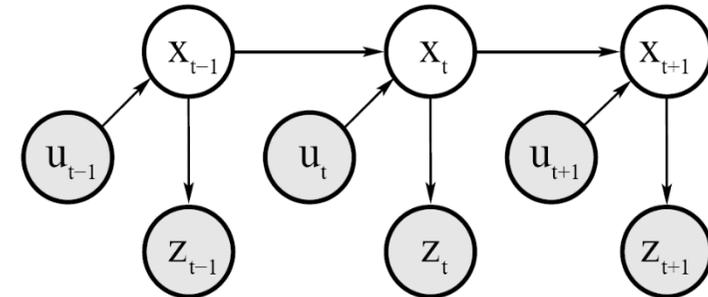


- Incertezza nel movimento
  - Modello Probabilistico
  - Filtro Bayesiano
  
- Un modello possibile sono le reti bayesiane
- Azioni, Osservazioni, Transizioni





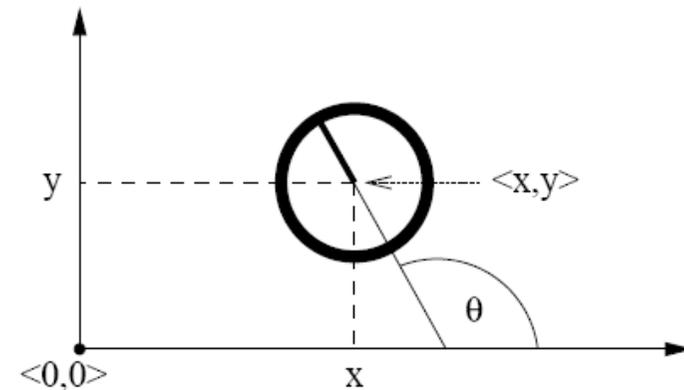
- **Locale vs Globale:**  
Location Tracking (posa iniziale nota); Global localization (posa iniziale non nota); kidnapped robot (riposizionamento)
- **Statica vs Dinamica:**  
oggetti statici o dinamici
- **Passiva vs Attiva:**  
movimenti del robot per aiutare la localizzazione
- **Robot vs Multi-Robot:**  
più robot possono cooperare per localizzarsi reciprocamente



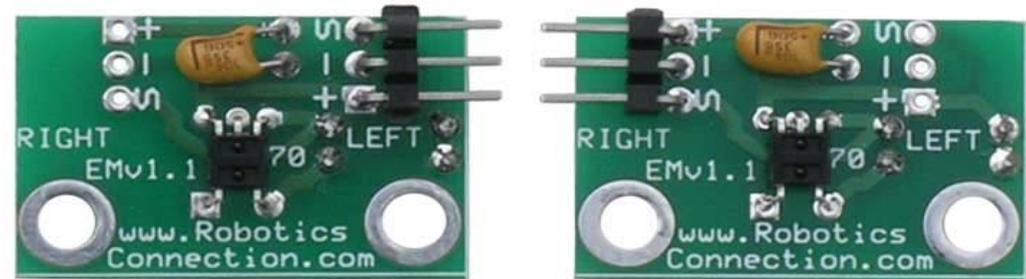
- Variante del Filtro di Bayes
- Richiede una mappa  $m$
- Modello di misura:  $P(z_t \mid x_t, m)$
- Modello di moto:  $P(x_t \mid u_t, x_{t-1}, m)$

Vediamo come si istanziano il modello di moto e il modello di misura

- Configurazione di un robot mobile (posa) descritta da 6 parametri. Coordinate 3D più 3 angoli di Eulero: pitch, roll, and tilt.
- Assumendo superficie planare lo spazio di stato è 3-dimensionale  $(x, y, \theta)$ .



These modules require +5V and GND to power them, and provide a 0 to 5V output. They provide +5V output when they "see" white, and a 0V output when they "see" black.





- Due tipi di modelli di moto:
  - Basato su odometria
  - Basato su velocità (dead reckoning)
- **Odometry-based** usati se il sistema è dotato di wheel encoders.
- **Velocity-based** usati senza wheel encoders.  
La nuova posa calcolata con velocità e il tempo passato.



L'odometria è sensore, non controllo

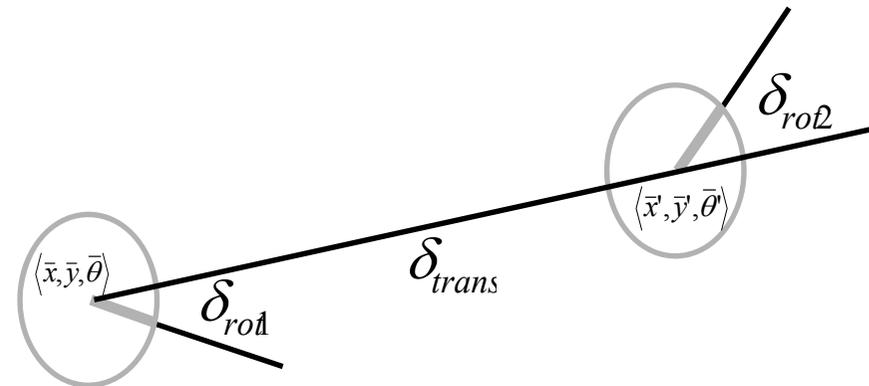
In questo modello si assume odometria come controllo

$$u_t = \begin{pmatrix} \bar{x}_{t-1} \\ \bar{x}_t \end{pmatrix} \quad \text{spostamento}$$

