Robotica Probabilistica

Filtri Bayesiani

Filtri discreti e Particle filters

Filtri ad Istogrammi e Filtri Discreti

- Decompongono lo spazio in regioni e rappresentano il post cumulativo di ogni regione con un valore di probabilità
- Se applicati a spazio finito sono detti Filtri Discreti: X_t prende valori in un insieme finito (es. occupancy grid X_{ij} ha 2 valori)

Filtro Discreto: Algoritmo

Algorithm **Discrete_Bayes_filter**(*Bel(x),d*):

η=0

If *d* is a perceptual data item *z* then

  For all *x* do

    $$Bel'(x) = P(z \mid x) Bel(x)$$

    $$\eta = \eta + Bel'(x)$$

  For all *x* do

    $$Bel'(x) = \eta^{-1} Bel'(x)$$

Else if *d* is an action data item *u* then

  For all *x* do

    $$Bel'(x) = \sum_{x'} P(x \mid u, x')\, Bel(x')$$

Return *Bel′(x)*

Filtro ad Istogramma

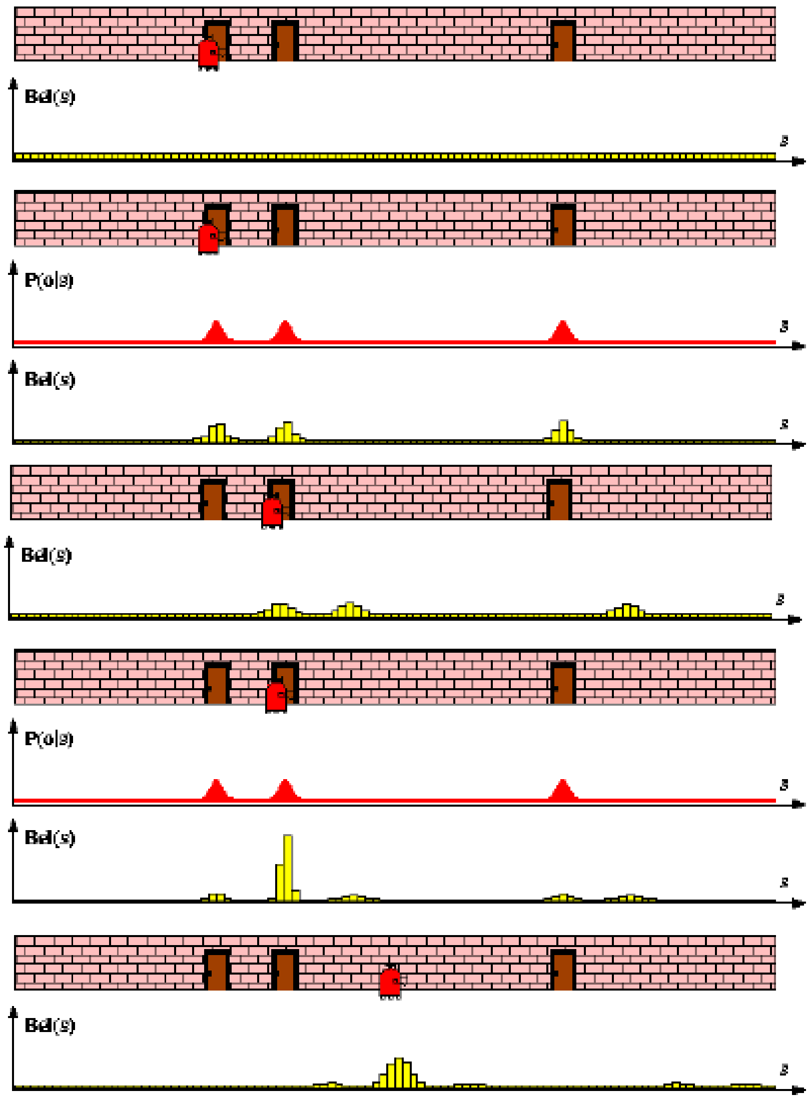
- Se spazio continuo Filtri ad Istogramma (X_t prende valori in spazio continuo)

- Spazio diviso in regioni finite e convesse (bins) che partizionano lo stato: $X_t = x_{1,t} \vee \cdots \vee x_{n,t}$

- Per ogni regione è associata la probabilità $p_{k,t}$ quindi $p(x_t) = \frac{p_{k,t}}{|x_{k,t}|}$

Filtro ad Istogramma

Costante a tratti

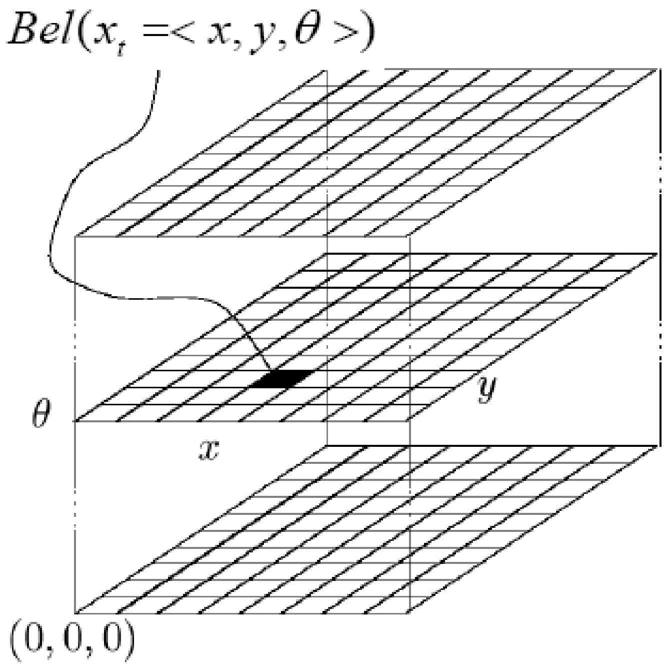


Implementazione

Statica: partizione statica dello spazio

Dinamica: partizione dello spazio dipendente dal contesto

Rappresentazione costante: occupancy grid



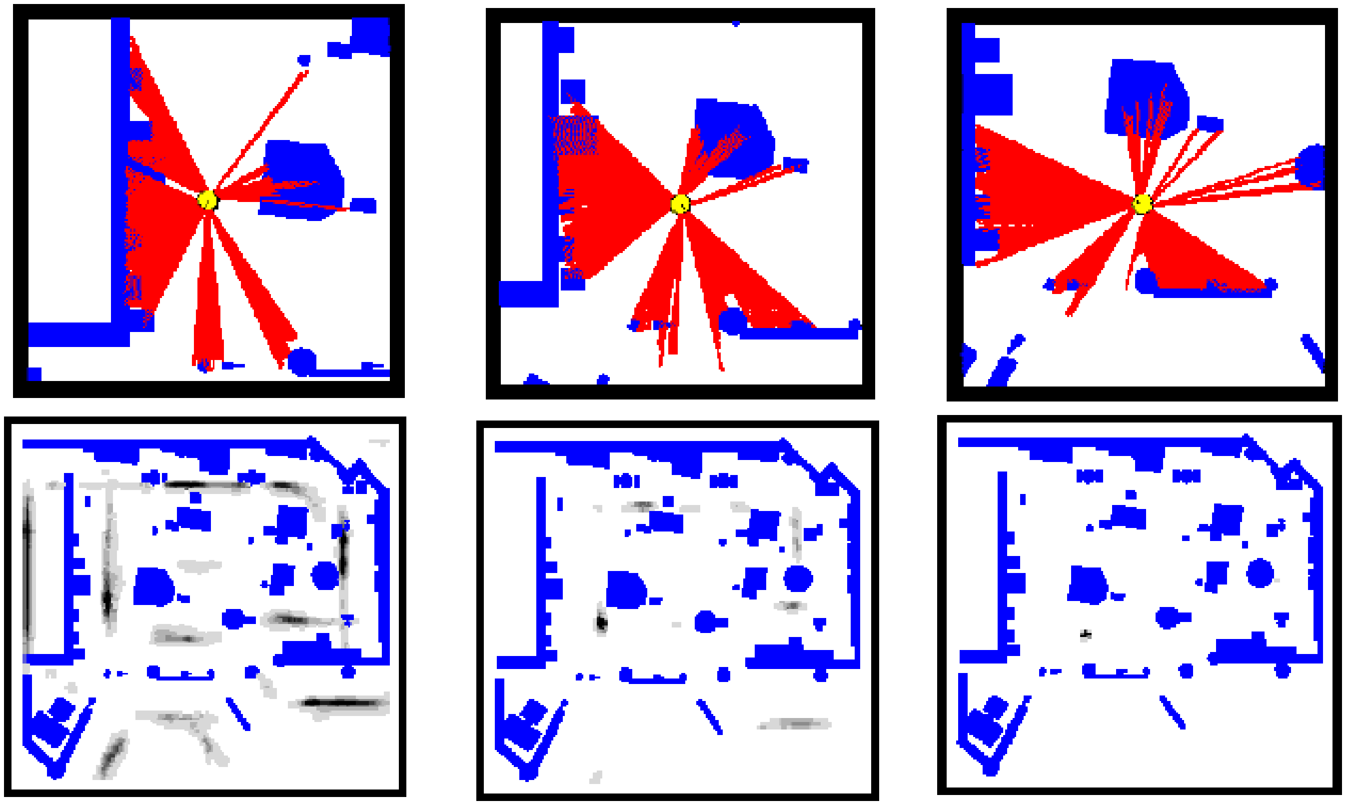
Rappresentazione Statica

Utili quando lo stato è binario

Problemi:

- Per update e normalizzazione occorre lo scan di tutta la griglia
- Se le credenze sono concentrate si vorrebbe evitare
- Si può fare l'update della sottogriglia "attiva", ma non gestisce la delocalizzazione

Localizzazione Grid-based

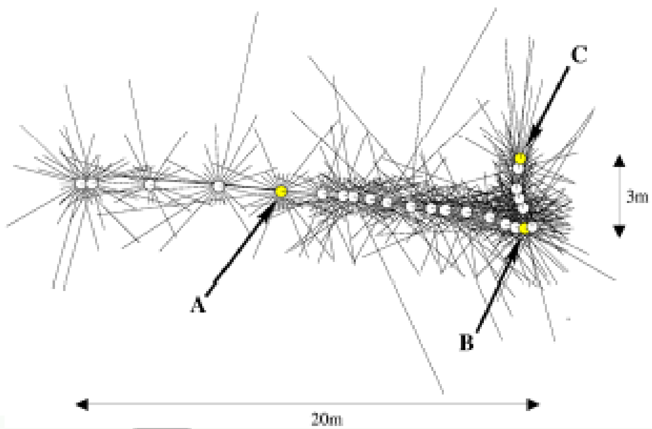


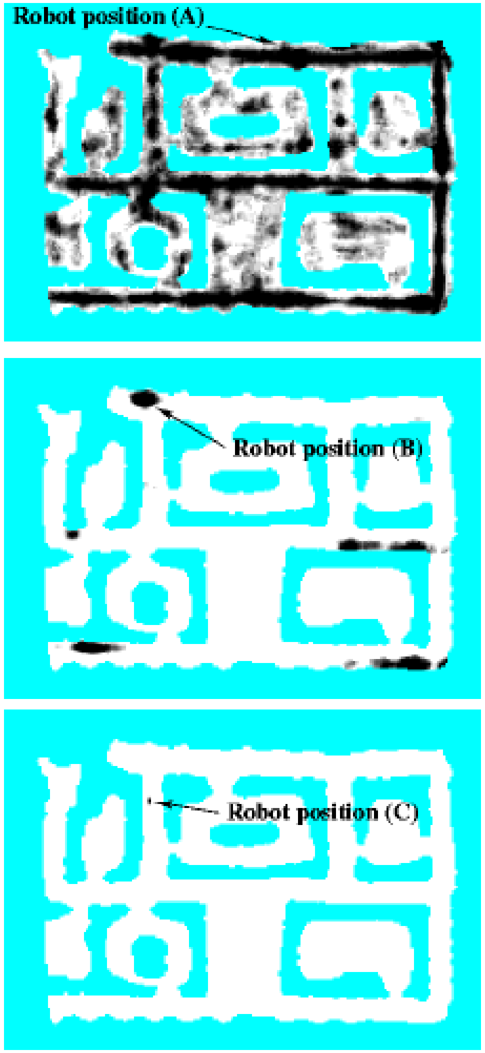
Gridmap risoluzione 15 cm, 15 g, 2LRF, beam model

Posizione dopo 3 scan

Localizzazione Grid-based

Sonars and Occupancy Grid Map





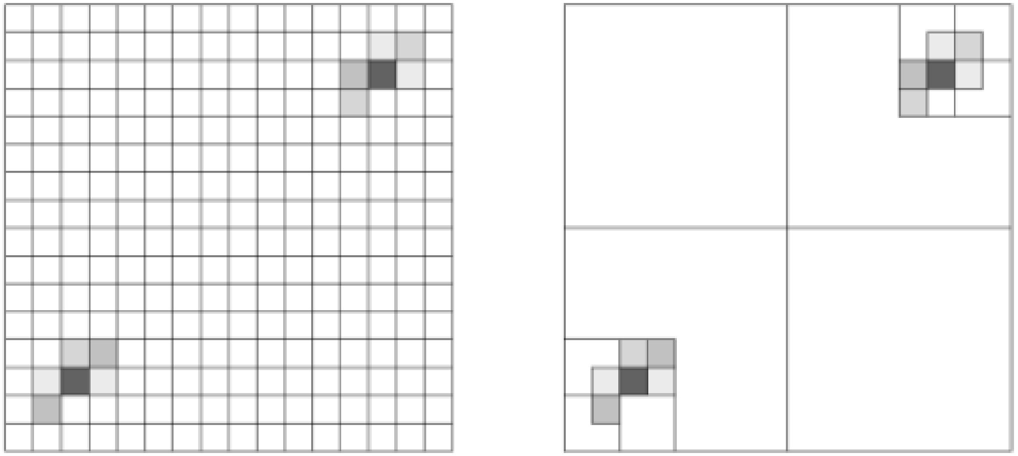
Decomposizione Dinamica

Density Tree: decomposizione ricorsiva adatta alla risoluzione del post

Meno è probabile una regione, minore è la risoluzione

Più precisa ed efficiente (ordini di grandezza)

Quadtree
o Octree



Particle Filters

- Filtri ad Istogrammi:

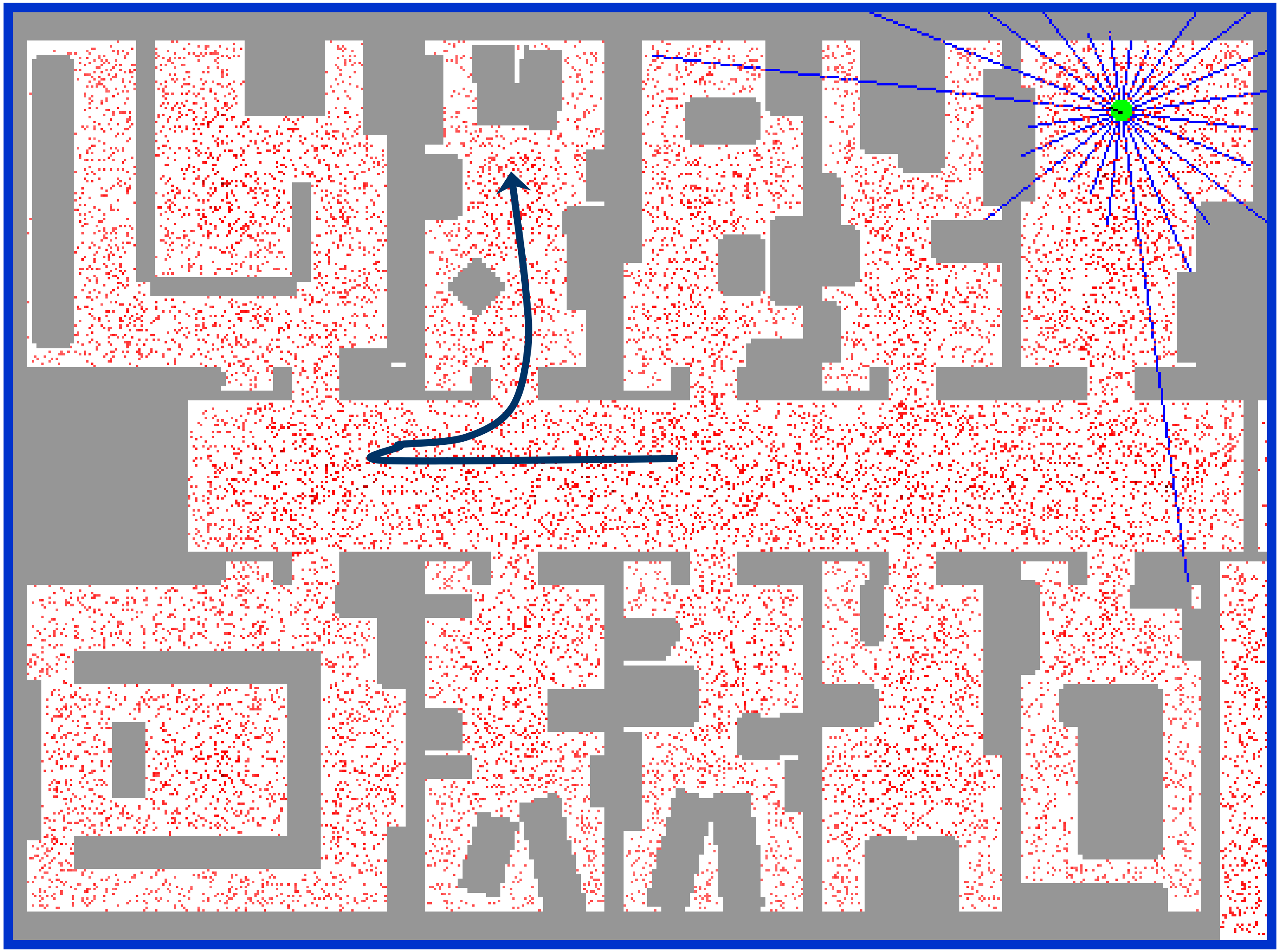
  - Discretizzazione dello spazio (statica o dinamica)
- Problemi di risoluzione ed efficienza

Alternativa più efficiente: **Particle Filters**

Approssimano il post con un numero finito di parametri, ma tecnica diversa:

- Insiemi di ipotesi (particles) rappresentano il post
- Sopravvivono le migliori

Localizzazione basata su Campioni (sonar)



Particle Filters

- Rappresenta i belief con campioni random
- Stima di non-Gaussiane, processi nonlineari

- Monte Carlo filter, Survival of the fittest, Condensation, Bootstrap filter, Particle filter
- Adattivi: numero di campioni dipendenti dalle risorse e dalla complessità del task.

- Filtering: [Rubin, 88], [Gordon et al., 93], [Kitagawa 96]
- Computer vision: [Isard and Blake 96, 98]
- Dynamic Bayesian Networks: [Kanazawa et al., 95]d

Particle Filter

- Lo stato al tempo t è rappresentato da un insieme di campioni (particles):

$$X_t = x_{t,1}, x_{t,2}, \ldots, x_{t,M}$$

- Ogni $x_{t,i}$ è una istanza dello stato al tempo t, M e' il numero delle particles

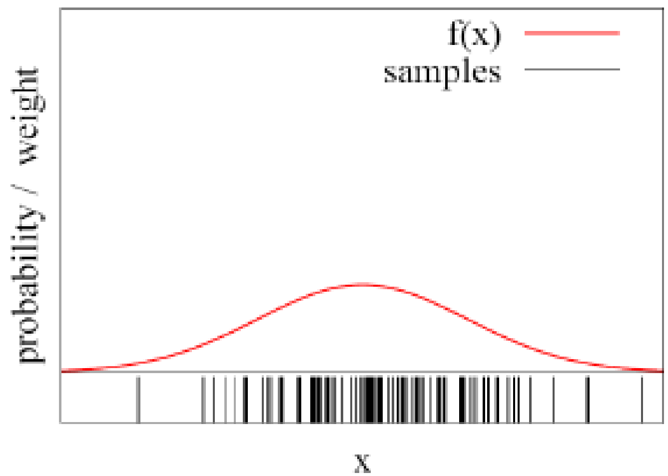
- Ogni particle è una ipotesi sullo stato

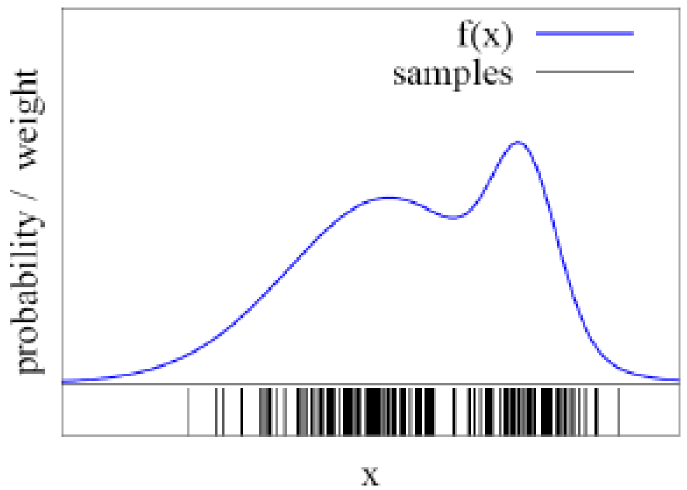
- Prob di "$x_{t,m}$ appartenente ad X_t " approssima $bel(x_{t,m})$: $x_{t,m} \sim p(x_t \mid z_{1:t}, u_{1:t})$

Particle Filter

Gli insiemi di particles X_t approssimano una funzione di distribuzione

Prob di "$x_{t,m}$ appartenente ad X_t" approssima $bel(x_t,m)$: $x_{t,m} \sim p(x_t \mid z_{1:t}, u_{1:t})$





Più particle cadono in un intervallo più è probabile l'intervallo

Campionamento per Rifiuto (rejection sampling)

