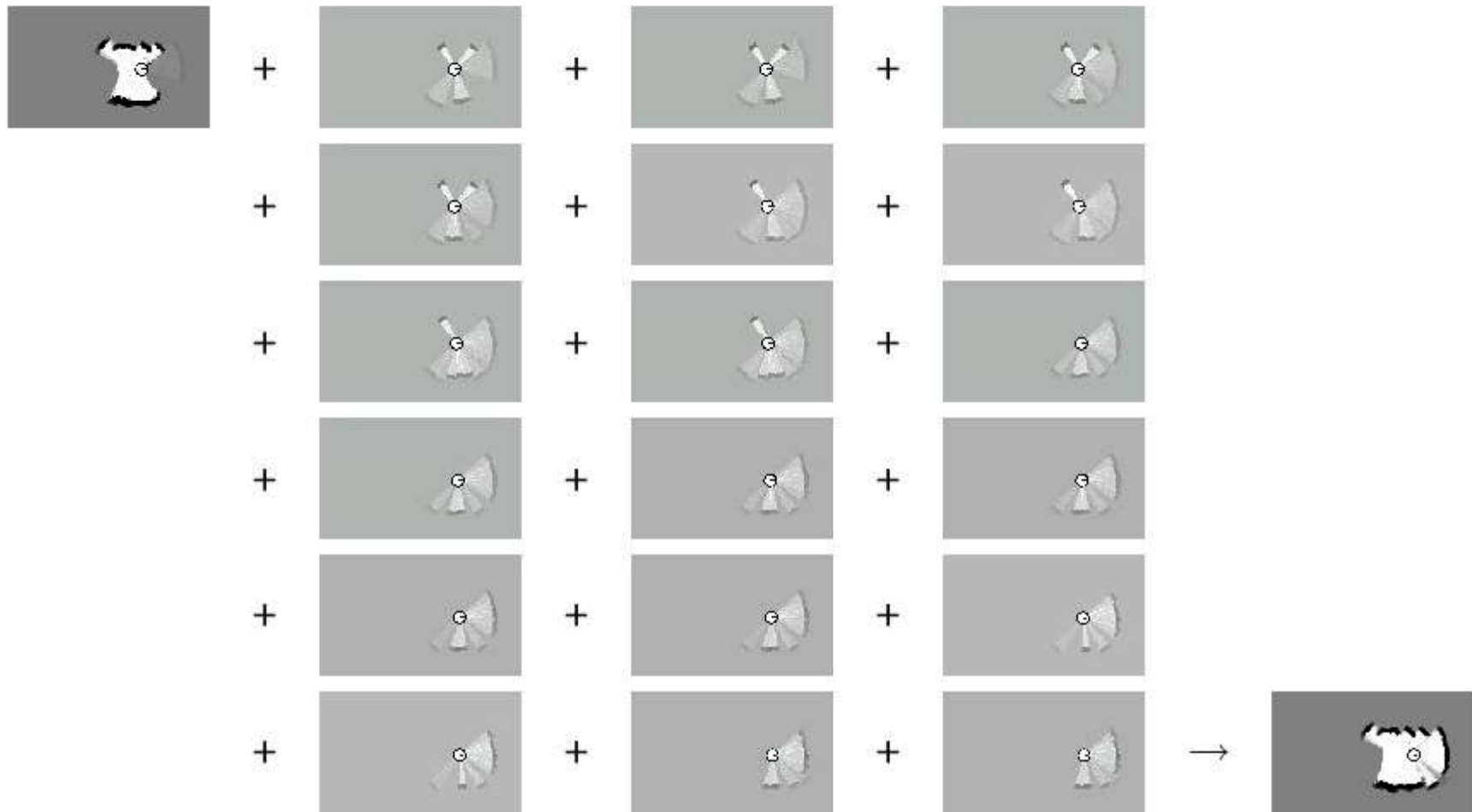
endfor

return $\{l_{t,i}\}$

$$\text{inverse_sensor_model}(m_i, x_t, z_t) = p(m_i \mid z_t, x_t)$$

$$l_0 = \log \frac{p(m_i = 1)}{p(m_i = 0)} = \log \frac{p(m_i)}{1 - p(m_i)}$$

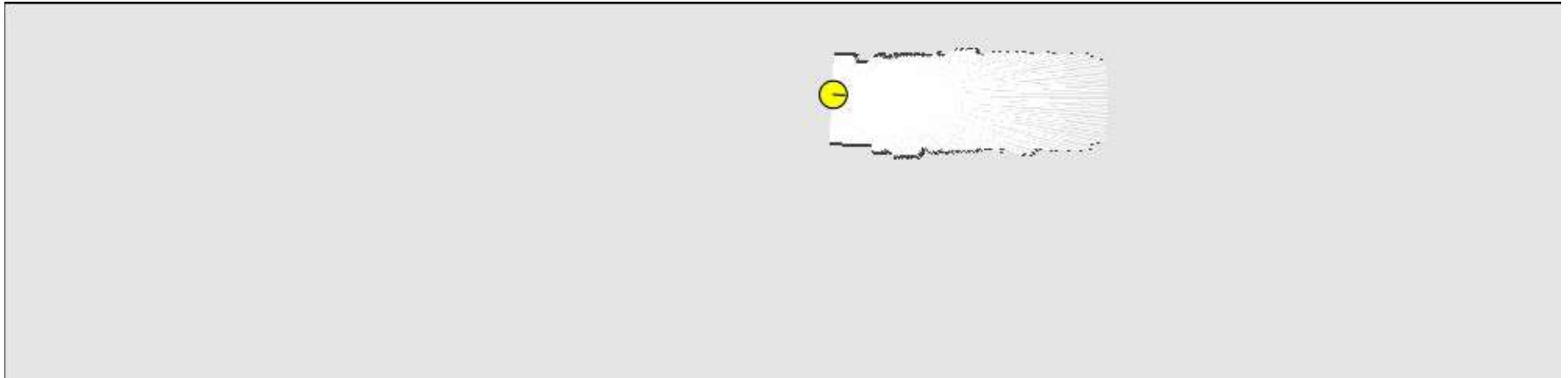
Incremental Updating of Occupancy Grids (Example)



Mapping usando Raw Odometry



Mapping con pose note



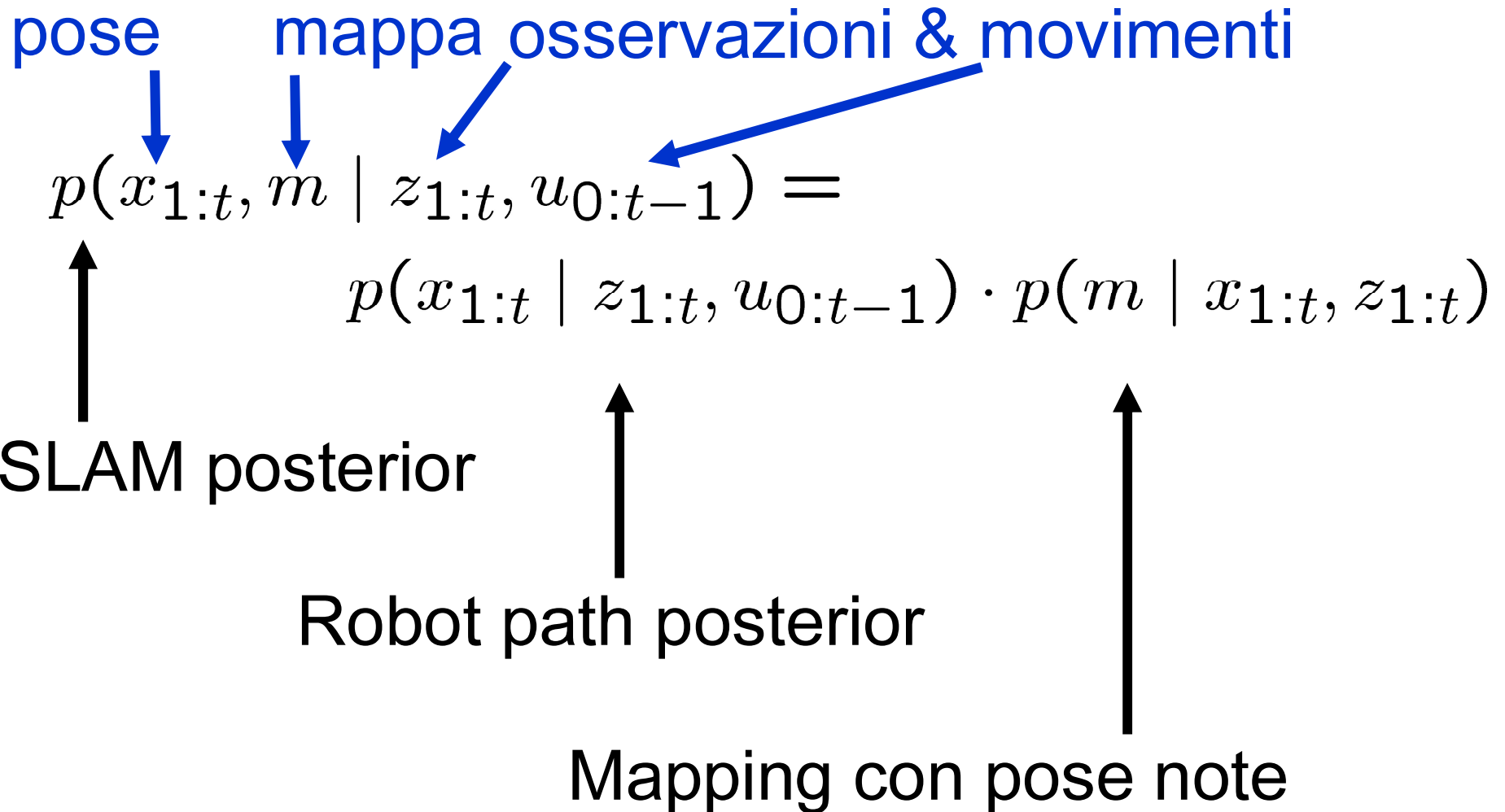
- Mapping con pose note usando dati di laser range

Rao-Blackwellization

pose mappa osservazioni & movimenti

$$p(x_{1:t}, m \mid z_{1:t}, u_{0:t-1}) = p(x_{1:t} \mid z_{1:t}, u_{0:t-1}) \cdot p(m \mid x_{1:t}, z_{1:t})$$

Rao-Blackwellization



Rao-Blackwellization

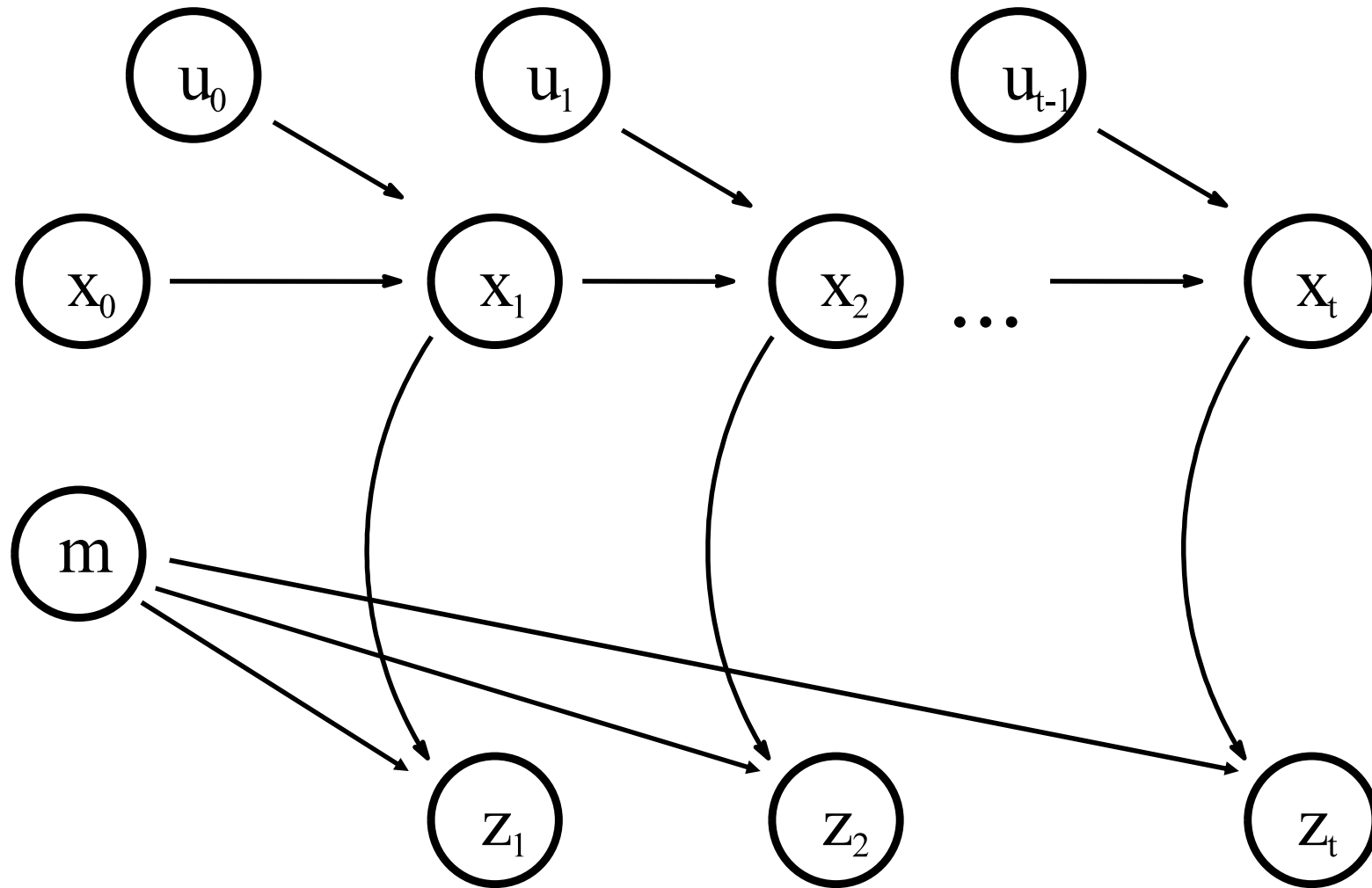
$$p(x_{1:t}, m \mid z_{1:t}, u_{0:t-1}) = p(x_{1:t} \mid z_{1:t}, u_{0:t-1}) \cdot p(m \mid x_{1:t}, z_{1:t})$$

Localizzazione, usa MCLoc



Usa la stima di posa dalla MCLoc e applica il mapping con pose note

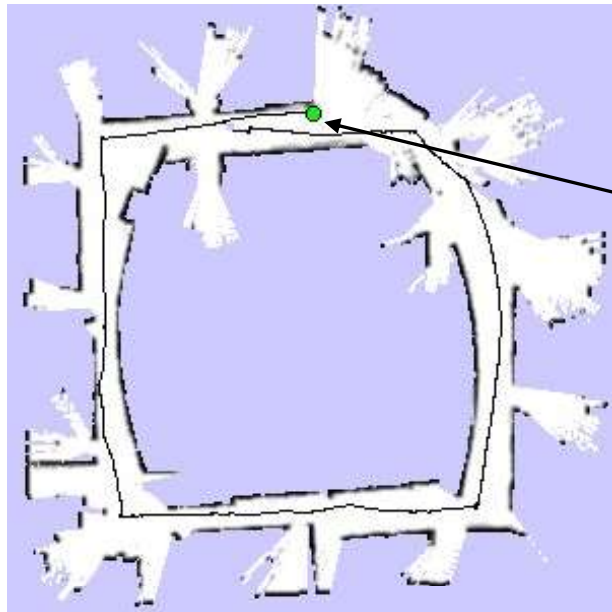
Un modello grafico del Mapping Rao-Blackwellized