Modello usato in diversi sistemi di localizzazione e mapping

- Robot va dalla posa  $\langle \bar{x}, \bar{y}, \bar{\theta} \rangle$  alla posa  $\langle \bar{x}', \bar{y}', \bar{\theta}' \rangle$
- Informazione Odometrica:

$$u = \langle \delta_{rot1}, \delta_{rot2}, \delta_{trans} \rangle$$



$$\delta_{trans} = \sqrt{(\bar{x}' - \bar{x})^2 + (\bar{y}' - \bar{y})^2}$$

$$\delta_{rot1} = \text{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta}$$

$$\delta_{rot2} = \bar{\theta}' - \bar{\theta} - \delta_{rot1}$$



Con la funzione arcotangente modificata:

$$\text{atan2}(y, x) = \begin{cases} \text{atan}(y/x) & \text{if } x > 0 \\ \text{sign}(y) (\pi - \text{atan}(|y/x|)) & \text{if } x < 0 \\ 0 & \text{if } x = y = 0 \\ \text{sign}(y) \pi/2 & \text{if } x = 0, y \neq 0 \end{cases}$$



- Il movimento stimato è dato dal movimento reale più un disturbo.

rotazione

$$\hat{\delta}_{rot1} = \delta_{rot1} + \varepsilon_{\alpha_1 |\delta_{rot1}| + \alpha_2 |\delta_{trans}|}$$

traslazione

$$\hat{\delta}_{trans} = \delta_{trans} + \varepsilon_{\alpha_3 |\delta_{trans}| + \alpha_4 |\delta_{rot1} + \delta_{rot2}|}$$

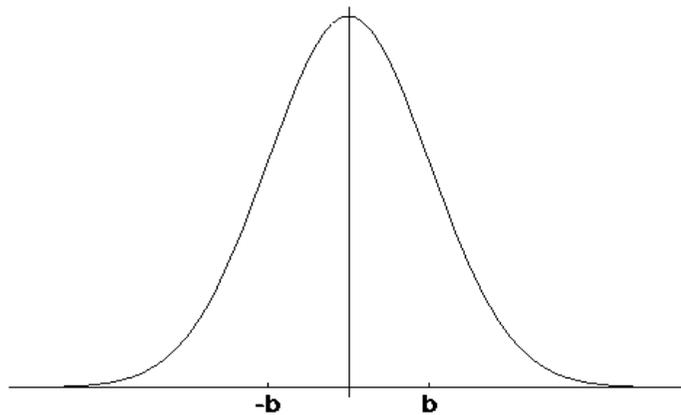
rotazione

$$\hat{\delta}_{rot2} = \delta_{rot2} + \varepsilon_{\alpha_1 |\delta_{rot2}| + \alpha_2 |\delta_{trans}|}$$

Parametri dipendenti dal robot definiscono l'errore

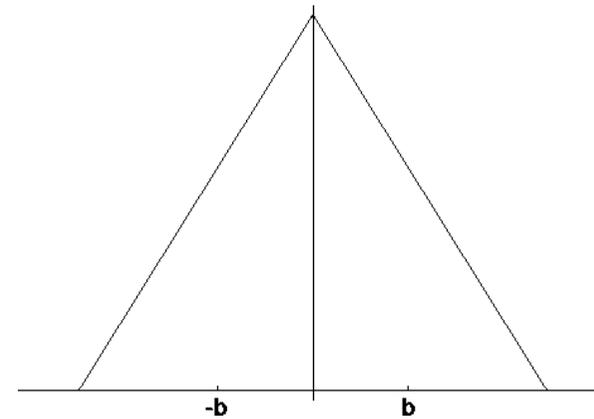
Distribuzioni di errore tipicamente impiegate

- Normale



$$\varepsilon_{\sigma^2}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{x^2}{\sigma^2}}$$

- Triangolare



$$\varepsilon_{\sigma^2}(x) = \begin{cases} 0 & \text{if } |x| > \sqrt{6\sigma^2} \\ \frac{\sqrt{6\sigma^2} - |x|}{6\sigma^2} & \text{otherwise} \end{cases}$$