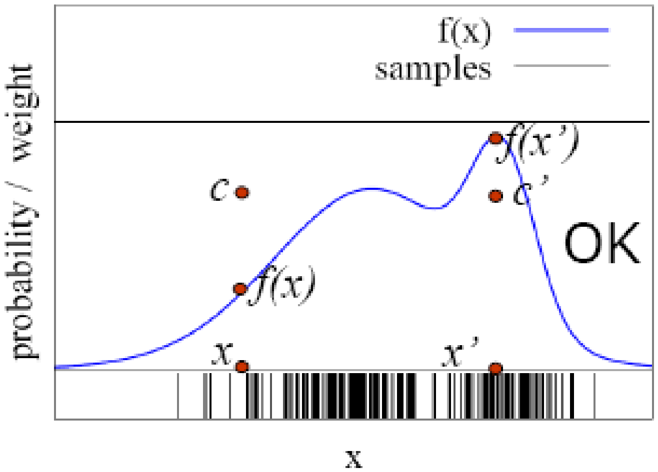
Si assume f(x) < 1 per ogni x

Campionamento da distribuzione uniforme

Si campiona c da [0,1]

Se f(x) > c allora si mantiene il campione

altrimenti si scarta

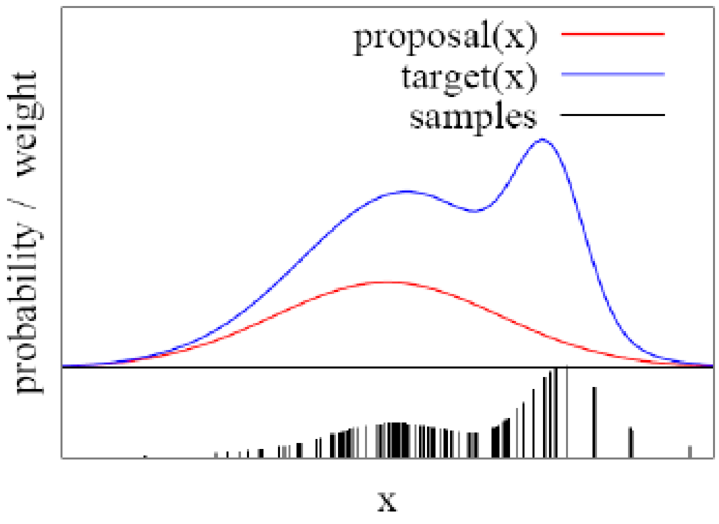


Importance Sampling

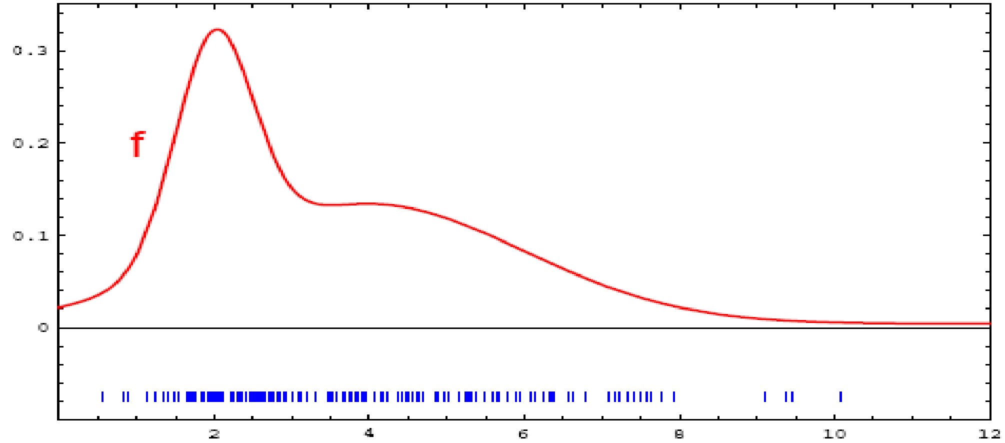
Si vuole fare sampling dalla target(x) ma si può fare da proposal(x)

Un sample da proposal(x) viene pesato con:
w = target(x)/proposal(x)

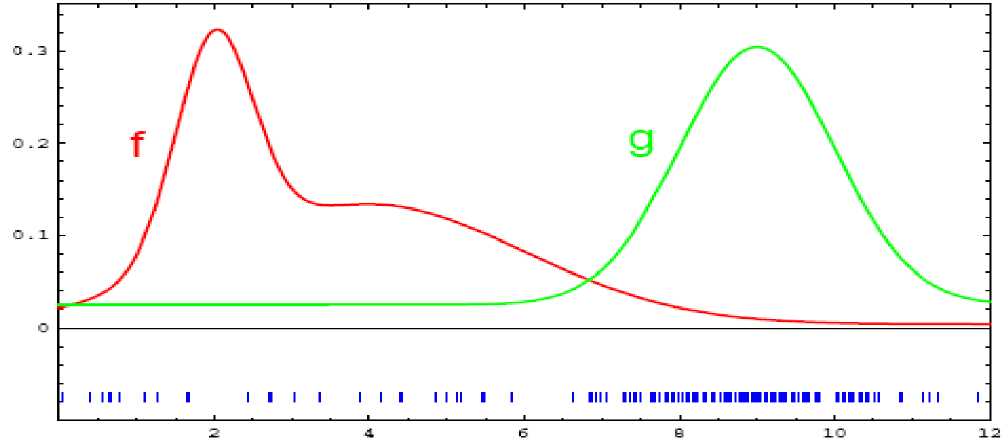
Condizione:
target(x) -> proposal(x)



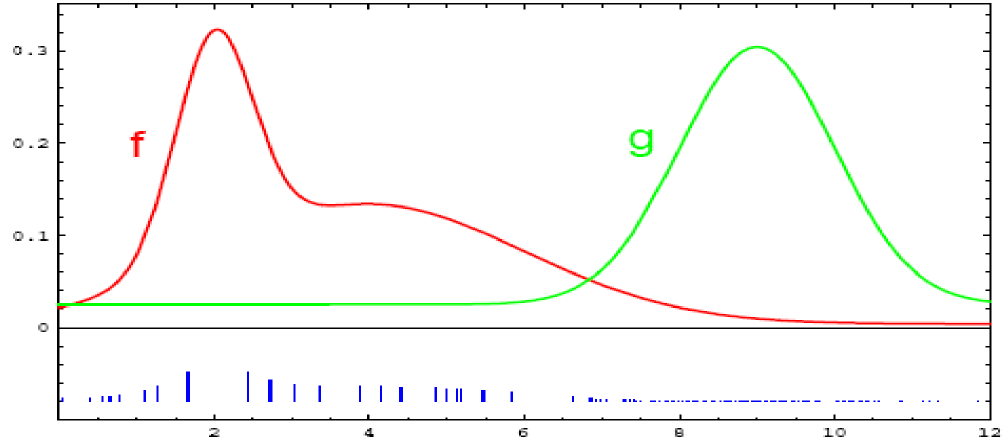
Campioni da f(x),
ma non possibile



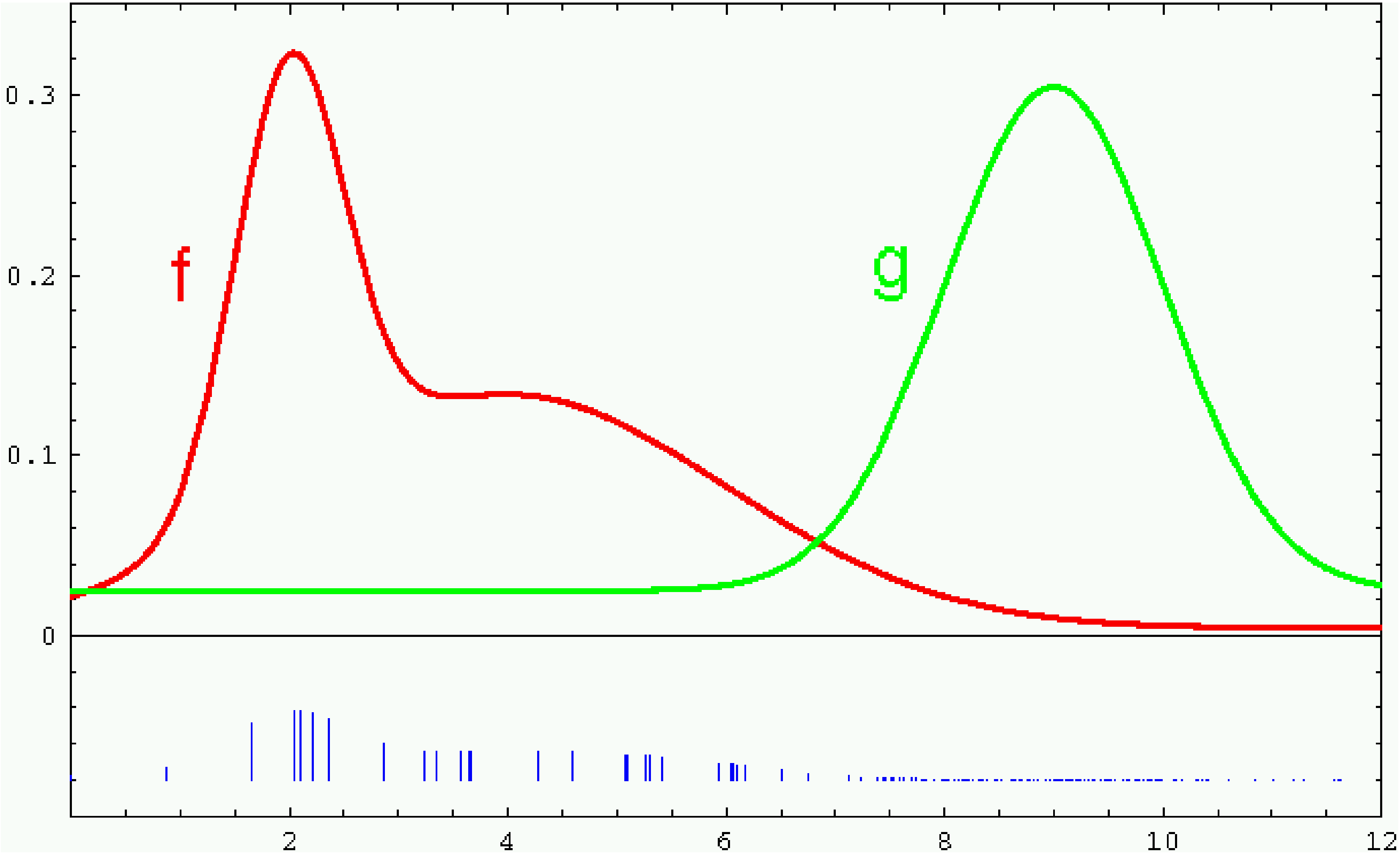
Campioni da g(x)



Campioni pesati
con f(x)/g(x)



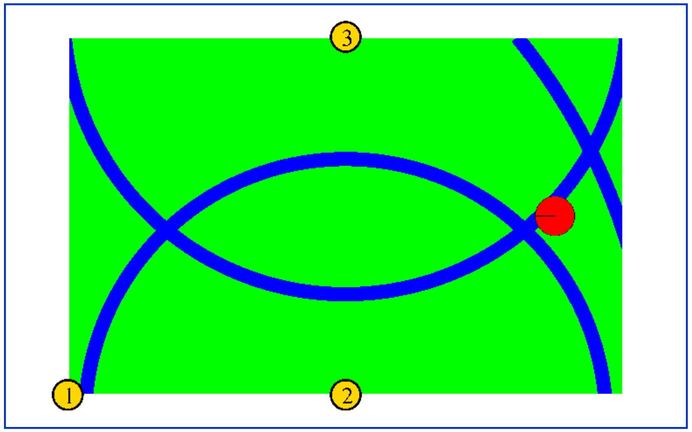
Importance Sampling

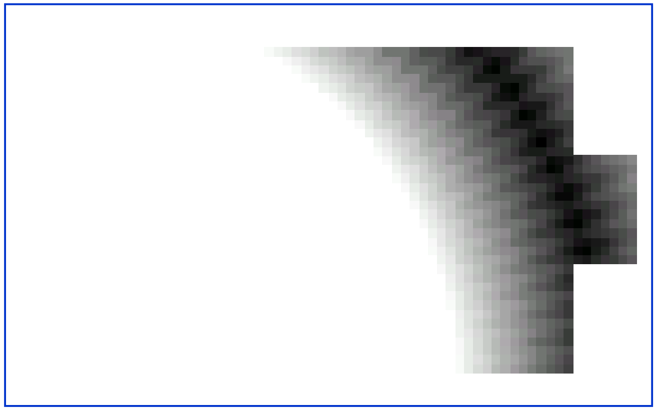


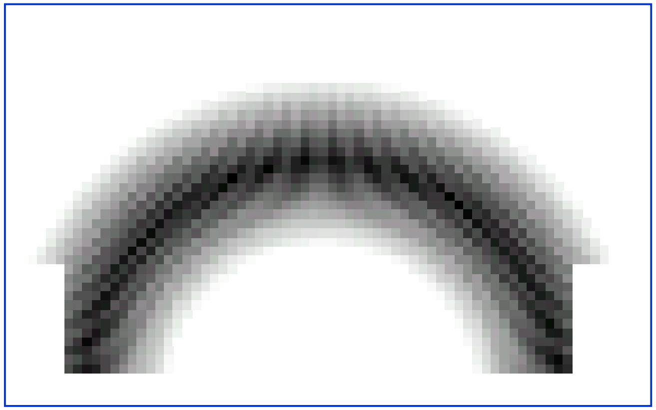
Peso dei campioni: $w = f / g$

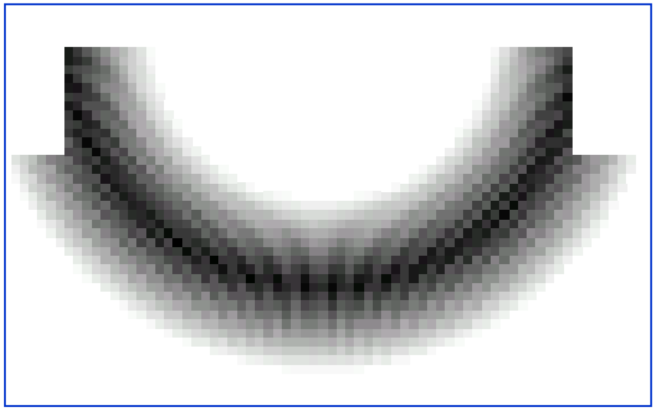
Importance Sampling con Resampling: Esempio

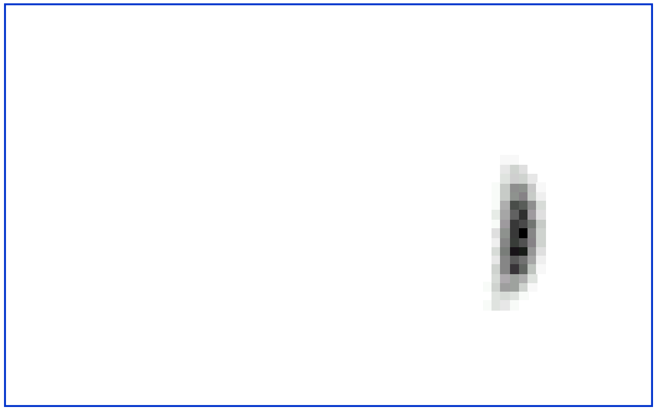
Distribuzioni



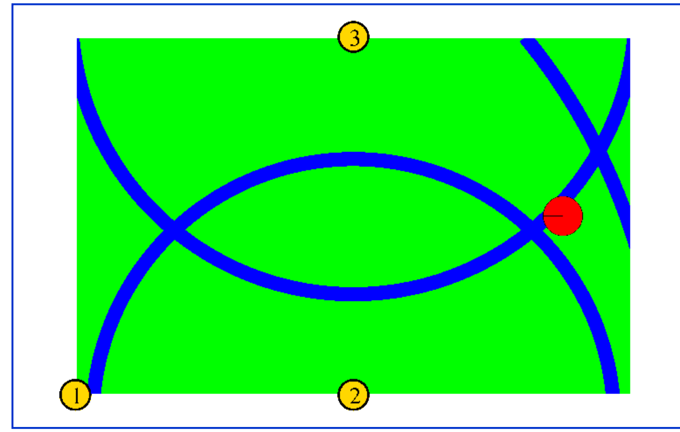


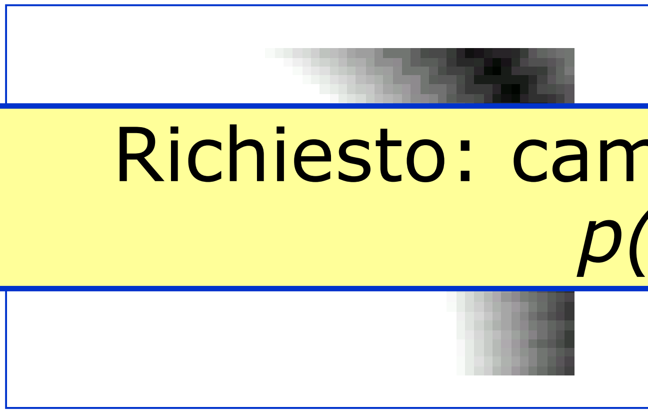


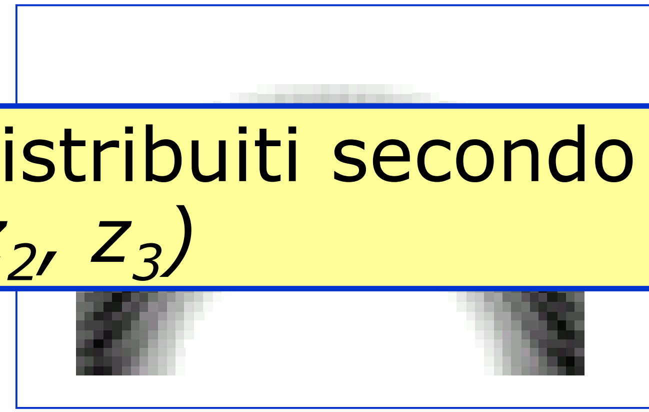




Distribuzioni

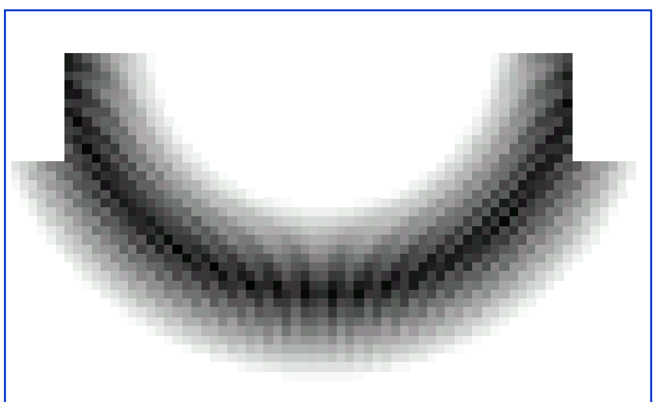


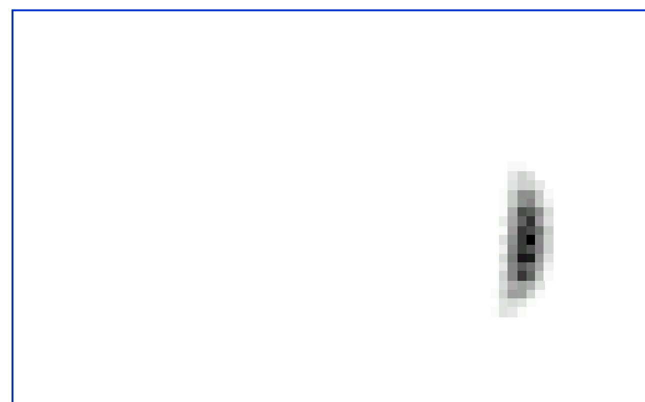




Richiesto: campioni distribuiti secondo

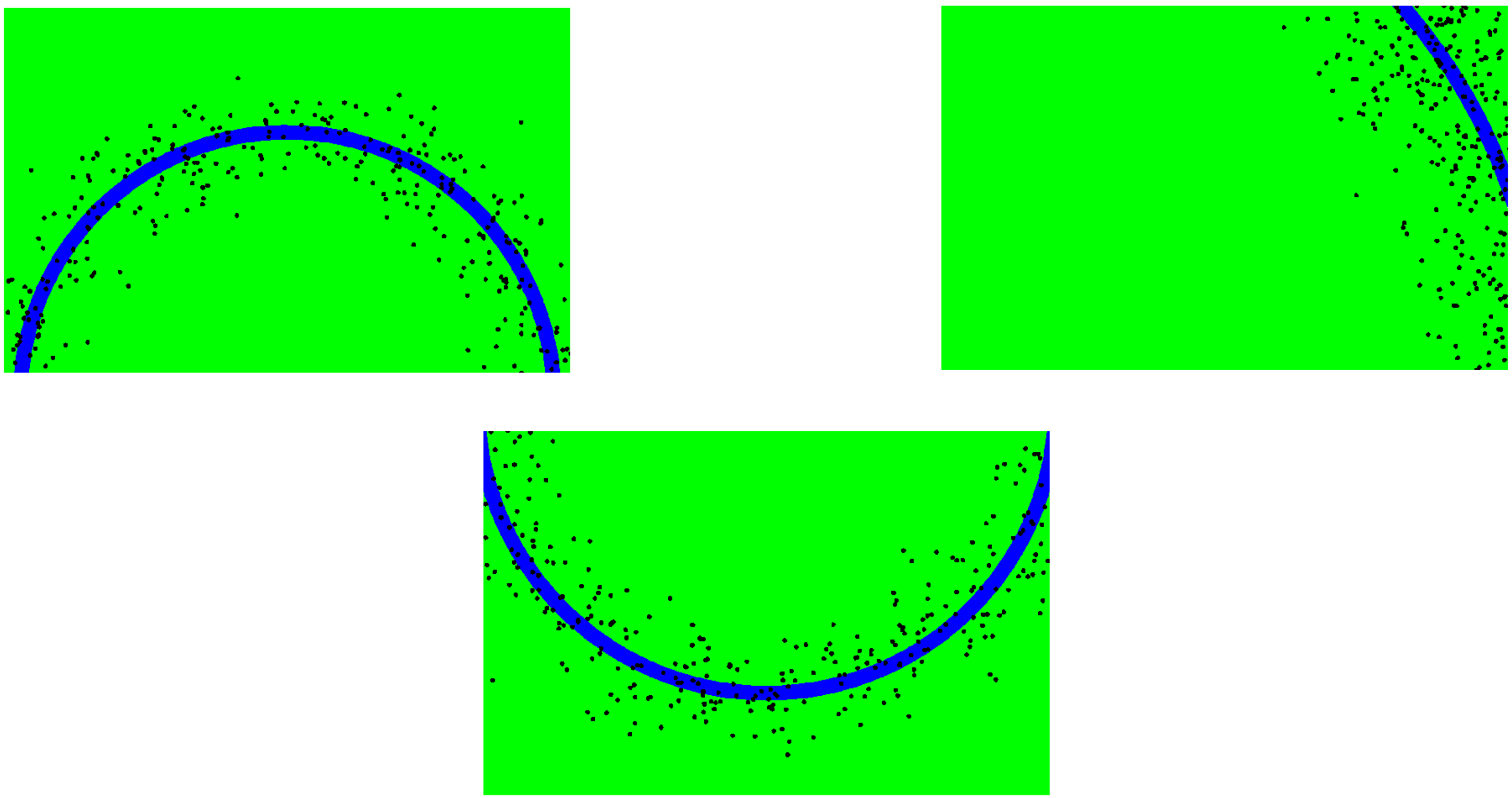
$$p(x|\ z_1, z_2, z_3)$$





Campionamento

Estrarre campioni da $p(x|z_l)$



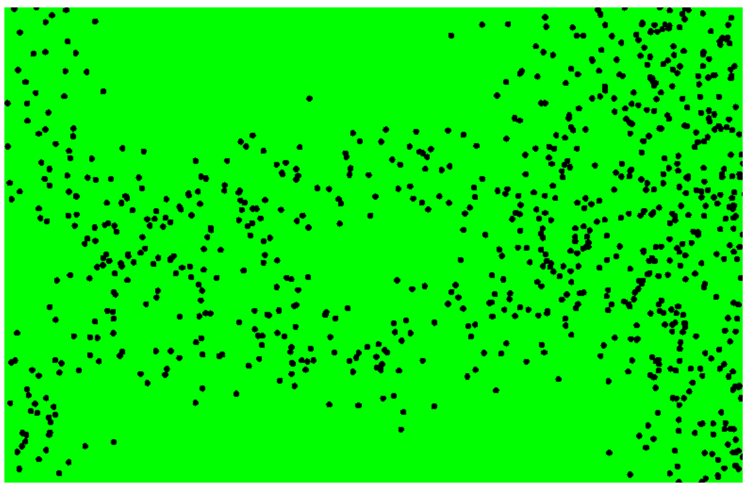
Importance Sampling con Resampling

$$\text{Target distribution f}: p(x \mid z_1, z_2, \ldots, z_n) = \frac{\prod_k p(z_k \mid x) \quad p(x)}{p(z_1, z_2, \ldots, z_n)}$$