Mapping Rao-Blackwellized

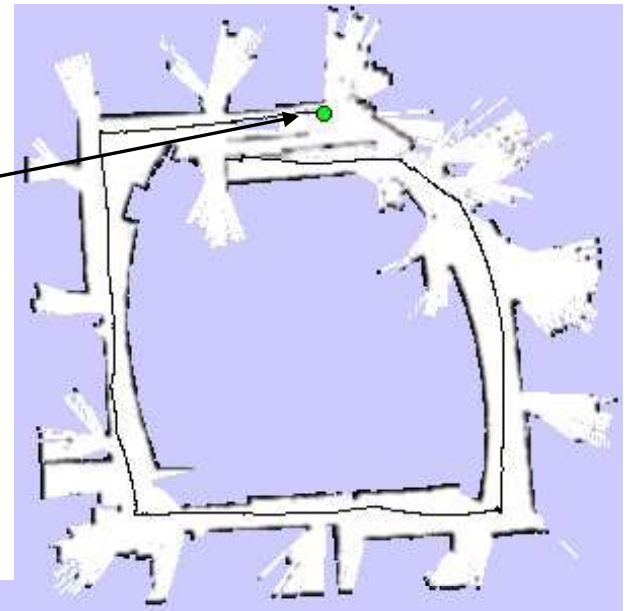
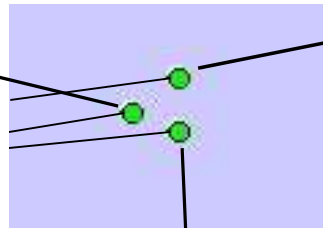
- Ogni particella rappresenta una possibile traiettoria del robot
- Ogni particella
 - mantiene la sua propria mappa e
 - aggiorna con “mapping con pose note”
- Ogni particella sopravvive con una probabilità proporzionale al likelihood di osservazione relativa alla sua propria mappa

Esempio Particle Filter

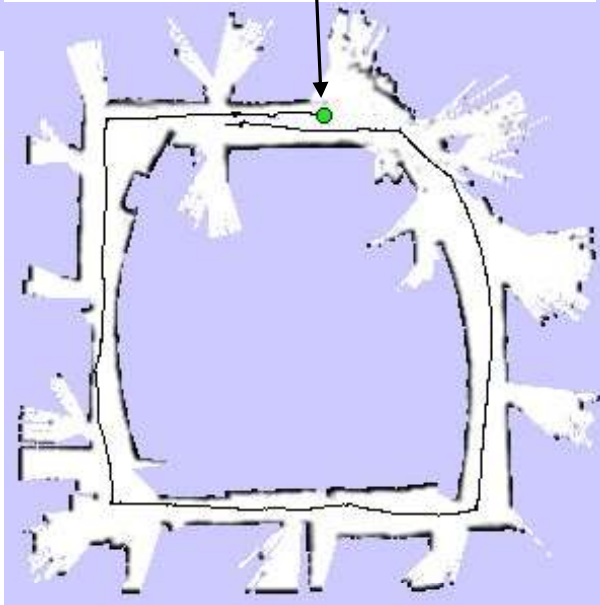


mappa particella 1

3 particelle



mappa particella 3



mappa particella 2

Problema

- Ogni mappa è grande (grid maps)
- Poichè ogni particella mantiene la sua mappa
- Il numero di particelle va mantenuto piccolo (poche ma buone)
- **Soluzione:**
Calcolo di migliori distribuzioni
- **Idea:**
Migliorare la stima di posa **prima** di applicare il particle filter

Correzione di posa con Scan Matching

Massimizza il likelihood della i -esima posa e mappa relativa alla $(i-1)$ -esima posa e mappa

$$\hat{x}_t = \underset{x_t}{\operatorname{argmax}} \left\{ p(z_t | x_t, \hat{m}_{t-1}) \cdot p(x_t | u_{t-1}, \hat{x}_{t-1}) \right\}$$

misura corrente

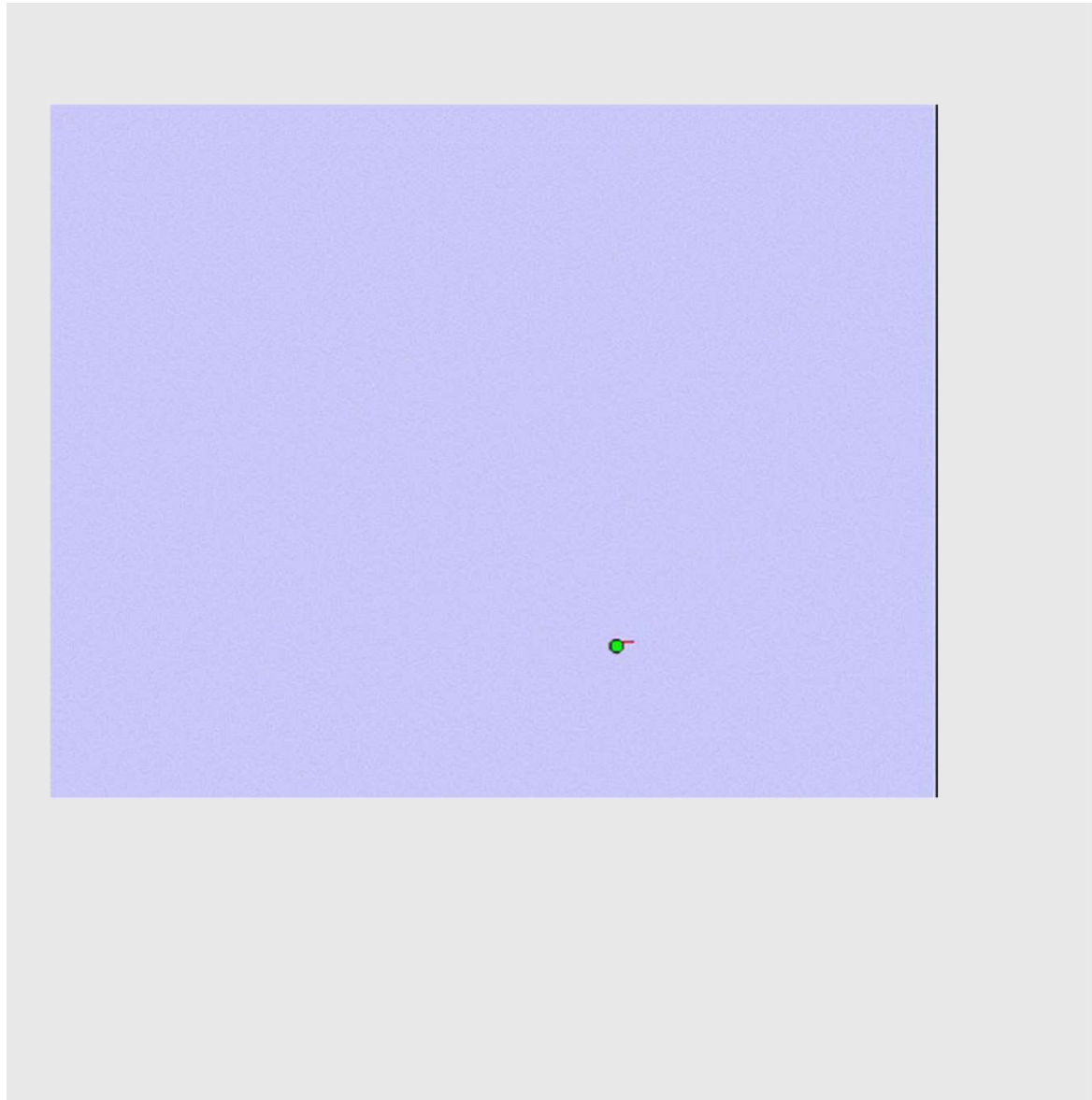
Movimento robotico

mappa costruita fino ad ora

FastSLAM con Odometria Migliorata

- Scan-matching fornisce una correzione di posa **localmente consistente**
- Sequenze odometriche pre-corrette usando scan-matching e uso come input per FastSLAM
- Minori particelle sono necessarie perchè l'errore di input è ridotto

Mapping con Scan Matching

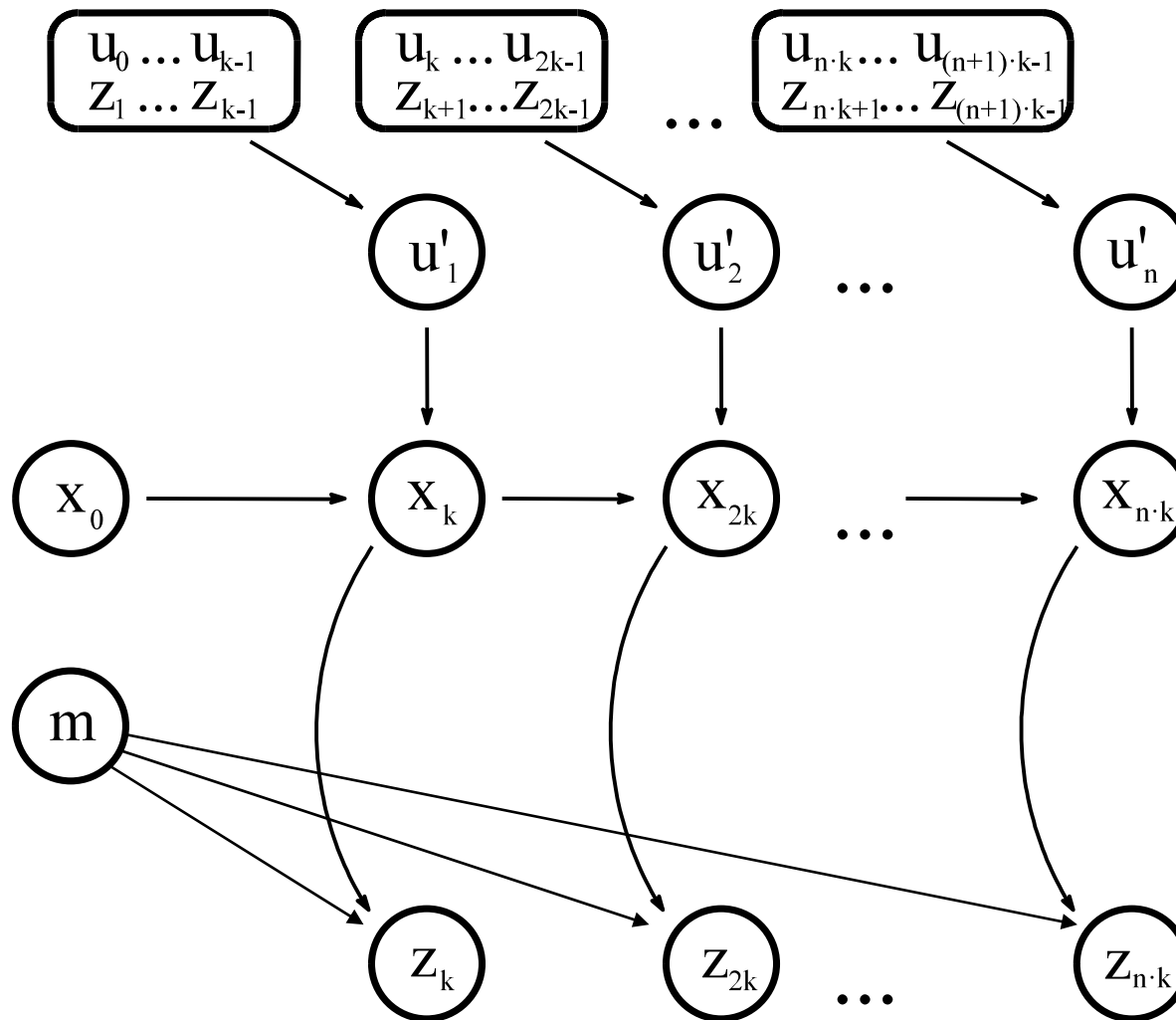


Confronto con FastSLAM Standard

- Stesso modello per le osservazioni
- Odometria invece di scan matching come input
- Numero di particelle variano da 500 a 2.000
- Risultato tipico:



Modello Grafico per Mapping con Odometria Migliorata

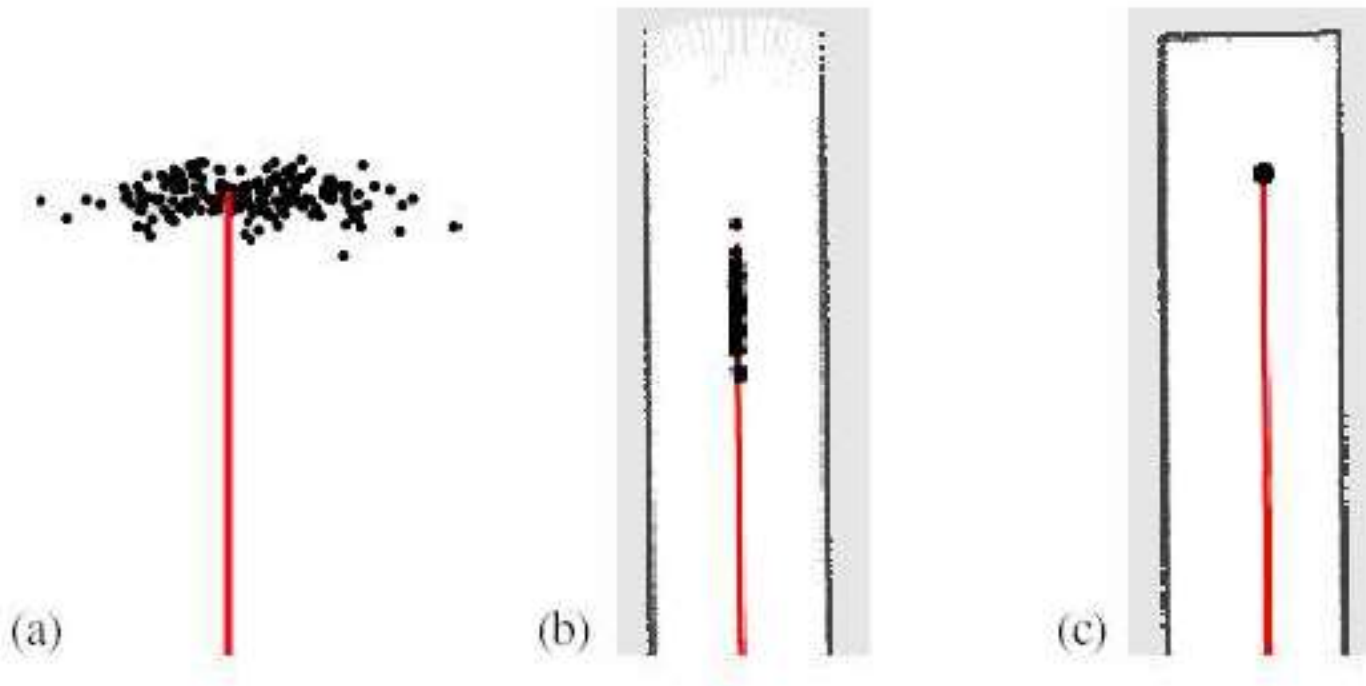


Ulteriori Miglioramenti

- Distribuzione proposta migliorata porta ad una mappa più accurata;
- Ottenuta adattando la distribuzione proposta considerando le osservazioni più recenti;
- Passi di re-sampling flessibile possono ulteriormente migliorare l'accuratezza.

Proposta Migliorata

- La proposta si adatta alla struttura dell'ambiente



Re-sampling selettivo

- Re-sampling è pericoloso, dato che campioni importanti possono andare perduti (particle depletion problem)
- Nel caso di proposte sub-ottime il re-sampling della distribuzione è necessario per arrivare alla convergenza.
- Questione chiave: quando fare il re-sampling?