- **Normale**

**prob\_normal**( $a, b$ ):

$$\frac{1}{\sqrt{2\pi} b^2} \exp \left\{ -\frac{1}{2} \frac{a^2}{b^2} \right\}$$

- **Triangolare**

**prob\_triangular**( $a, b$ ):

$$\max \left\{ 0, \frac{1}{\sqrt{6} b} - \frac{|a|}{6 b^2} \right\}$$

Update  $P(x_t \mid x_{t-1} \text{ e } u_t)$  a partire da  $x_{t-1}$  e  $u_t$

1. Algorithm **motion\_model\_odometry(x,x',u)**

2.  $\delta_{trans} = \sqrt{(\bar{x}' - \bar{x})^2 + (\bar{y}' - \bar{y})^2}$

3.  $\delta_{rot1} = \text{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta}$

4.  $\delta_{rot2} = \bar{\theta}' - \bar{\theta} - \delta_{rot1}$

5.  $\hat{\delta}_{trans} = \sqrt{(x' - x)^2 + (y' - y)^2}$

6.  $\hat{\delta}_{rot1} = \text{atan2}(y' - y, x' - x) - \bar{\theta}$

7.  $\hat{\delta}_{rot2} = \theta' - \theta - \hat{\delta}_{rot1}$

8.  $p_1 = \text{prob}(\delta_{rot1} - \hat{\delta}_{rot1}, \alpha_1 \mid \hat{\delta}_{rot1} \mid + \alpha_2 \hat{\delta}_{trans})$

9.  $p_2 = \text{prob}(\delta_{trans} - \hat{\delta}_{trans}, \alpha_3 \hat{\delta}_{trans} + \alpha_4 (|\hat{\delta}_{rot1}| + |\hat{\delta}_{rot2}|))$

10.  $p_3 = \text{prob}(\delta_{rot2} - \hat{\delta}_{rot2}, \alpha_1 \mid \hat{\delta}_{rot2} \mid + \alpha_2 \hat{\delta}_{trans})$

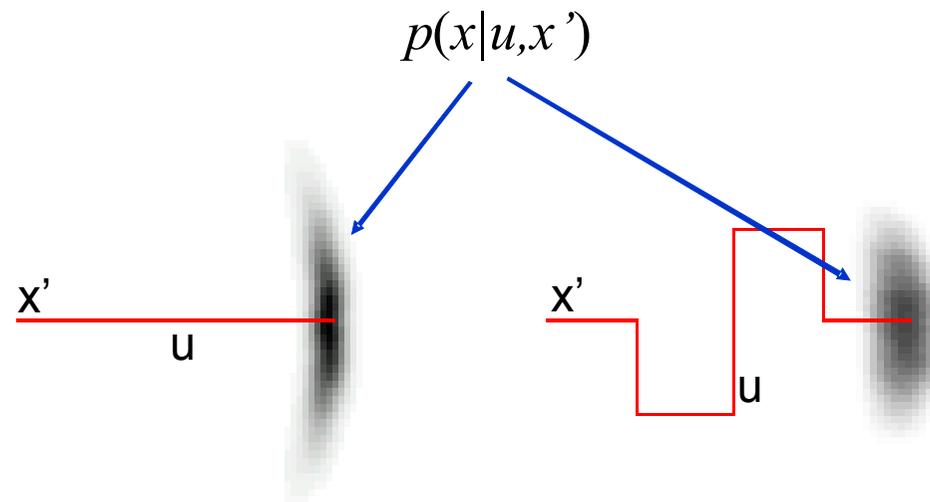
11. return  $p_1 \cdot p_2 \cdot p_3$

odometry values (u)  
spostamento odometrico

values of interest (x,x')  
spostamento reale

Differenza spostamento odometrico/reale

- Andamento tipico:
- Distribuzione sullo stato  $x$  dopo l'esecuzione delle azioni  $u$  a partire dallo stato  $x'$