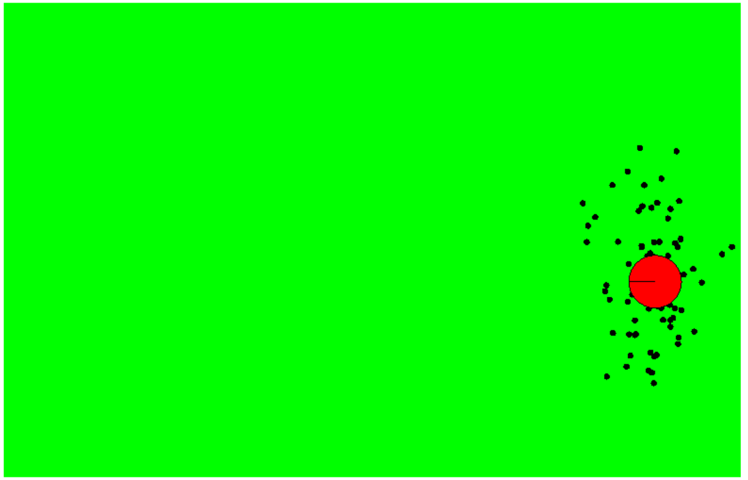
$$\text{Sampling distribution g}: p(x \mid z_l) = \frac{p(z_l \mid x) p(x)}{p(z_l)}$$

$$\text{Importance weights w}: \frac{f}{g} = \frac{p(x \mid z_1, z_2, \ldots, z_n)}{p(x \mid z_l)} = \frac{p(z_l) \prod_{k \neq l} p(z_k \mid x)}{p(z_1, z_2, \ldots, z_n)}$$

Importance Sampling con Resampling



Campioni pesati



Dopo il resampling

Importance Sampling

Campionamento di g

$$\frac{1}{M}\sum_{m=1}^{M} I(x^{[m]} \in A) \quad \longrightarrow \quad \int_A g(x)\, dx$$

Rapporto tra g ed f

$$w^{[m]} \quad = \quad \frac{f(x^{[m]})}{g(x^{[m]})}$$

Permette di ricostruire f

$$\left[\sum_{m=1}^{M} w^{[m]}\right]^{-1} \sum_{m=1}^{M} I(x^{[m]} \in A)\, w^{[m]} \quad \longrightarrow \quad \int_A f(x)\, dx$$

Importance Sampling

Nel caso del filtro particellare:

Target(x) = $Bel(x_t)$ → X_t

Proposal(x) = $p(x_t | u_t, x_{t-1})\, Bel(x_{t-1})$ → X'_t

w = $p(z_t | x_t)$

Particle Filter

Come bel(x_t) si aggiorna da bel(x_{t-1})

Così X_t si aggiorna da X_{t-1}

Le ipotesi sono pesate:

$$S = \left\{ \left\langle s^{[i]}, w^{[i]} \right\rangle \mid i = 1, \ldots, N \right\}$$

State hypothesis (→ $s^{[i]}$)   Importance weight (→ $w^{[i]}$)

I campioni pesati usati per rappresentare il post:

$$p(x) = \sum_{i=1}^{N} w_i \cdot \delta_{s^{[i]}}(x)$$

Particle Filter: Algoritmo

Prima crea X'_t dal moto u_t rappresenta bel'(t), quindi corregge con le osservazioni z_t

Algorithm Particle_filter$(\mathcal{X}_{t-1}, u_t, z_t)$**:**
    $\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$
    for $m = 1$ to M do
        sample $x_t^{[m]} \sim p(x_t \mid u_t, x_{t-1}^{[m]})$
        $w_t^{[m]} = p(z_t \mid x_t^{[m]})$
        $\bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle x_t^{[m]}, w_t^{[m]} \rangle$
    endfor
    for $m = 1$ to M do
        draw i with probability $\propto w_t^{[i]}$
        add $x_t^{[i]}$ to $\mathcal{X}_t$
    endfor
    return $\mathcal{X}_t$

Generation

Importance factor

Resampling

Particle Filter: Algoritmo

Si parte da X_{t-1} che rapp $Bel_{t-1}(x)$

Si applica il modello di moto
$p(x_{t,i} \mid u_t, x_{t-1,i})$ e si campiona X'_t

Si pesano i campioni rispetto a $P(z|x)$

Si effettua il ricampionamento per avere X_t

Particle Filter: Algoritmo

Target

$$
\begin{aligned}
& p(x_{0:t} \mid z_{1:t}, u_{1:t}) \\
& \stackrel{\text{Bayes}}{=} \quad \eta\, p(z_t \mid x_{0:t}, z_{1:t-1}, u_{1:t})\, p(x_{0:t} \mid z_{1:t-1}, u_{1:t}) \\
& \stackrel{\text{Markov}}{=} \quad \eta\, p(z_t \mid x_t)\, p(x_{0:t} \mid z_{1:t-1}, u_{1:t}) \\
& = \quad \eta\, p(z_t \mid x_t)\, p(x_t \mid x_{0:t-1}, z_{1:t-1}, u_{1:t})\, p(x_{0:t-1} \mid z_{1:t-1}, u_{1:t}) \\
& \stackrel{\text{Markov}}{=} \quad \eta\, p(z_t \mid x_t)\, p(x_t \mid x_{t-1}, u_t)\, p(x_{0:t-1} \mid z_{1:t-1}, u_{1:t-1})
\end{aligned}
$$

Proposal

$$
\begin{aligned}
& p(x_t \mid x_{t-1}, u_t)\, bel(x_{0:t-1}) \\
& = \quad p(x_t \mid x_{t-1}, u_t)\, p(x_{0:t-1} \mid z_{0:t-1}, u_{0:t-1})
\end{aligned}
$$

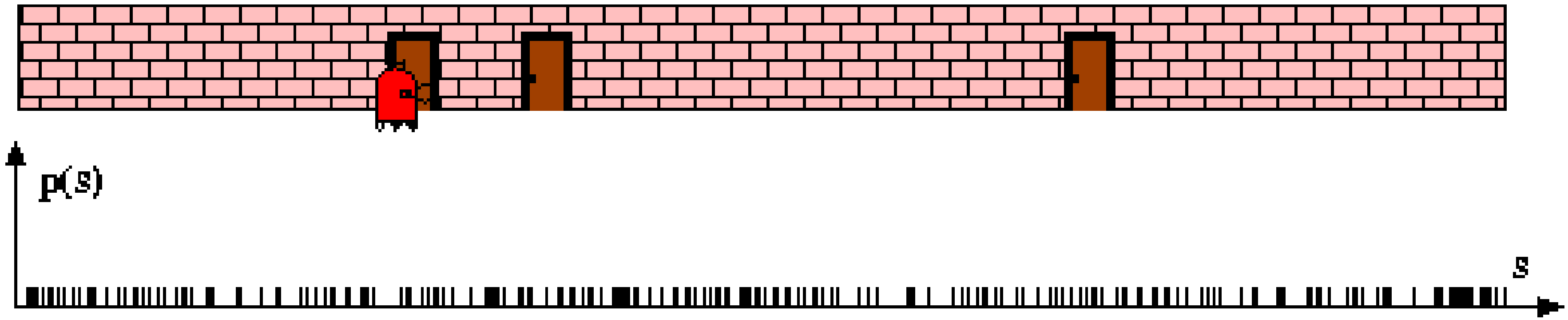
Peso

$$
\begin{aligned}
w_t^{[m]} & = \frac{\text{target distribution}}{\text{proposal distribution}} \\
& = \frac{\eta\, p(z_t \mid x_t)\, p(x_t \mid x_{t-1}, u_t)\, p(x_{0:t-1} \mid z_{1:t-1}, u_{1:t-1})}{p(x_t \mid x_{t-1}, u_t)\, p(x_{0:t-1} \mid z_{0:t-1}, u_{0:t-1})} \\
& = \eta\, p(z_t \mid x_t)
\end{aligned}
$$

Target

$$
\eta\, w_t^{[m]}\, p(x_t \mid x_{t-1}, u_t)\, p(x_{0:t-1} \mid z_{0:t-1}, u_{0:t-1}) \quad = \quad bel(x_{0:t})
$$

Particle Filters

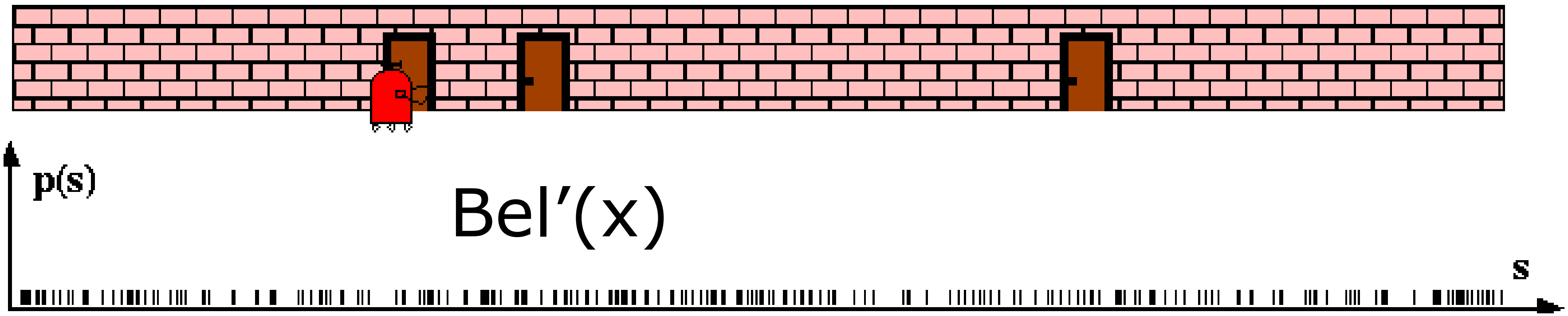


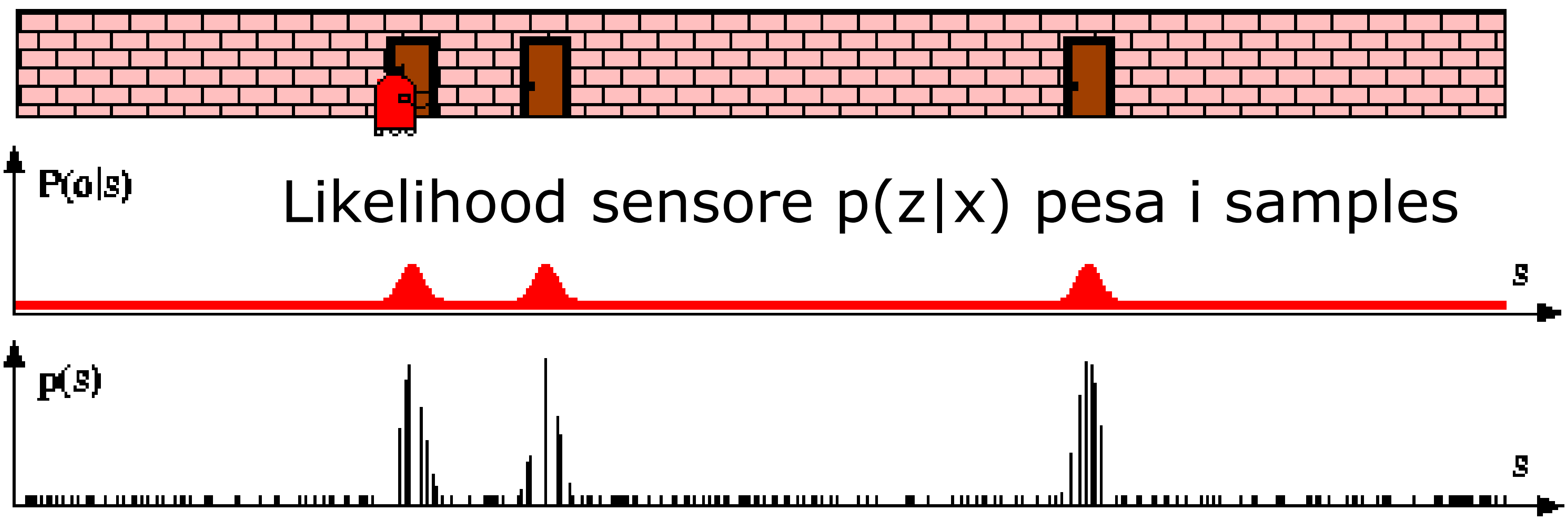
La posa iniziale sono particelle prese in modo random

Informazione Sensori: Importance Sampling

$$Bel(x) \leftarrow \alpha\ p(z \mid x)\ Bel^{-}(x)$$

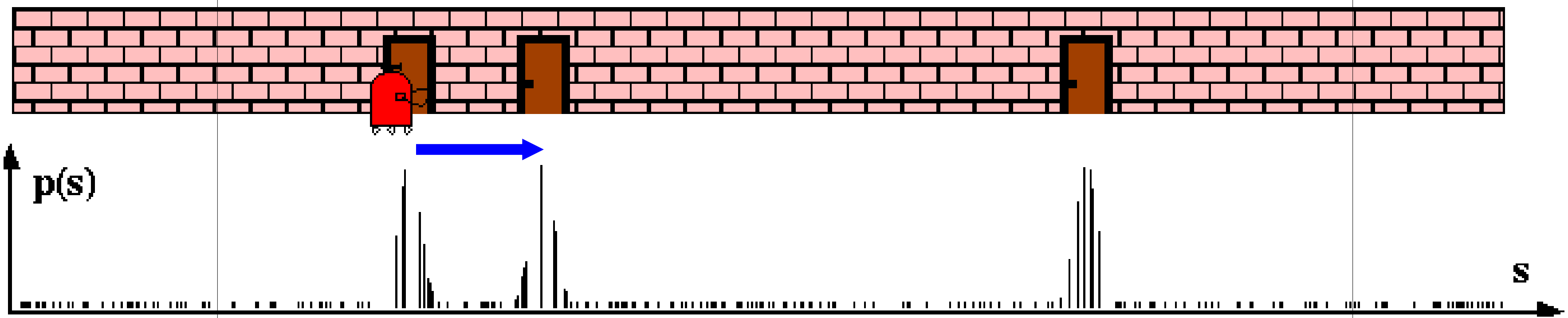
$$w \leftarrow \frac{\alpha\ p(z \mid x)\ Bel^{-}(x)}{Bel^{-}(x)} = \alpha\ p(z \mid x)$$



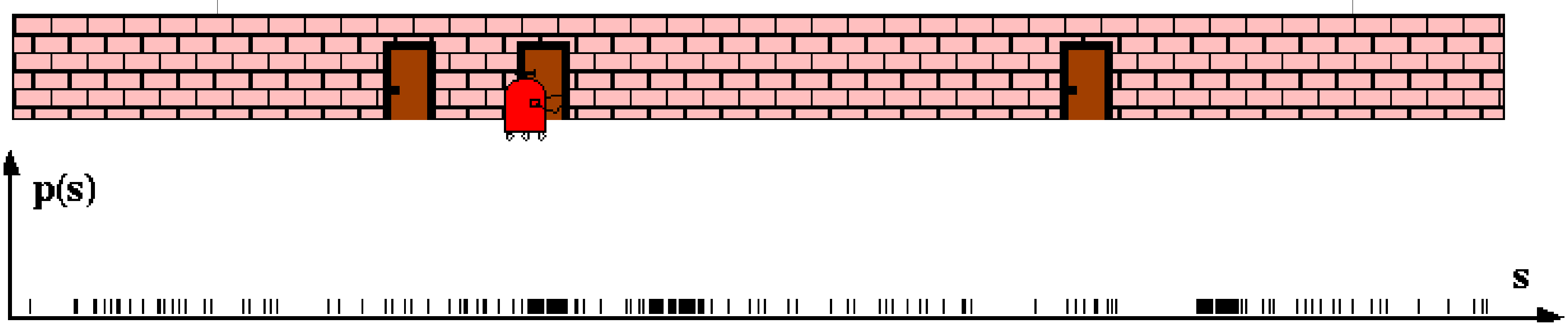


Robot Motion

$$Bel^{-}(x) \leftarrow \int p(x \mid u, x')\, Bel(x')\; \mathrm{d}x'$$



Ricampionamento e spostamento Bel’(x)

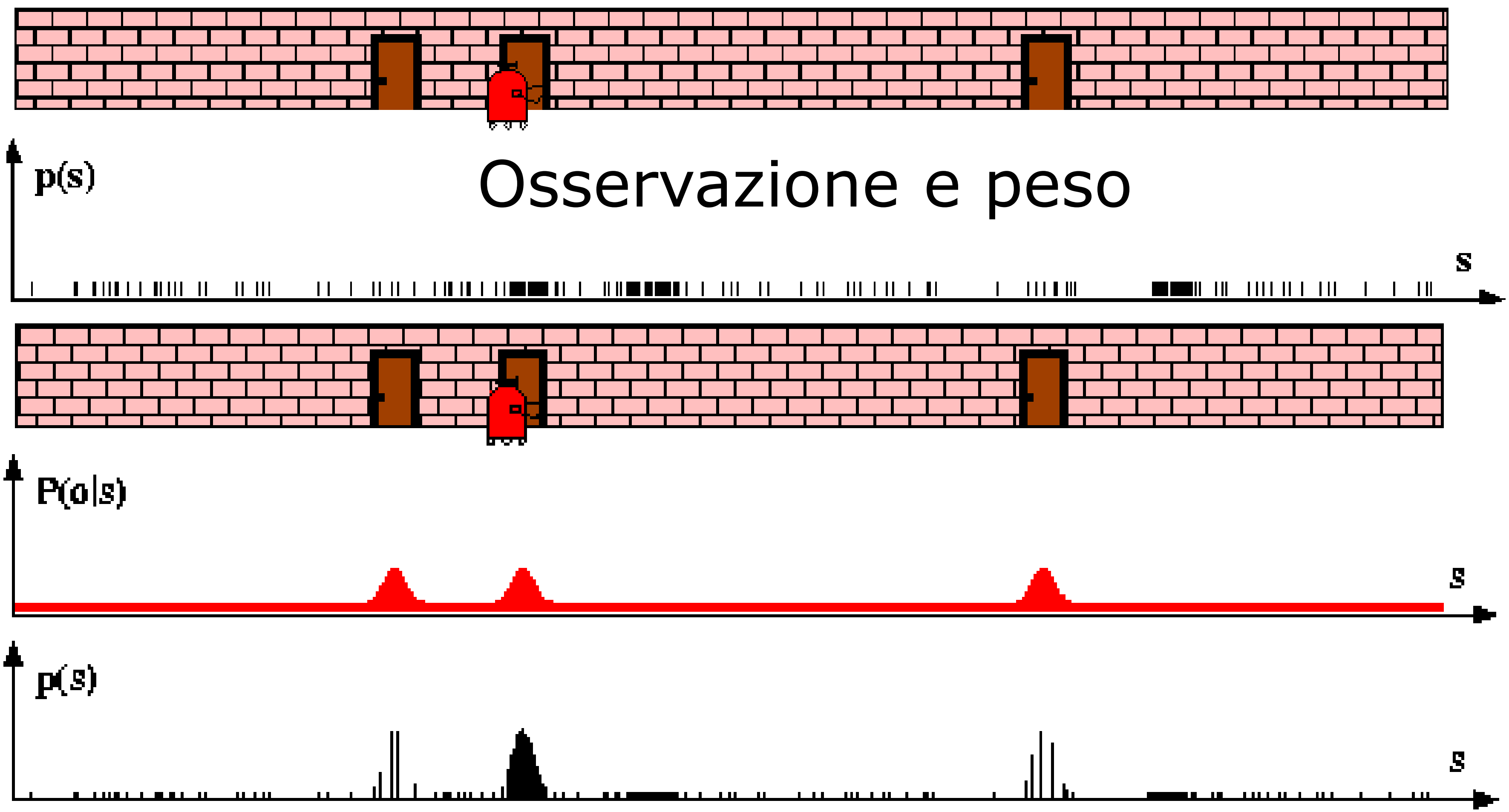


Informazione Sensori: Importance Sampling

$$Bel(x) \quad \leftarrow \quad \alpha \ p(z\,|\,x)\, Bel^{-}(x)$$

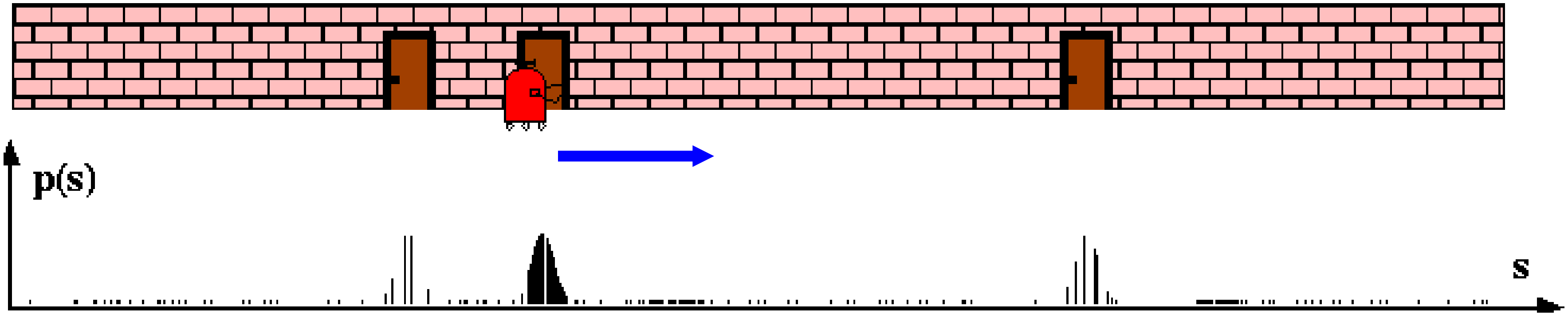
$$w \quad \leftarrow \quad \frac{\alpha \ p(z\,|\,x)\, Bel^{-}(x)}{Bel^{-}(x)} \quad = \quad \alpha \ p(z\,|\,x)$$

