Robotica Probabilistica

Filtri Bayesiani

Filtri discreti e Particle filters

Filtri ad Istogrammi e Filtri Discreti

- Decompongono lo spazio in regioni e rappresentano il post cumulativo di ogni regione con un valore di probabilità

- Se applicati a spazio finito sono detti Filtri Discreti: X_t prende valori in un insieme finito (es. occupancy grid X_{ij} ha 2 valori)

- Filtro Discreto molto comune: forward pass di un HMM

Filtro Discreto: Algoritmo

1. Algorithm **Discrete_Bayes_filter**(*Bel(x),d*):
2. η=0
3. If *d* is a perceptual data item *z* then
4. For all *x* do
5. $Bel'(x) = P(z \mid x) Bel(x)$
6. $\eta = \eta + Bel'(x)$
7. For all *x* do
8. $Bel'(x) = \eta^{-1} Bel'(x)$
9. Else if *d* is an action data item *u* then
10. For all *x* do
11. $Bel'(x) = \sum_{x'} P(x \mid u, x')\, Bel(x')$
12. Return *Bel'(x)*

Filtro ad Istogramma

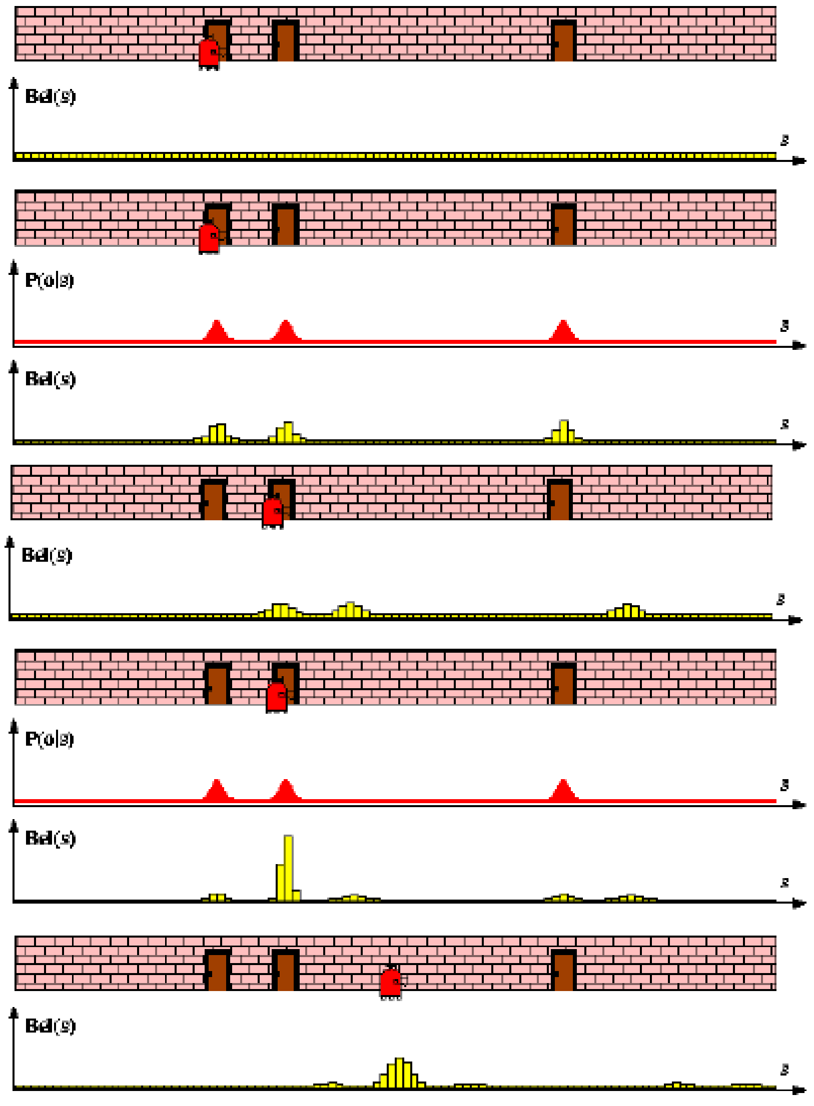
- Se spazio continuo Filtri ad Istogramma (**X_t** prende valori in spazio continuo)

- Spazio diviso in regioni finite e convesse (bins) che partizionano lo stato: $\mathbf{X_t = x_{1,t} \vee \ldots \vee x_{n,t}}$

- Per ogni regione è associata la probabilità $\mathbf{p_{k,t}}$ quindi $\mathbf{p(x_t) = p_{k,t} / |x_{k,t}|}$

Filtro ad Istogramma

Costante a tratti

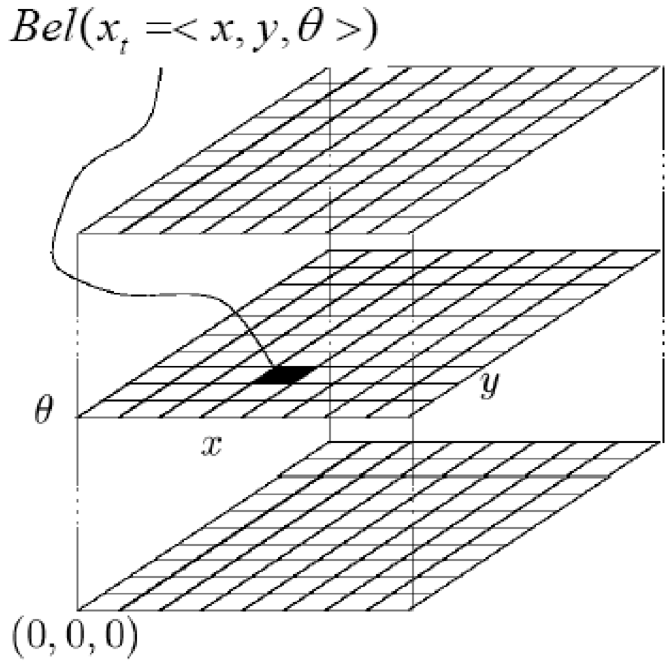


Implementazione

Statica: partizione statica dello spazio

Dinamica: partizione dello spazio dipendente dal contesto

Rappresentazione costante: occupancy grid



Rappresentazione Statica

Utili quando lo stato è binario

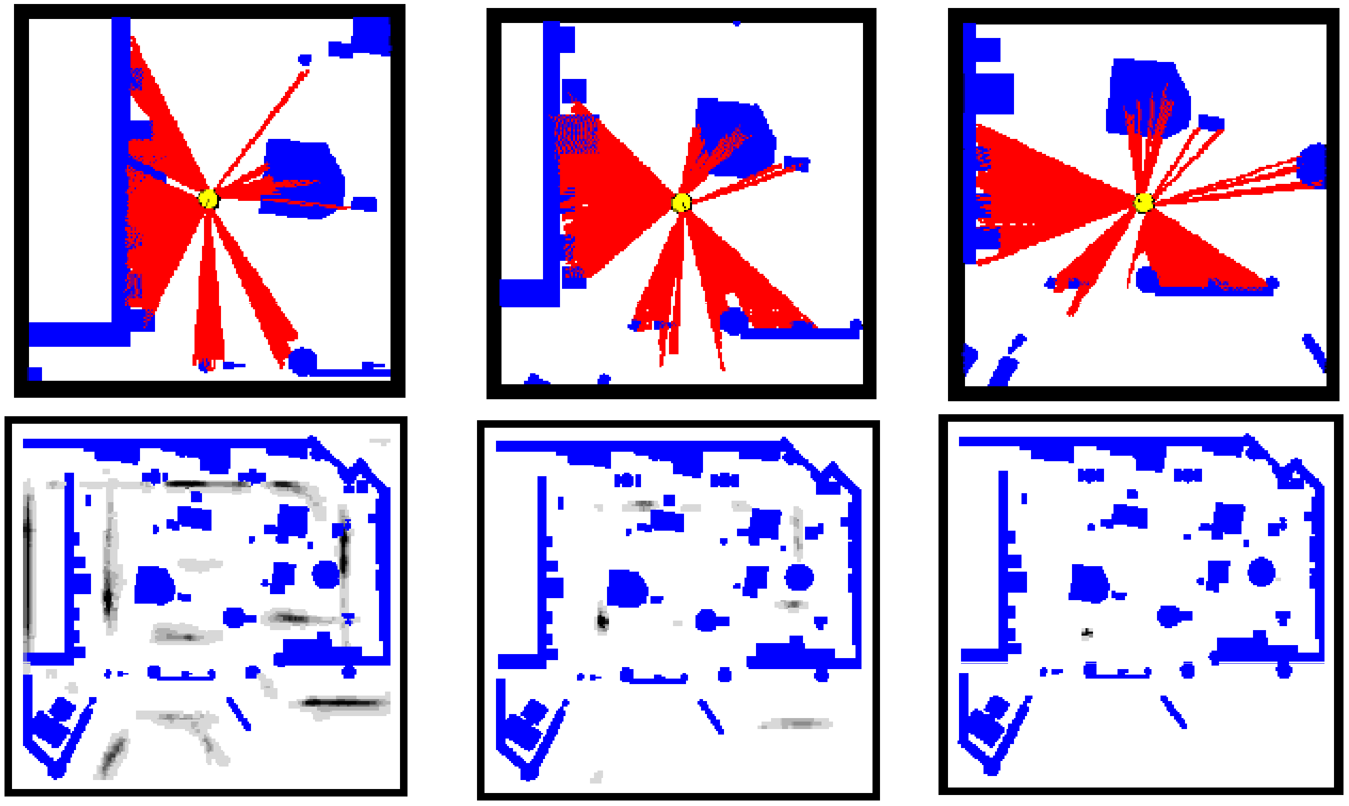
Problemi:

-Per update e normalizzazione occorre lo scan di tutta la griglia

- Se le credenze sono concentrate si vorrebbe evitare

- Si può fare l'update della sottogriglia "attiva", ma non gestisce la delocalizzazione