## 1. Algorithm `sample_motion_model`( $u, x$ ):

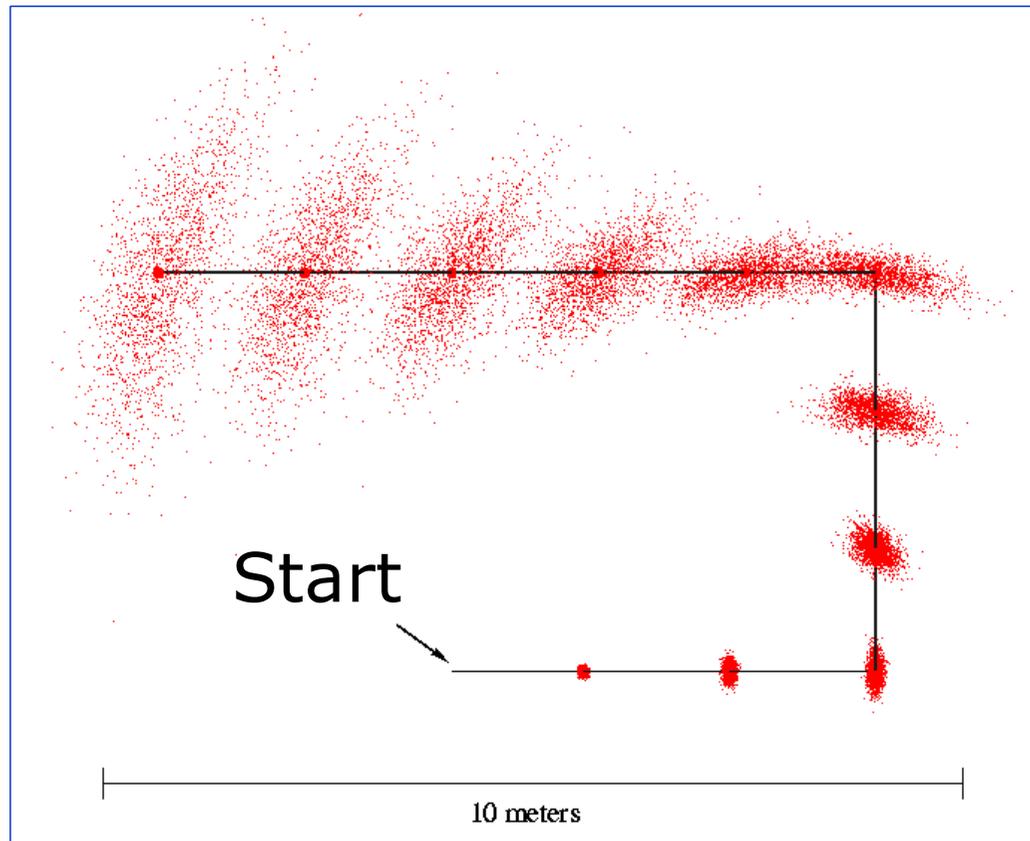
$$u = \langle \delta_{rot1}, \delta_{rot2}, \delta_{trans} \rangle, x = \langle x, y, \theta \rangle$$

1.  $\hat{\delta}_{rot1} = \delta_{rot1} + \text{sample}(\alpha_1 | \delta_{rot1} | + \alpha_2 \delta_{trans})$
2.  $\hat{\delta}_{trans} = \delta_{trans} + \text{sample}(\alpha_3 \delta_{trans} + \alpha_4 (| \delta_{rot1} | + | \delta_{rot2} |))$
3.  $\hat{\delta}_{rot2} = \delta_{rot2} + \text{sample}(\alpha_1 | \delta_{rot2} | + \alpha_2 \delta_{trans})$
4.  $x' = x + \hat{\delta}_{trans} \cos(\theta + \hat{\delta}_{rot1})$
5.  $y' = y + \hat{\delta}_{trans} \sin(\theta + \hat{\delta}_{rot1})$
6.  $\theta' = \theta + \hat{\delta}_{rot1} + \hat{\delta}_{rot2}$

`sample_normal_distribution`

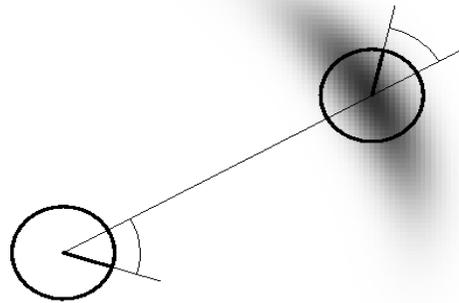
7. Return  $\langle x', y', \theta' \rangle$

- Esempio campionamento distribuzione dei campioni nel modello odometrico

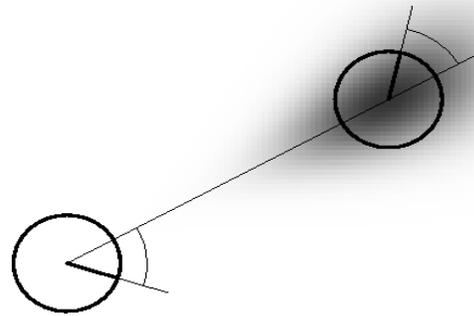


- Da distribuzione normale
  1. Algorithm **sample\_normal\_distribution**( $b$ ):
  2. return  $\frac{1}{2} \sum_{i=1}^{12} \text{rand}(-b, b)$
- Da distribuzione triangolare
  1. Algorithm **sample\_triangular\_distribution**( $b$ ):
  2. return  $\frac{\sqrt{6}}{2} [\text{rand}(-b, b) + \text{rand}(-b, b)]$

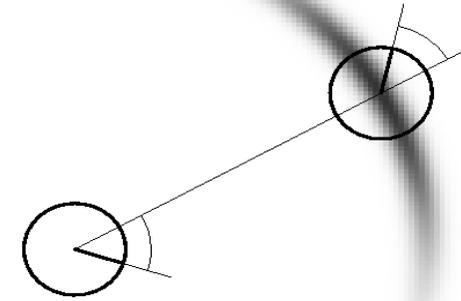
# Esempi (Odometry-Based) per diversi setting dei parametri