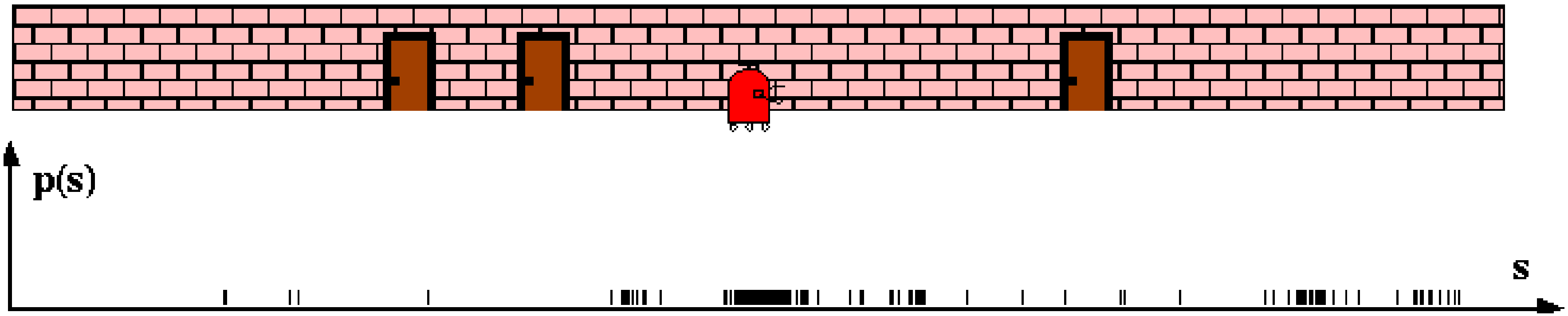
Osservazione e peso



Robot Motion

$$Bel^{-}(x) \quad \leftarrow \quad \int p(x \mid u, x')\, Bel(x') \; \mathrm{d}x'$$





Algoritmo del Particle Filter

1. Algorithm **particle_filter**(S_{t-1}, u_{t-1} z_t):
2. $S_t = \emptyset, \quad \eta = 0$
3. **For** $i = 1 \ldots n$ — ***Generate new samples***
4. Sample index *j(i)* from the discrete distribution given by w_{t-1}
5. Sample x_t^i from $p(x_t \mid x_{t-1}, u_{t-1})$ using $x_{t-1}^{j(i)}$ and u_{t-1}
6. $w_t^i = p(z_t \mid x_t^i)$ — ***Compute importance weight***
7. $\eta = \eta + w_t^i$ — ***Update normalization factor***
8. $S_t = S_t \cup \{< x_t^i, w_t^i >\}$ — ***Insert***
9. **For** $i = 1 \ldots n$
10. $w_t^i = w_t^i / \eta$ — ***Normalize weights***

Algoritmo Particle Filter

$$Bel(x_t) = \eta\, p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_{t-1})\, Bel(x_{t-1})\, dx_{t-1}$$

draw x^i_{t-1} from $Bel(x_{t-1})$

draw x^i_t from $p(x_t \mid x^i_{t-1}, u_{t-1})$

Importance factor for x^i_t:

$$w^i_t = \frac{\text{target distribution}}{\text{proposal distribution}}$$

$$= \frac{\eta\, p(z_t \mid x_t)\; p(x_t \mid x_{t-1}, u_{t-1})\; Bel(x_{t-1})}{p(x_t \mid x_{t-1}, u_{t-1})\; Bel(x_{t-1})}$$

$$\propto p(z_t \mid x_t)$$

Sampling da Bel’

Peso da P(z|x)

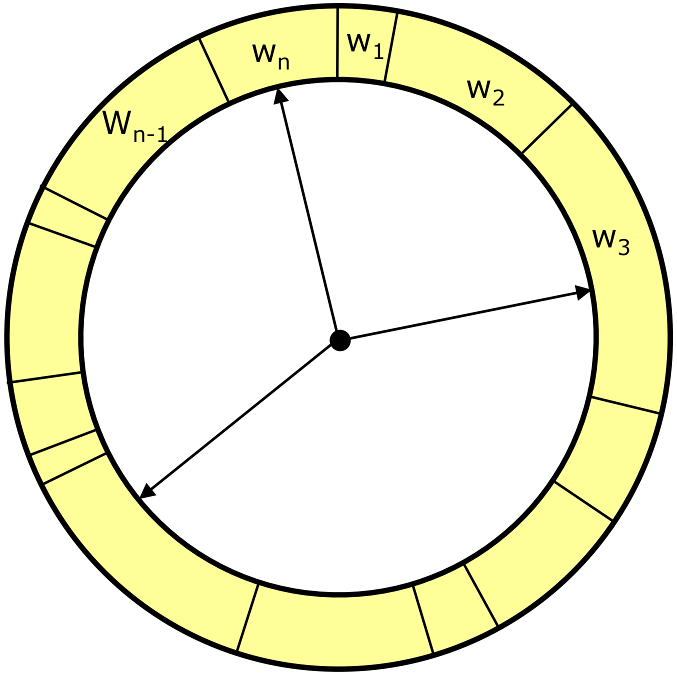
Resampling

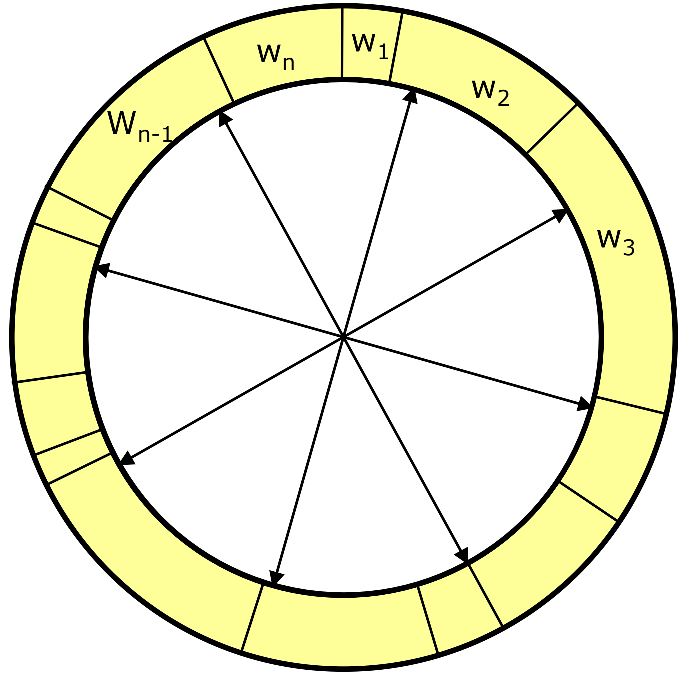
- **Dato**: Insieme *S* di campioni pesati.

- **Desiderata** : Campione Random, dove la prob di estrarre x_i è proporzionale a w_i.

- Fatto *n* volte con rimpiazzamento per generare il nuovo insieme di campioni *S'*.

Resampling

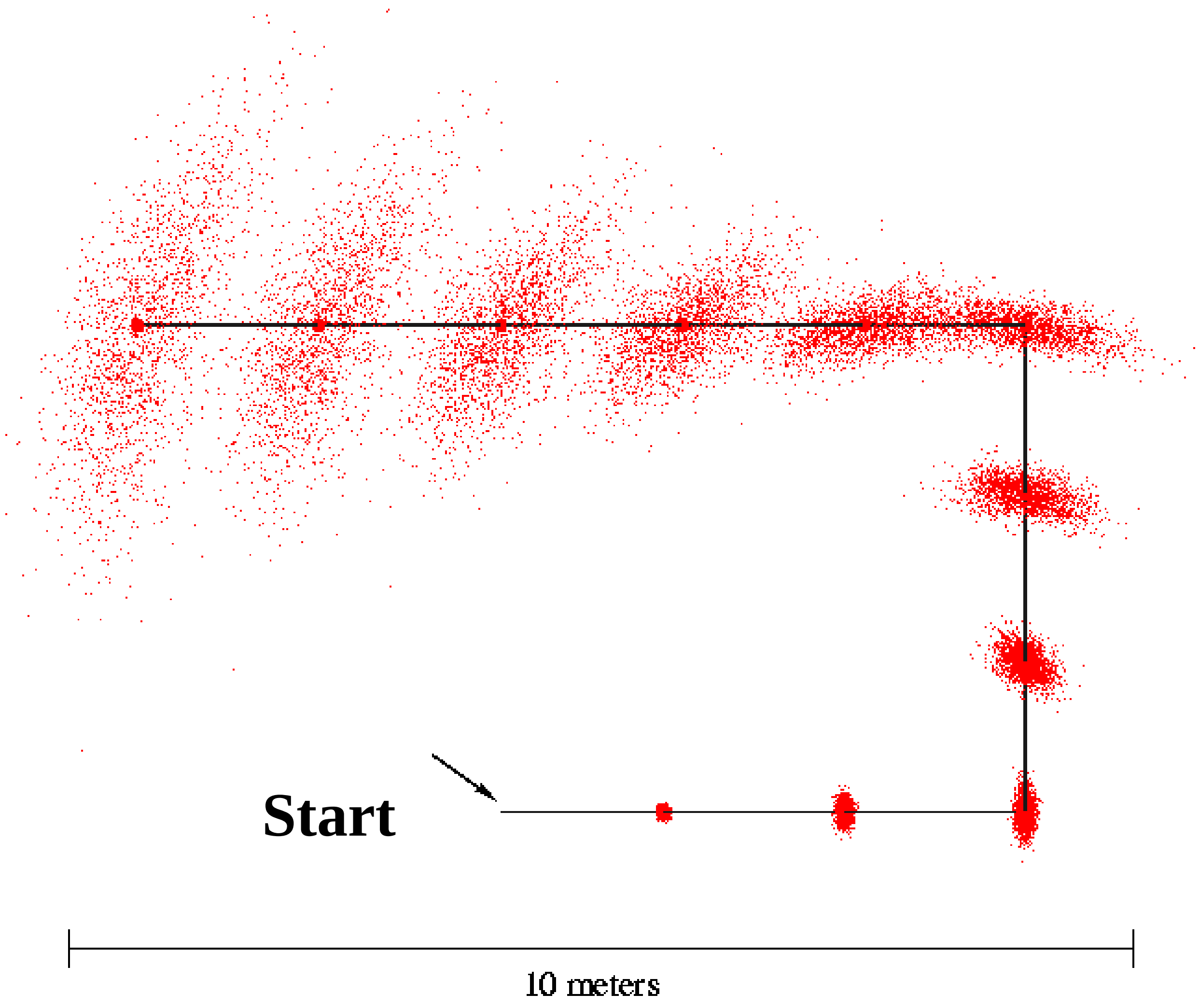




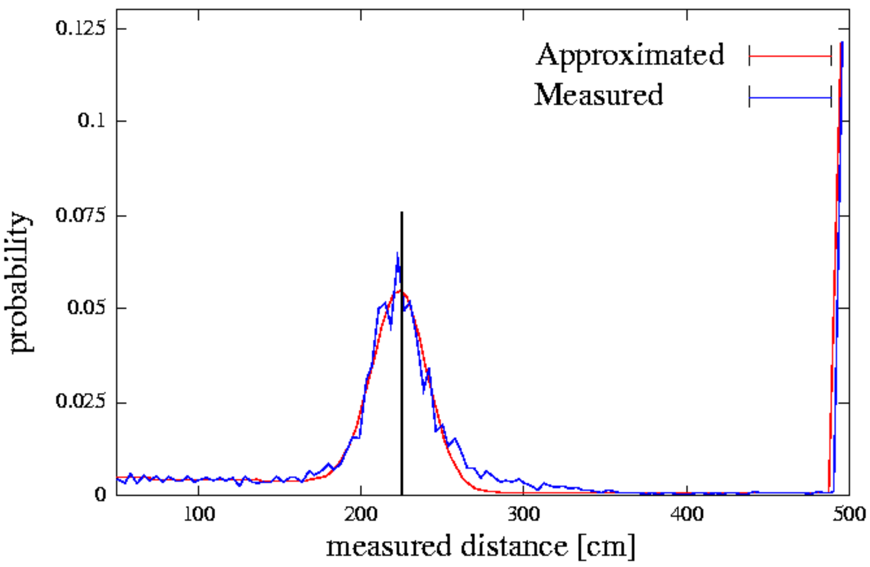
- Roulette wheel
- Binary search, n log n

- Stochastic universal sampling
- Systematic resampling
- Linear time complexity
- Easy to implement, low variance