Localizzazione Grid-based

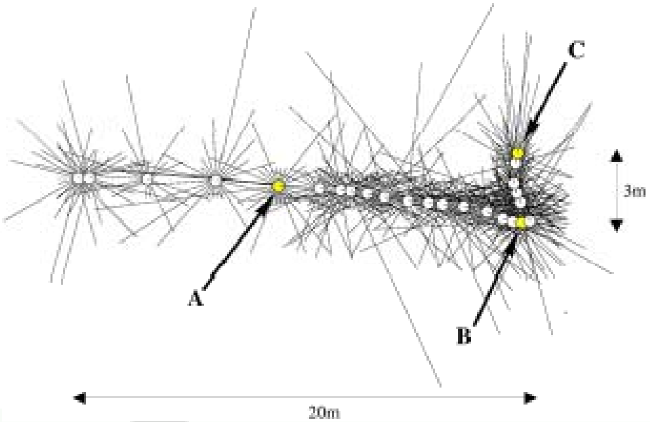


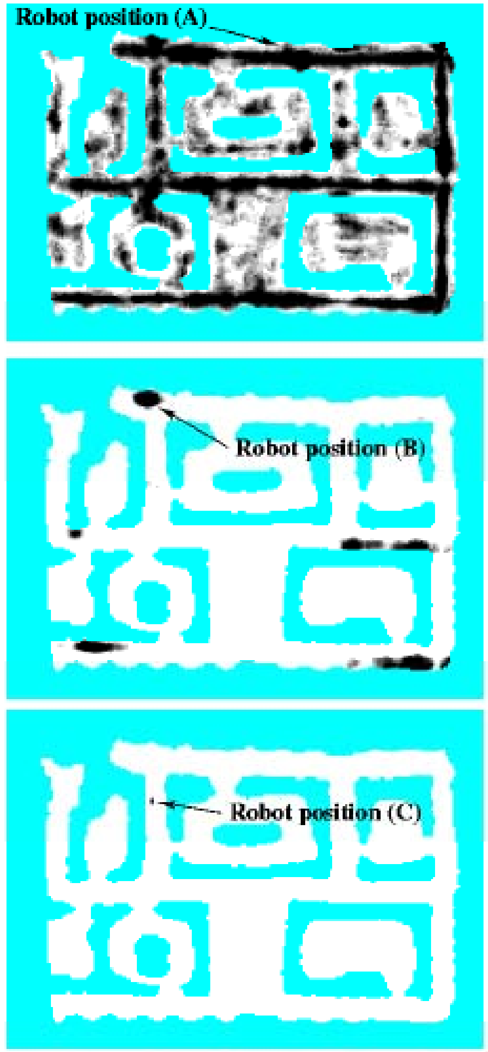
Gridmap risoluzione 15 cm, 15 g, 2LRF, beam model

Posizione dopo 3 scan

Localizzazione Grid-based

Sonars and Occupancy Grid Map





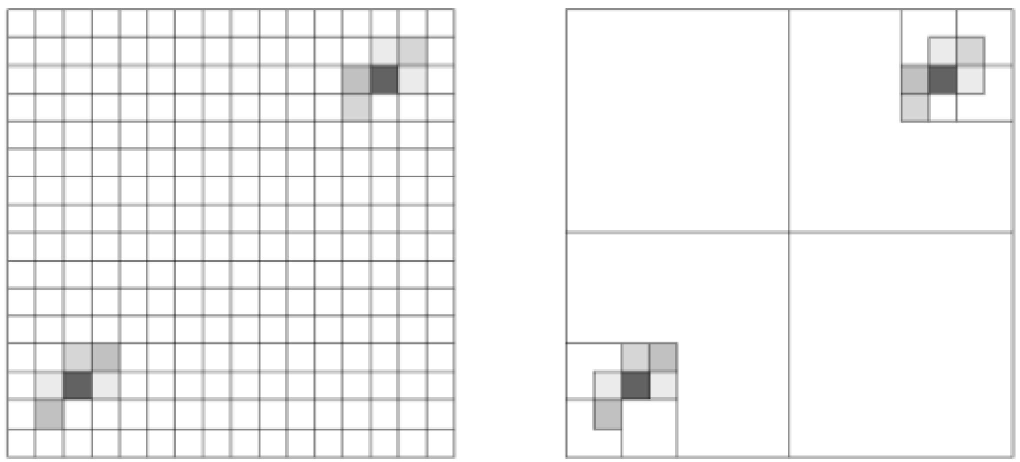
Decomposizione Dinamica

Density Tree: decomposizione ricorsiva adatta alla risoluzione del post

Meno è probabile una regione, minore è la risoluzione

Più precisa ed efficiente (ordini di grandezza)

Quadtree
o Octree



Particle Filters

Filtri ad Istogrammi:

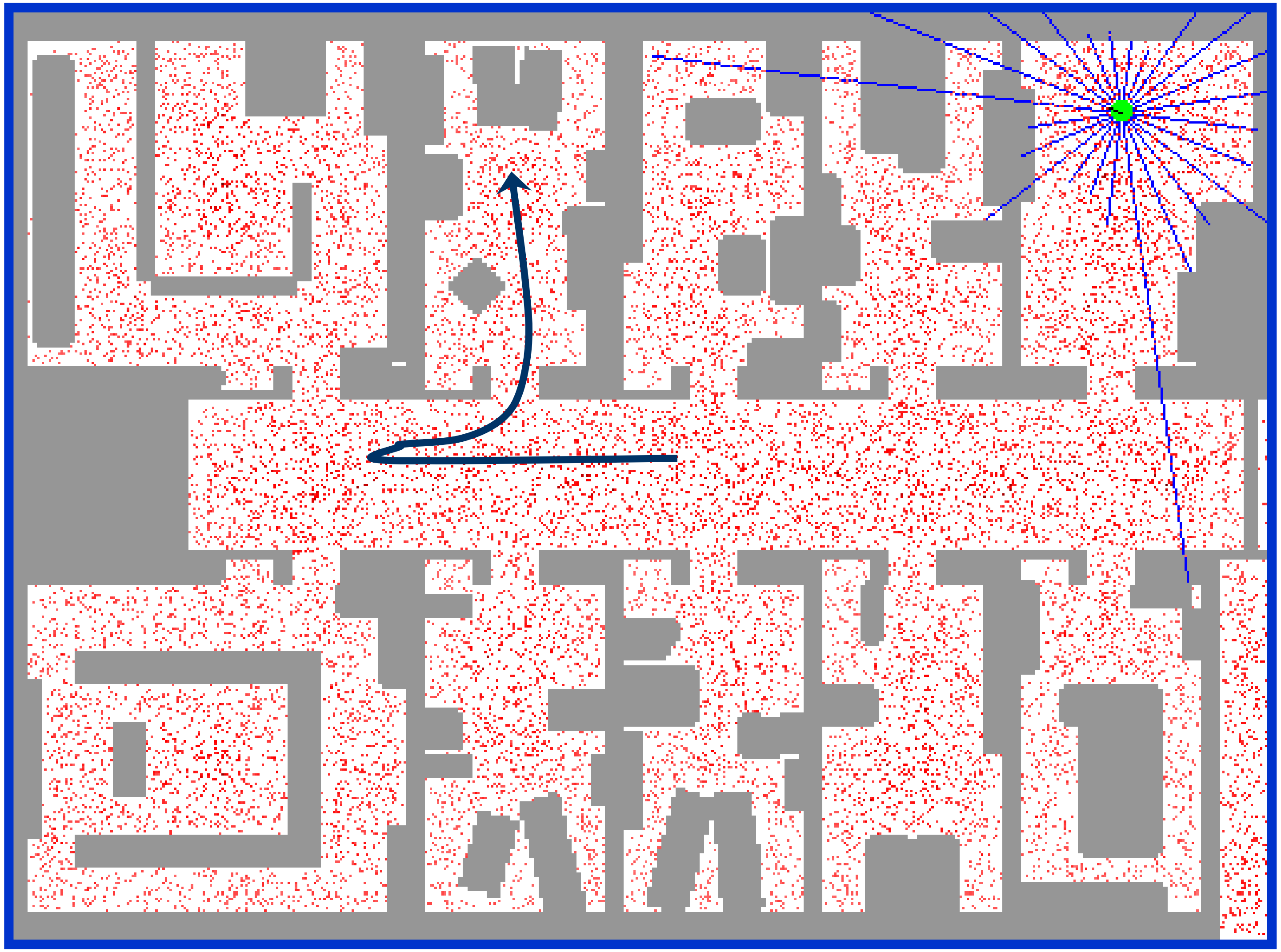
- Discretizzazione dello spazio (statica o dinamica)
- Problemi di risoluzione ed efficienza

Alternativa più efficiente: **Particle Filters**

Approssimano il post con un numero finito di parametri, ma tecnica diversa:

- Insiemi di ipotesi (particles) rappresentano il post
- Sopravvivono le migliori

Localizzazione basata su Campioni (sonar)



Particle Filters

- Rappresenta i belief con campioni random
- Stima di non-Gaussiane, processi nonlineari

- Monte Carlo filter, Survival of the fittest, Condensation, Bootstrap filter, Particle filter
- Adattivi: numero di campioni dipendenti dalle risorse e dalla complessità del task.

- Filtering: [Rubin, 88], [Gordon et al., 93], [Kitagawa 96]
- Computer vision: [Isard and Blake 96, 98]
- Dynamic Bayesian Networks: [Kanazawa et al., 95]d

Particle Filter

1. Lo stato al tempo t è rappresentato da un insieme di campioni (particles):

$X_t = x_{t,1}, x_{t,2}, \ldots , x_{tM}$

2. Ogni $x_{t,i}$ è una istanza dello stato al tempo t, M e' il numero delle particles

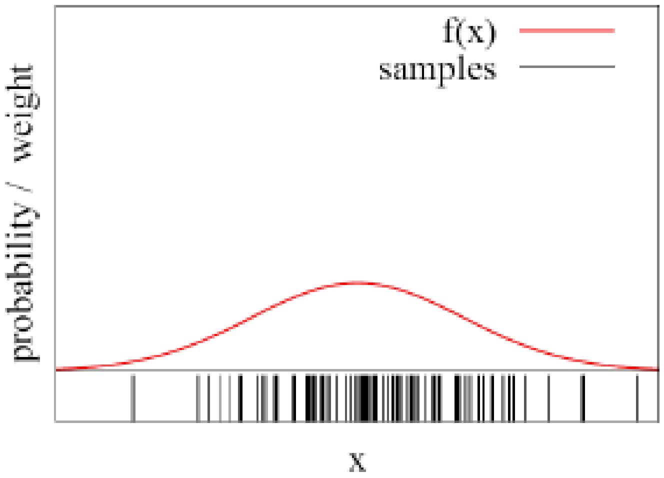
3. Ogni particle e' una ipotesi sullo stato

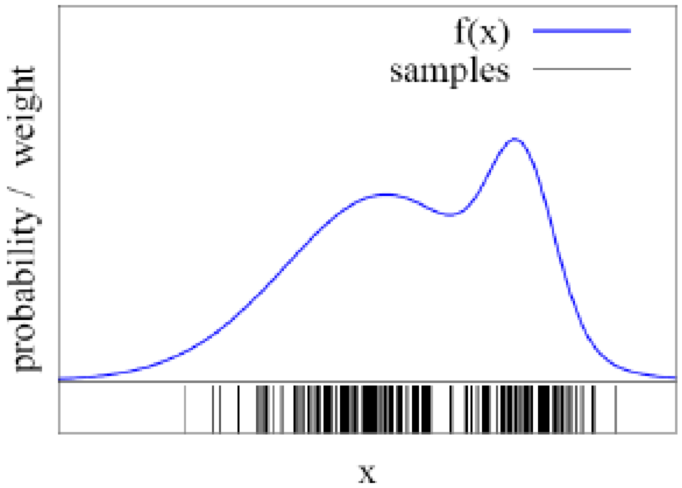
4. Prob di "$x_{t,m}$ appartenente ad X_t " approssima $bel(x_{t,m})$: $x_{t,m} \sim p(x_t \mid z_{1:t}, u_{1:t})$