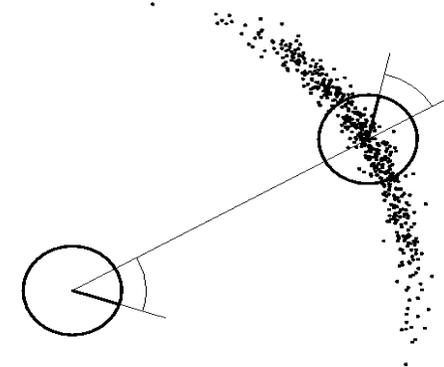
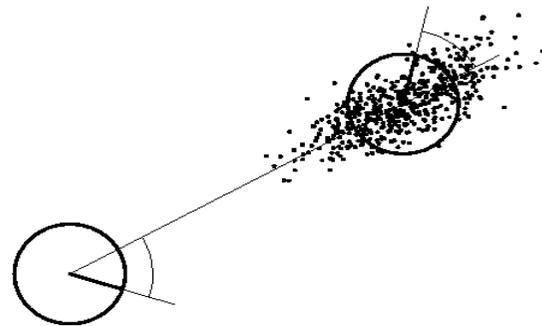
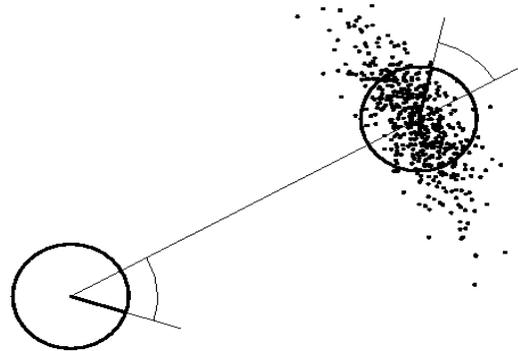
Classico



Errore Traslazione



Errore Rotazione



## Velocity-Based Model

Sistema controllato in velocità

$$u_t = \begin{pmatrix} v_t \\ \omega_t \end{pmatrix}$$

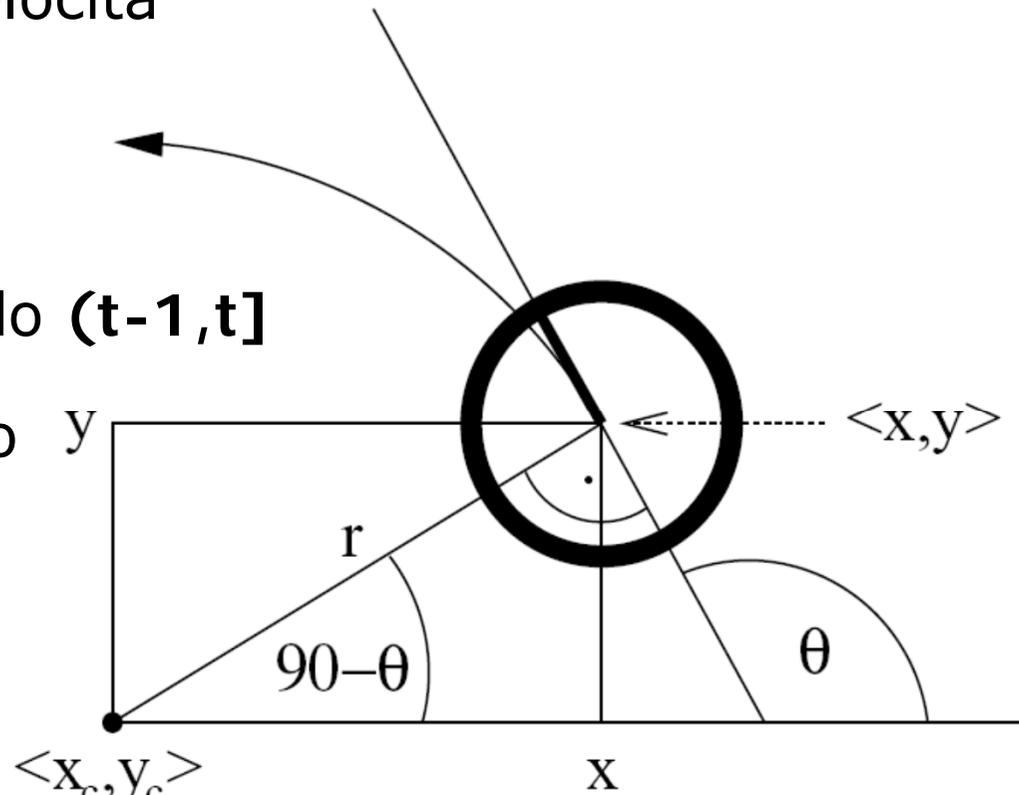
$(v, \omega)$  Fisse nell'intervallo  $(t-1, t]$