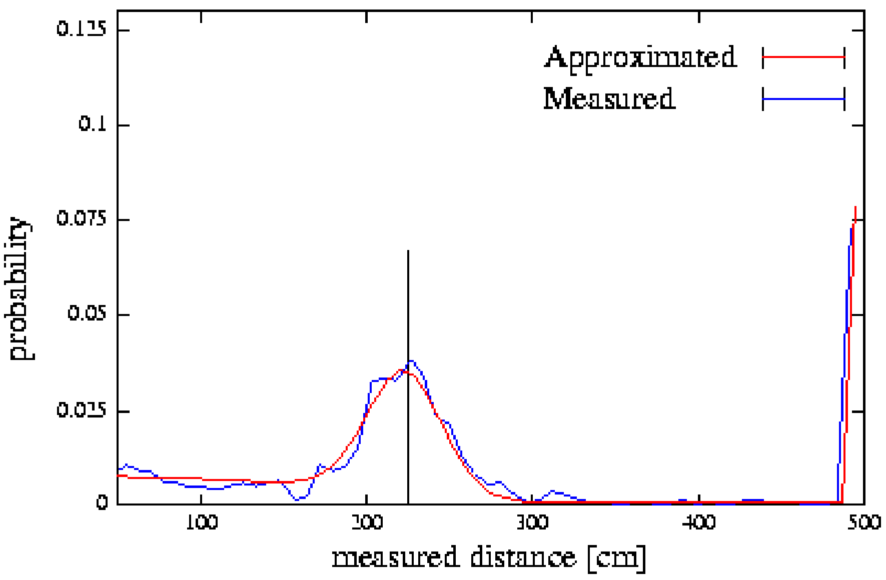
Modello di Moto



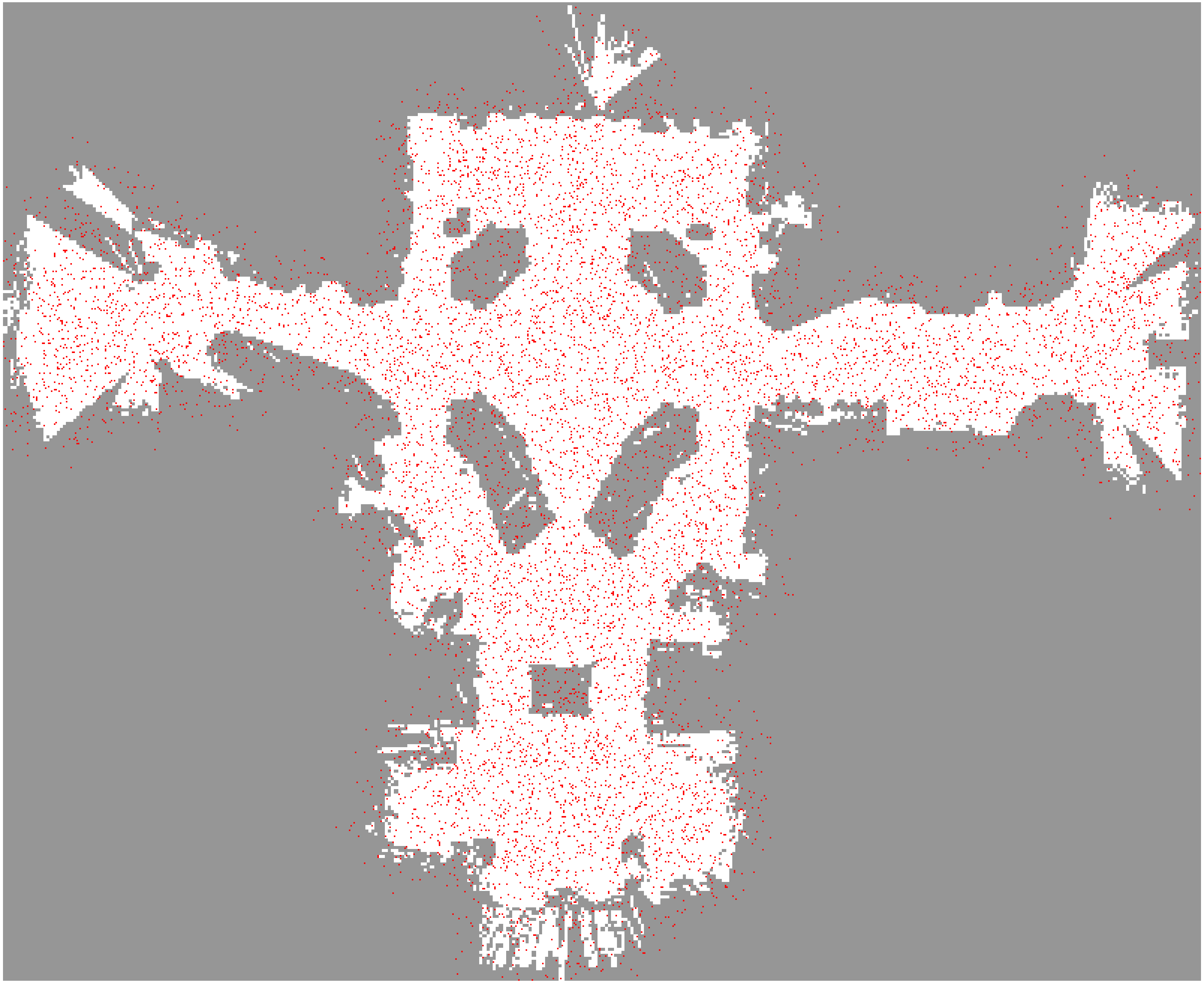
Proximity Sensor Model

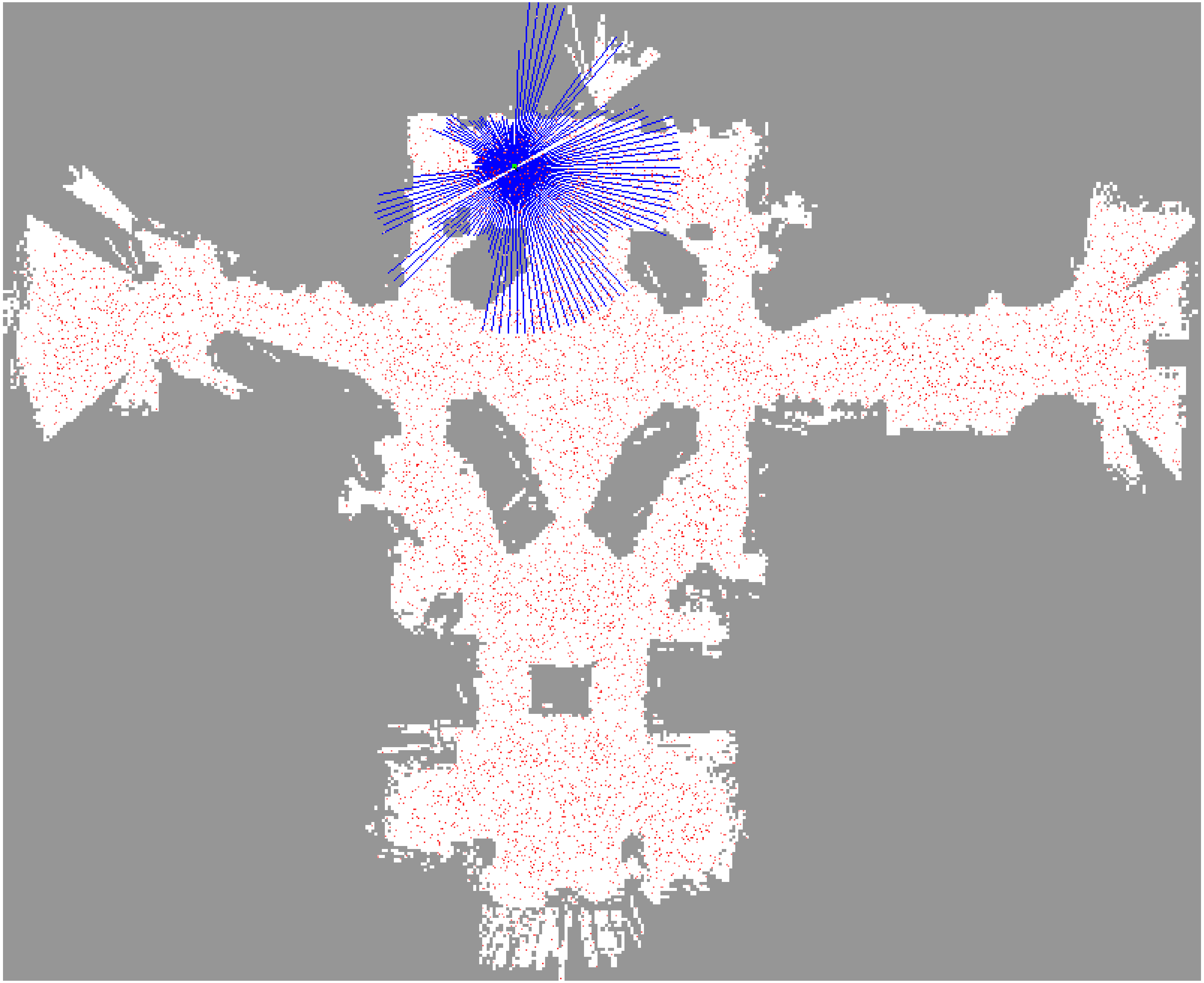


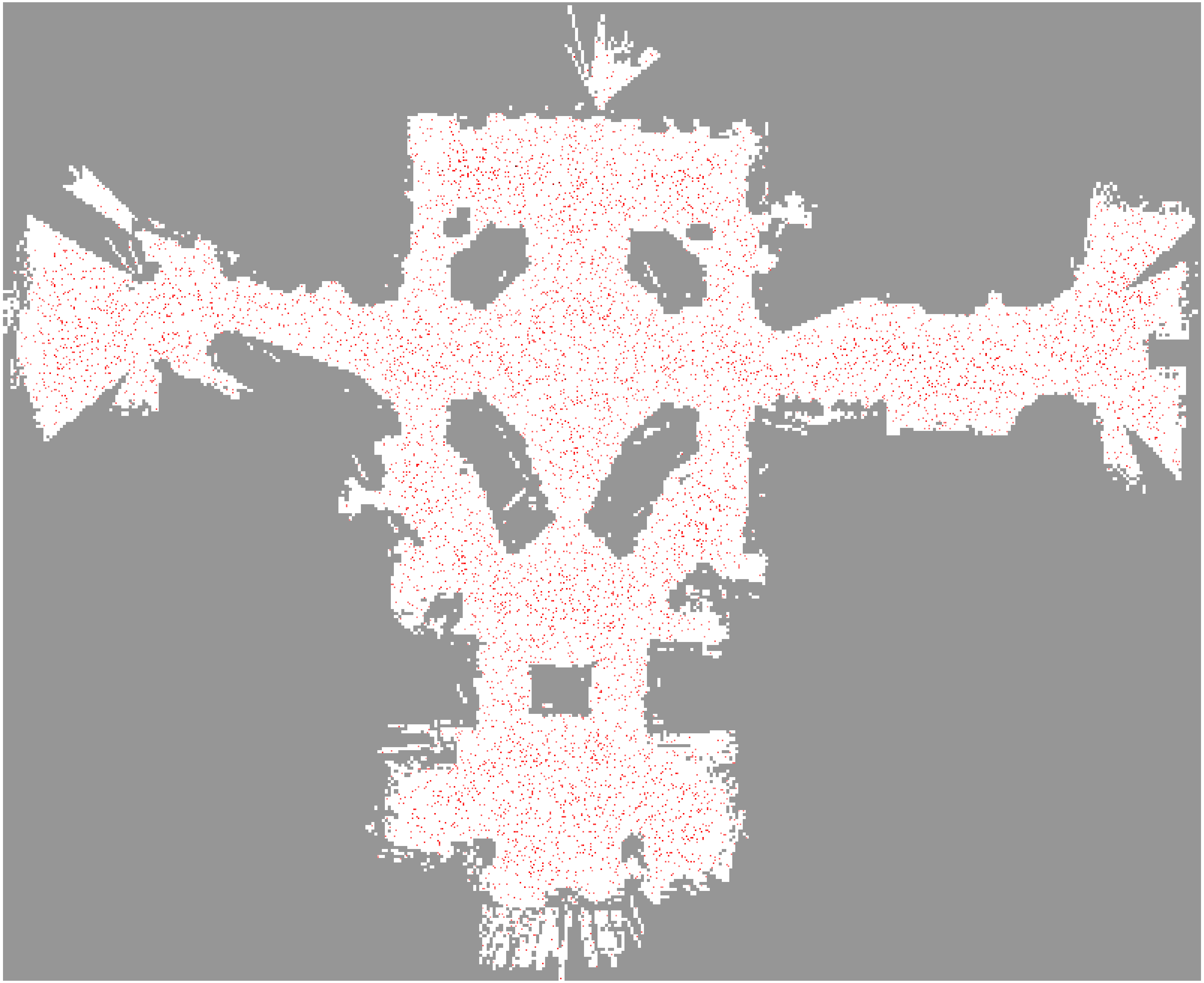
Laser sensor

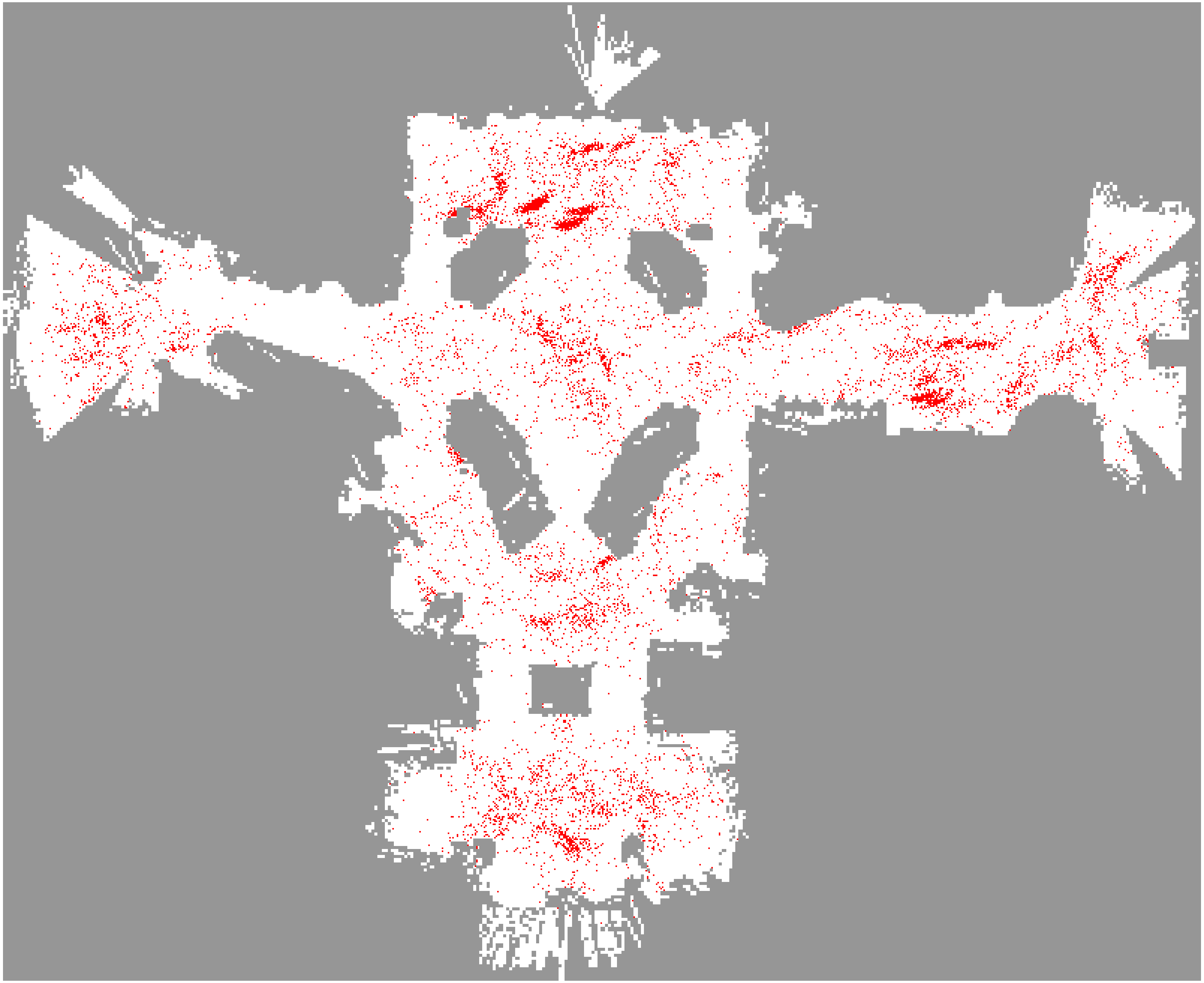


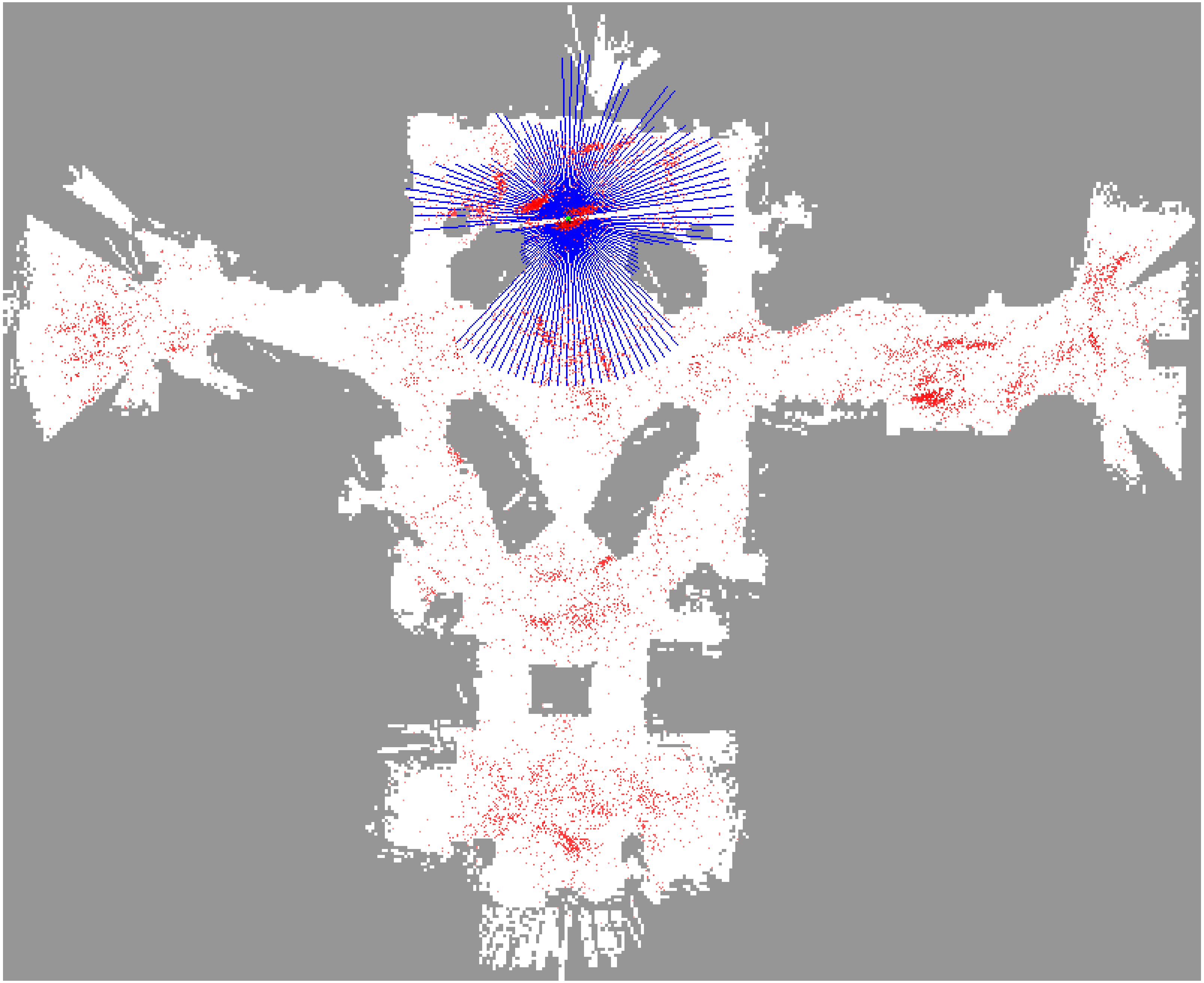
Sonar sensor