Particle Filter

Gli insiemi di particles X_t approssimano una funzione di distribuzione

Prob di “$x_{t,m}$ appartenente ad X_t “ approssima $bel(x_{t,m})$: $x_{t,m} \sim p(x_t \mid z_{1:t}, u_{1:t})$





Più particle cadono in un intervallo più è probabile l’intervallo

Campionamento per Rifiuto (rejection sampling)

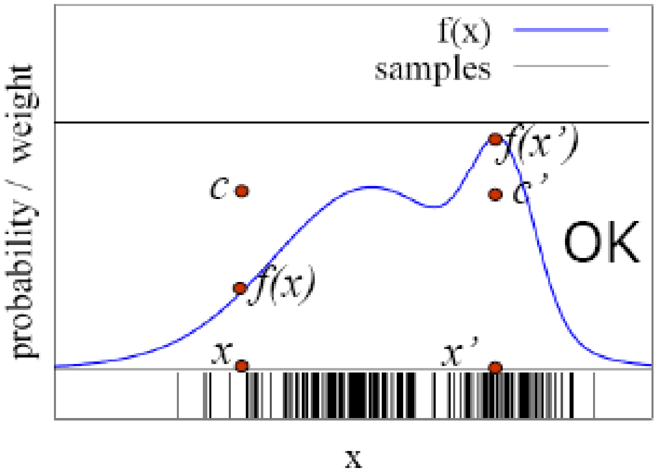
Si assume f(x) < 1 per ogni x

Campionamento da distribuzione uniforme

Si campiona c da [0,1]

Se f(x) > c allora si mantiene il campione

altrimenti si scarta

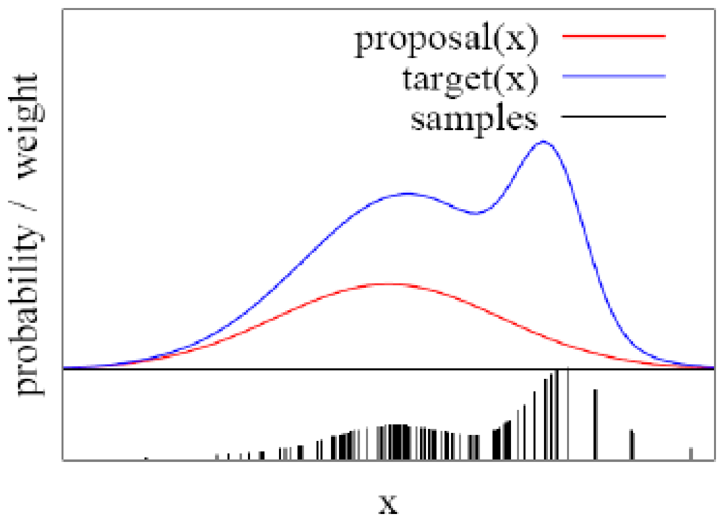


Importance Sampling

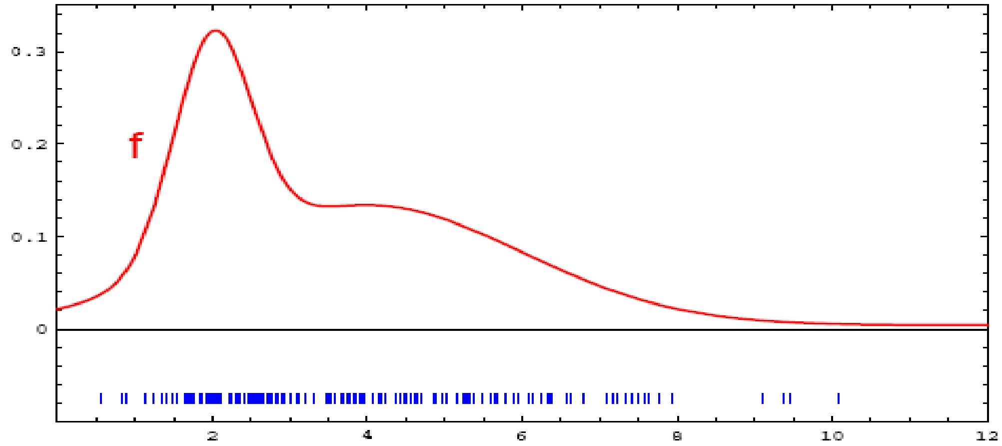
Si vuole fare sampling dalla target(x) ma si può fare da proposal(x)

Un sample da proposal(x) viene pesato con:
w = target(x)/proposal(x)

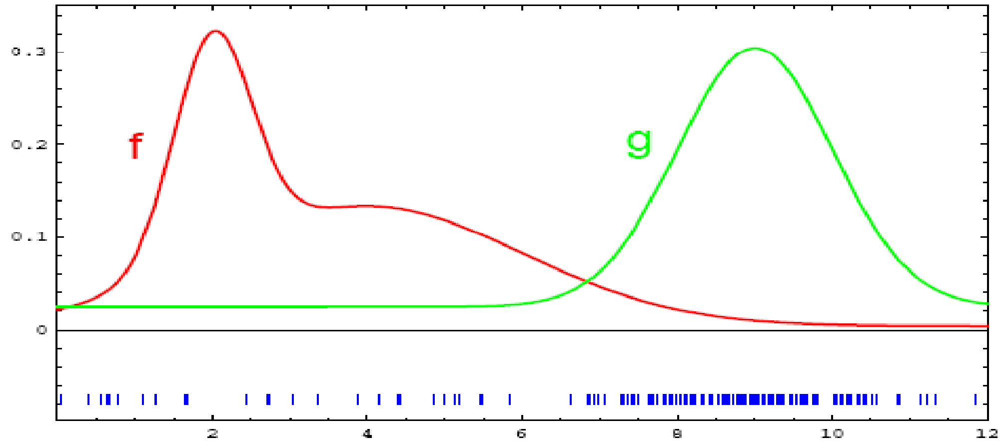
Condizione:
target(x) -> proposal(x)



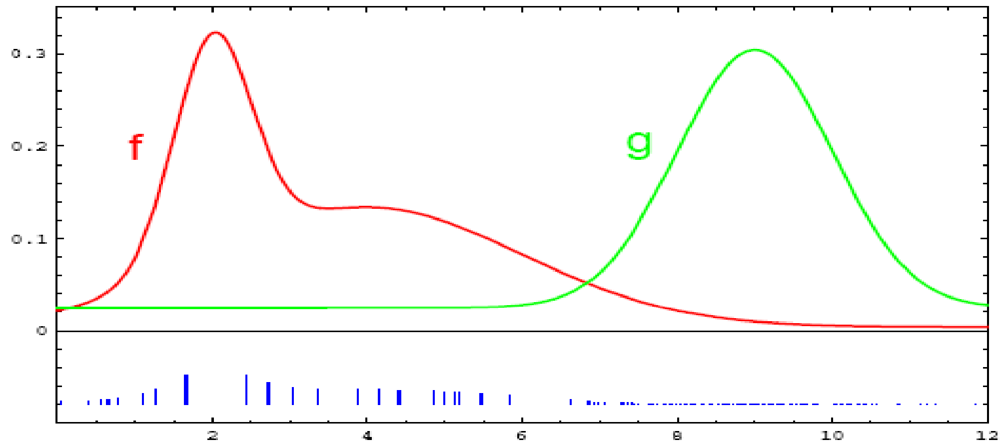
Campioni da f(x),
ma non possibile



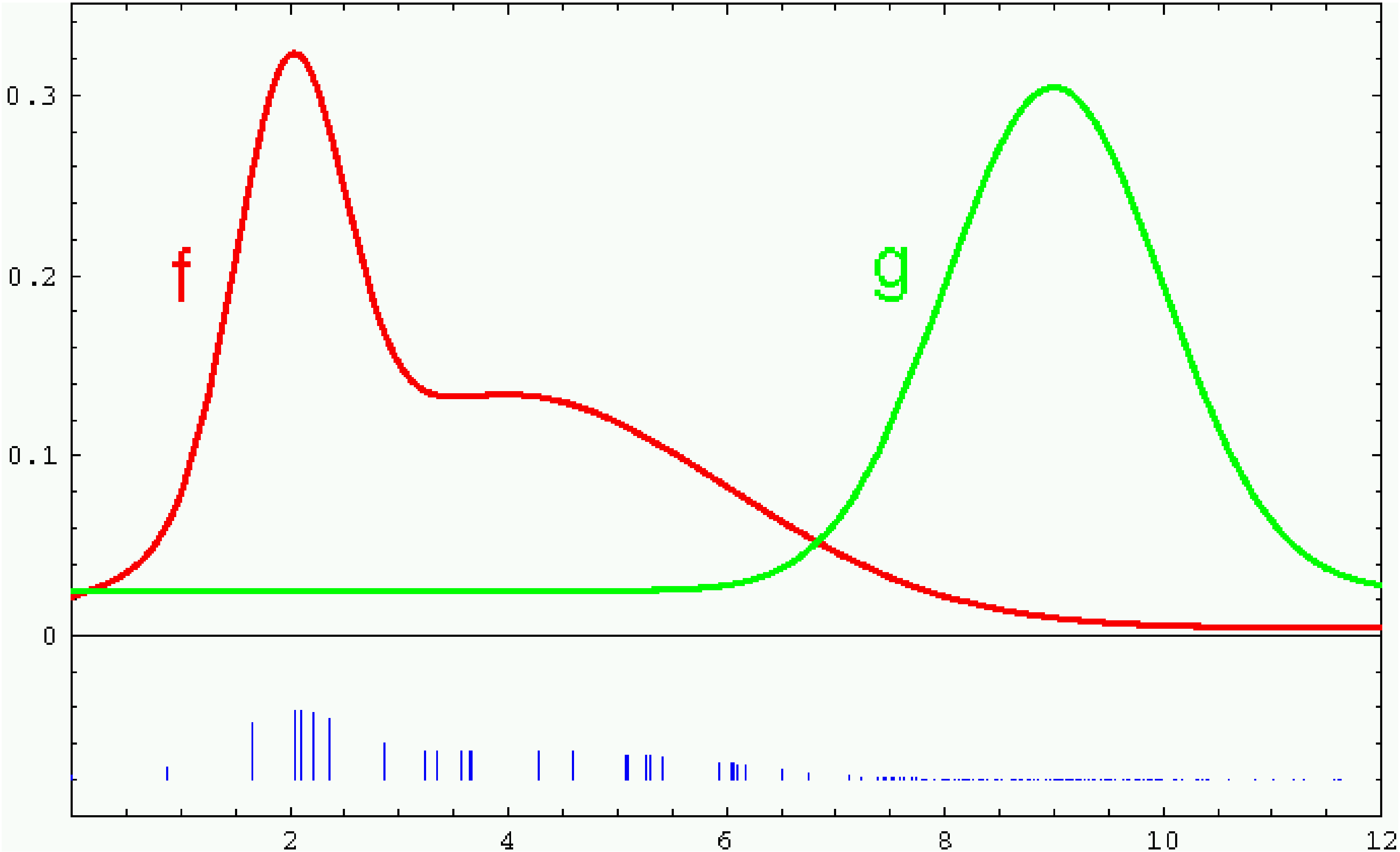
Campioni da g(x)