Mov. circolare con raggio

$$r = \left| \frac{v}{\omega} \right|$$

$$x_c = x - \frac{v}{\omega} \sin \theta$$

$$y_c = y + \frac{v}{\omega} \cos \theta$$



Dopo  $\Delta t$  le nuove coordinate sono:

$$\begin{aligned} \begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} &= \begin{pmatrix} x_c + \frac{v}{\omega} \sin(\theta + \omega \Delta t) \\ y_c - \frac{v}{\omega} \cos(\theta + \omega \Delta t) \\ \theta + \omega \Delta t \end{pmatrix} \\ &= \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} -\frac{v}{\omega} \sin \theta + \frac{v}{\omega} \sin(\theta + \omega \Delta t) \\ \frac{v}{\omega} \cos \theta - \frac{v}{\omega} \cos(\theta + \omega \Delta t) \\ \omega \Delta t \end{pmatrix} \end{aligned}$$

Ma c'è il rumore, quindi le coordinate reali sono:

$$\begin{pmatrix} \hat{v} \\ \hat{\omega} \end{pmatrix} = \begin{pmatrix} v \\ \omega \end{pmatrix} + \begin{pmatrix} \varepsilon_{\alpha_1} |v| + \alpha_2 |\omega| \\ \varepsilon_{\alpha_3} |v| + \alpha_4 |\omega| \end{pmatrix}$$

Quindi:

$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} -\frac{\hat{v}}{\hat{\omega}} \sin \theta + \frac{\hat{v}}{\hat{\omega}} \sin(\theta + \hat{\omega} \Delta t) \\ \frac{\hat{v}}{\hat{\omega}} \cos \theta - \frac{\hat{v}}{\hat{\omega}} \cos(\theta + \hat{\omega} \Delta t) \\ \hat{\omega} \Delta t \end{pmatrix}$$



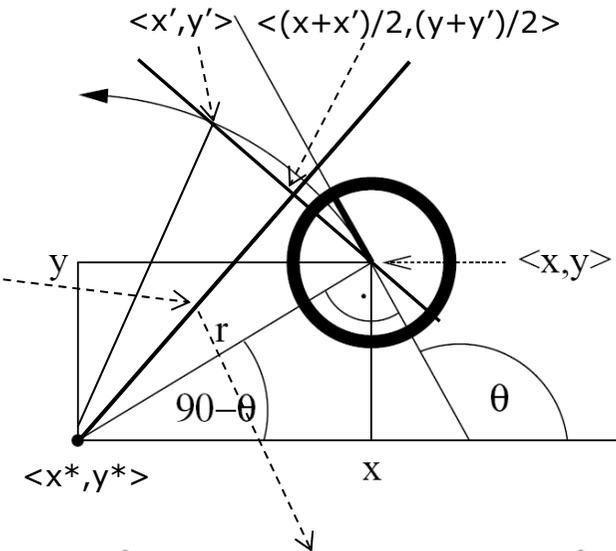
Ma errore solo per  $v$  e  $w$  (assunzione di moto circolare),  
introduzione di ulteriore rotazione:

$$\theta' = \theta + \hat{\omega}\Delta t + \hat{\gamma}\Delta t \qquad \hat{\gamma} = \varepsilon_{\alpha_5|v} + \alpha_6|\omega|$$

$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} -\frac{\hat{v}}{\hat{\omega}} \sin \theta + \frac{\hat{v}}{\hat{\omega}} \sin(\theta + \hat{\omega}\Delta t) \\ \frac{\hat{v}}{\hat{\omega}} \cos \theta - \frac{\hat{v}}{\hat{\omega}} \cos(\theta + \hat{\omega}\Delta t) \\ \hat{\omega}\Delta t + \hat{\gamma}\Delta t \end{pmatrix}$$

Occorre calcolare  $P(x_t \mid u_t \text{ e } x_{t-1})$

Retta passante per  $\langle (x+x')/2, (y+y')/2 \rangle$   
ed ortogonale alla retta che collega  $\langle x, y \rangle$  e  $\langle x', y' \rangle$



- Centro del circolo

$$\begin{pmatrix} x^* \\ y^* \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -\lambda \sin \theta \\ \lambda \cos \theta \end{pmatrix} = \begin{pmatrix} \frac{x+x'}{2} + \mu(y - y') \\ \frac{y+y'}{2} + \mu(x' - x) \end{pmatrix}$$

- dove

$$\mu = \frac{1}{2} \frac{(x - x') \cos \theta + (y - y') \sin \theta}{(y - y') \cos \theta - (x - x') \sin \theta}$$



Quindi ... centro del circolo

$$\begin{pmatrix} x^* \\ y^* \end{pmatrix} = \begin{pmatrix} \frac{x+x'}{2} + \frac{1}{2} \frac{(x-x') \cos \theta + (y-y') \sin \theta}{(y-y') \cos \theta - (x-x') \sin \theta} (y-y') \\ \frac{y+y'}{2} + \frac{1}{2} \frac{(x-x') \cos \theta + (y-y') \sin \theta}{(y-y') \cos \theta - (x-x') \sin \theta} (x'-x) \end{pmatrix}$$