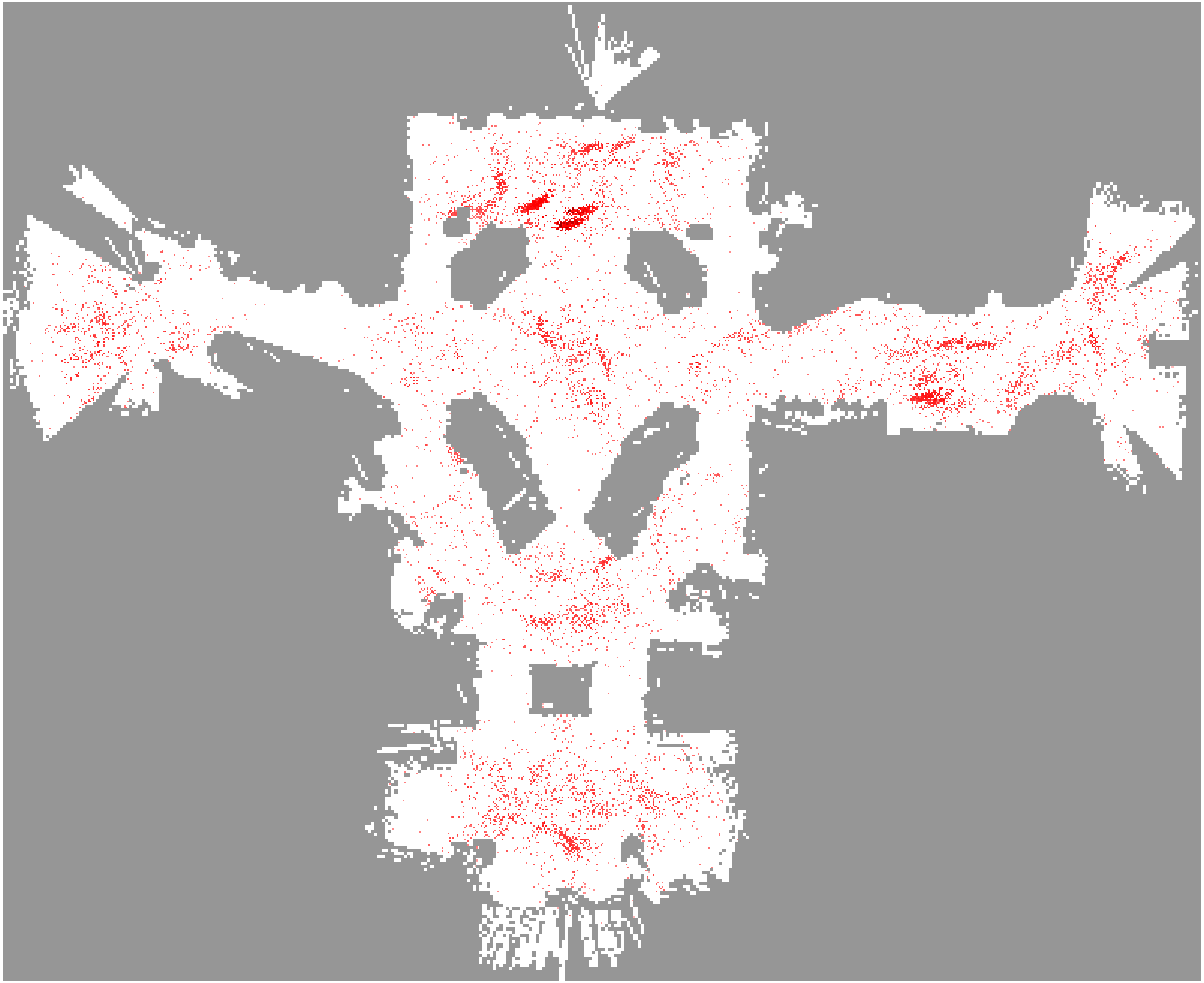


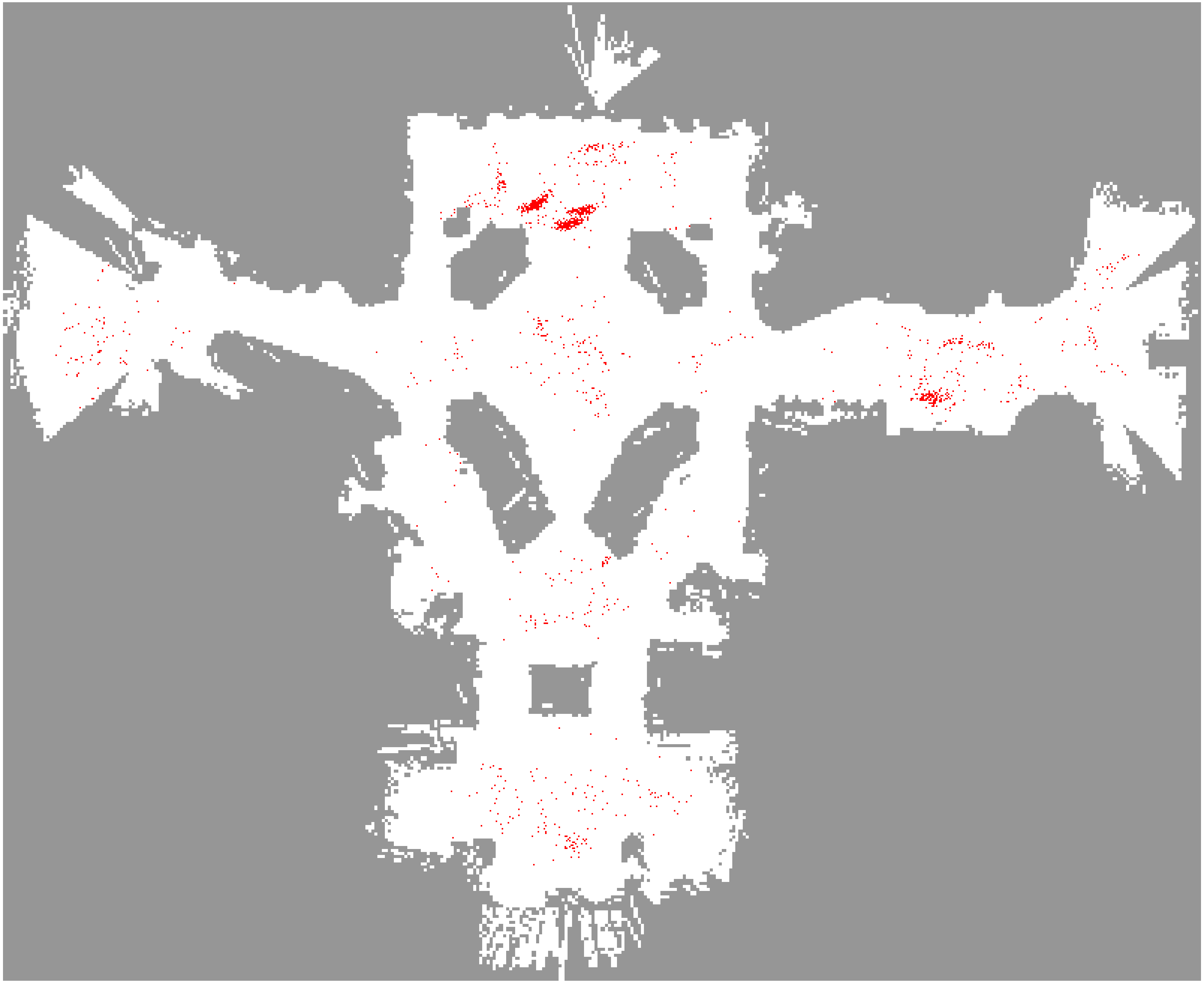


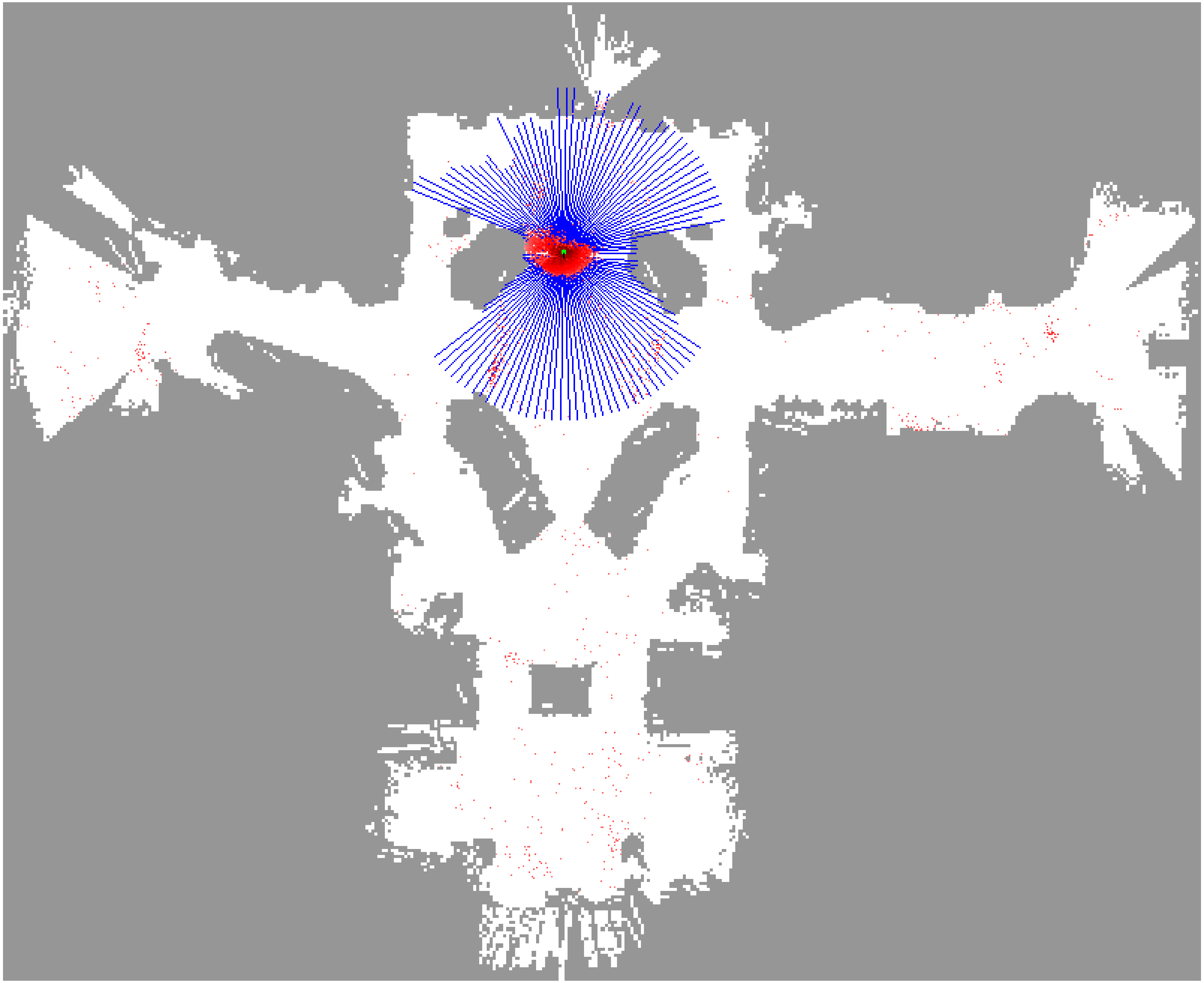


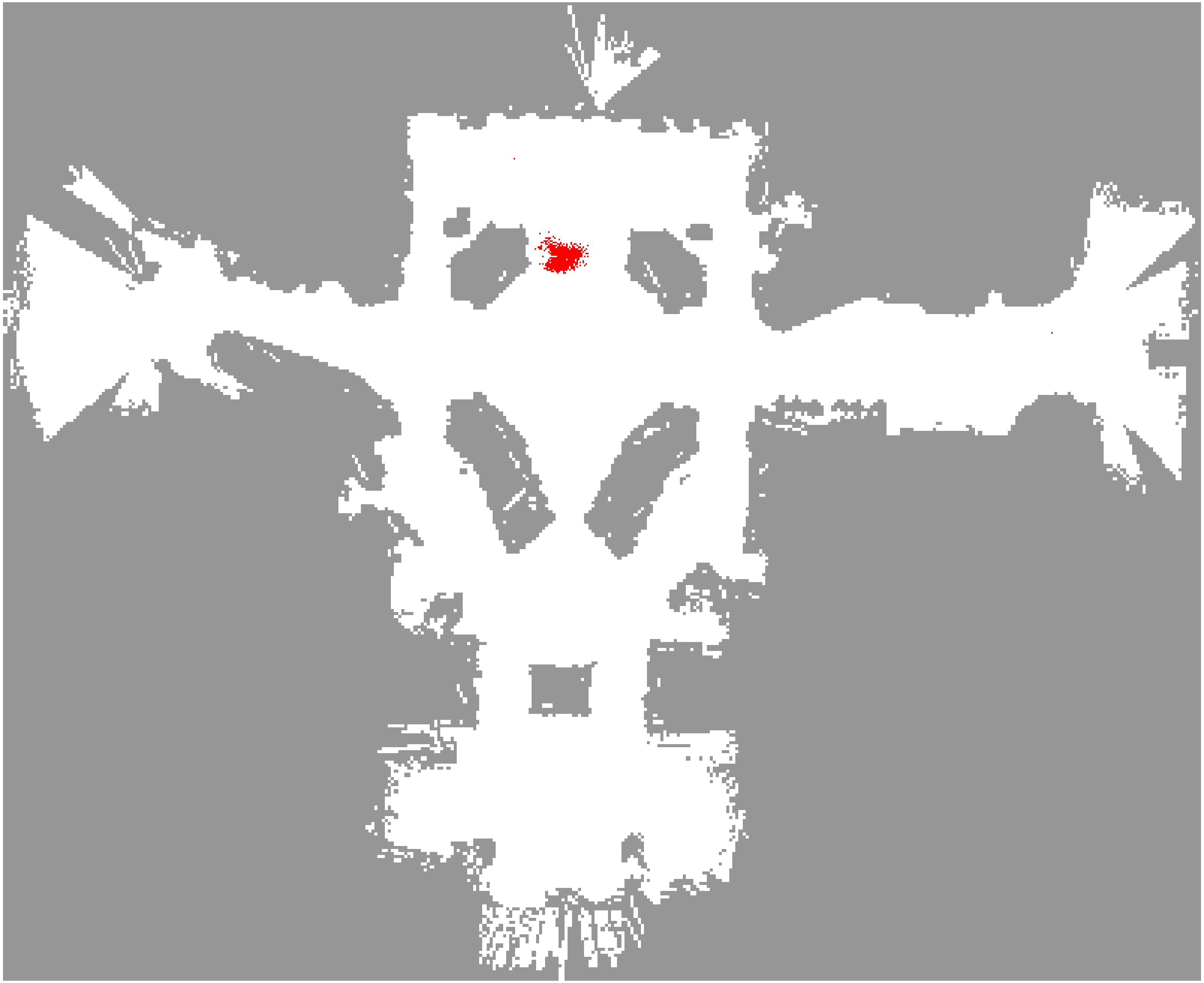


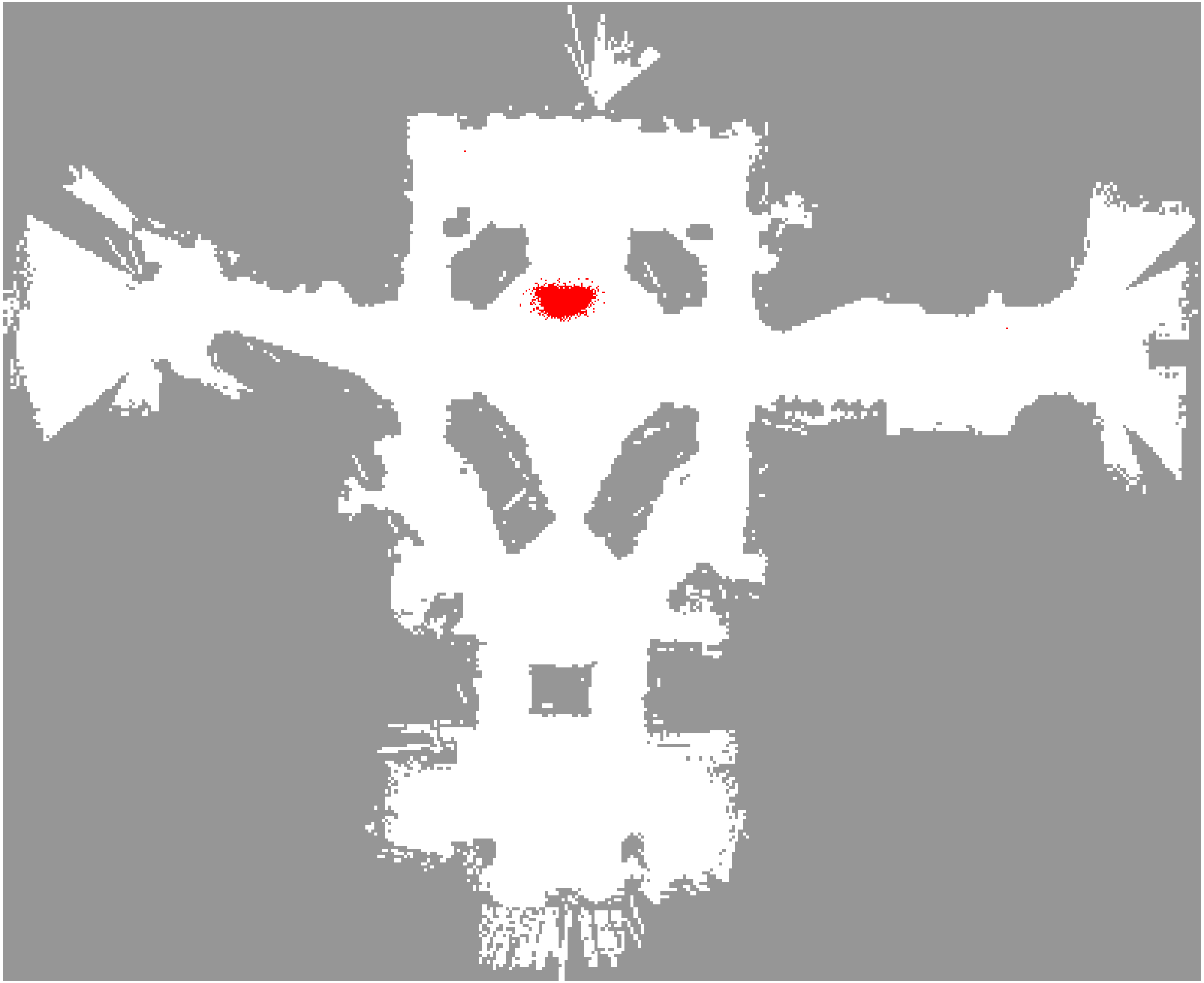


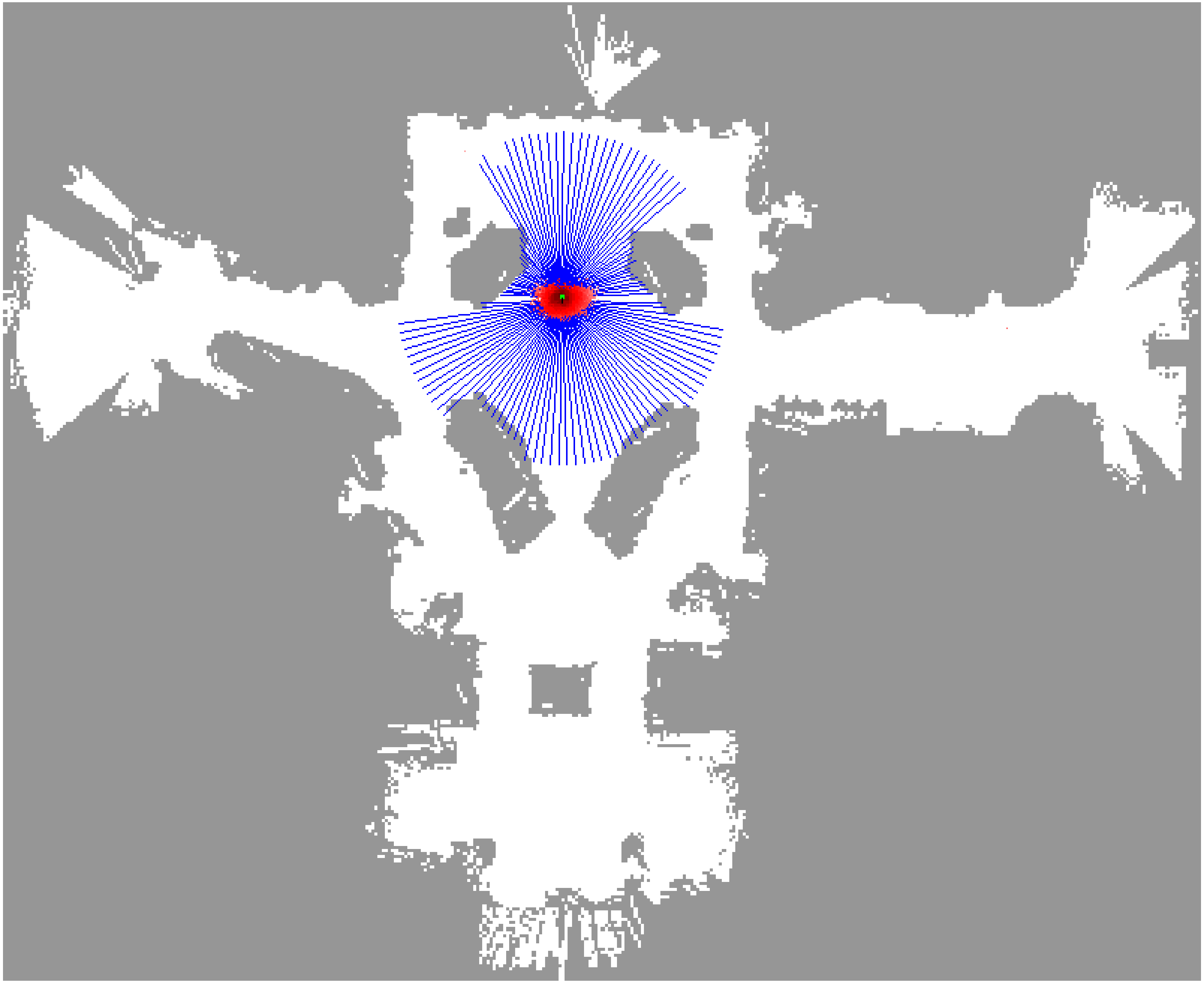


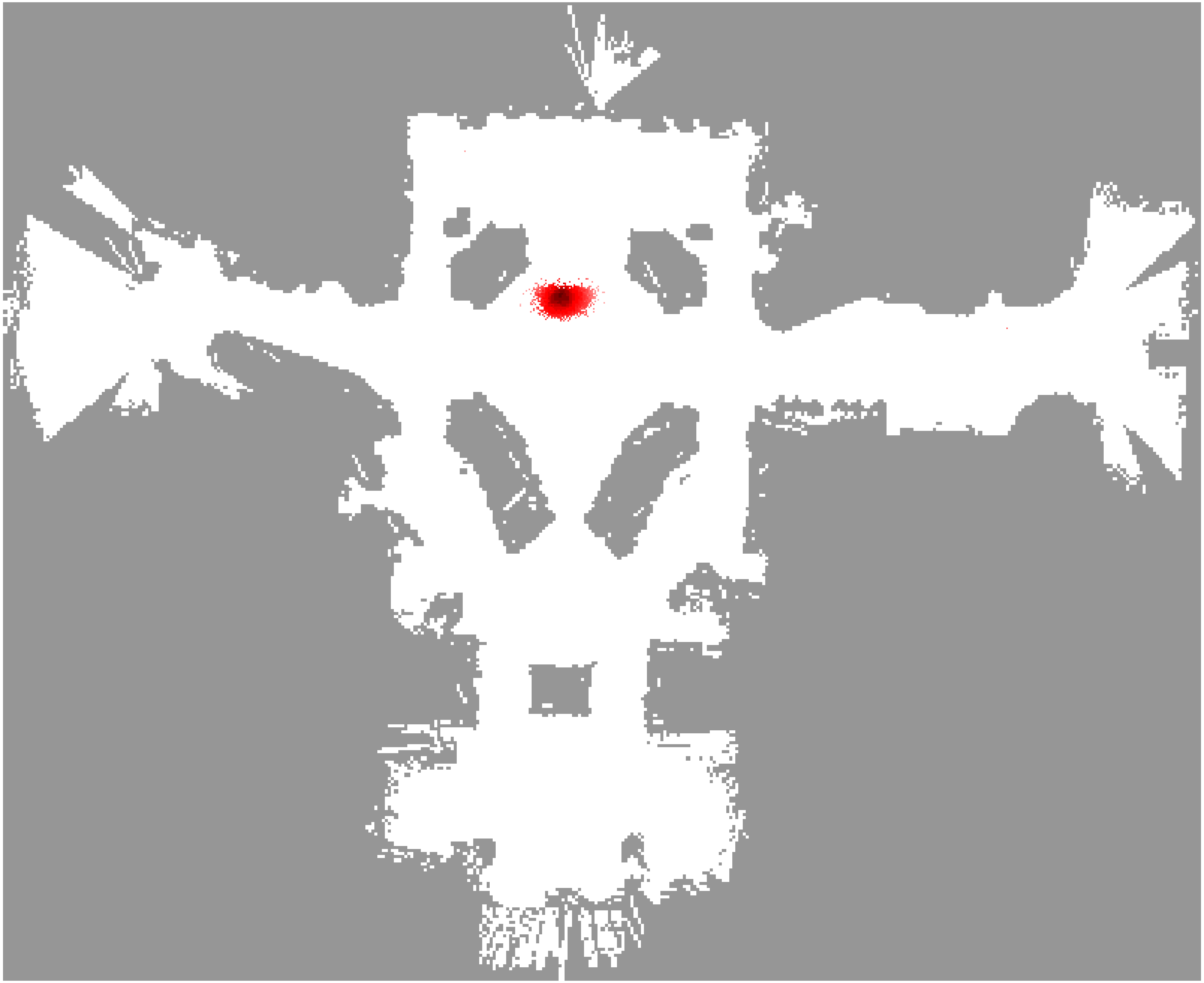


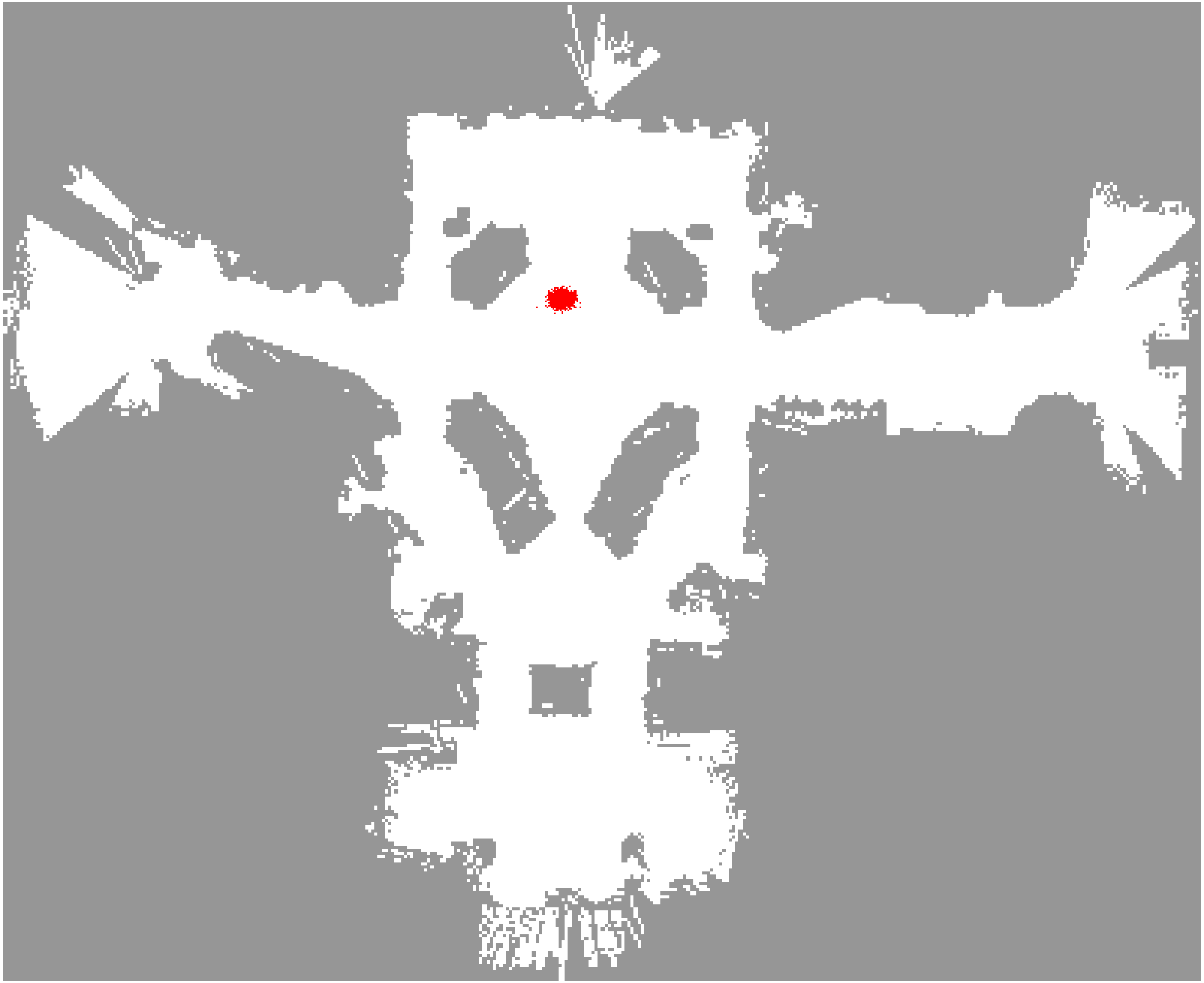