Numero di Particelle Effettive

$$n_{eff} = \frac{1}{\sum_i \left(w_t^{(i)}\right)^2}$$

- Misure empiriche di quanto la distribuzione obiettivo è approssimata da campioni estratti dalla proposta;
- n_{eff} descrive “la varianza del peso delle particelle”: peggiore è l’approssimazione maggiore è la varianza;
- n_{eff} con pesi uguali massima: la distribuzione è vicina a quella proposta

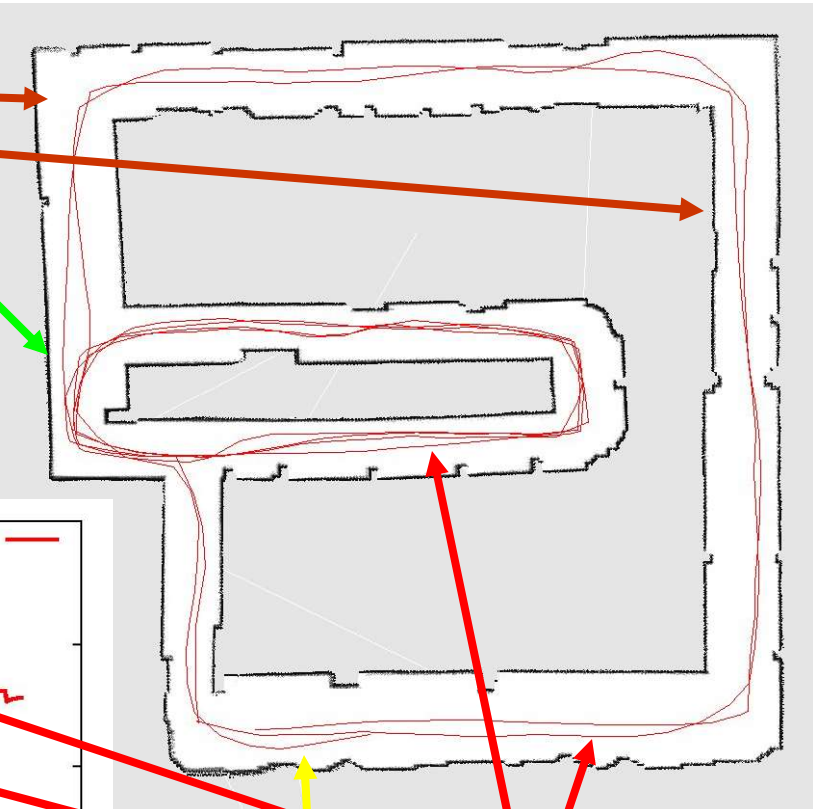
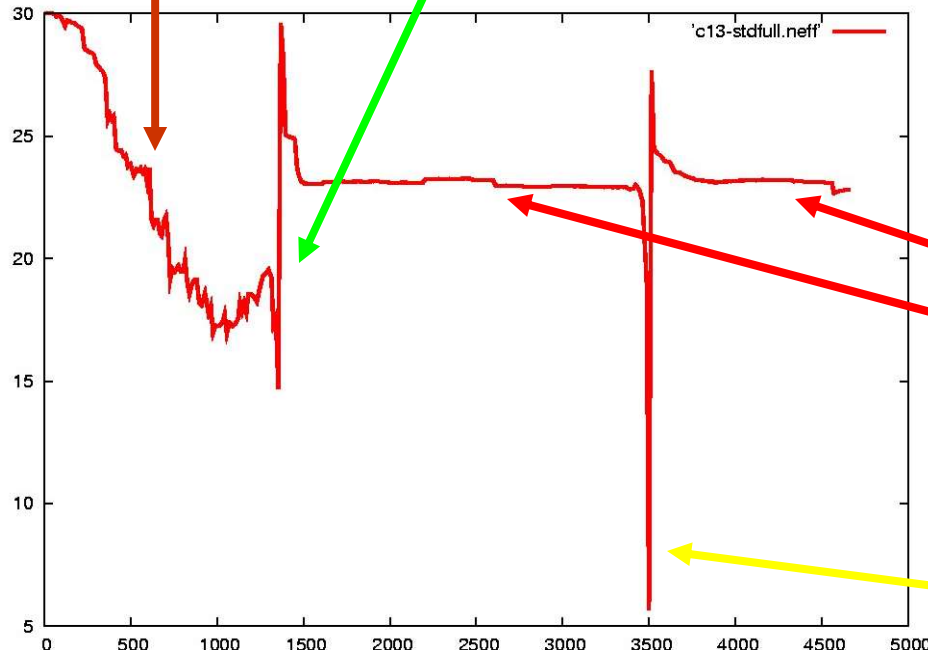
Resampling con Neff

- Re-sampling ogni volta che n_{eff} va sotto una soglia ($n/2$)
- Vedi [Doucet, '98; Arulampalam, '01]

Evoluzione tipica di n_{eff}

Visita di nuove aree

chiusura del primo loop



Visita di area note

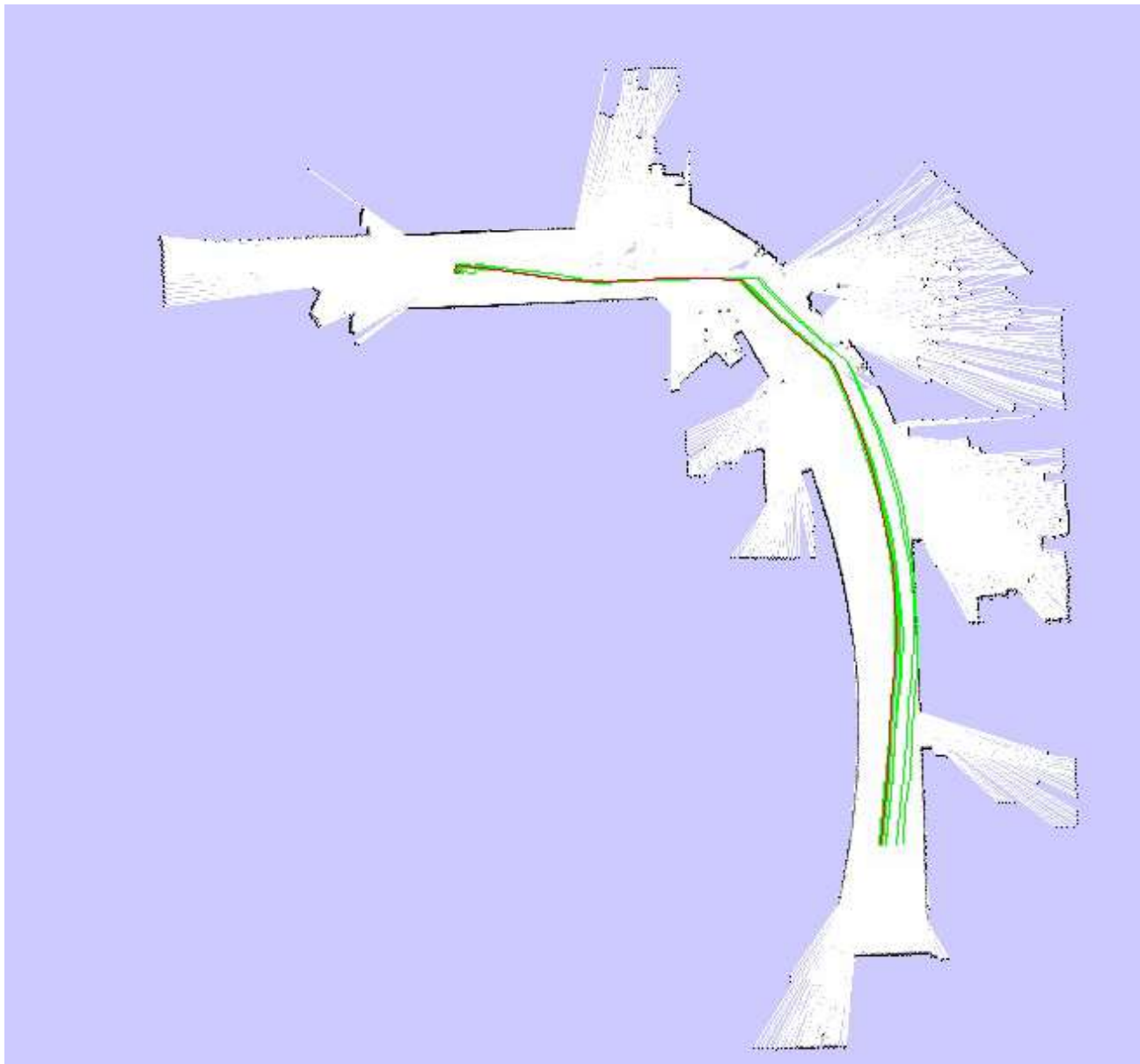
Seconda chiusura loop

Intel Lab



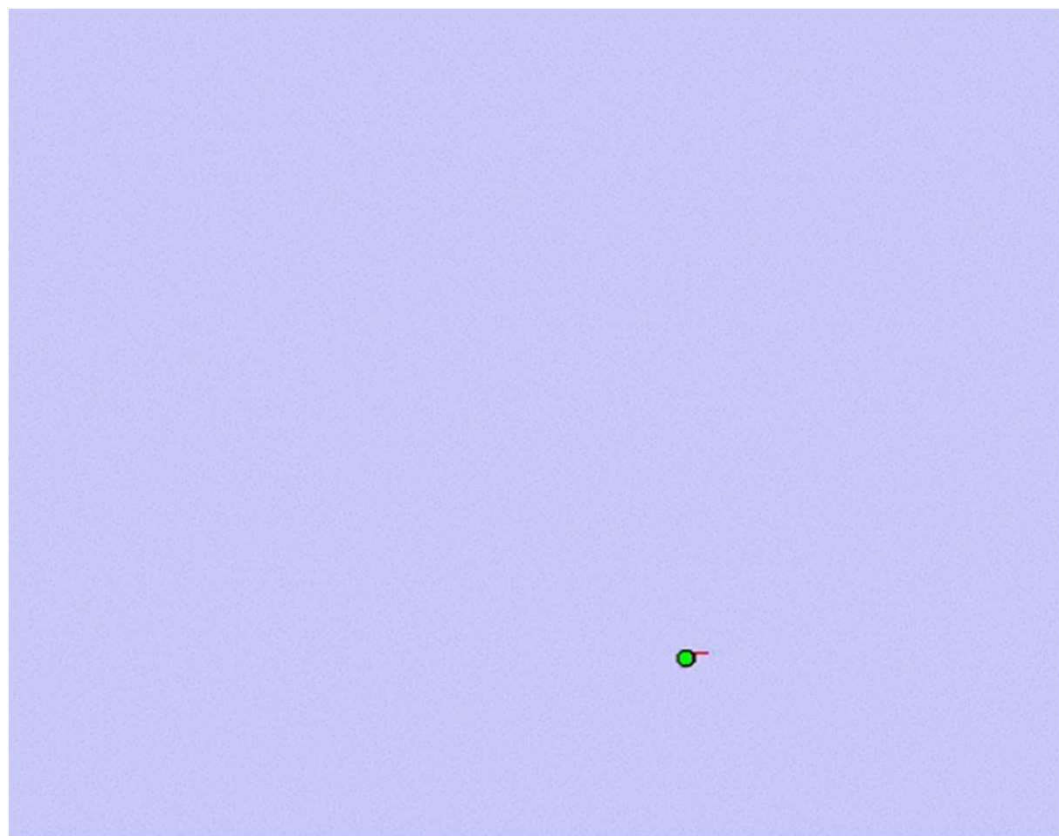
- **15 particelle**
- Quattro volte più veloce del real-time P4, 2.8GHz
- 5cm di risoluzione durante lo scan matching
- 1cm di risoluzione nella mappa finale

Intel Lab



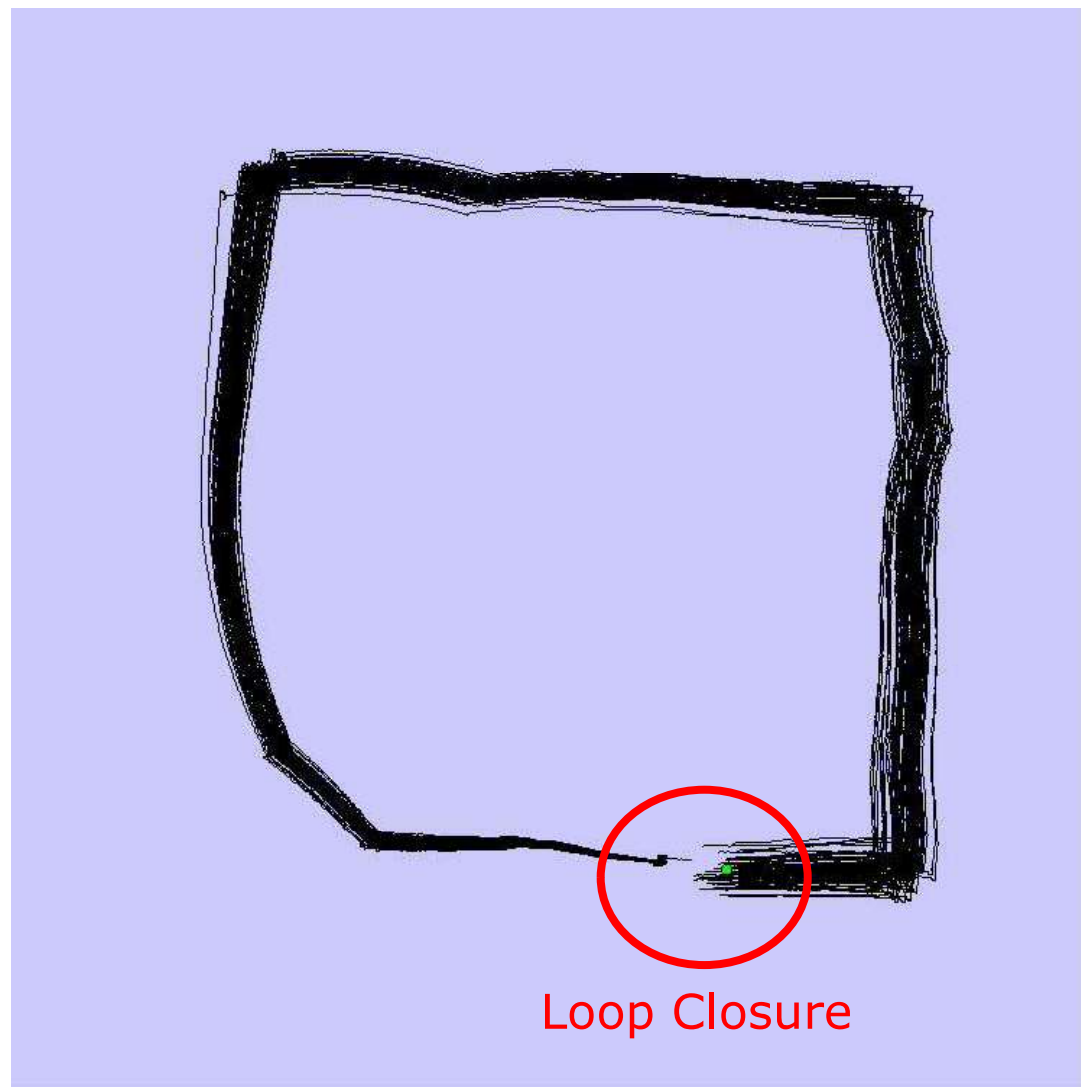
- **15 particelle**
- Confrontato a FastSLAM con Scan-Matching, le particelle sono propagate più vicine alla vera distribuzione