... raggio del circolo

$$r^* = \sqrt{(x-x^*)^2 + (y-y^*)^2} = \sqrt{(x'-x^*)^2 + (y'-y^*)^2}$$

... variazione angolare

$$\Delta \theta = \text{atan2}(y' - y^*, x' - x^*) - \text{atan2}(y - y^*, x - x^*)$$

Rispetto a  $x, y$  e  $x', y'$



1: **Algorithm motion\_model\_velocity**( $x_t, u_t, x_{t-1}$ ):

2: 
$$\mu = \frac{1}{2} \frac{(x - x') \cos \theta + (y - y') \sin \theta}{(y - y') \cos \theta - (x - x') \sin \theta}$$

3: 
$$x^* = \frac{x + x'}{2} + \mu(y - y')$$

4: 
$$y^* = \frac{y + y'}{2} + \mu(x' - x)$$

$x', x$  stanno per  
 $x_t, x_{t-1}$

5: 
$$r^* = \sqrt{(x - x^*)^2 + (y - y^*)^2}$$

6: 
$$\Delta\theta = \text{atan2}(y' - y^*, x' - x^*) - \text{atan2}(y - y^*, x - x^*)$$

7: 
$$\hat{v} = \frac{\Delta\theta}{\Delta t} r^*$$

8: 
$$\hat{\omega} = \frac{\Delta\theta}{\Delta t}$$

9: 
$$\hat{\gamma} = \frac{\theta' - \theta}{\Delta t} - \hat{\omega}$$

10: **return**  $\text{prob}(v - \hat{v}, \alpha_1|v| + \alpha_2|\omega|) \cdot \text{prob}(\omega - \hat{\omega}, \alpha_3|v| + \alpha_4|\omega|)$   
 $\cdot \text{prob}(\hat{\gamma}, \alpha_5|v| + \alpha_6|\omega|)$



## Campionamento dal Velocity Model

1:       **Algorithm** `sample_motion_model_velocity`( $u_t, x_{t-1}$ ):

2:            $\hat{v} = v + \text{sample}(\alpha_1|v| + \alpha_2|\omega|)$

3:            $\hat{\omega} = \omega + \text{sample}(\alpha_3|v| + \alpha_4|\omega|)$

4:            $\hat{\gamma} = \text{sample}(\alpha_5|v| + \alpha_6|\omega|)$

5:            $x' = x - \frac{\hat{v}}{\hat{\omega}} \sin \theta + \frac{\hat{v}}{\hat{\omega}} \sin(\theta + \hat{\omega}\Delta t)$

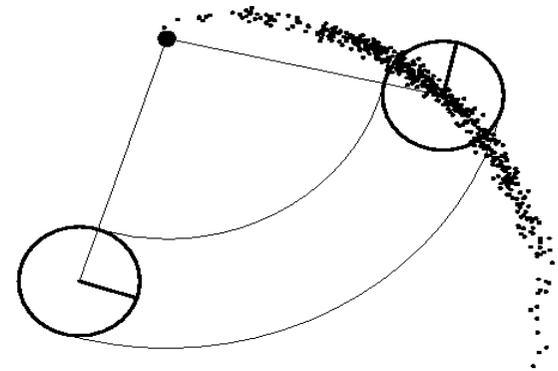
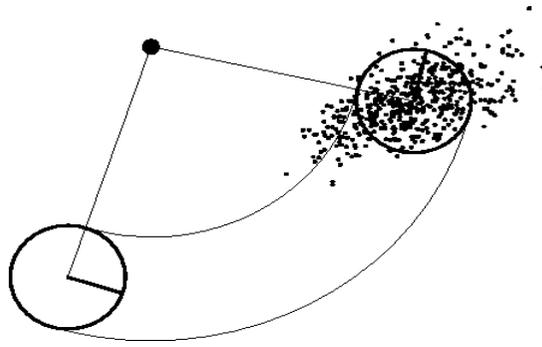
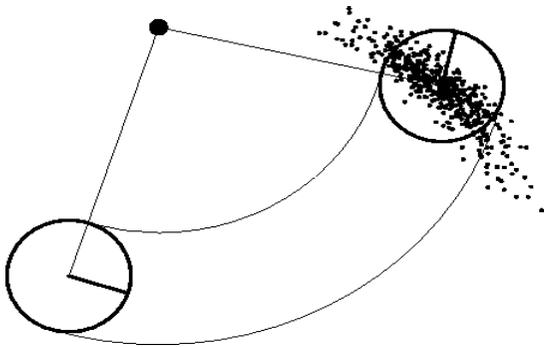
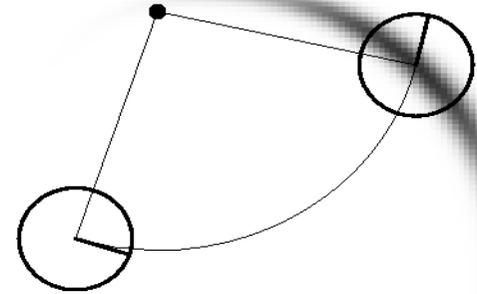
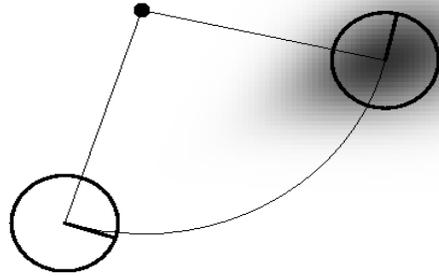
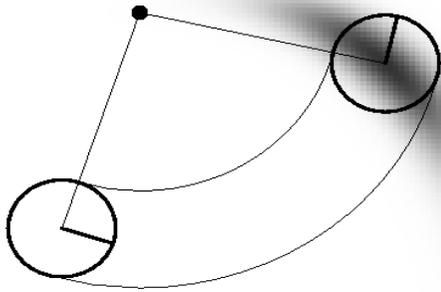
6:            $y' = y + \frac{\hat{v}}{\hat{\omega}} \cos \theta - \frac{\hat{v}}{\hat{\omega}} \cos(\theta + \hat{\omega}\Delta t)$