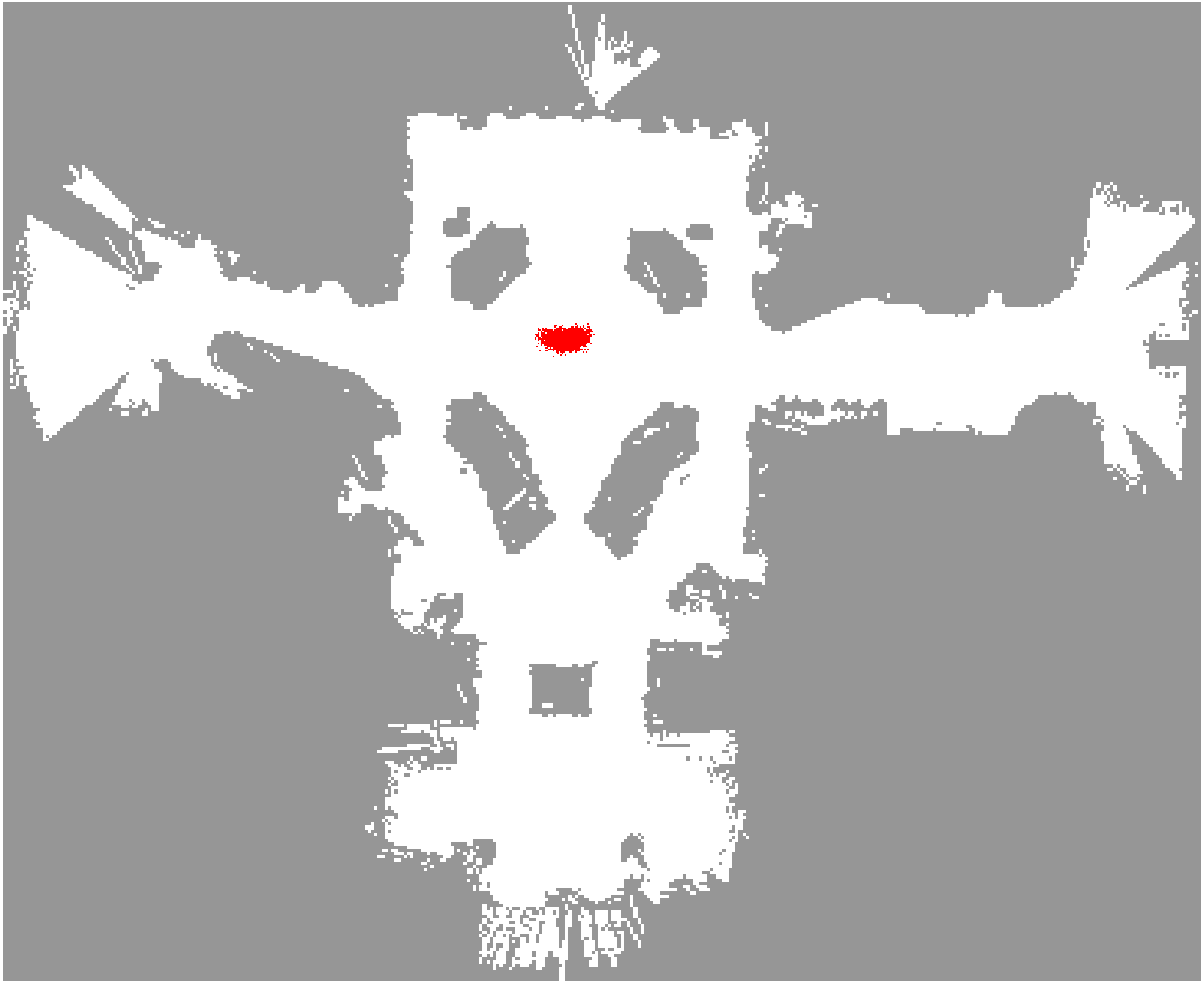


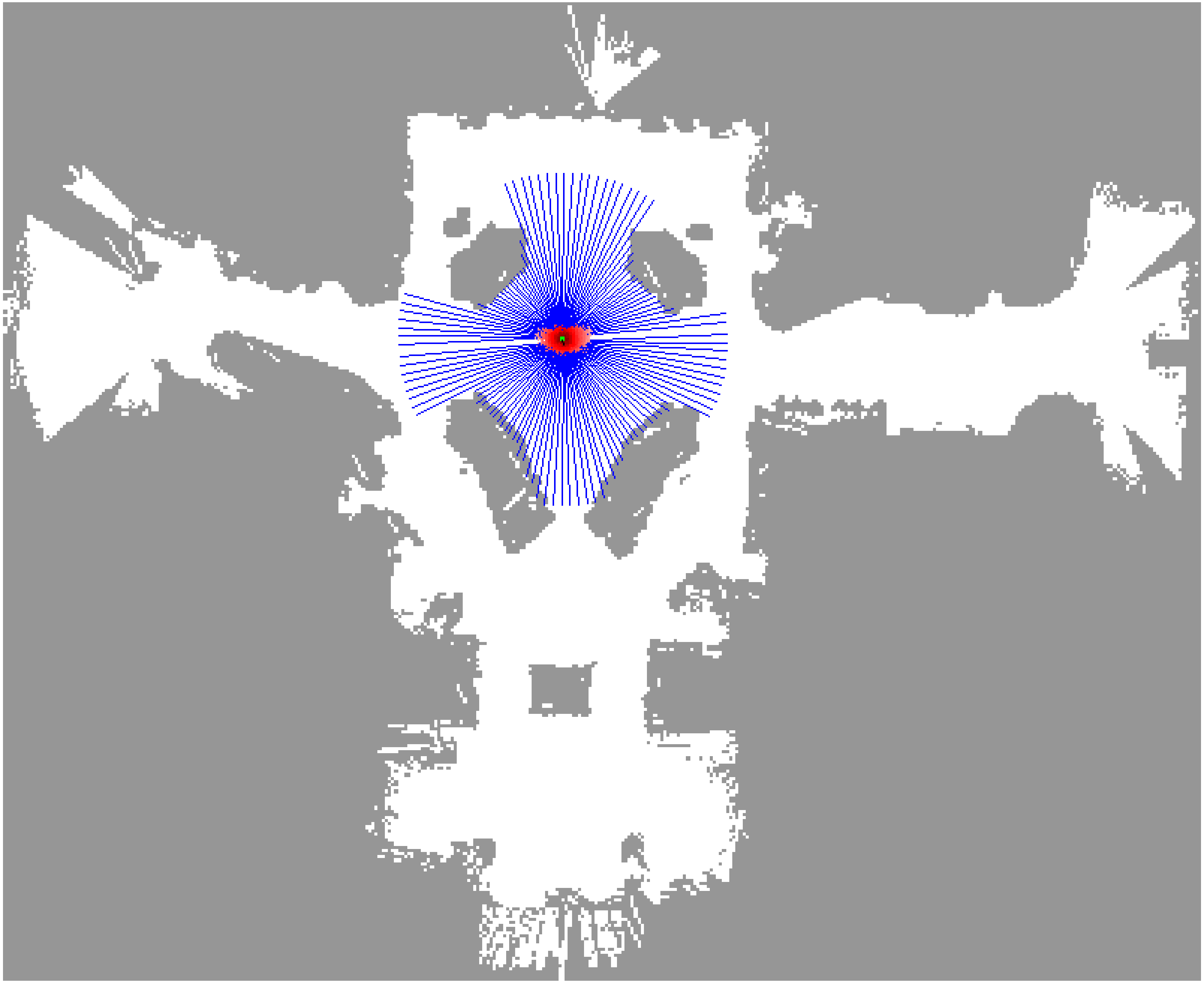


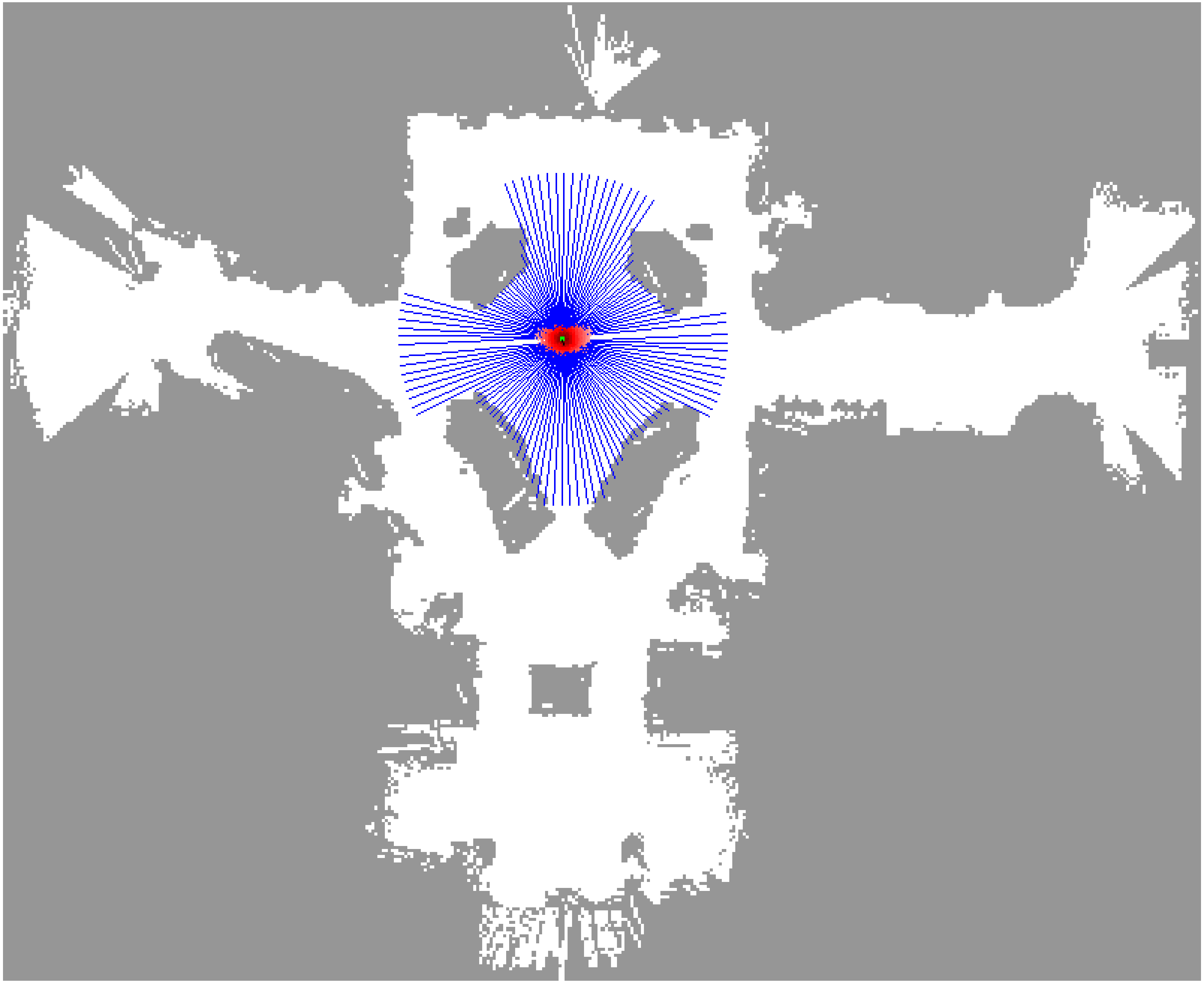




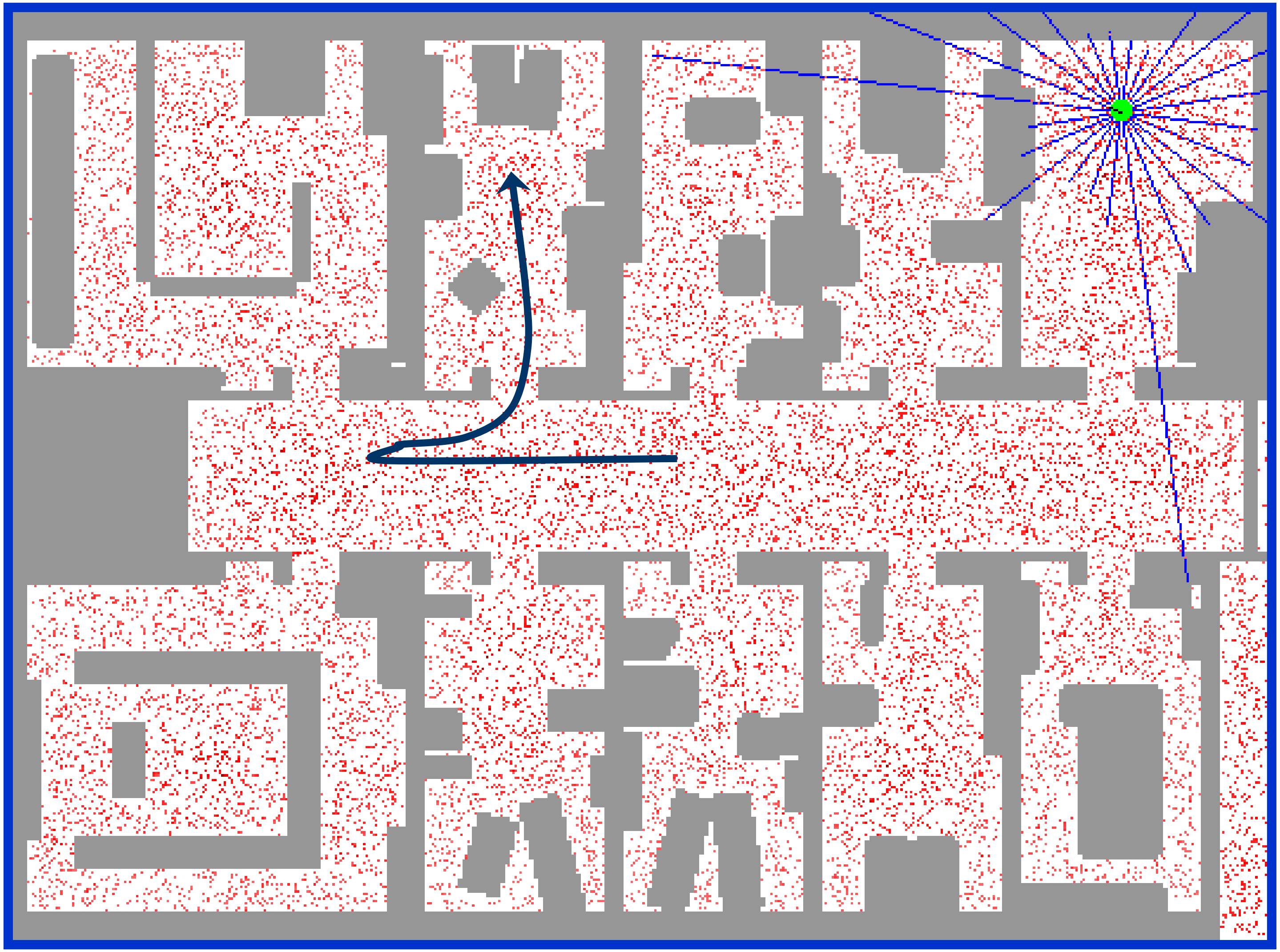




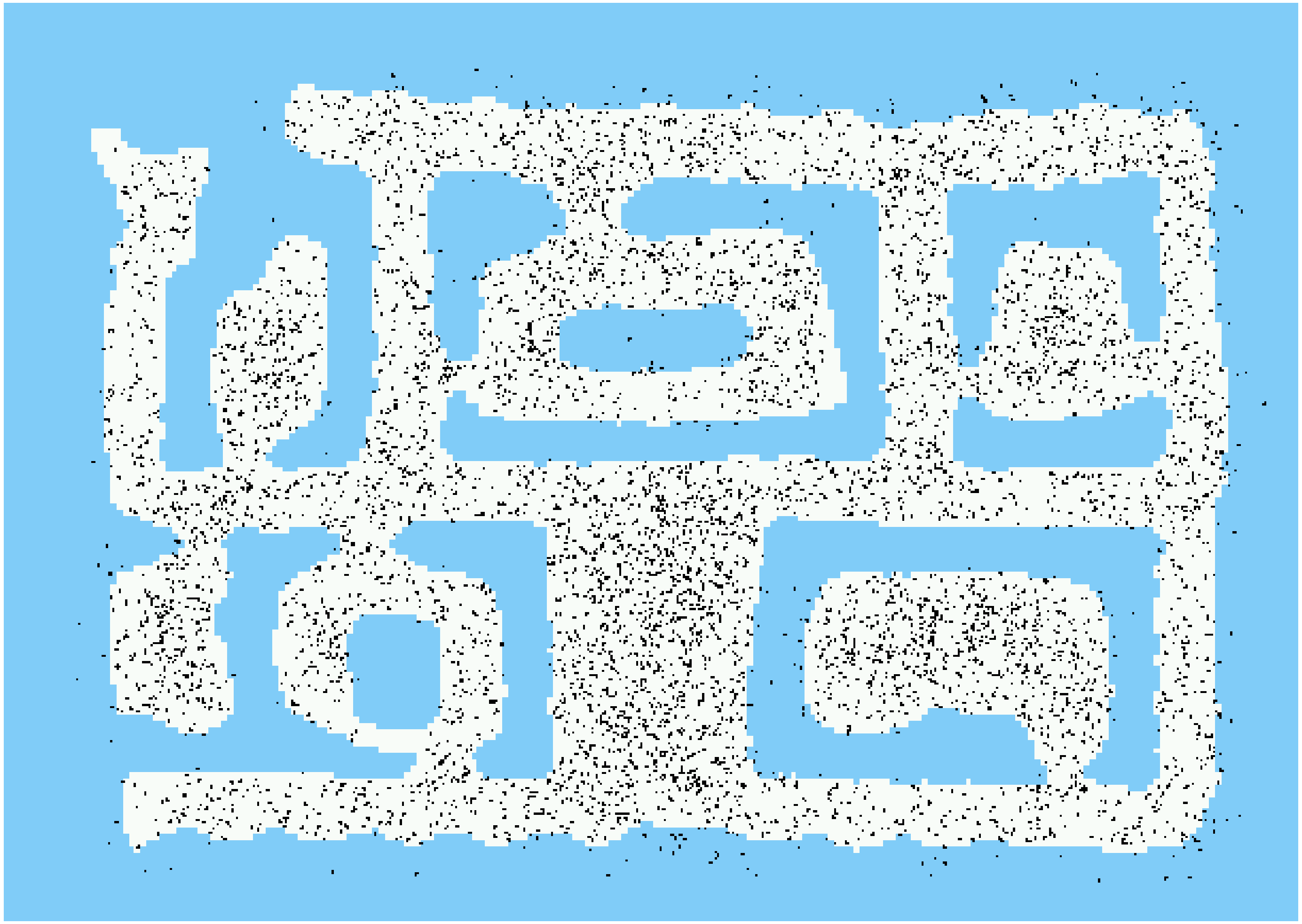




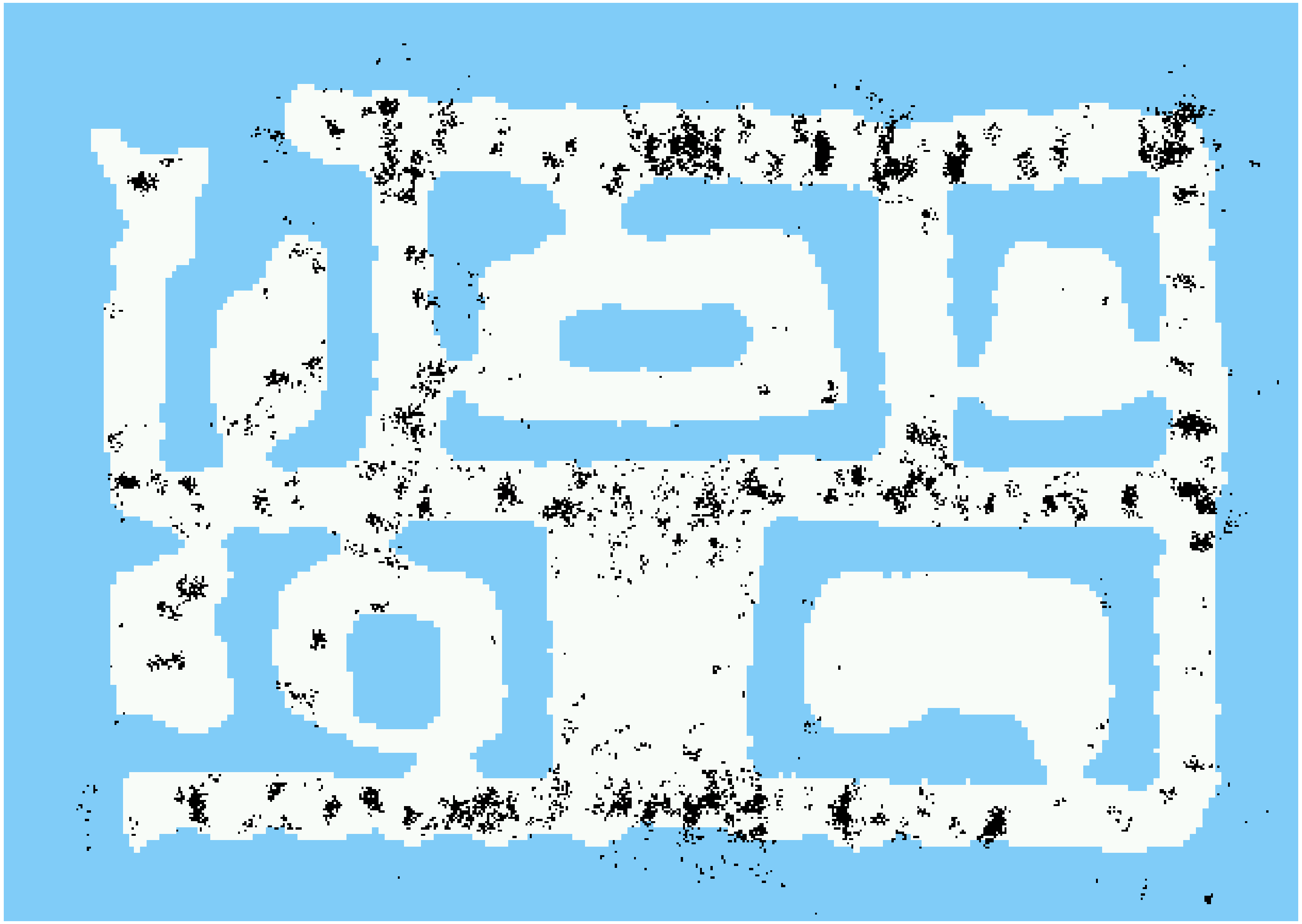
Localizzazione Sample based (sonar)