FastSLAM con Scan-Matching

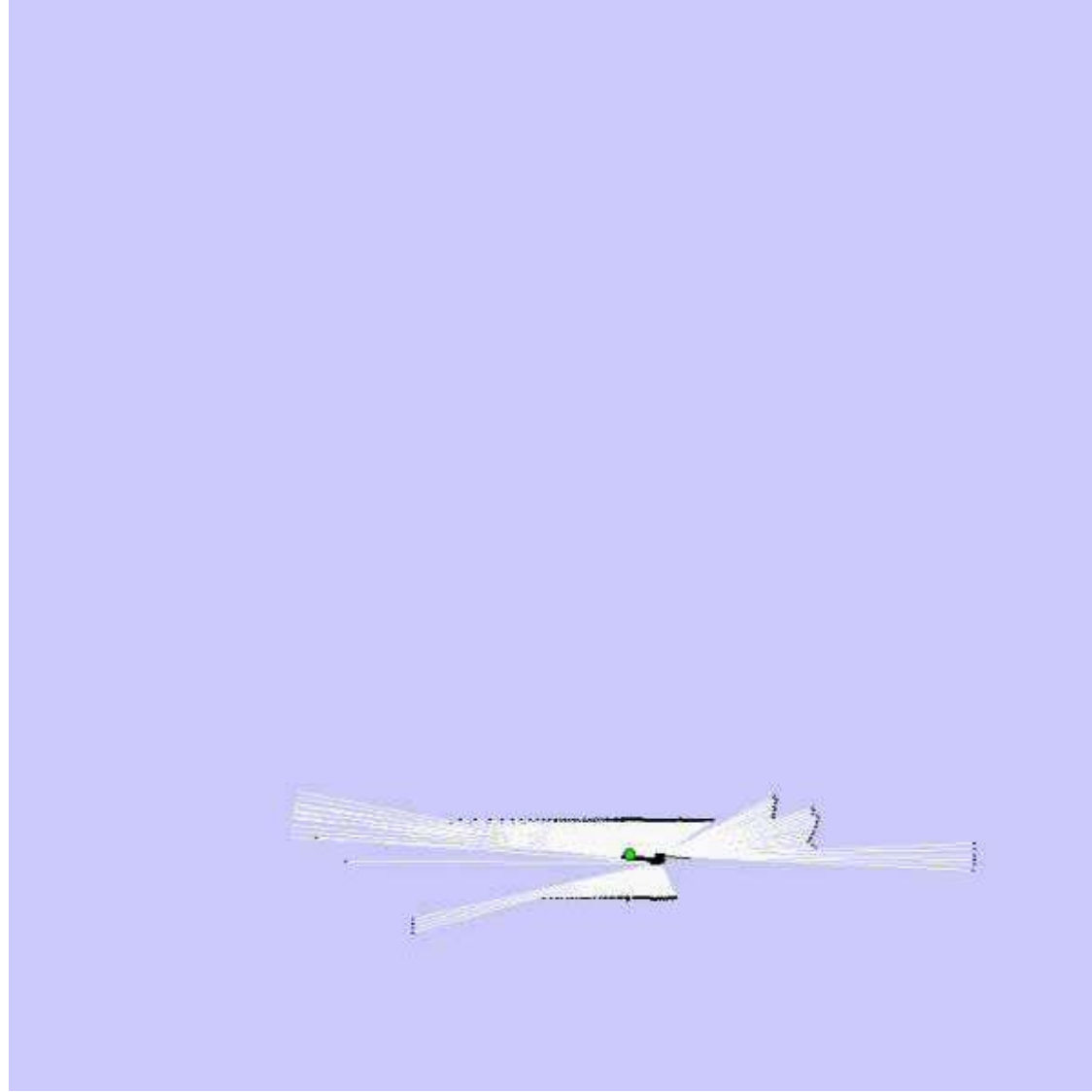


by Dirk Haehne

FastSLAM con Scan-Matching



FastSLAM con Scan-Matching

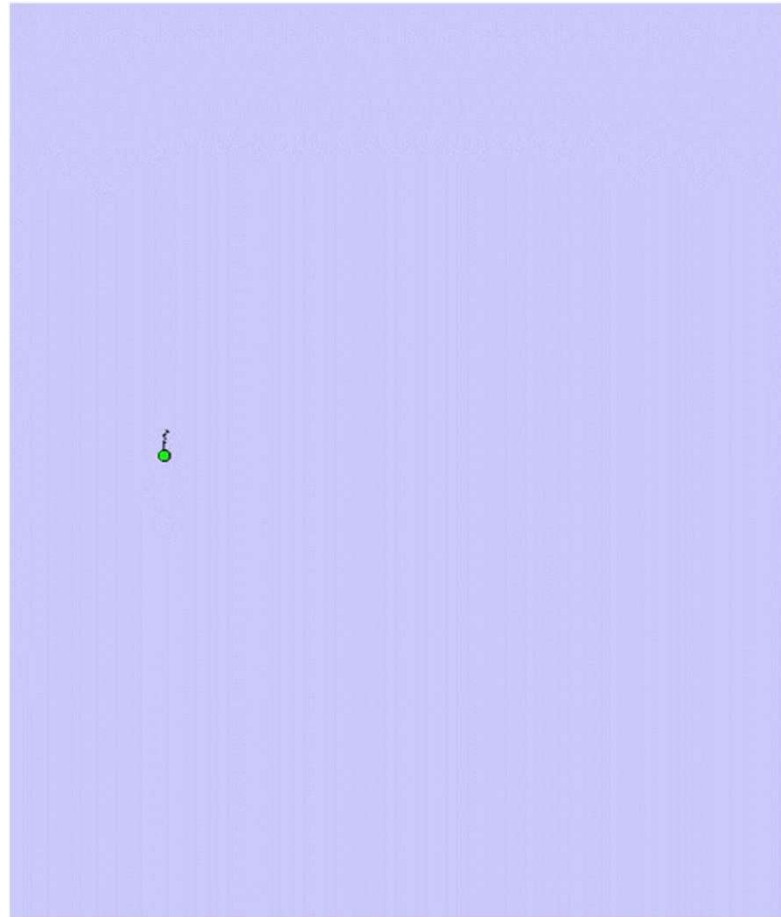


Outdoor Campus Map

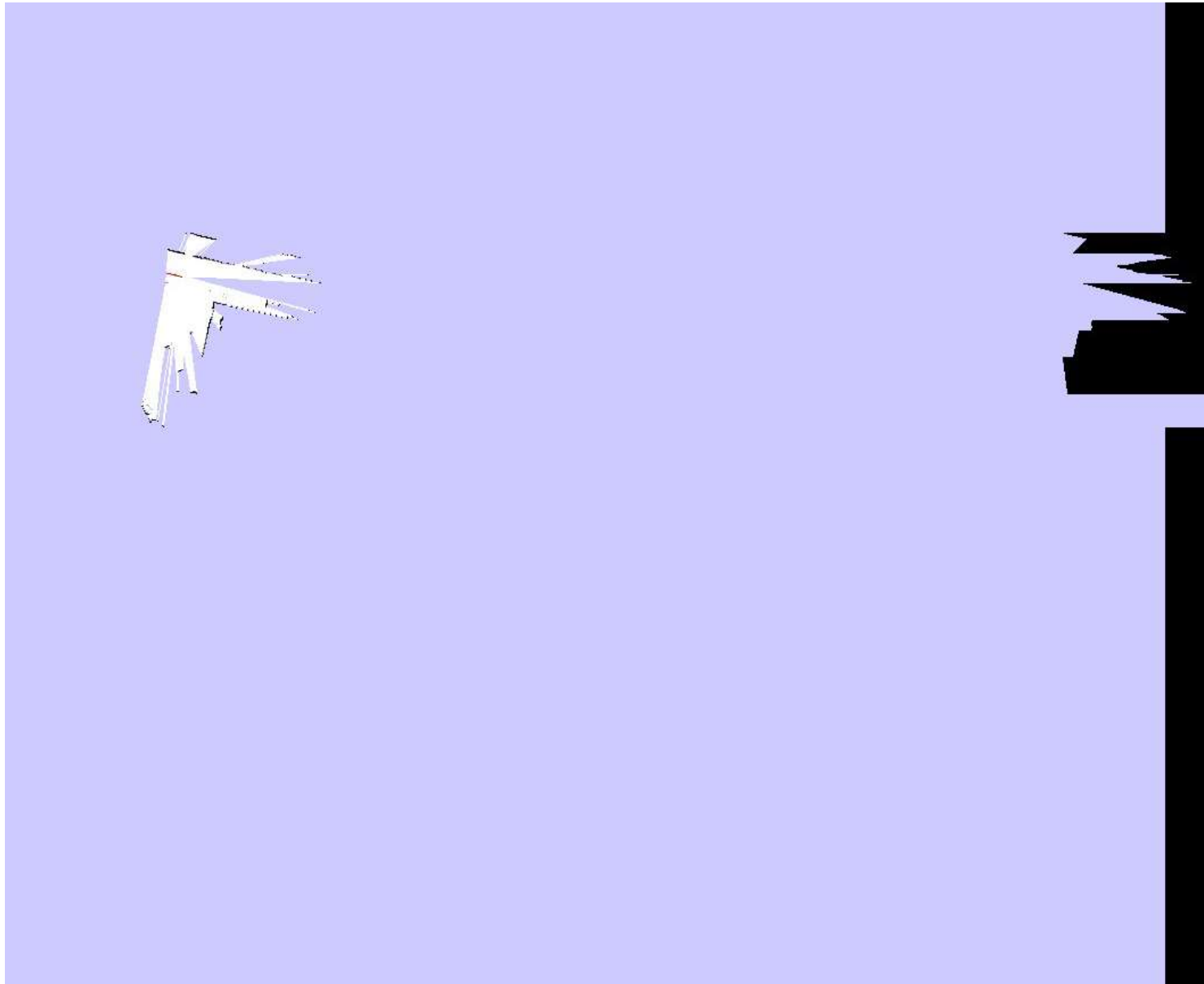


- **30 particelle**
- 250x250m²
- 1.088 miglia (odometria)
- 20cm risoluzione durante lo scan matching
- 30cm risoluzione nella mappa finale

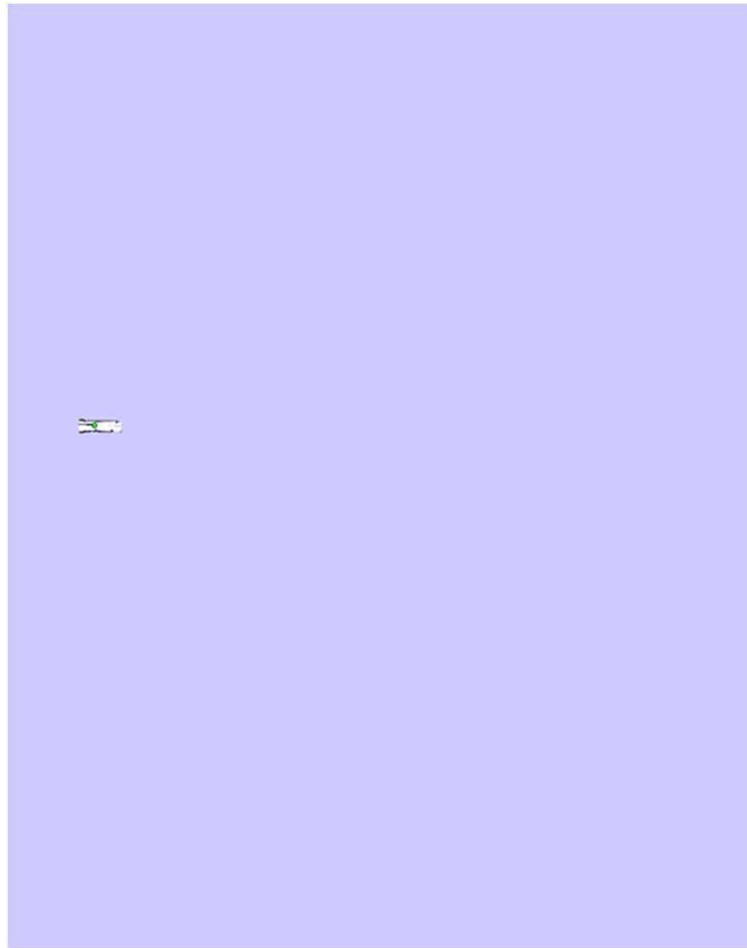
FastSLAM con Scan-Matching



Outdoor Campus Map - Video



FastSLAM con Scan-Matching

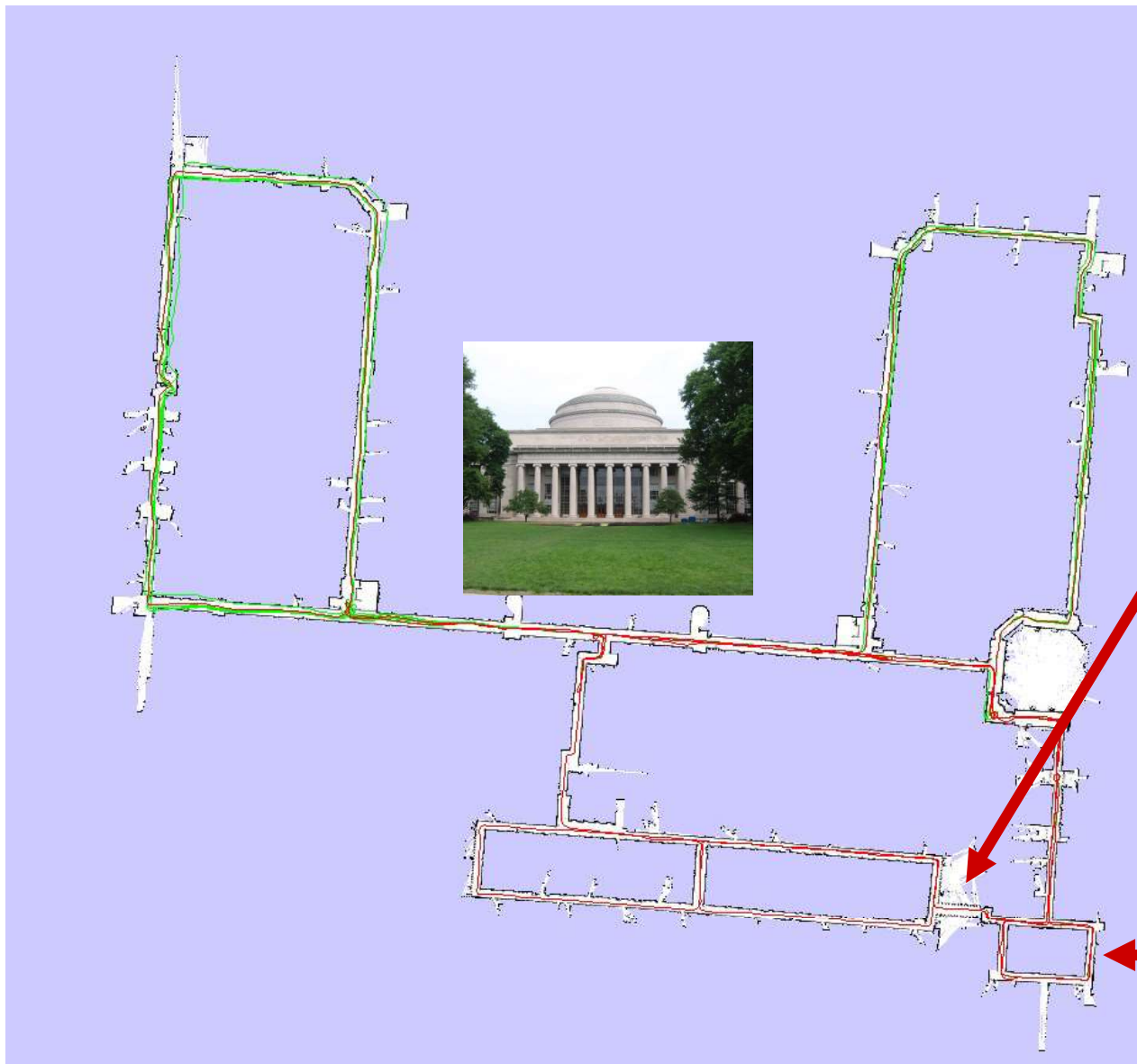


MIT Killian Court

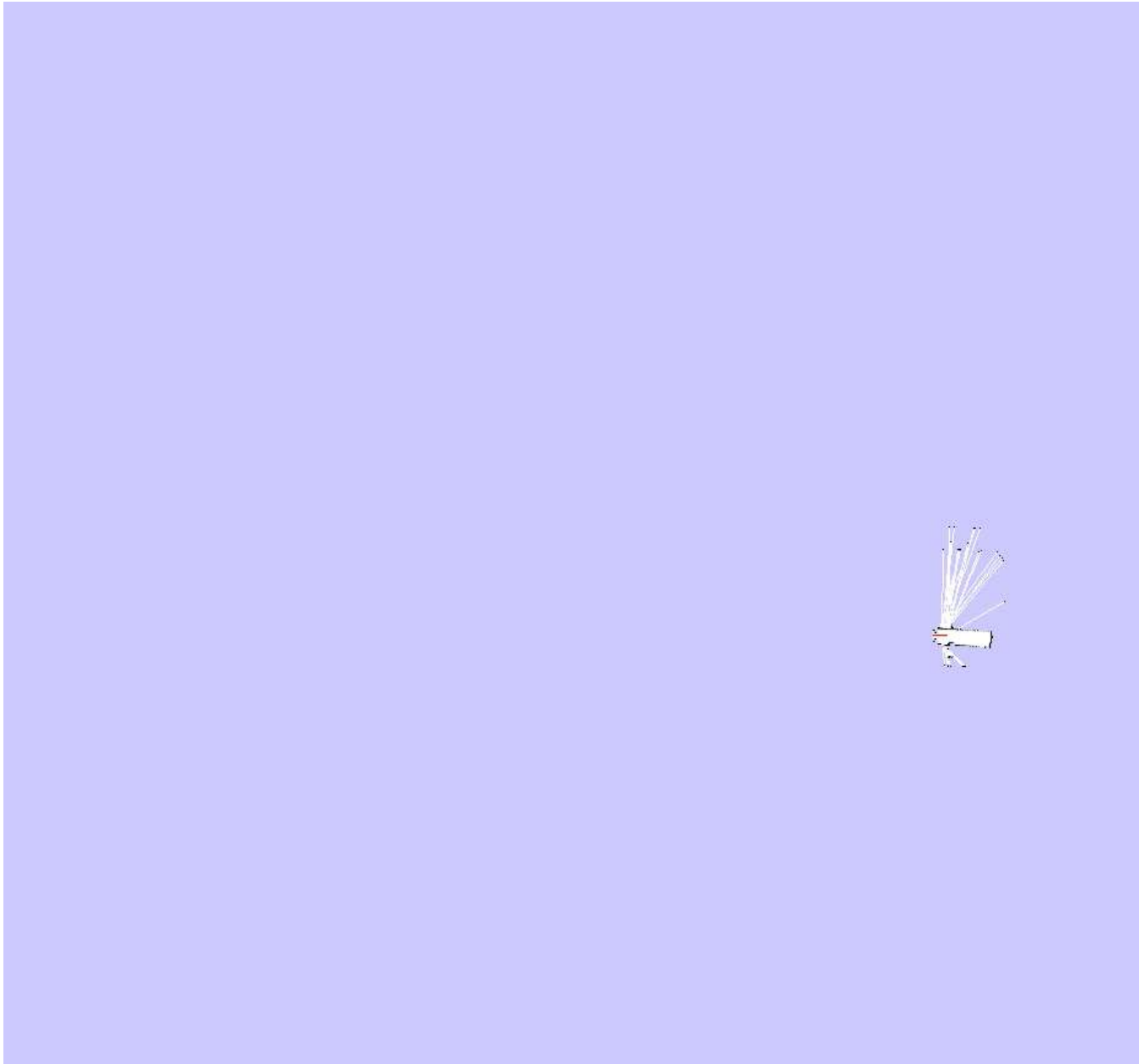


- **“infinite-corridor-dataset”** at MIT

MIT Killian Court



MIT Killian Court - Video



Conclusioni

- Le idee di FastSLAM possono essere applicate alle grid maps
- Usando sensori accurati per le osservazioni porta a buone proposte e filtri altamente efficienti
- Simile allo scan-matching su base per-particle
- Il numero di particelle necessarie e passi di re-sampling può essere seriamente ridotto
- Versioni migliorate di FastSLAM grid-based può gestire ambienti più larghi delle implementazioni naïve in "real time" perché richiedono un ordine di grandezza minore di campioni