7:            $\theta' = \theta + \hat{\omega}\Delta t + \hat{\gamma}\Delta t$

8:           *return*  $x_t = (x', y', \theta')^T$

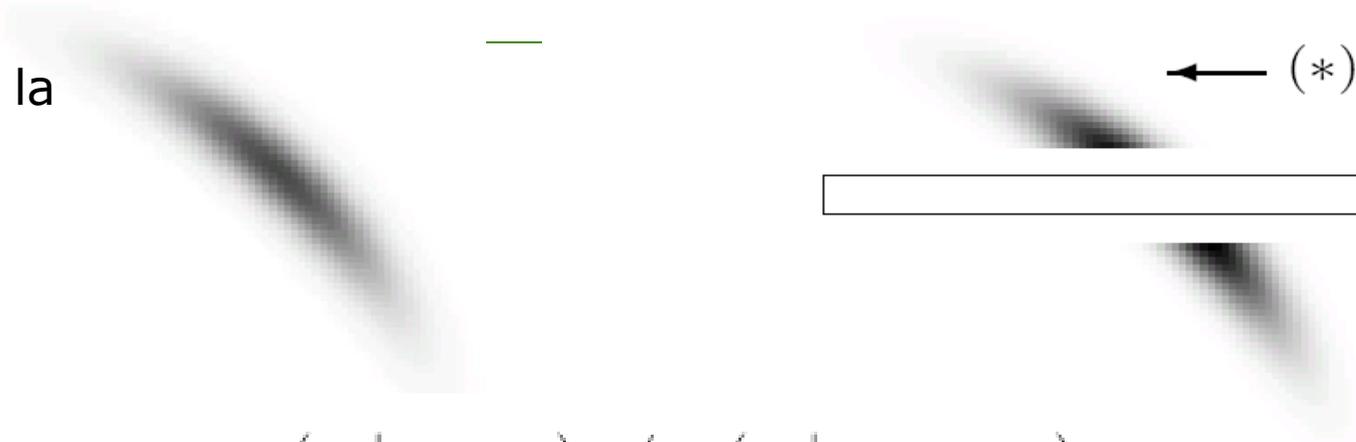
sample genera i campioni da una dist. con la varianza specificata

## Esempi distribuzioni (velocity based)



## Map-Consistent Motion Model

Se si considera la mappa occorre integrare gli ostacoli.



$$p(x_t | u_t, x_{t-1}) \neq p(x_t | u_t, x_{t-1}, m)$$



$$p(x | u, x')$$



$$p(x | u, x', m)$$

Approssimazione: 
$$p(x | u, x', m) = \eta p(x | m) p(x | u, x')$$



Però la relazione è la seguente:

$$p(x_t | u_t, x_{t-1}, m) = \eta p(m | x_t, u_t, x_{t-1}) p(x_t | u_t, x_{t-1})$$

Approssimazione:

$$p(x_t | u_t, x_{t-1}, m) = \eta p(m | x_t) p(x_t | u_t, x_{t-1})$$

Solo per brevi spostamenti si può assumere:

$$p(m | x_t, u_t, x_{t-1}) = p(m | x_t)$$



- Modelli basati su odometria e velocità
- Calcolo della posterior probability  $p(x| x', u)$ .
- Campionamento da  $p(x| x', u)$ .
- Calcolo da intervalli di tempo fissi  $t$ .
- Parameteri del modello devono essere appresi.
- Modello esteso che considera la mappa.