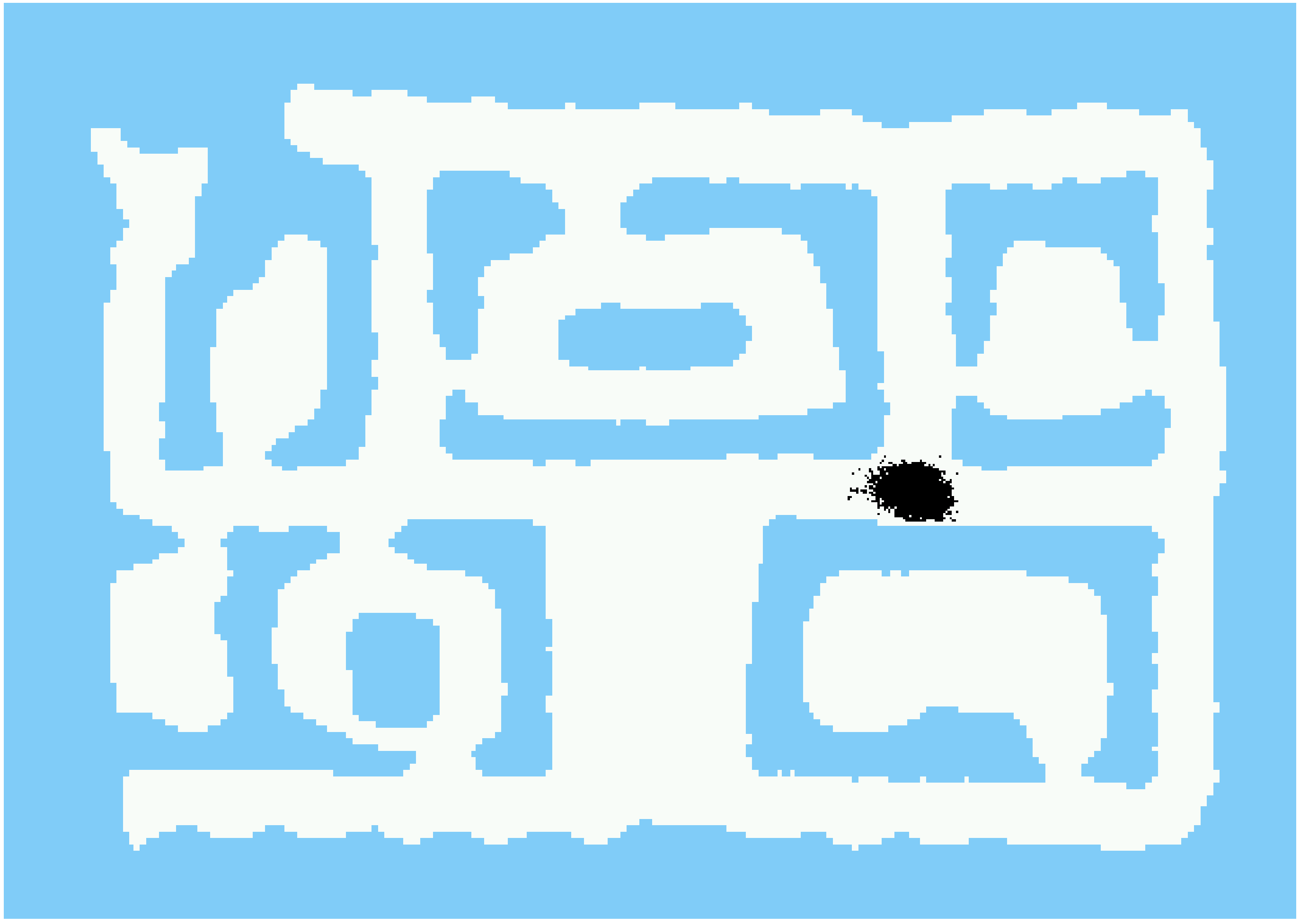
Initial Distribution



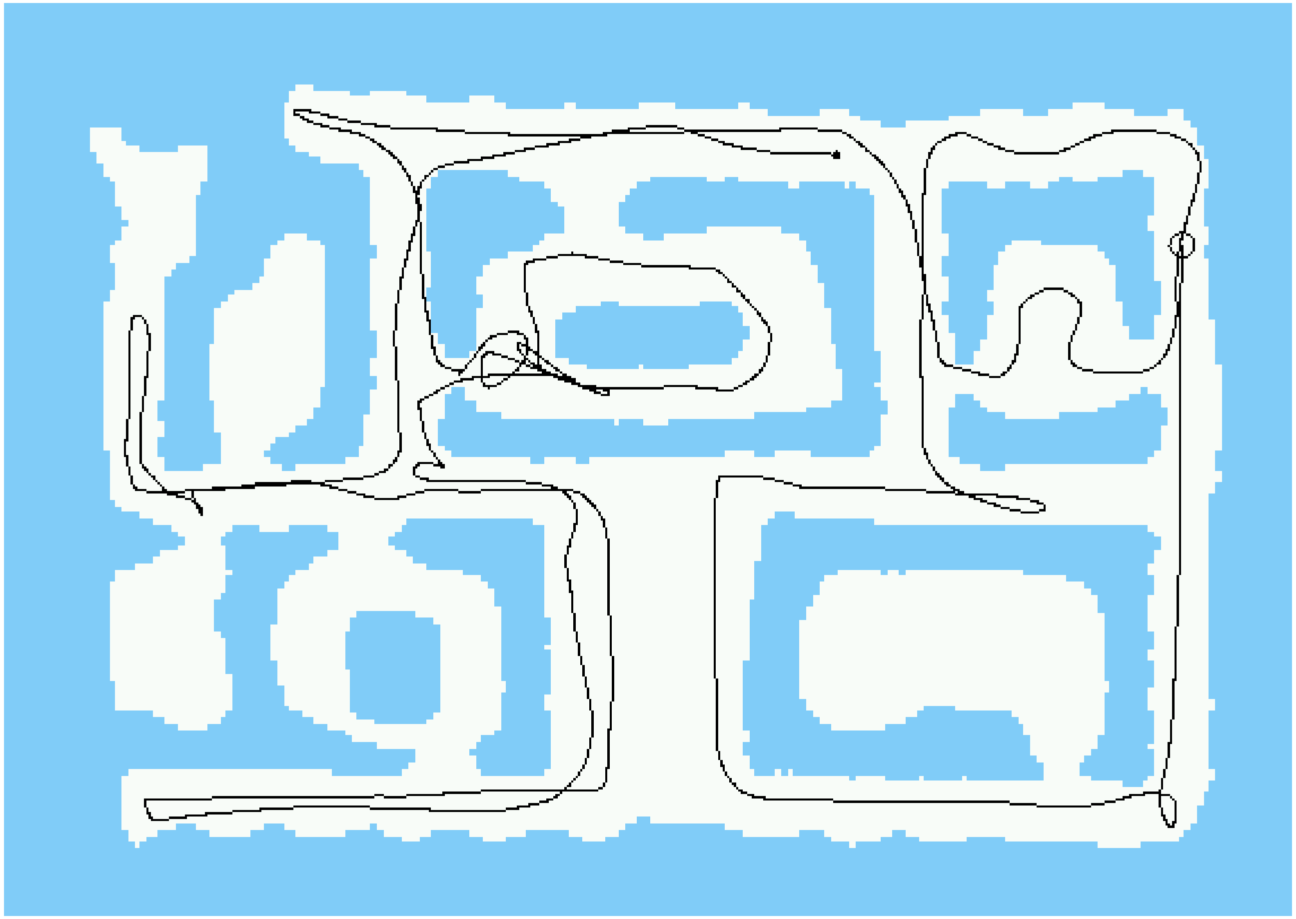
After Incorporating Ten Ultrasound Scans



After Incorporating 65 Ultrasound Scans



Percorso stimato



Limiti

- L'approaccio funziona per
  - tracciare la posa del robot mobile
  - Localizzazione globale.

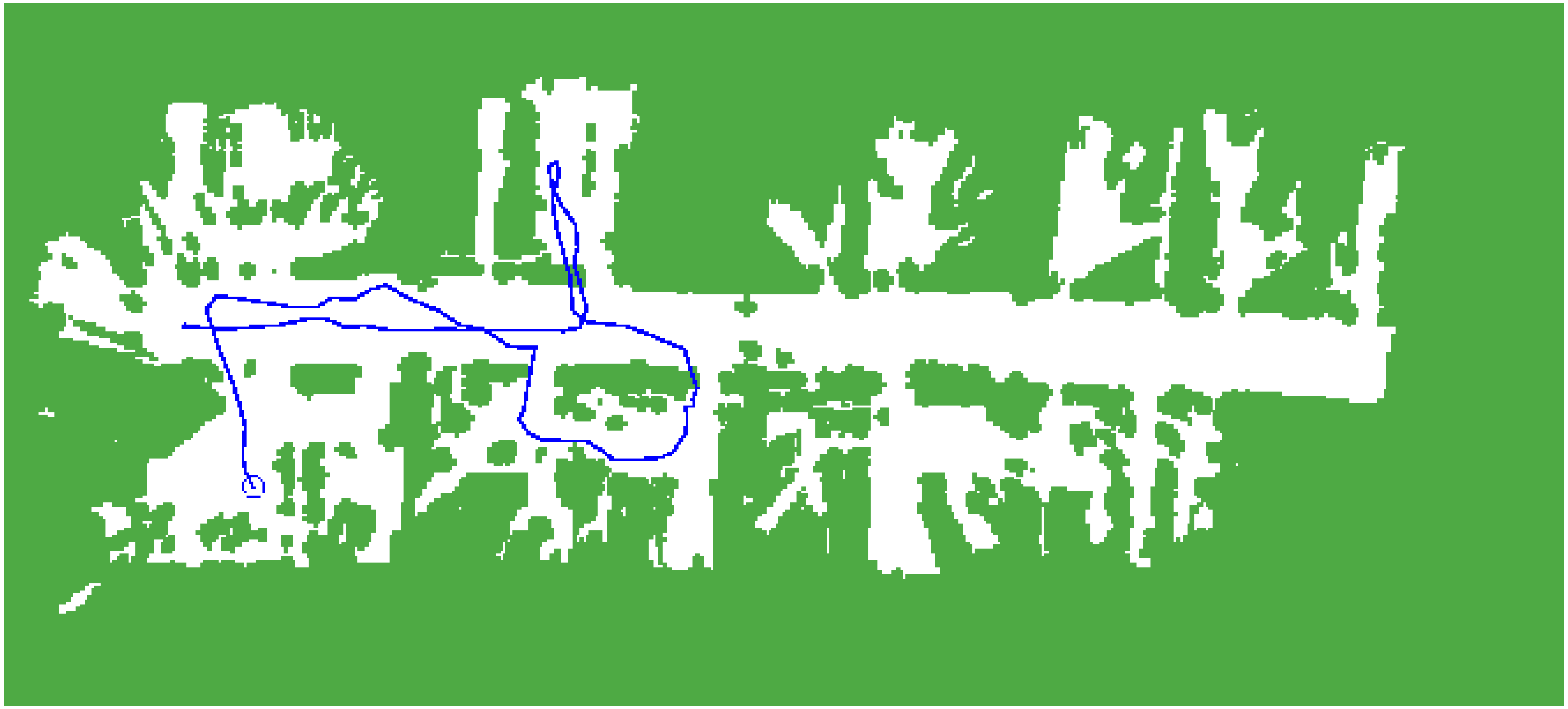
- Errori di localizzazione (i.e., kidnapped robot)?

Approcci

- Inserire campioni random (robot può essere teletrasportato in ogni punto).
- Inserire campioni random proporzionali al likelihood medio delle particelle (il robot è teletrasportato con più probabilità quando il likelihood delle osservazioni decade).

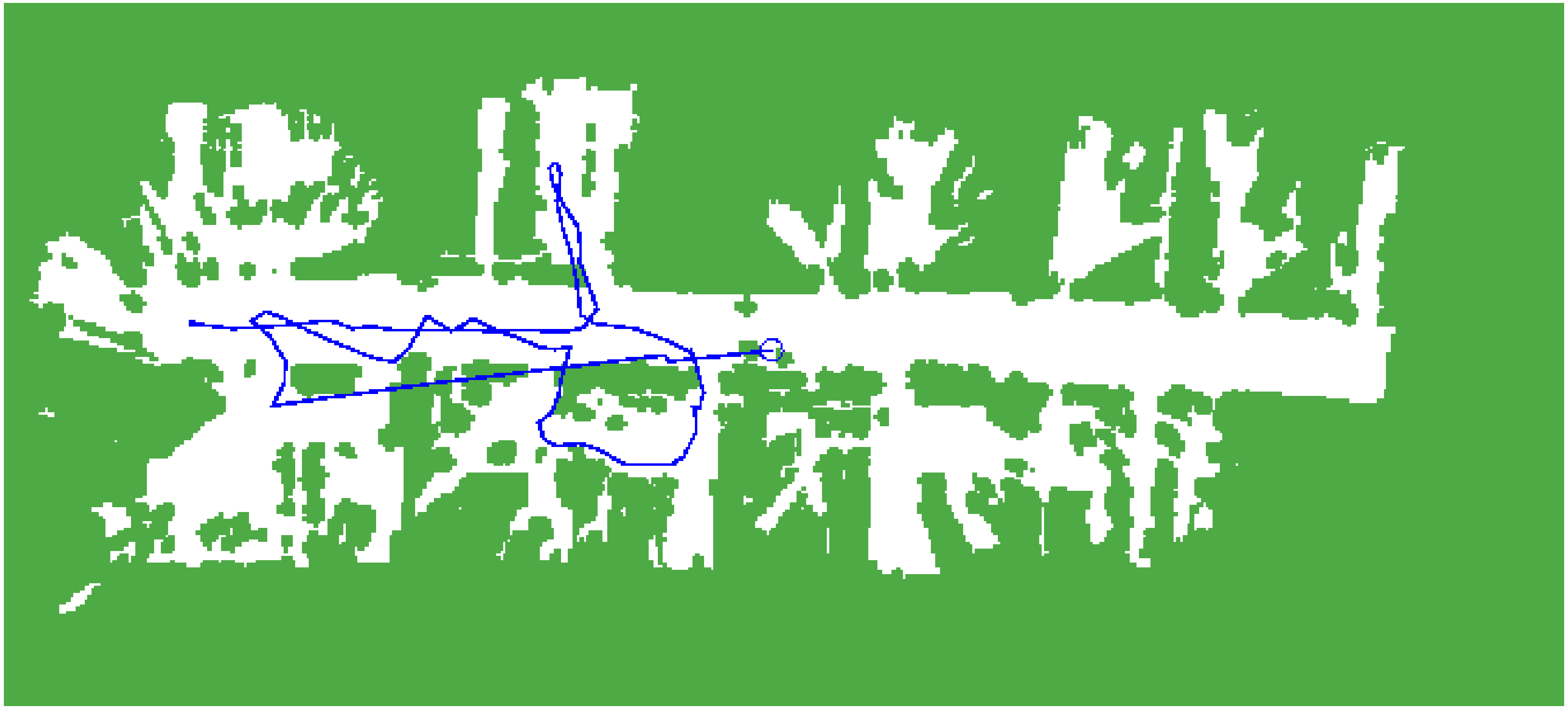
Traiettoria

Position tracking:

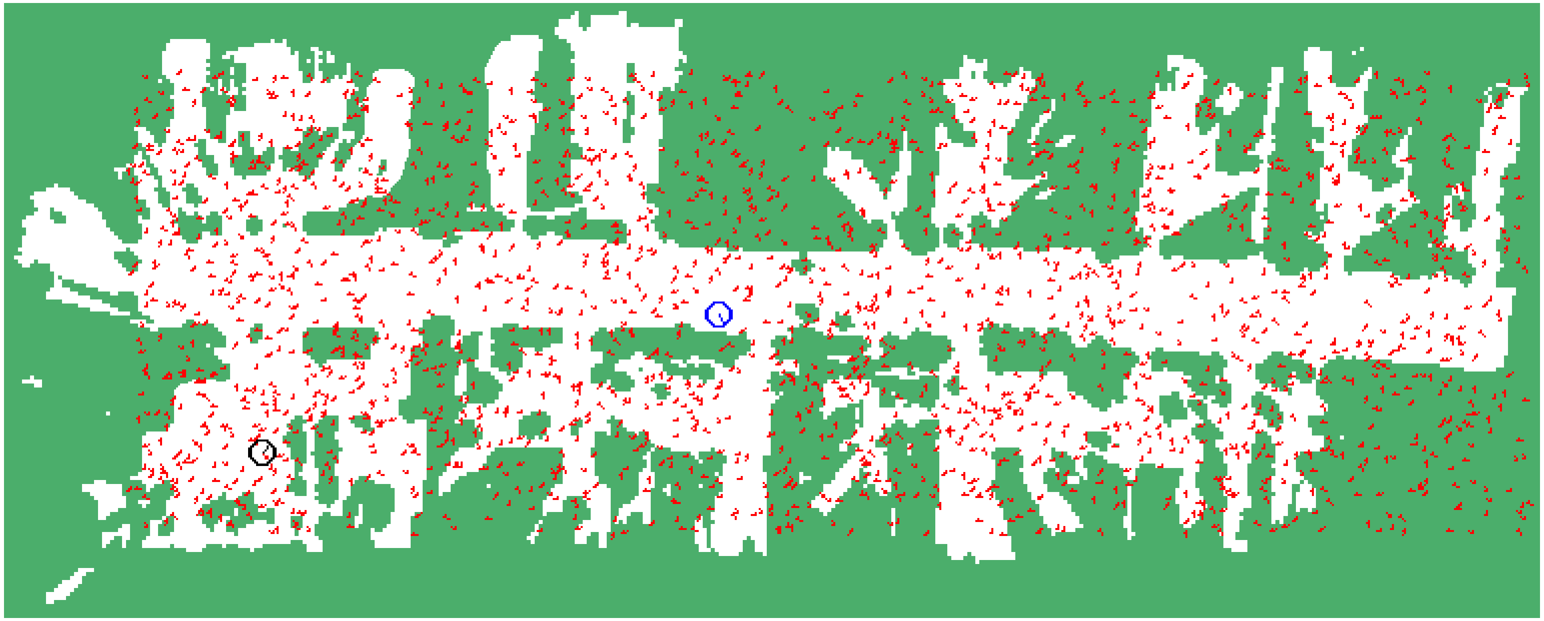


Resulting Trajectories

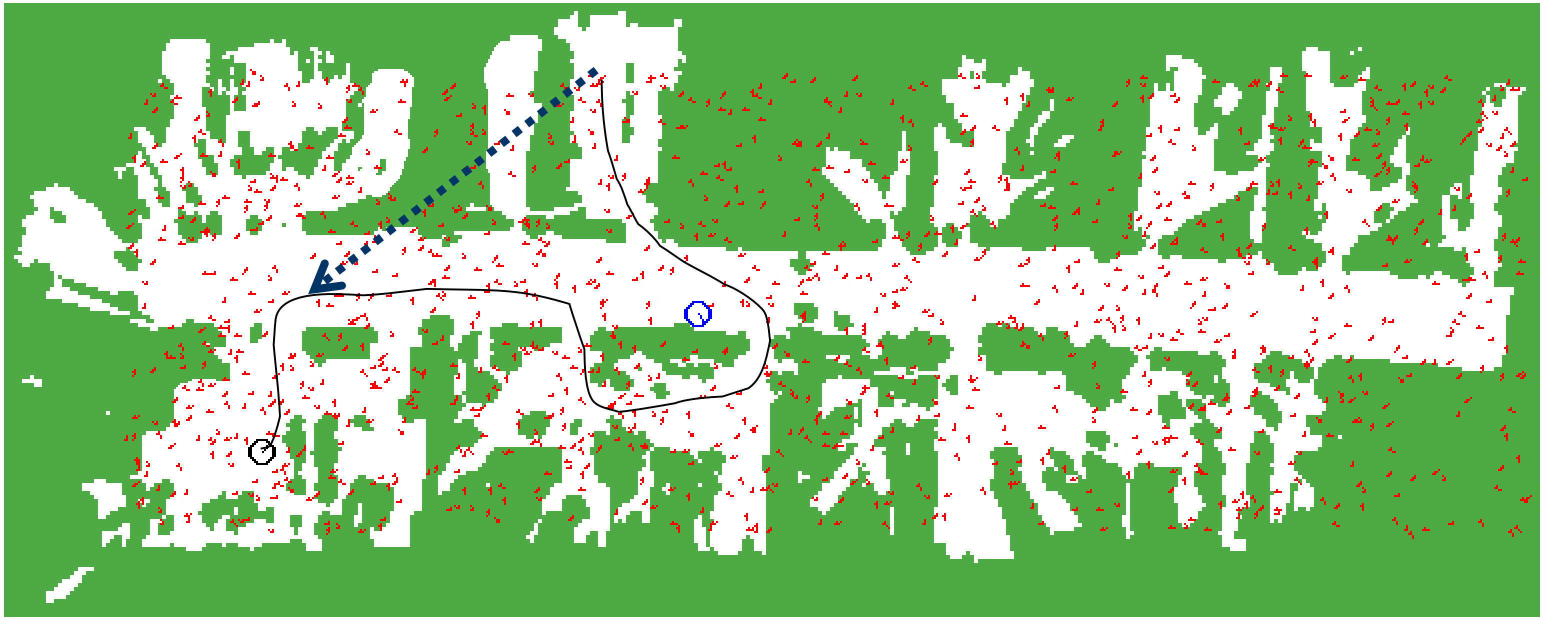
Global localization:



Global Localization



Kidnapping the Robot



Sommario

- Particle filter implementazione del recursive Bayesian filter
- Rapresentano il post con un insieme di campioni pesati.
- Per la localizzazione le particelle sono propagate secondo il motion model.
- Pesate secondo il likelihood delle osservazioni.
- Nel passo di re-sampling si estraggono nuove particelle proporzionali al likelihood dell'osservazione.