Riferimenti su FastSLAM

- M. Montemerlo, S. Thrun, D. Koller, and B. Wegbreit. FastSLAM: A factored solution to simultaneous localization and mapping, *AAAI02*
- D. Haehnel, W. Burgard, D. Fox, and S. Thrun. An efficient FastSLAM algorithm for generating maps of large-scale cyclic environments from raw laser range measurements, *IROS03*
- M. Montemerlo, S. Thrun, D. Koller, B. Wegbreit. FastSLAM 2.0: An Improved particle filtering algorithm for simultaneous localization and mapping that provably converges. *IJCAI-2003*
- G. Grisetti, C. Stachniss, and W. Burgard. Improving grid-based slam with rao-blackwellized particle filters by adaptive proposals and selective resampling, *ICRA05*
- A. Eliazar and R. Parr. DP-SLAM: Fast, robust simultaneous localization and mapping without predetermined landmarks, *IJCAI03*

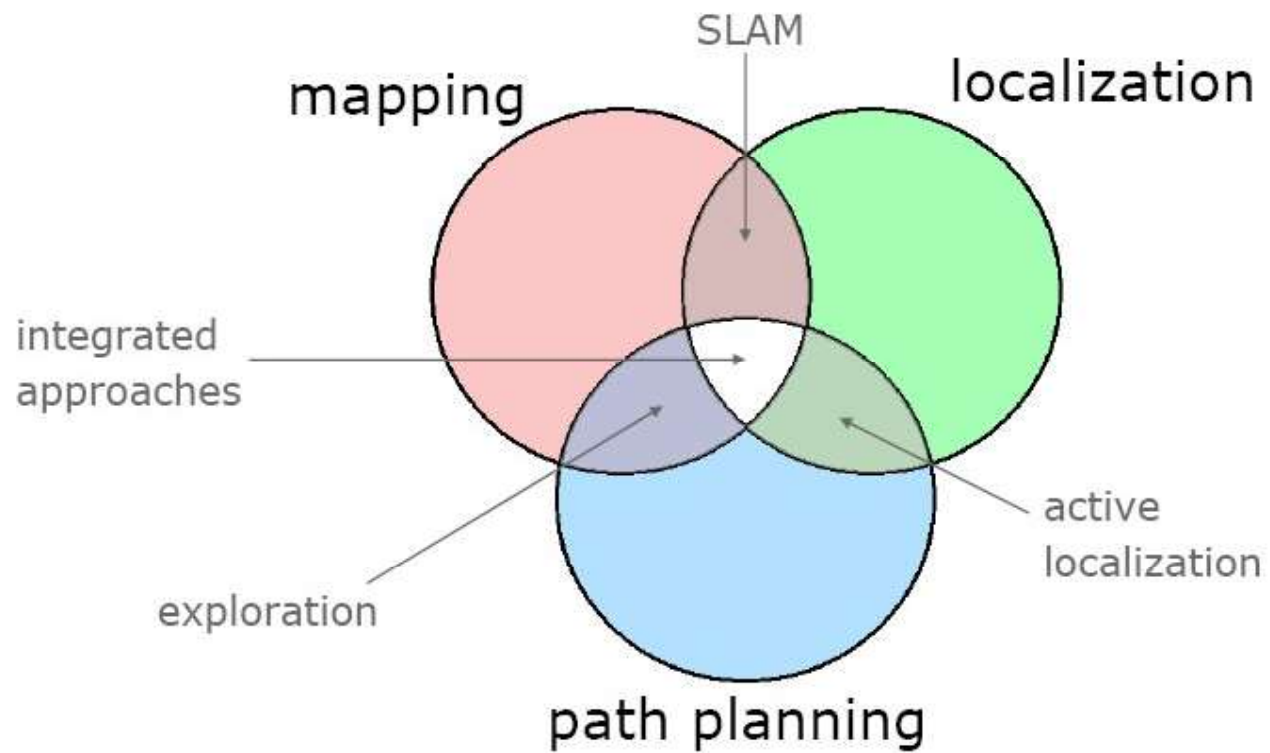
Implementazione

- Implementazione open source
GMapping www.OpenSlam.org
- Implementato in C++
- Utilizzabile con libreria black box

Robotica Probabilistica

Information Gain-Based Exploration

Task dei Robot Mobili



Esplorazione e SLAM

- SLAM is typically **passive**, because it consumes incoming sensor data
- Exploration **actively guides the robot** to cover the environment with its sensors
- Exploration in combination with SLAM: **Acting under pose and map uncertainty**
- Uncertainty should/needs to be taken into account when selecting an action

Filtri Particellari

- Each particle represents a possible trajectory of the robot
- Each particle
 - maintains its own map and
 - updates it upon “mapping with known poses”
- Each particle survives with a probability proportional to the likelihood of the observations relative to its own map

Fattorizzazione

poses map observations & odometry

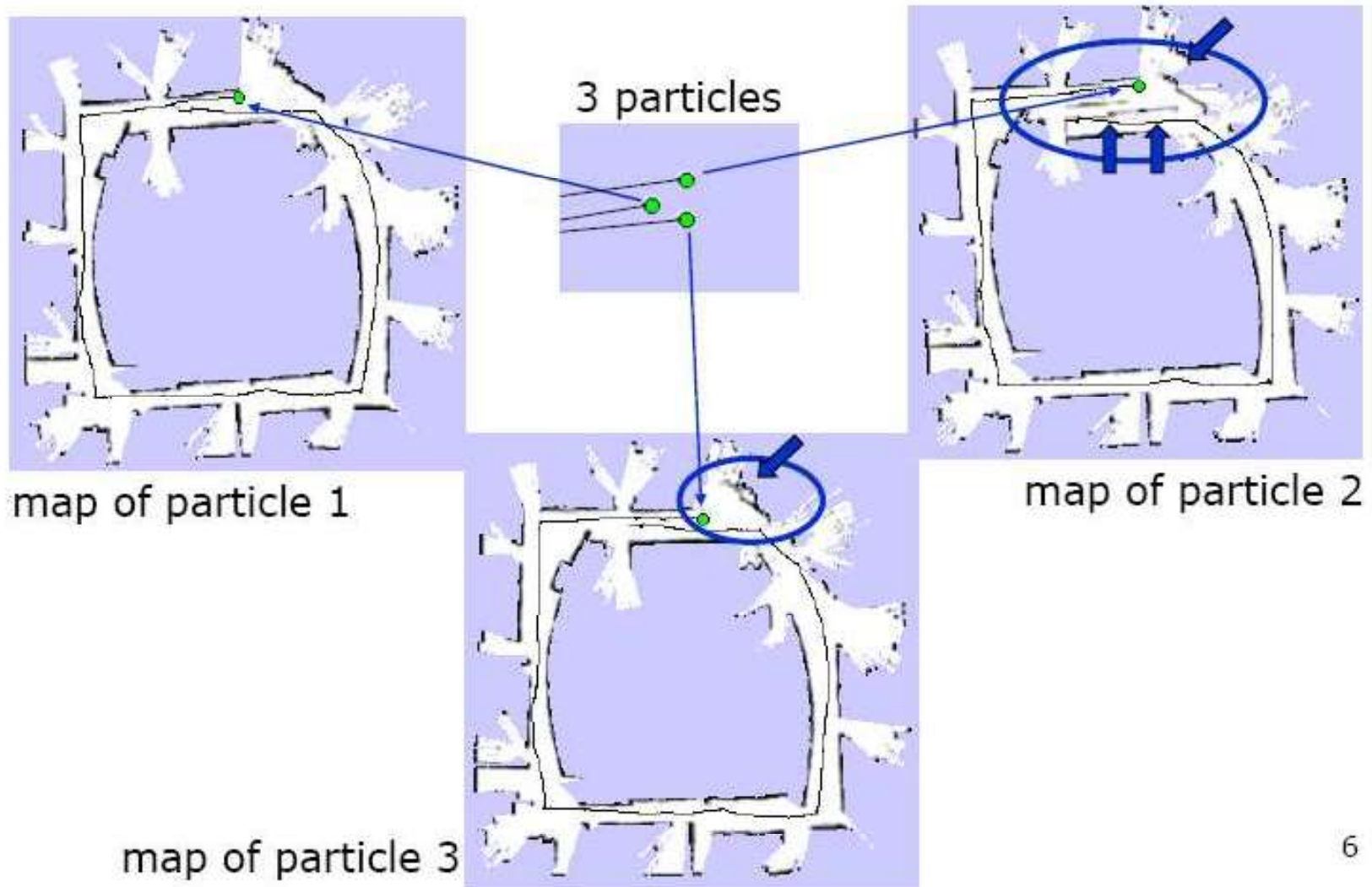
$$p(x, m \mid z, u)$$

$$= p(m \mid x, z, u) p(x \mid z, u)$$

Mapping with known poses

Particle filter representing trajectory hypotheses

Filtri Particellari



FastSLAM

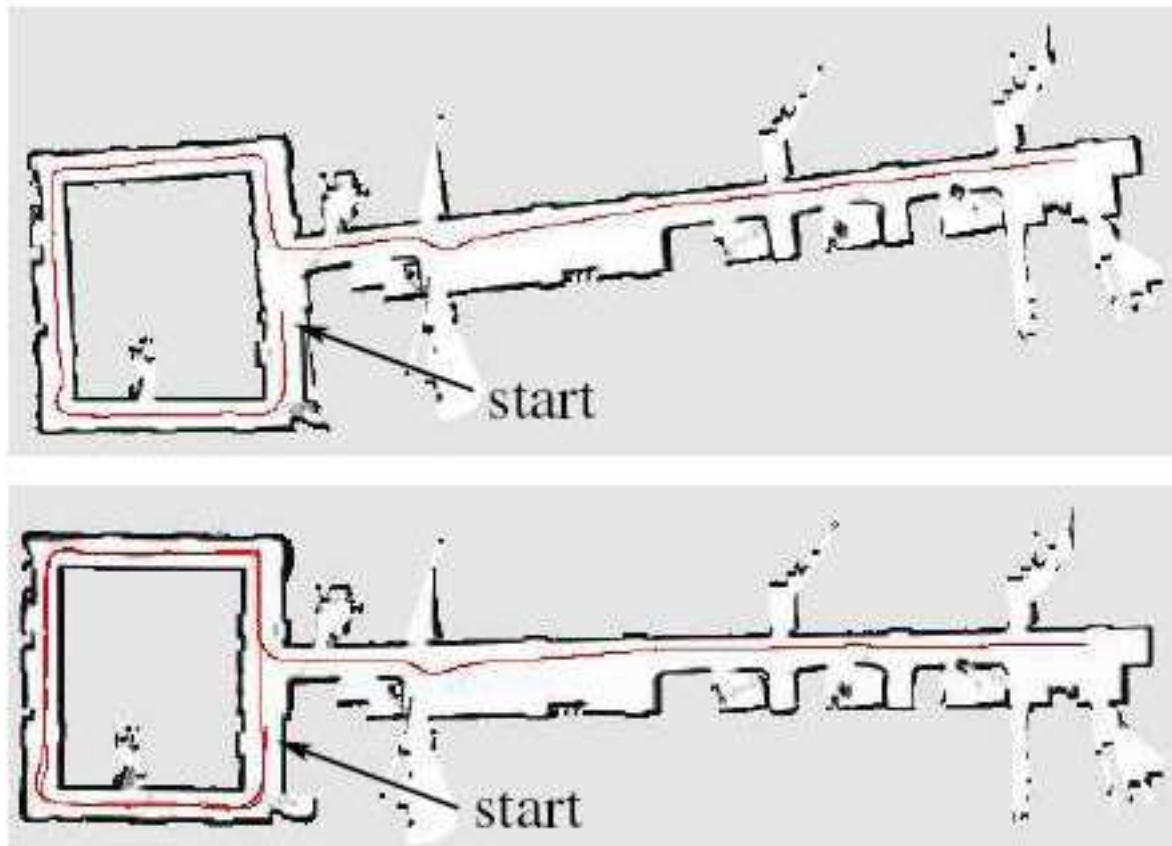


- **30 particles**
- 250x250m²
- 1.75 km (odometry)
- 20cm resolution during scan matching
- 30cm resolution in final map

Esplorazione

- The approaches seen so far are purely passive
- By reasoning about control, the mapping process can be made much more effective
- Question: **Where to move next?**

Esplorazione vs SLAM



La strategia di esplorazione determina la qualità della mappa

Approccio Decision-Teoretico

- Learn the map using a Rao-Blackwellized particle filter
- Consider a set of potential actions
- Apply an exploration approach that minimizes the overall uncertainty

Utility = uncertainty reduction - cost

Incertezza Posterior

- Entropy is a general measure for the uncertainty of a posterior

$$\begin{aligned} H(p(x)) &= - \int_x p(x) \log p(x) dx \\ &= E_x[-\log(p(x))] \end{aligned}$$

- Information Gain = Uncertainty Reduction

$$I(t + 1 | t) = H(p(x_t)) - H(p(x_{t+1}))$$

Calcolo Entropia

$$\begin{aligned} H(p(x, y)) &= E_{x,y}[-\log p(x, y)] \\ &= E_{x,y}[-\log(p(x) p(y | x))] \\ &= E_{x,y}[-\log p(x)] + E_{x,y}[-\log p(y | x)] \\ &= H(p(x)) + \int_{x,y} -p(x, y) \log p(y | x) dx dy \\ &= H(p(x)) + \int_{x,y} -p(y | x)p(x) \log p(y | x) dx dy \\ &= H(p(x)) + \int_x p(x) \int_y -p(y | x) \log p(y | x) dy dx \\ &= H(p(x)) + \int_x p(x) H(p(y | x)) dx \end{aligned}$$

Calcolo incertezza della Mappa e della Posa

$$H(p(x_{1:t}, m | d_t)) = H(p(x_{1:t} | d_t)) + \int_{x_{1:t}} p(x_{1:t} | d_t) H(p(m | x_{1:t}, d_t)) dx_{1:t}$$

Data la rappresentazione approssimata

$$H(p(m, x_{1:t} | d_t)) \approx H(p(x_{1:t} | d_t)) + \sum_{i=1}^{\#particles} \omega_t^{[i]} H(p(m^{[i]} | x_{1:t}^{[i]}, d_t))$$