Campioni pesati
con f(x)/g(x)



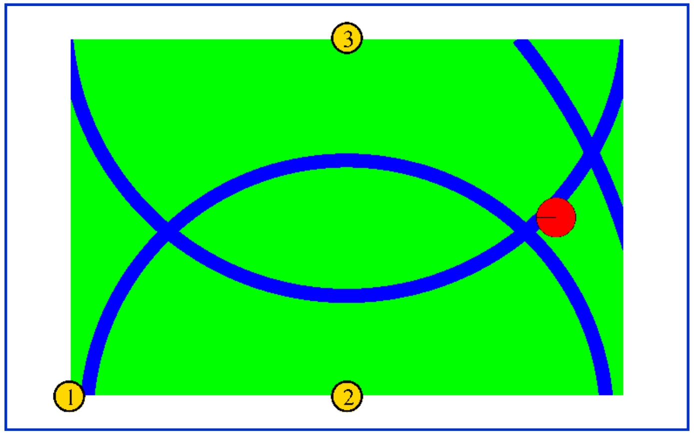
Importance Sampling

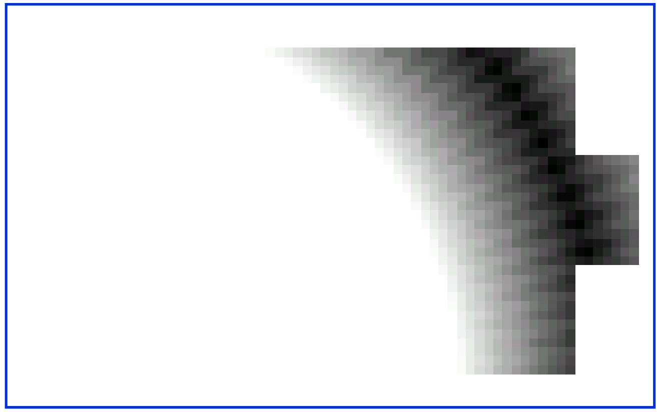


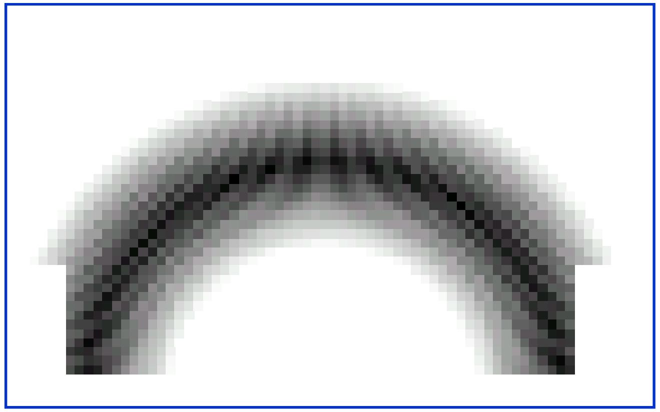
Peso dei campioni: $w = f / g$

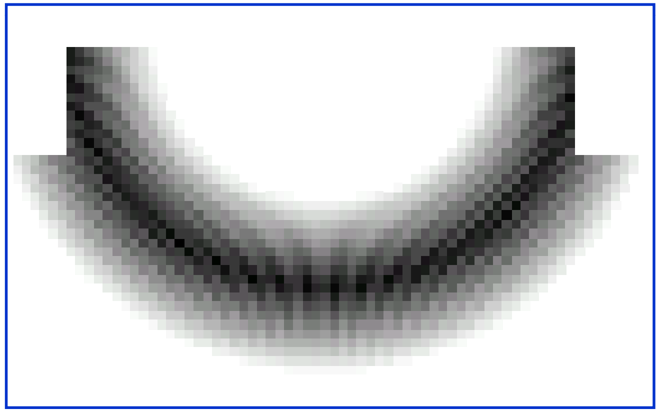
Importance Sampling con Resampling: Esempio

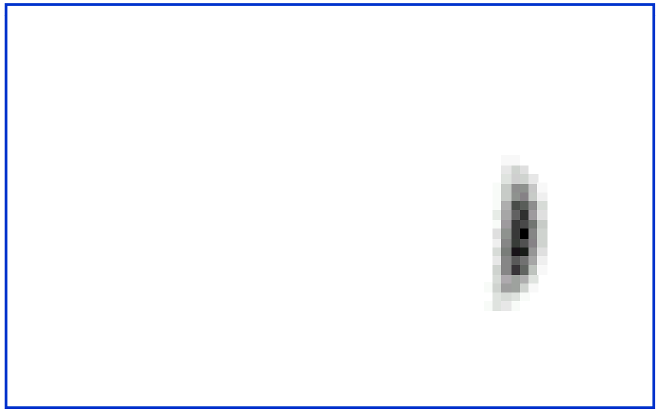
Distribuzioni



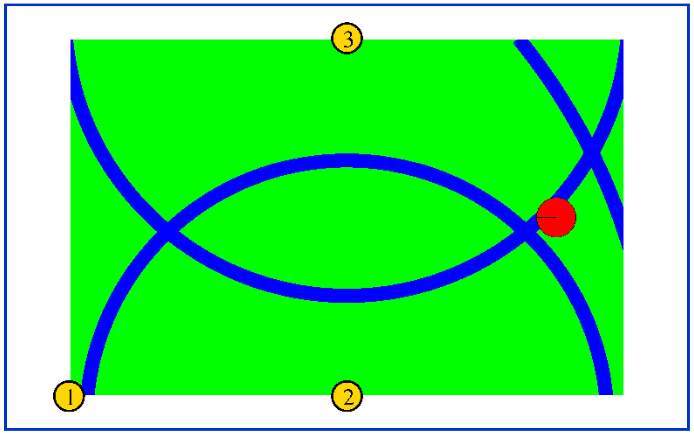


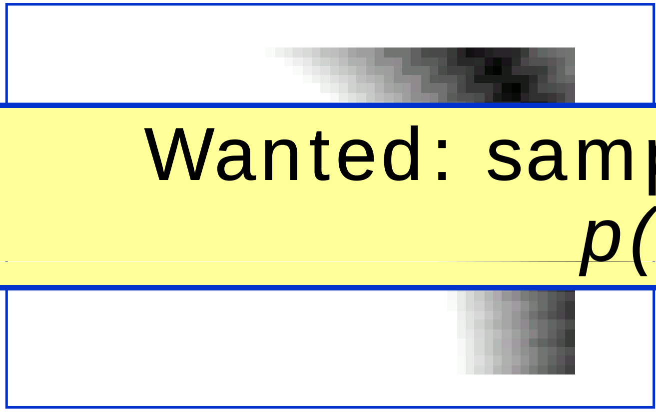


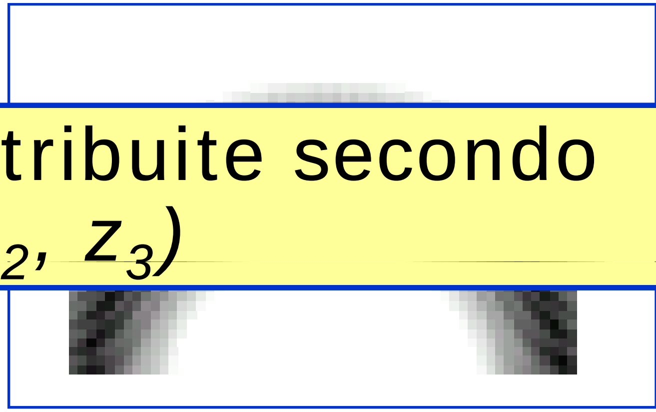




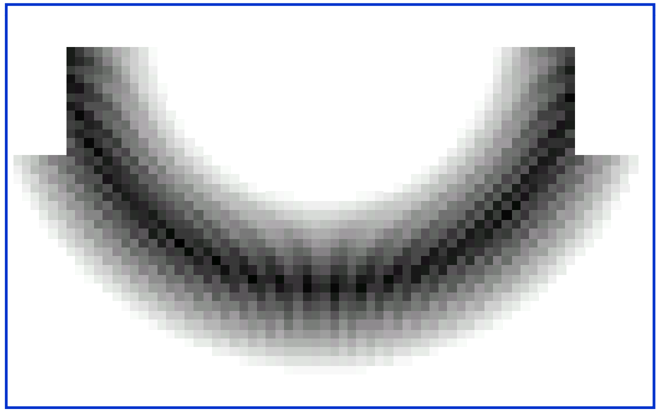
Distribuzioni

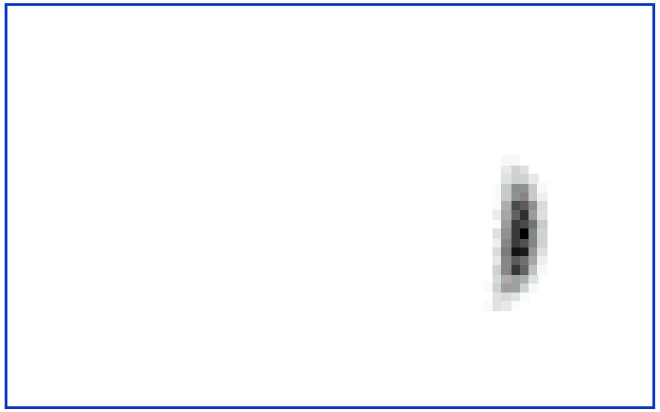






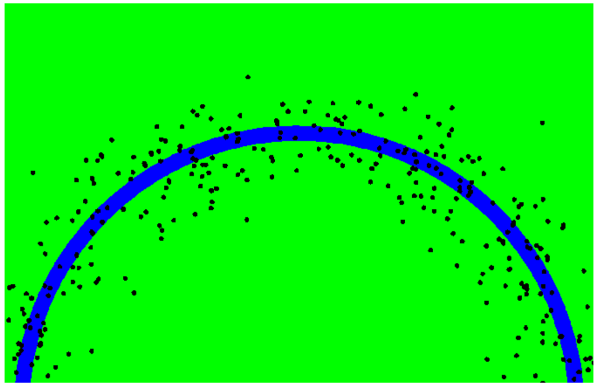
Wanted: samples distribuite secondo
$p(x|\ z_1,\ z_2,\ z_3)$