Incertezza percorso + incertezza mappa

Incertezza del posterior nelle Grid Map

Ogni cella è una variabile aleatoria binaria

Occupancy Grid map m :

$$H(p(m)) = - \sum_{c \in m} p(c) \log p(c) + (1 - p(c)) \log(1 - p(c))$$

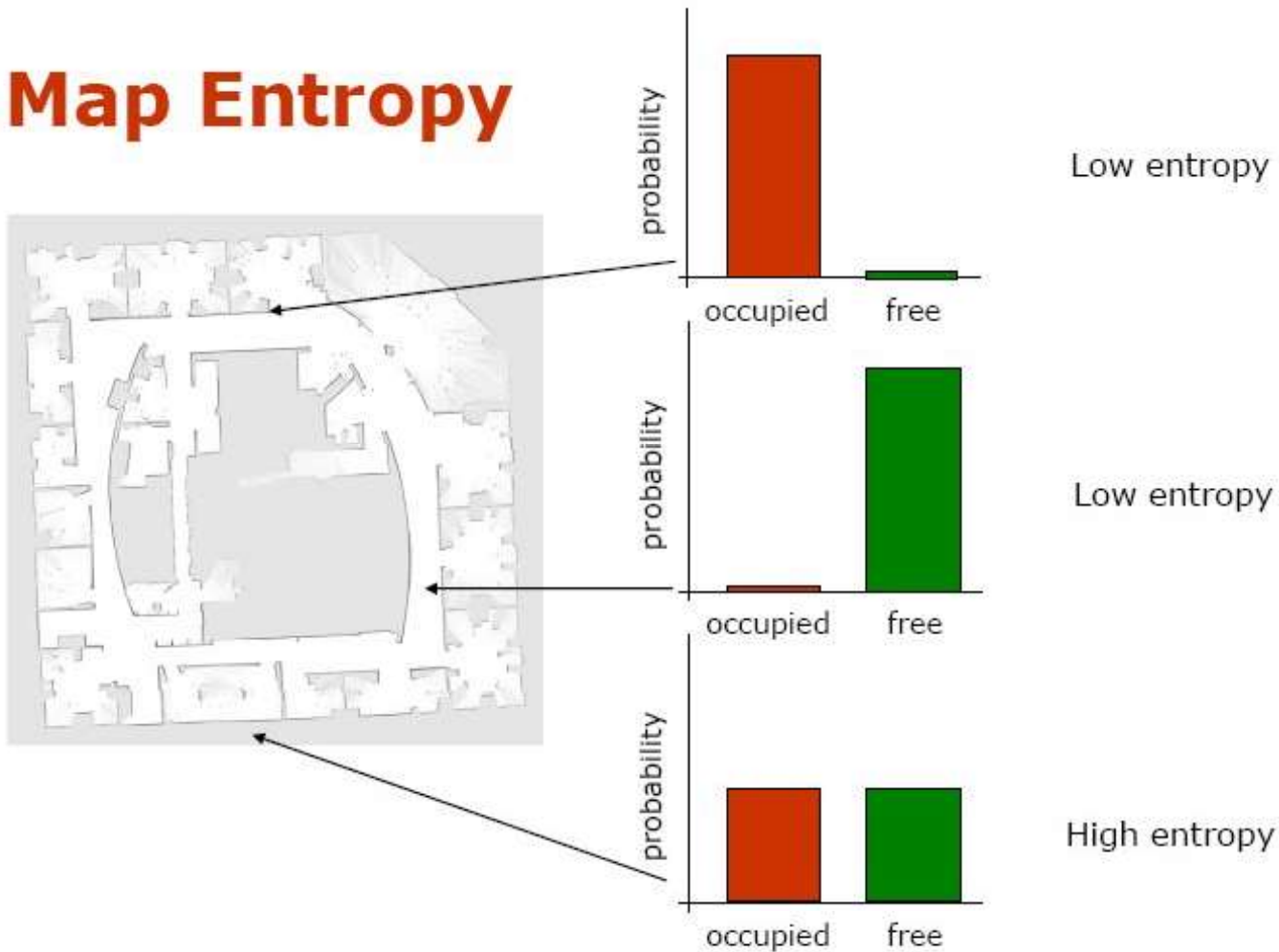
map
uncertainty

grid cells

probability that the
cell is occupied

Entropia della Mappa

Map Entropy



The overall entropy is the sum of the individual entropy values

Incerteza sulla traiettoria

- Ogni posa dipende dalle pose precedenti $0:t-1$
- Approssimazione con incerteza media sul percorso: $H(p(x_{1:t}|d_t))$
- Posterior su traiettoria rappresentato come gaussiana

Incerteza sulla traiettoria

1. High-dimensional Gaussian

$$H(\mathcal{G}(\mu, \Sigma)) = \log((2\pi e)^{(n/2)} |\Sigma|)$$

reduced rank for sparse particle sets

2. Grid-based approximation

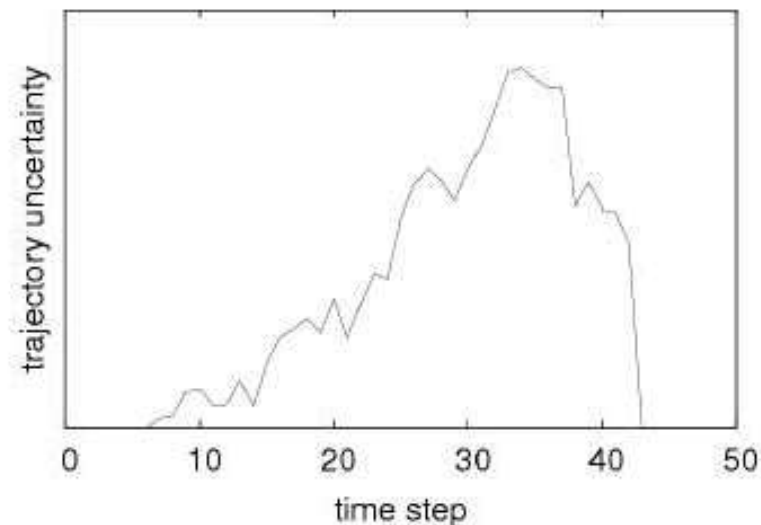
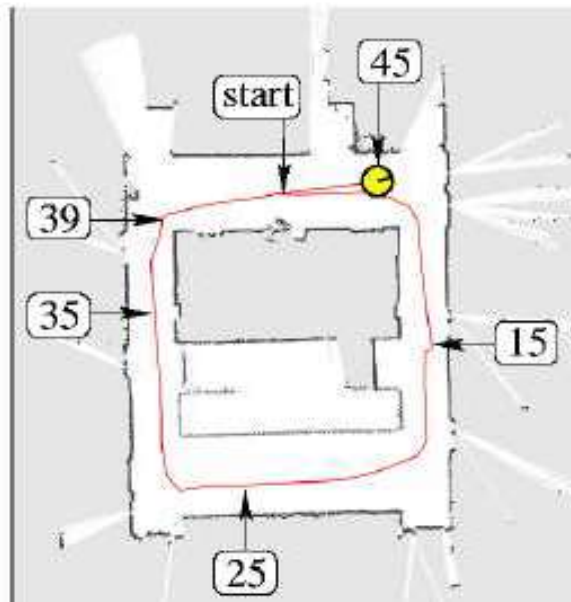
$$H(p(x | d)) \rightsquigarrow \text{const.}$$

for sparse particle clouds

Approssimazione incertezza sulla traiettoria

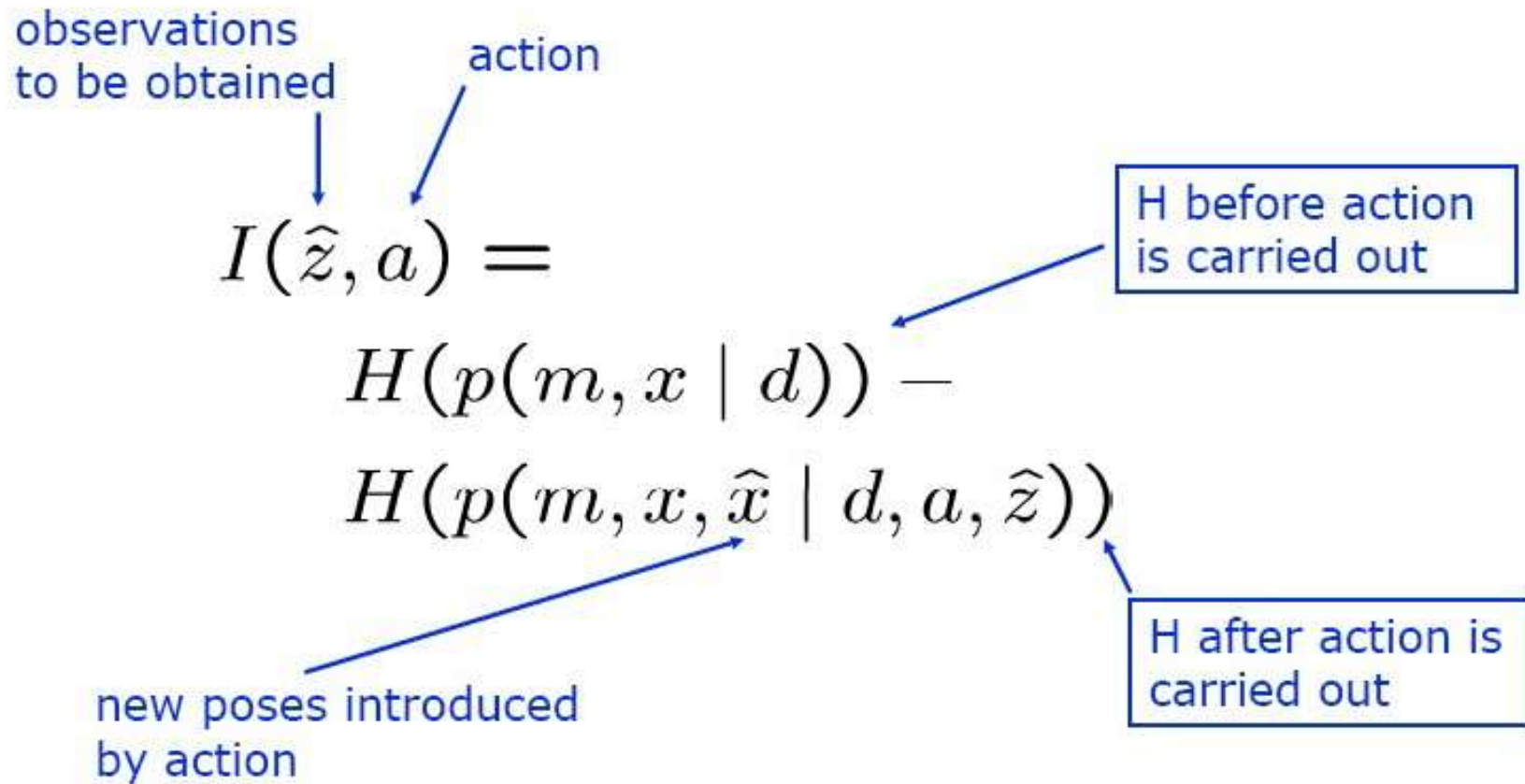
Average pose entropy over time:

$$H(p(x_{1:t} | d)) \approx \frac{1}{t} \sum_{t'=1}^t H(p(x_{t'} | d))$$



Guadagno di informazione con l'esecuzione delle azioni

- The reduction of entropy in the model



Guadagno di informazione atteso

- To compute the information gain one needs to know the observations obtained when carrying out an action
- This quantity is not known! Reason about potential measurements

$$E[I(a)] = \int_{\hat{z}} p(\hat{z} | a, d) \cdot I(\hat{z}, a) d\hat{z}$$

Sequenze di Misure

- The filter represents a posterior about possible maps
- Use these maps to reason about possible observation
- Simulate laser measurements in the maps of the particles

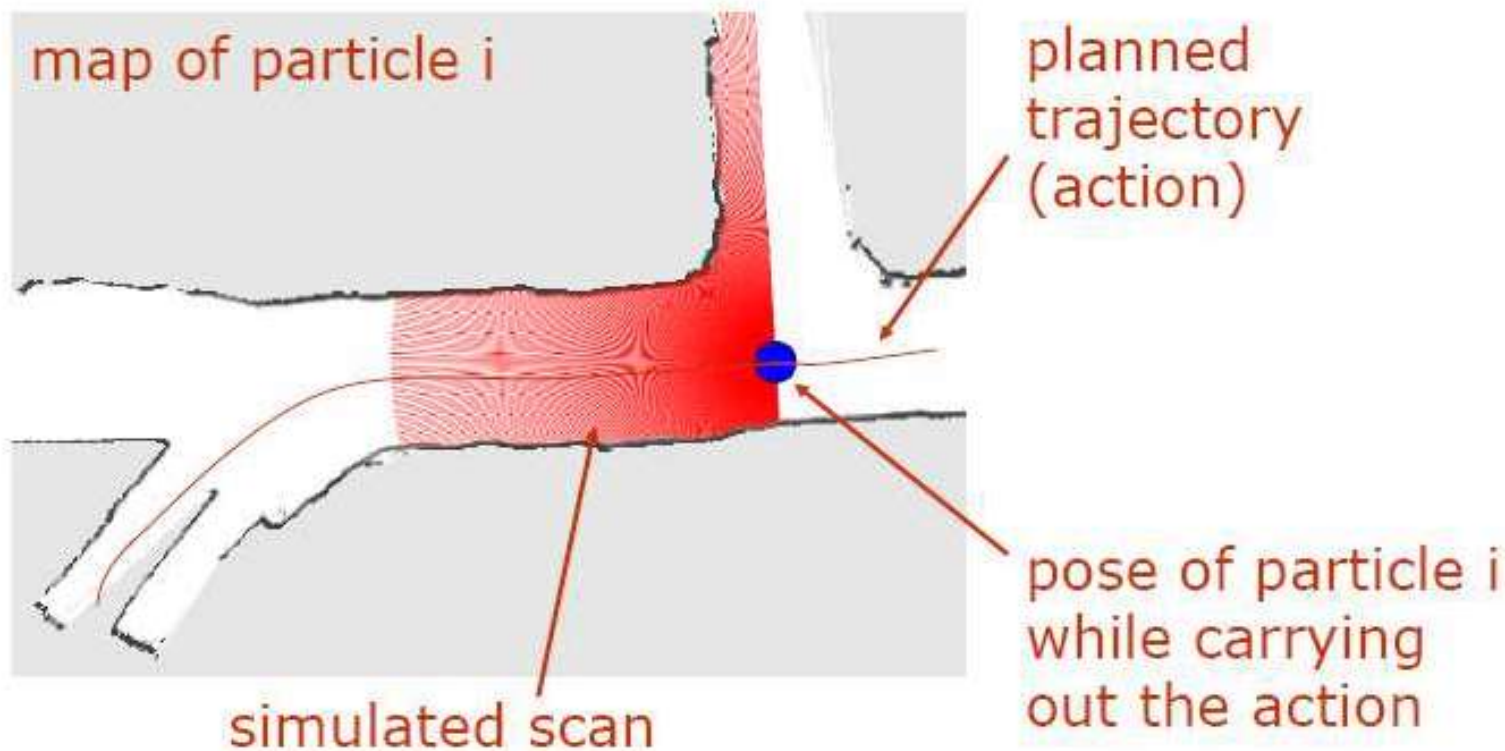
$$E[I(a)] = \int_{\hat{z}} p(\hat{z} | a, d) \cdot I(\hat{z}, a) d\hat{z}$$

measurement sequences
simulated in the maps

likelihood
(particle weight)

Sequenze di Misura

- Ray-casting in the map of each particle to generate observation sequences



$$p(\hat{z} | a_t, d_t) \approx \sum_{i=1}^{\#particles} p(\hat{z} | a_t, m^{[i]}, x_{1:t}^{[i]}, d_t) \cdot \omega_t^{[i]} p(m^{[i]} | x_{1:t}^{[i]}, d_t)$$