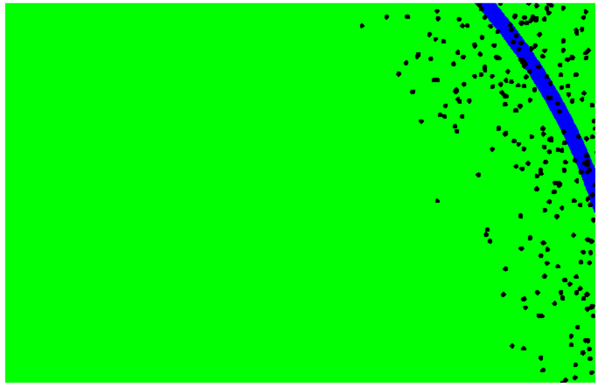


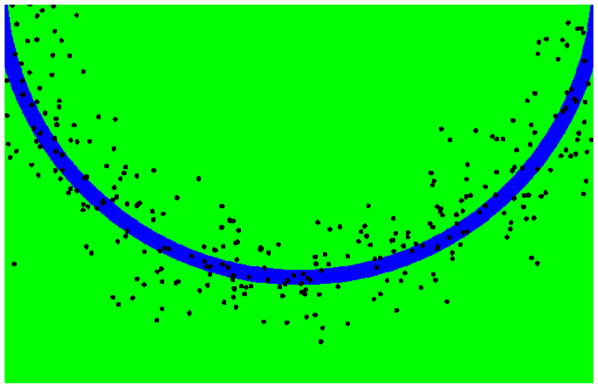


Campionamento

Estrarre campioni da $p(x|z_l)$







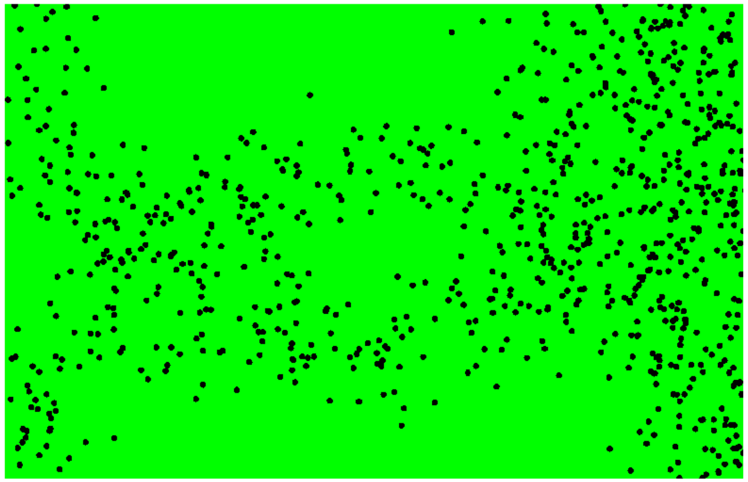
Importance Sampling con Resampling

$$\text{Target distribution f}: p(x \mid z_1, z_2, \ldots, z_n) = \frac{\prod_k p(z_k \mid x) \quad p(x)}{p(z_1, z_2, \ldots, z_n)}$$

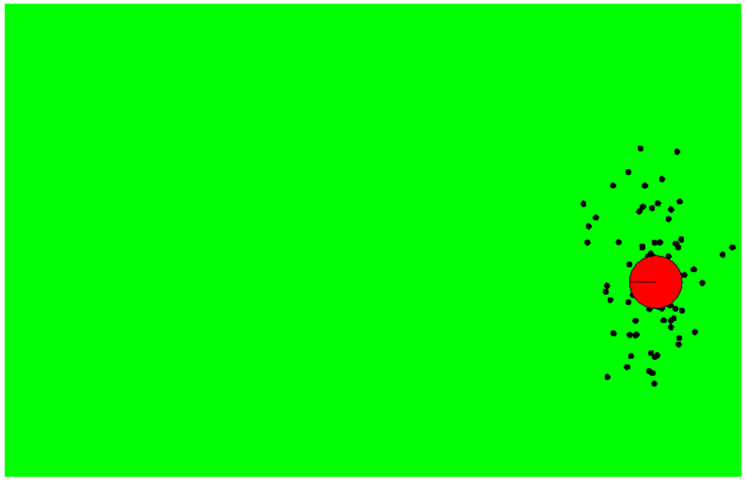
$$\text{Sampling distribution g}: p(x \mid z_l) = \frac{p(z_l \mid x) p(x)}{p(z_l)}$$

$$\text{Importance weights w}: \frac{f}{g} = \frac{p(x \mid z_1, z_2, \ldots, z_n)}{p(x \mid z_l)} = \frac{p(z_l) \prod_{k \neq l} p(z_k \mid x)}{p(z_1, z_2, \ldots, z_n)}$$

Importance Sampling con Resampling



Campioni pesati



Dopo il resampling

Importance Sampling

Campionamento di g

$$\frac{1}{M}\sum_{m=1}^{M} I(x^{[m]} \in A) \longrightarrow \int_A g(x)\, dx$$

Differenza tra g ed f

$$w^{[m]} = \frac{f(x^{[m]})}{g(x^{[m]})}$$

Permette di ricostruire f

$$\left[\sum_{m=1}^{M} w^{[m]}\right]^{-1} \sum_{m=1}^{M} I(x^{[m]} \in A)\, w^{[m]} \longrightarrow \int_A f(x)\, dx$$

Importance Sampling

Nel caso del filtro particellare:

$$\text{Target}(x) = Bel(x_t) \rightarrow X_t$$

$$\text{Proposal}(x) = p(x_t | u_t, x_{t-1})\, Bel(x_{t-1}) \rightarrow X'_t$$

$$w = p(z_t \mid x_t)$$

Particle Filter

Come bel(x_t) si aggiorna da bel(x_{t-1})

Così X_t si aggiorna da X_{t-1}

Le ipotesi sono pesate:

$$S = \left\{ \left\langle s^{[i]}, w^{[i]} \right\rangle \mid i = 1, \ldots, N \right\}$$

State hypothesis

Importance weight

I campioni pesati usati per rappresentare il post:

$$p(x) = \sum_{i=1}^{N} w_i \cdot \delta_{s^{[i]}}(x)$$

Particle Filter: Algoritmo

Prima crea X'_t dal moto u_t rappresenta bel'(t), quindi corregge con le osservazioni z_t

Algorithm Particle_filter($\mathcal{X}_{t-1}, u_t, z_t$):
    $\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$
    for $m = 1$ to M do
        sample $x_t^{[m]} \sim p(x_t \mid u_t, x_{t-1}^{[m]})$
        $w_t^{[m]} = p(z_t \mid x_t^{[m]})$
        $\bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle x_t^{[m]}, w_t^{[m]} \rangle$
    endfor
    for $m = 1$ to M do
        draw i with probability $\propto w_t^{[i]}$
        add $x_t^{[i]}$ to $\mathcal{X}_t$
    endfor
    return $\mathcal{X}_t$

Generation

Importance factor

Resampling

Particle Filter: Algoritmo

Si parte da X_{t-1} che rapp $Bel_{t-1}(x)$

Si applica il modello di moto
$p(x_{t,i} \mid ut, x_{t-1,i})$ e si campiona X'_t

Si pesano i campioni rispetto a $P(z|x)$

Si effettua il ricampionamento per avere X_t

Particle Filter: Algoritmo

Target

$$
\begin{aligned}
& p(x_{0:t} \mid z_{1:t}, u_{1:t}) \\
& \stackrel{\text{Bayes}}{=} \quad \eta \; p(z_t \mid x_{0:t}, z_{1:t-1}, u_{1:t}) \; p(x_{0:t} \mid z_{1:t-1}, u_{1:t}) \\
& \stackrel{\text{Markov}}{=} \quad \eta \; p(z_t \mid x_t) \; p(x_{0:t} \mid z_{1:t-1}, u_{1:t}) \\
& = \quad \eta \; p(z_t \mid x_t) \; p(x_t \mid x_{0:t-1}, z_{1:t-1}, u_{1:t}) \; p(x_{0:t-1} \mid z_{1:t-1}, u_{1:t}) \\
& \stackrel{\text{Markov}}{=} \quad \eta \; p(z_t \mid x_t) \; p(x_t \mid x_{t-1}, u_t) \; p(x_{0:t-1} \mid z_{1:t-1}, u_{1:t-1})
\end{aligned}
$$

Proposal

$$
\begin{aligned}
& p(x_t \mid x_{t-1}, u_t) \; bel(x_{0:t-1}) \\
& = \quad p(x_t \mid x_{t-1}, u_t) \; p(x_{0:t-1} \mid z_{0:t-1}, u_{0:t-1})
\end{aligned}
$$

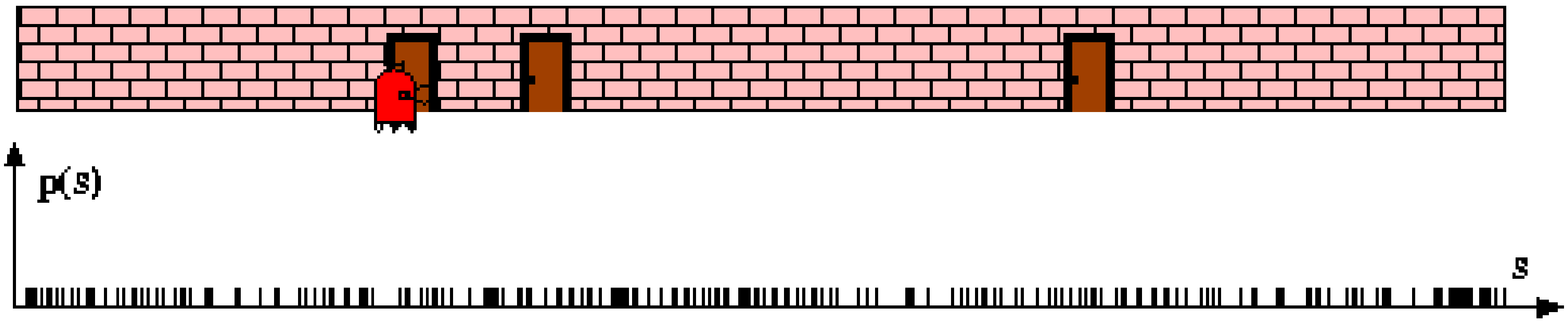
Peso

$$
\begin{aligned}
w_t^{[m]} & = \frac{\text{target distribution}}{\text{proposal distribution}} \\
& = \frac{\eta \; p(z_t \mid x_t) \; p(x_t \mid x_{t-1}, u_t) \; p(x_{0:t-1} \mid z_{1:t-1}, u_{1:t-1})}{p(x_t \mid x_{t-1}, u_t) \; p(x_{0:t-1} \mid z_{0:t-1}, u_{0:t-1})} \\
& = \eta \; p(z_t \mid x_t)
\end{aligned}
$$

Target

$$
\eta \; w_t^{[m]} \; p(x_t \mid x_{t-1}, u_t) \; p(x_{0:t-1} \mid z_{0:t-1}, u_{0:t-1}) \quad = \quad bel(x_{0:t})
$$

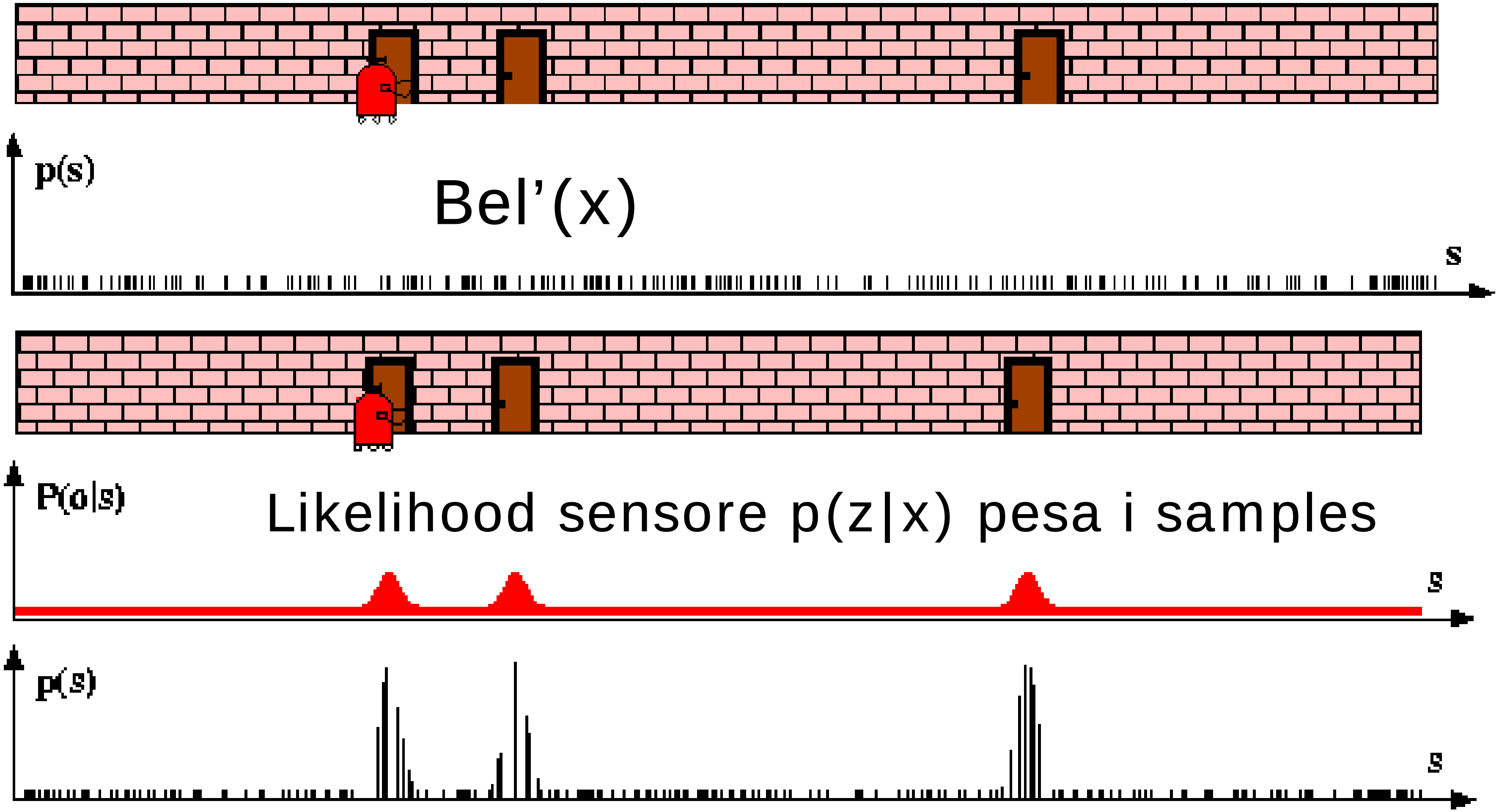
Particle Filters



La posa iniziale sono particelle prese in modo random

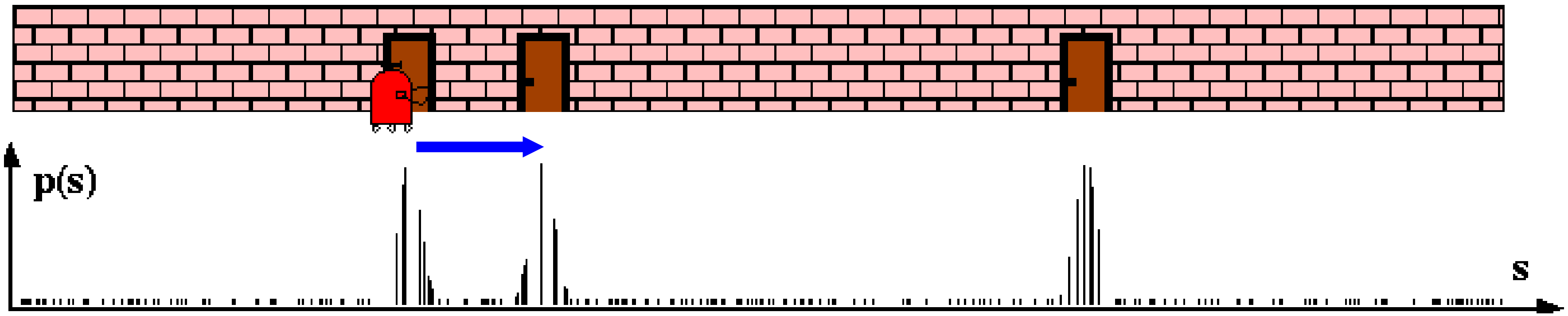
Informazione Sensori: Importance Sampling

$$
\begin{aligned}
Bel(x) &\leftarrow \alpha\ p(z\,|\,x)\,Bel^{-}(x) \\
w &\leftarrow \frac{\alpha\ p(z\,|\,x)\,Bel^{-}(x)}{Bel^{-}(x)} = \alpha\ p(z\,|\,x)
\end{aligned}
$$

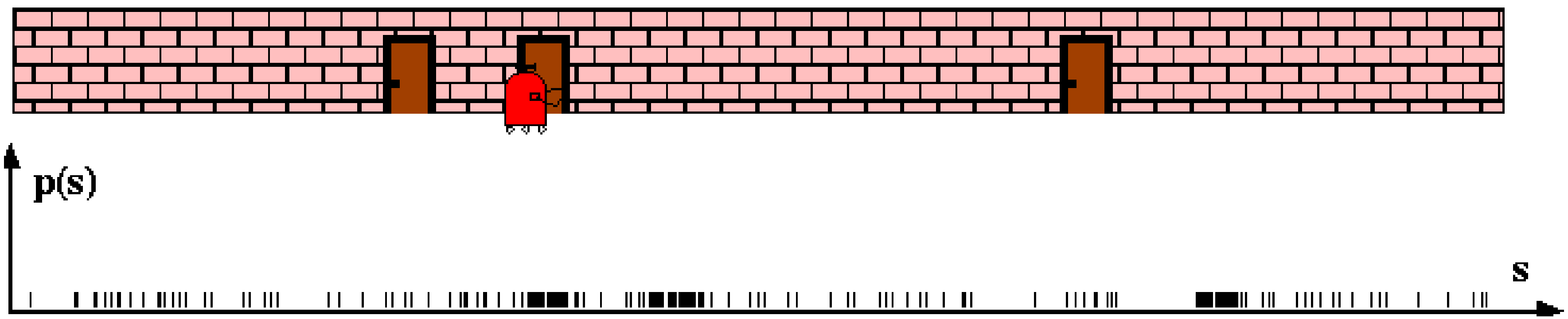


Robot Motion

$$\boxed{Bel^{-}(x) \quad \leftarrow \quad \int p(x \mid u, x')\, Bel(x')\ \mathrm{d}\, x'}$$



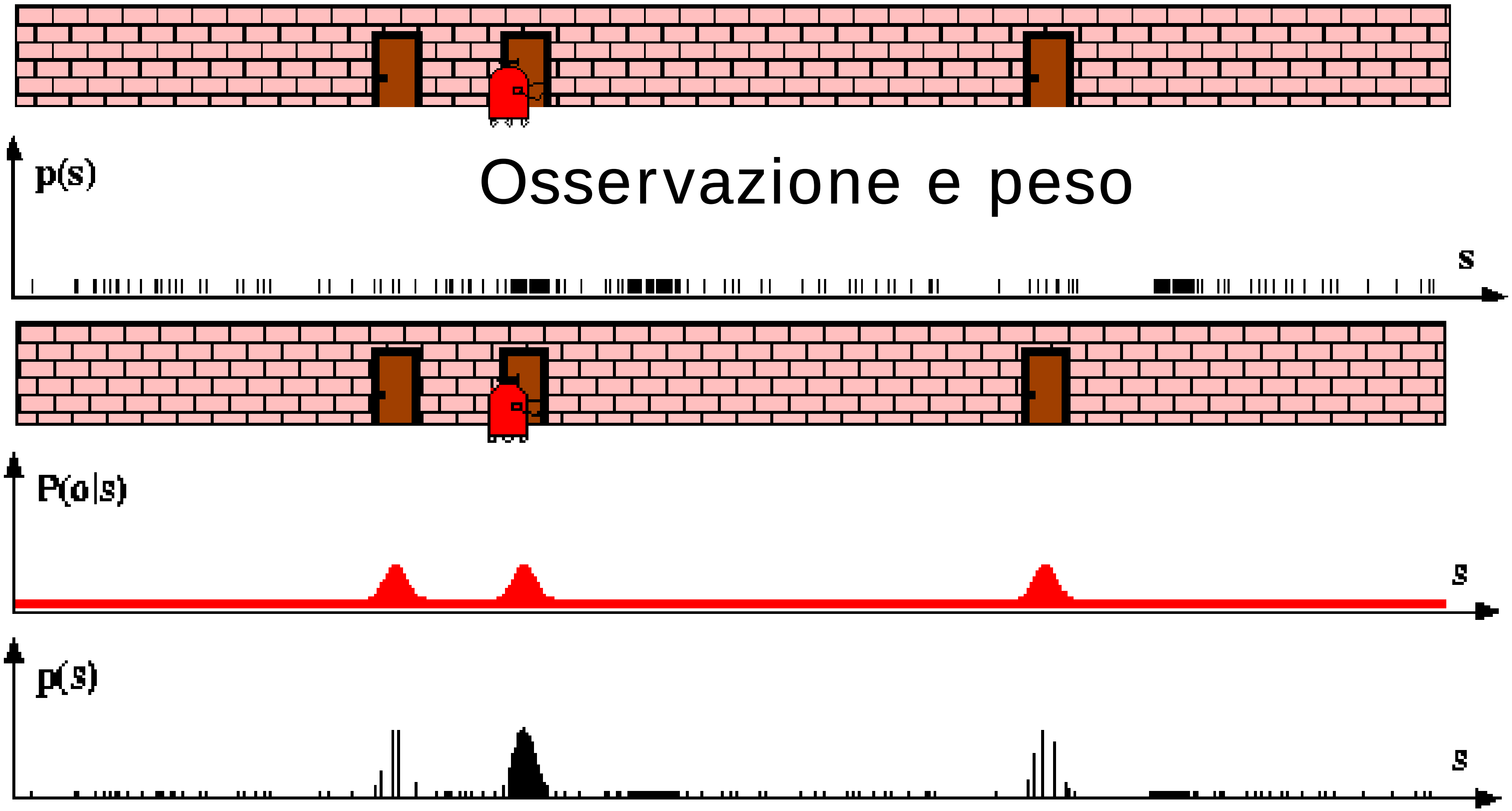
Ricampionamento e spostamento Bel’(x)