Utilità

- To take into account the cost of an action, we compute a utility

$$U(a) = I(a) - \alpha \cdot \text{cost}(a)$$

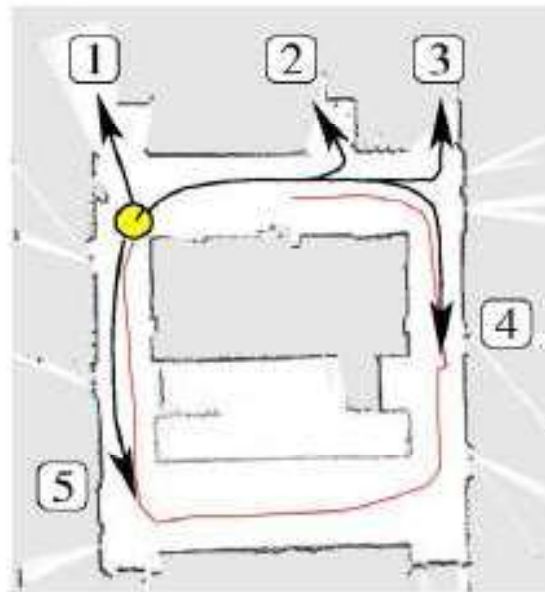
- Select the action with the highest expected utility

$$a^* = \underset{a}{\operatorname{argmax}} \{E[U(a)]\}$$

Azioni specifiche

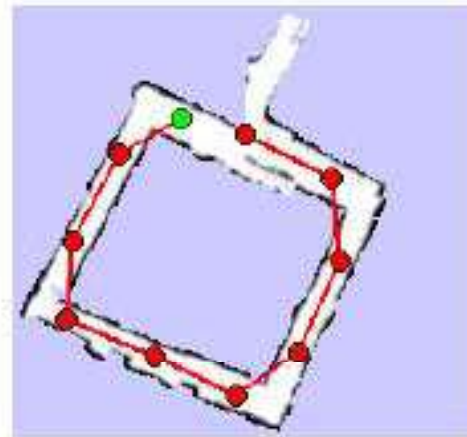
To efficiently sample actions we consider

- **exploratory actions (1-3)**
- **loop closing actions (4)** and
- **place revisiting actions (5)**

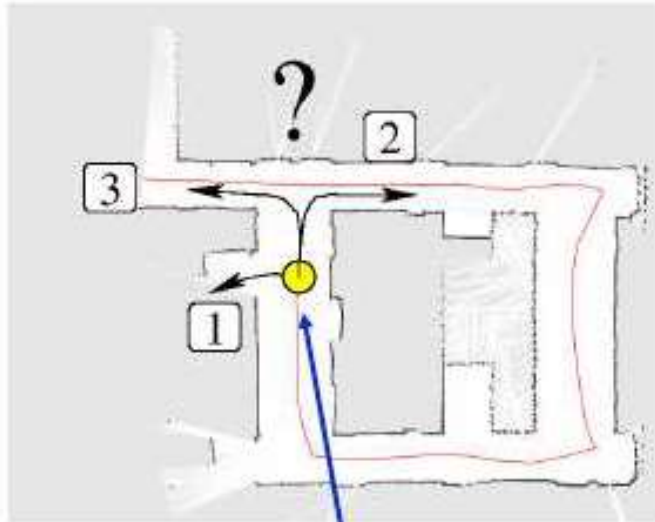


Doppia Rappresentazione per la rilevazione di cicli

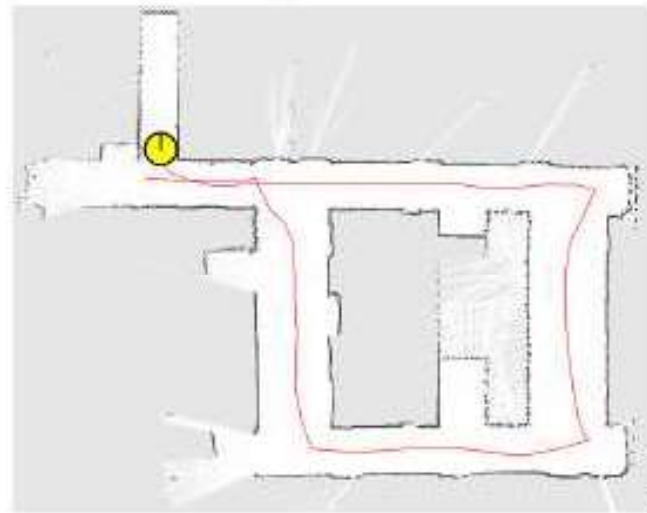
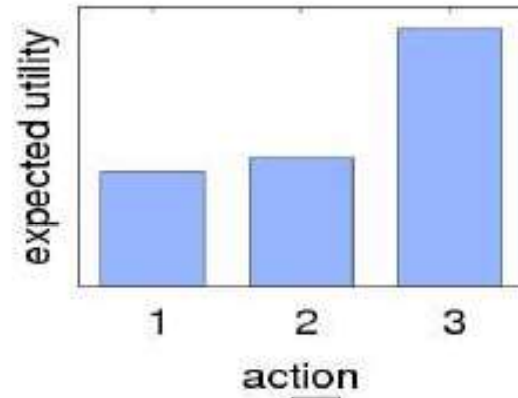
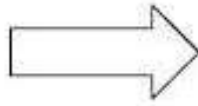
- **Trajectory graph** (“topological map”) stores the **path traversed by the robot**
- **Occupancy grid** map represents the **space covered by the sensors**
- **Loops** correspond to **long paths in the trajectory graph** and **short paths in the grid map**



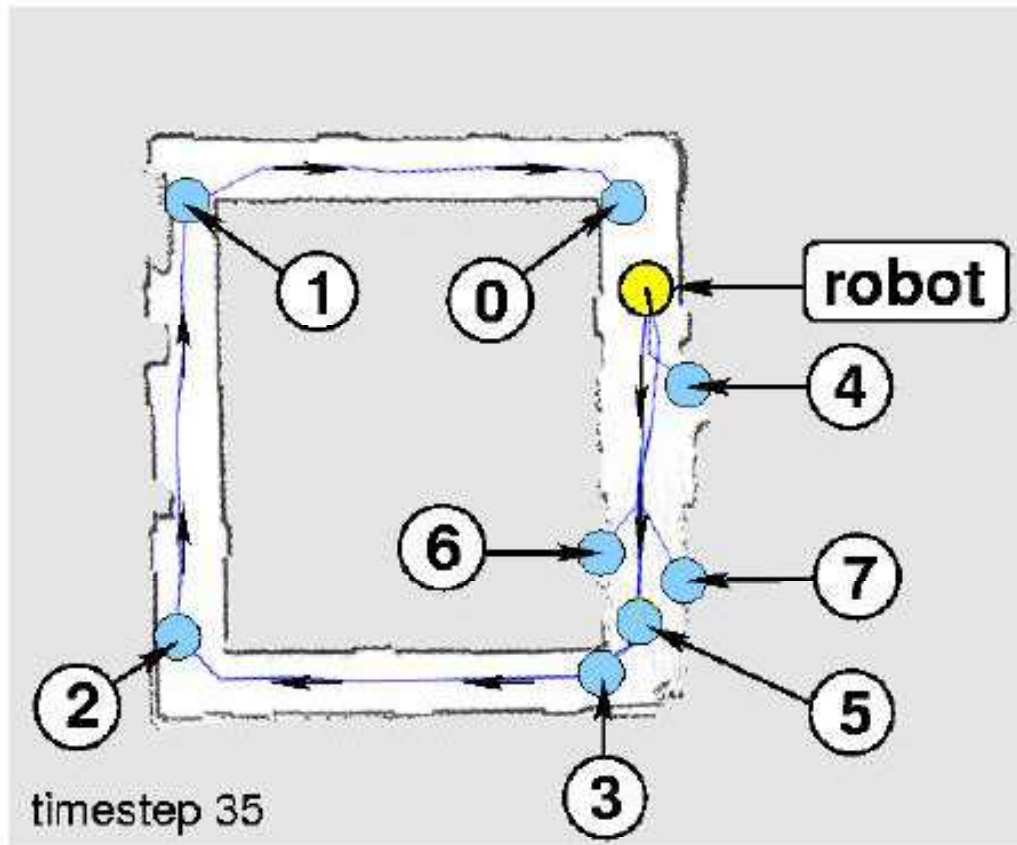
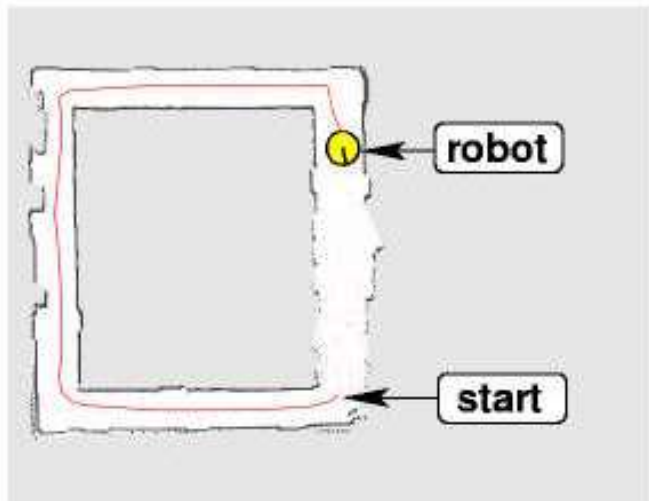
Esempio



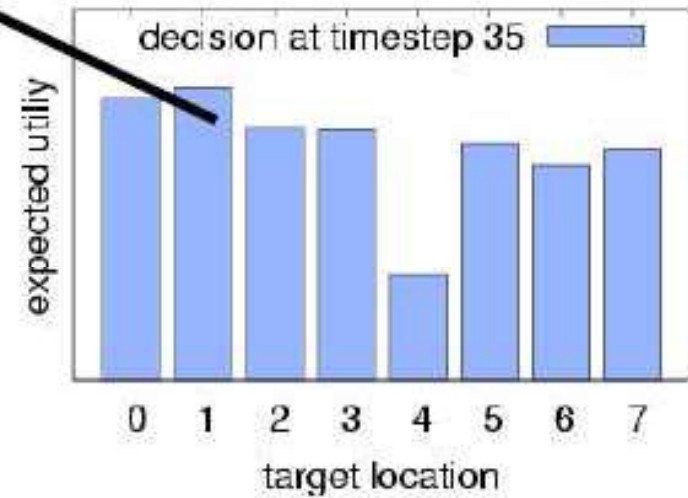
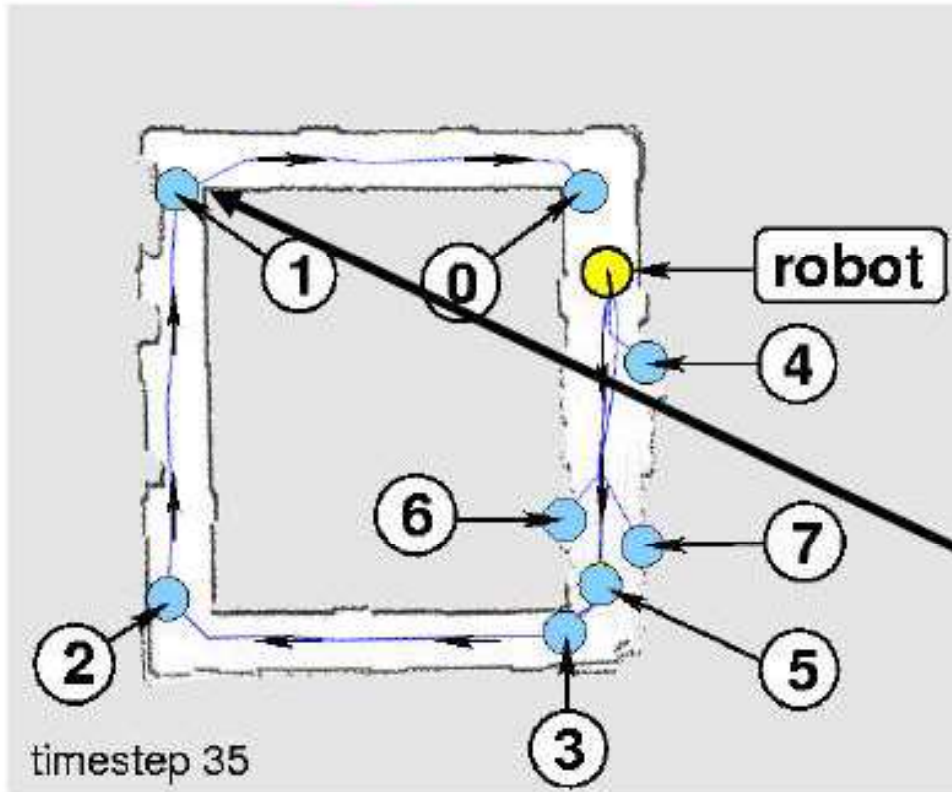
high pose uncertainty



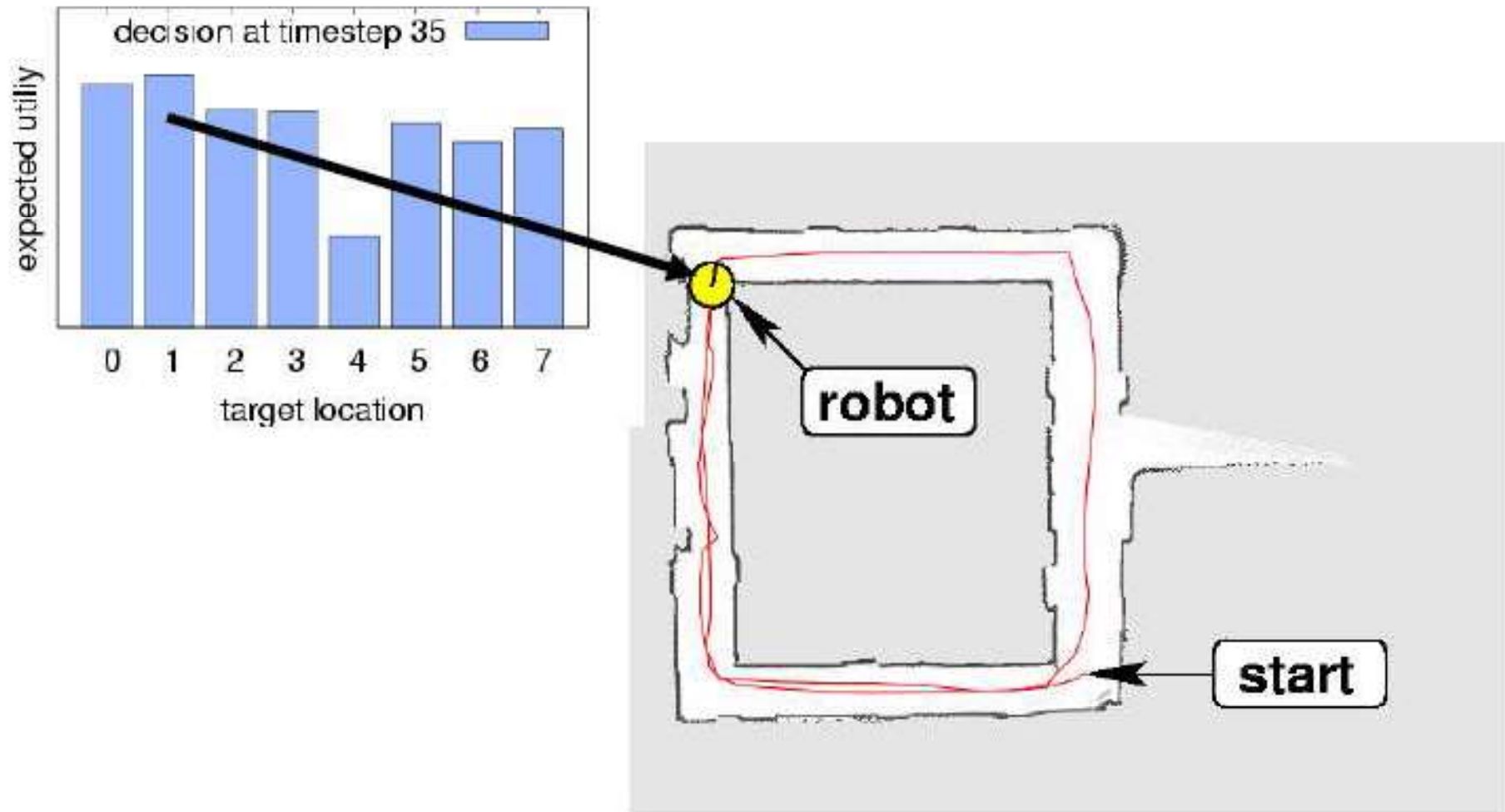
Possibili Target



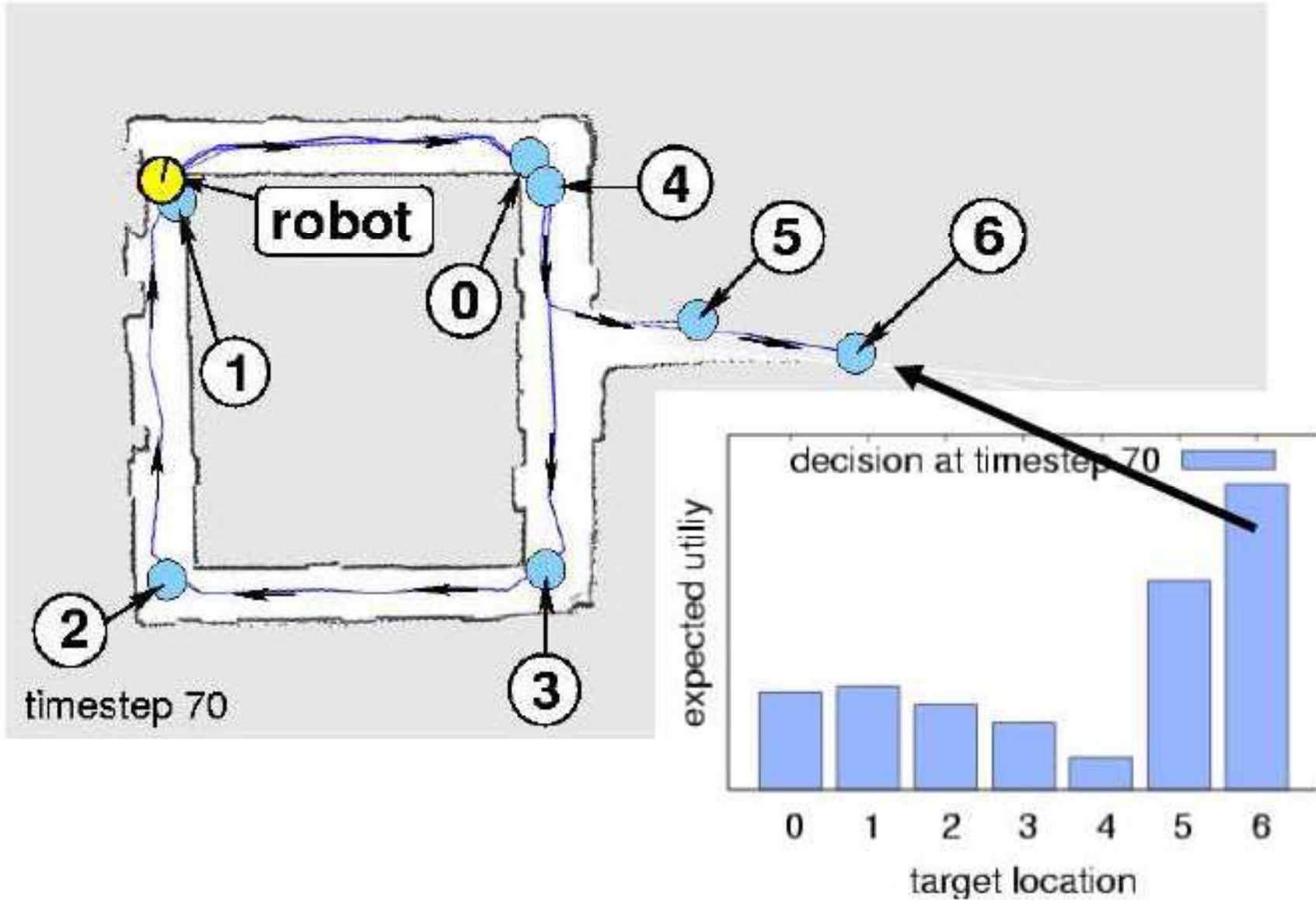
Valutazione dei Target



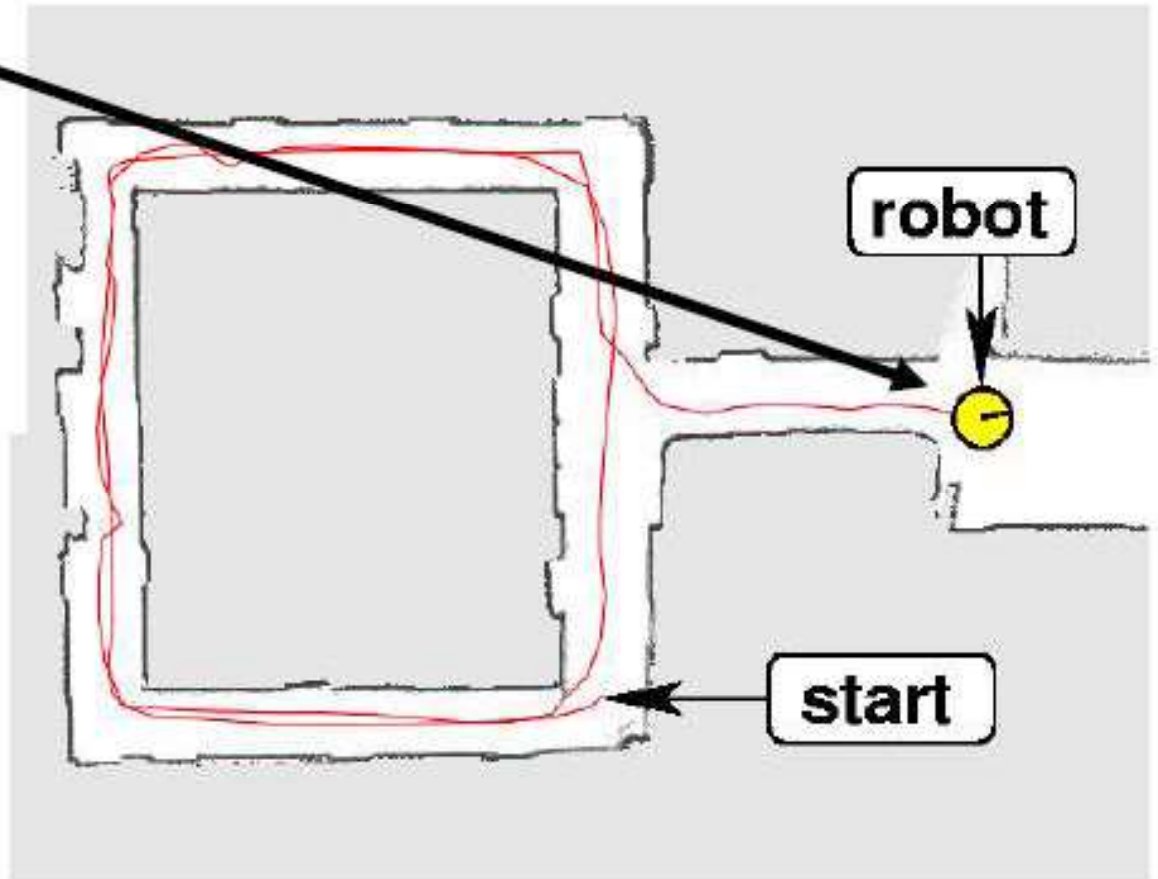
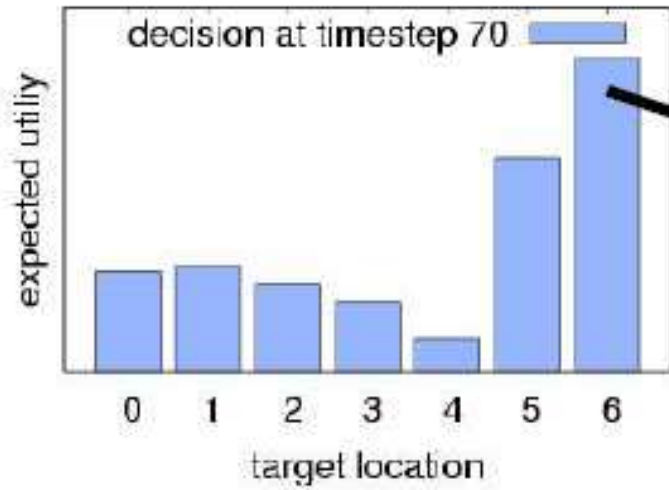
Spostamento sul Target



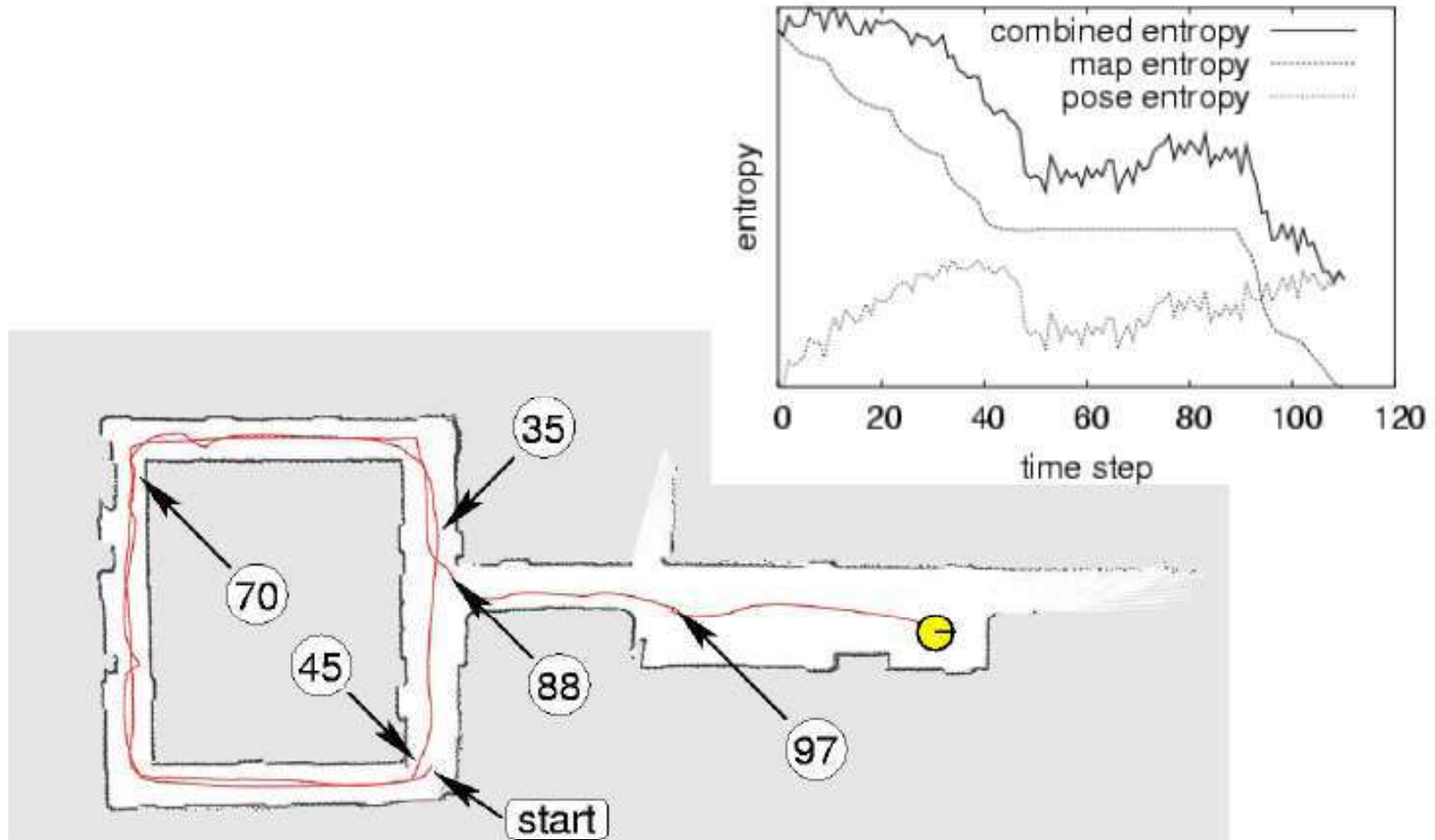
Valutazione dei Target



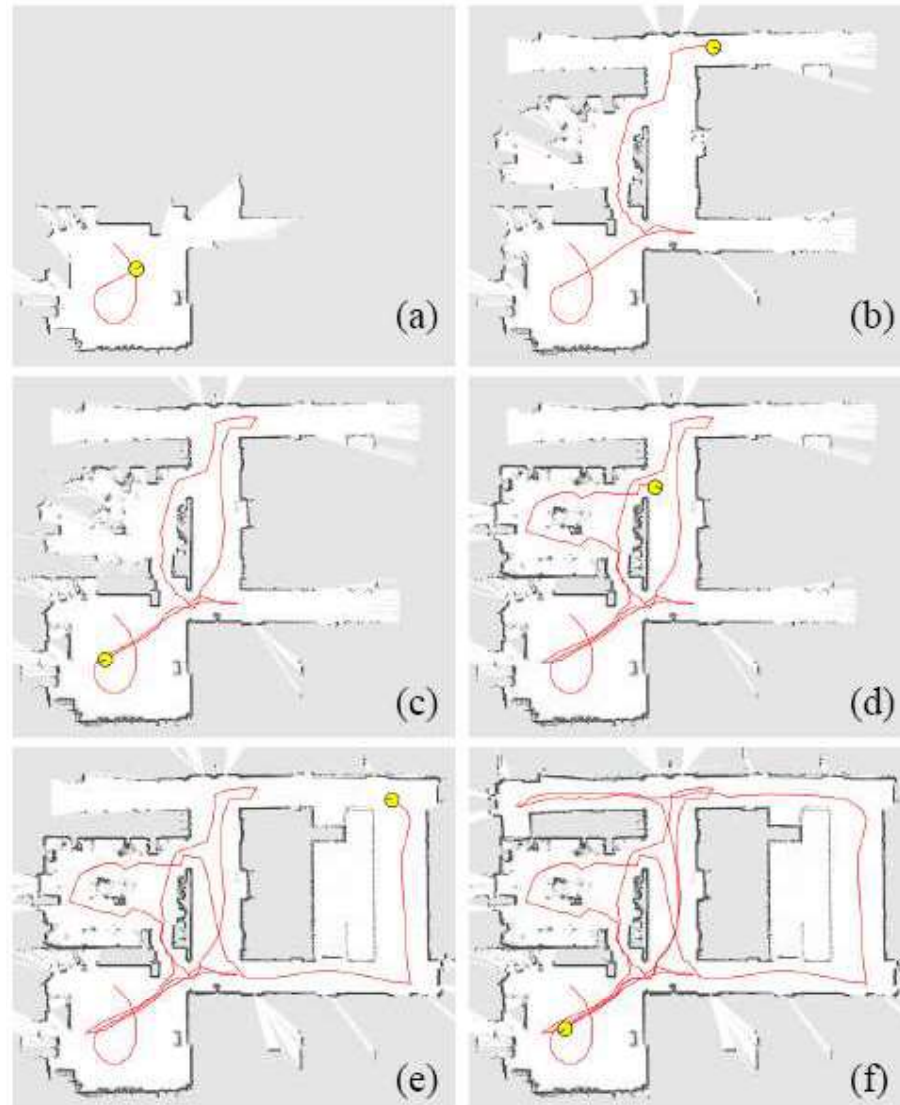
Movimento Robot



Evoluzione dell'entropia

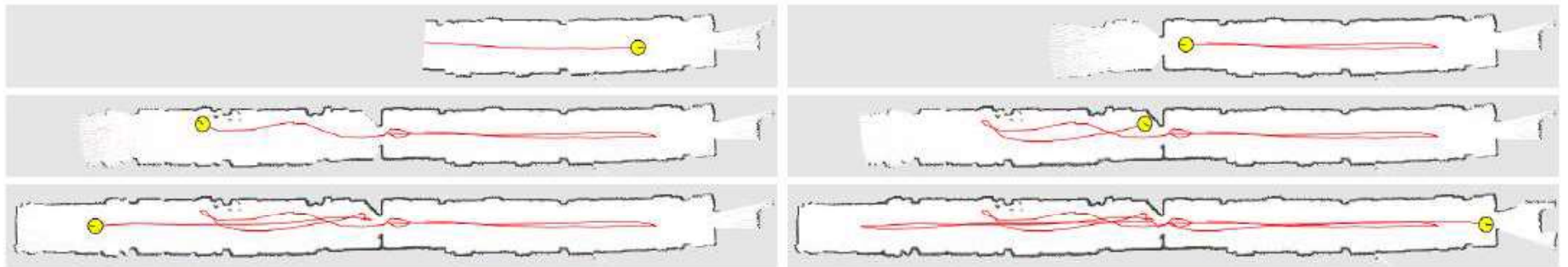


Esplorazione



Esplorazione Corridoio

Torna indietro per rilocalizzarsi



Riassunto

- A decision-theoretic approach to exploration in the context of RBPF-SLAM
- The approach utilizes the factorization of the Rao-Blackwellization to efficiently calculate the expected information gain
- Reasons about measurements obtained along the path of the robot
- Considers a reduced action set consisting of exploration, loop-closing, and place-revisiting actions
- Experimental results demonstrate the usefulness of the overall approach