Informazione Sensori: Importance Sampling

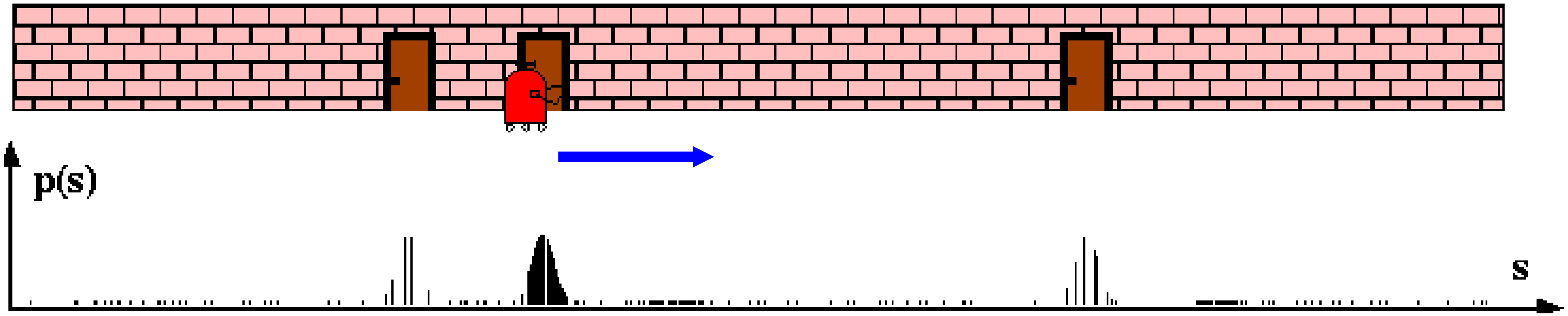
$$Bel(x) \quad \leftarrow \quad \alpha \; p(z \mid x) \, Bel^{-}(x)$$

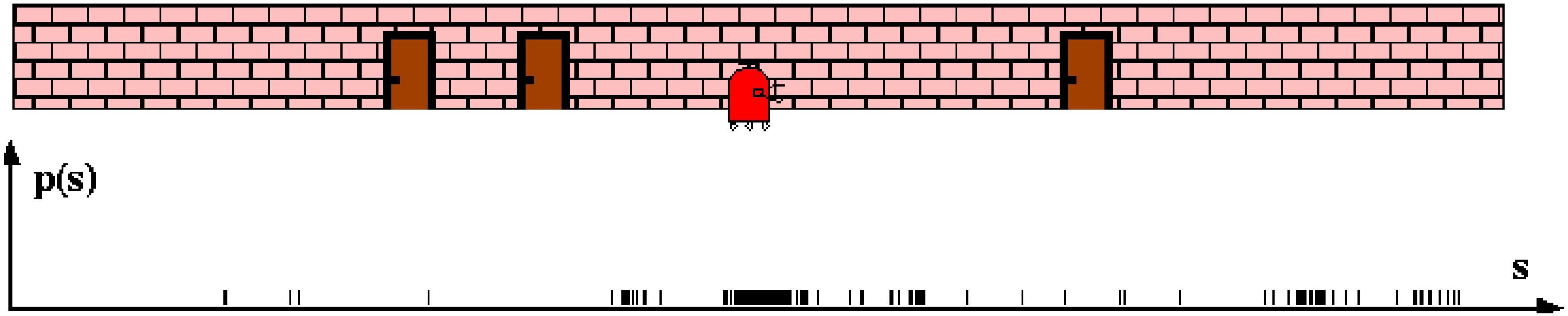
$$w \quad \leftarrow \quad \frac{\alpha \; p(z \mid x) \, Bel^{-}(x)}{Bel^{-}(x)} \quad = \quad \alpha \; p(z \mid x)$$



Robot Motion

$$Bel^-(x) \quad \leftarrow \quad \int p(x\,|\,u, x')\; Bel(x') \;\; \mathrm{d}\,x'$$





Algoritmo del Particle Filter

Algorithm **particle_filter**(S_{t-1}, u_{t-1} z_t):

$S_t = \emptyset, \quad \eta = 0$

For $i = 1 \dots n$ ***Generate new samples***

Sample index $j(i)$ from the discrete distribution given by w_{t-1}

Sample x_t^i from $p(x_t \mid x_{t-1}, u_{t-1})$ using $x_{t-1}^{j(i)}$ and u_{t-1}

$w_t^i = p(z_t \mid x_t^i)$ ***Compute importance weight***

$\eta = \eta + w_t^i$ ***Update normalization factor***

$S_t = S_t \cup \{ < x_t^i, w_t^i > \}$ ***Insert***

For $i = 1 \dots n$

$w_t^i = w_t^i / \eta$ ***Normalize weights***

Algoritmo Particle Filter

$$Bel(x_t) = \eta\, p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_{t-1})\, Bel(x_{t-1})\, dx_{t-1}$$

draw x^i_{t-1} from $Bel(x_{t-1})$

draw x^i_t from $p(x_t \mid x^i_{t-1}, u_{t-1})$

Importance factor for x^i_t:

$$
\begin{aligned}
w^i_t &= \frac{\text{target distribution}}{\text{proposal distribution}} \\
&= \frac{\eta\ p(z_t \mid x_t)\ p(x_t \mid x_{t-1}, u_{t-1})\ Bel\ (x_{t-1})}{p(x_t \mid x_{t-1}, u_{t-1})\ Bel\ (x_{t-1})} \\
&\propto p(z_t \mid x_t)
\end{aligned}
$$

Sampling da Bel’

Peso da P(z|x)

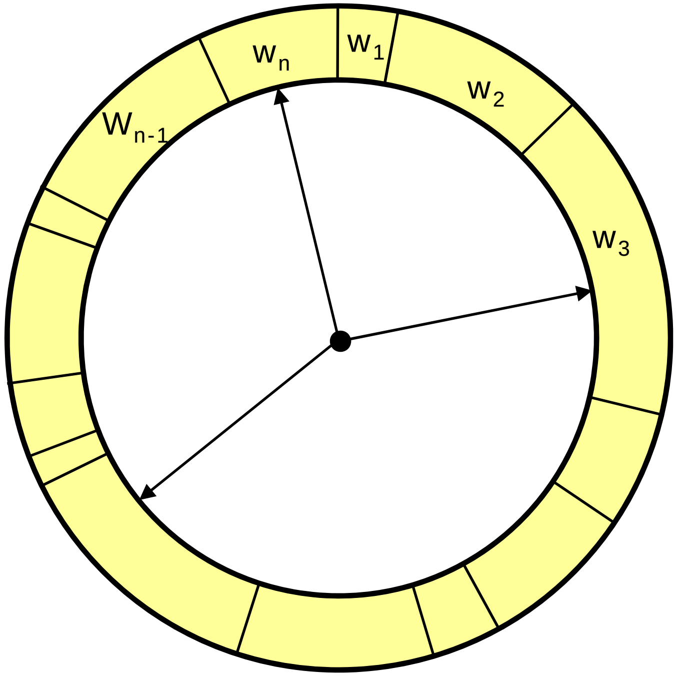
Resampling

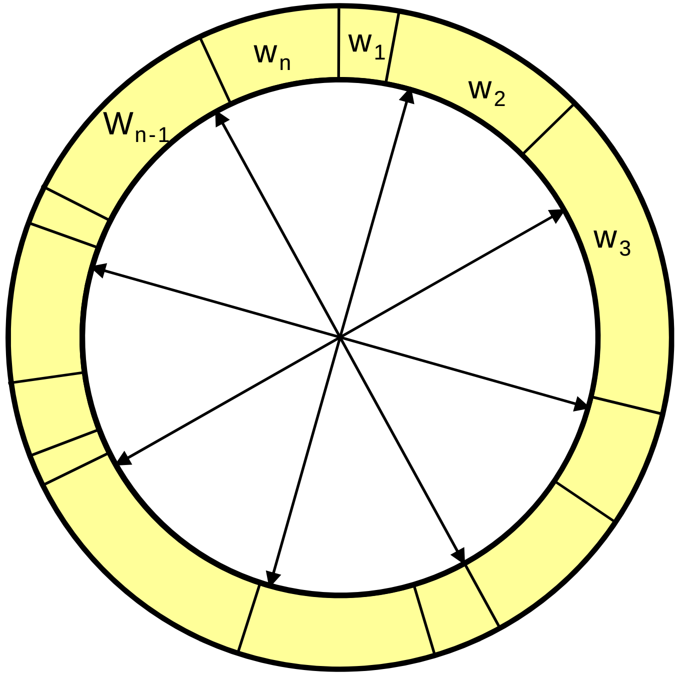
- **Dato**: Insieme S di campioni pesati.

- **Desiderata** : Campione Random, dove la prob di estrarre x_i è proporzionale a w_i.

- Fatto n volte con rimpiazzamento per generare il nuovo insieme di campioni S'.

Resampling





- Roulette wheel
- Binary search, n log n

- Stochastic universal sampling
- Systematic resampling
- Linear time complexity
- Easy to implement, low variance

Resampling

Algorithm **systematic_resampling**(S,n):

$S' = \emptyset, c_1 = w^1$

For $i = 2 \dots n$ — ***Generate cdf***

$\quad c_i = c_{i-1} + w^i$

$u_1 \sim U]0, n^{-1}], i = 1$ — ***Initialize threshold***

For $j = 1 \dots n$ — ***Draw samples …***

$\quad$ **While** ($u_j > c_i$) — ***Skip until next threshold reached***

$\quad\quad i = i + 1$

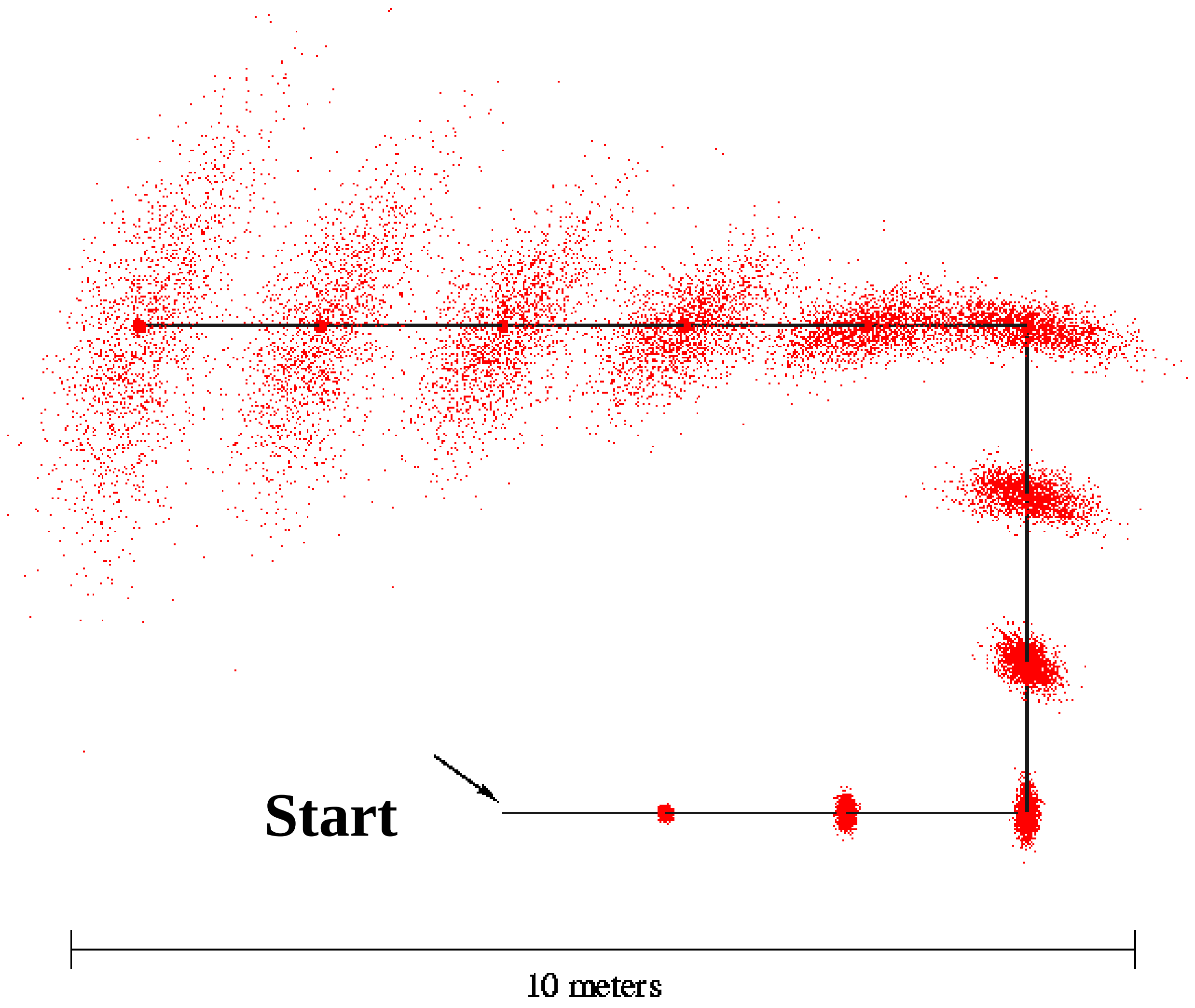
$\quad S' = S' \cup \{< x^i, n^{-1} >\}$ — ***Insert***

$\quad u_{j+1} = u_j + n^{-1}$ — ***Increment threshold***

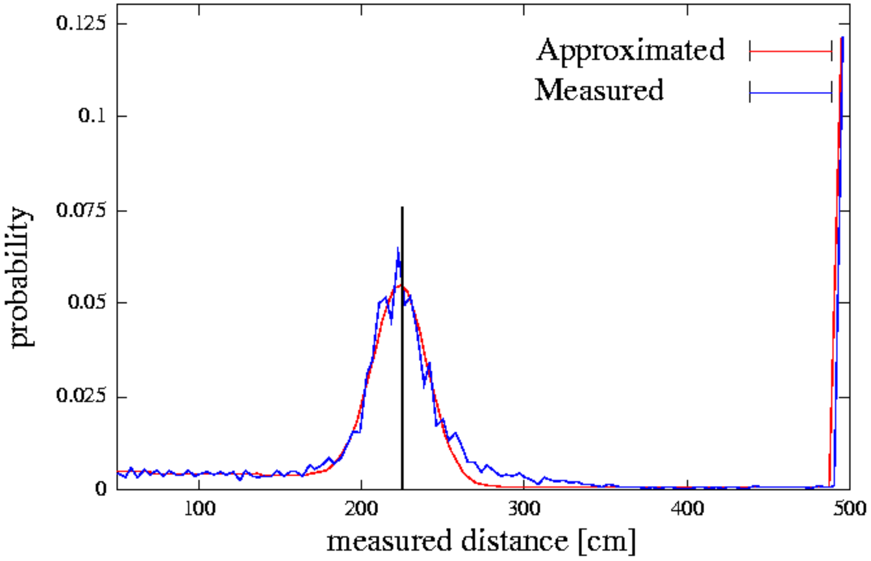
Return S'

Also called **stochastic universal sampling**

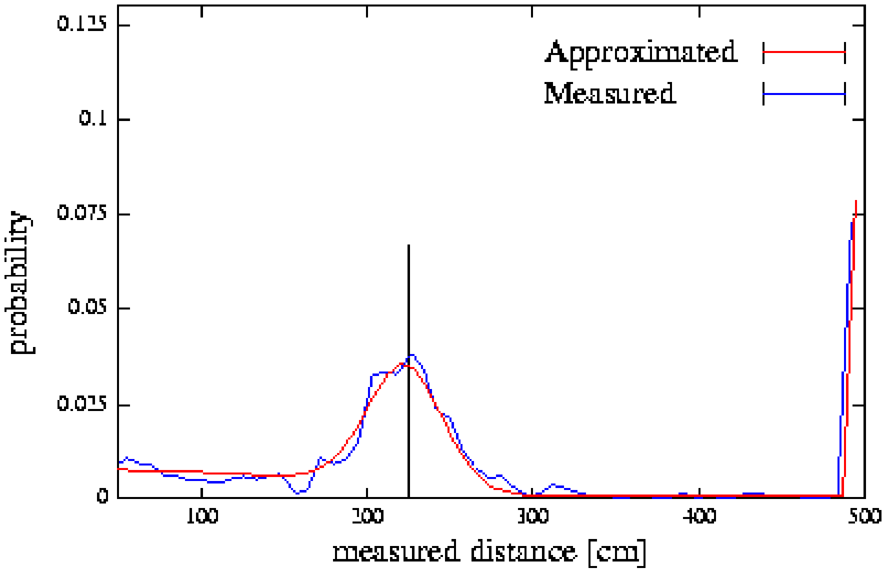
Modello di Moto



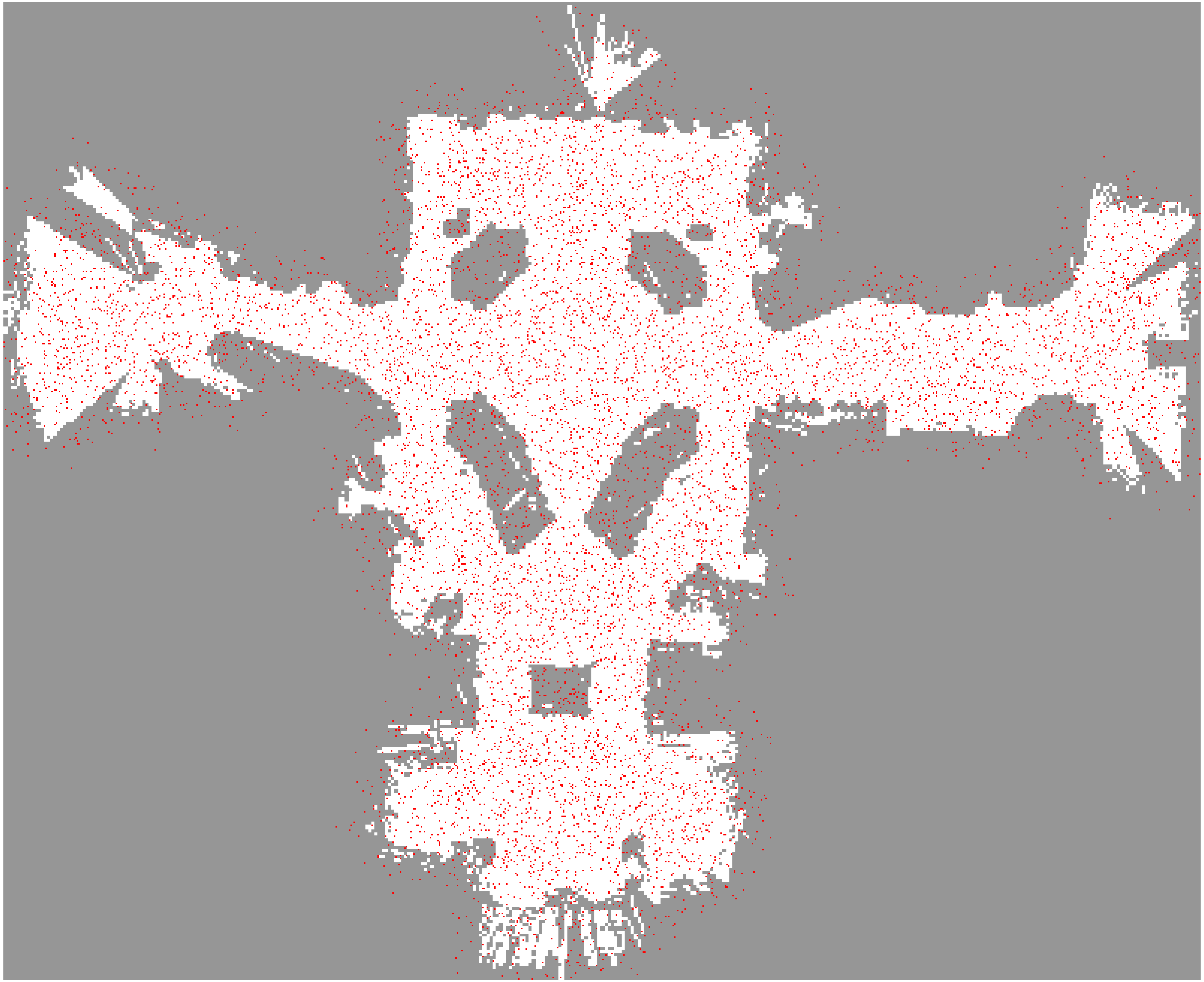
Proximity Sensor Model

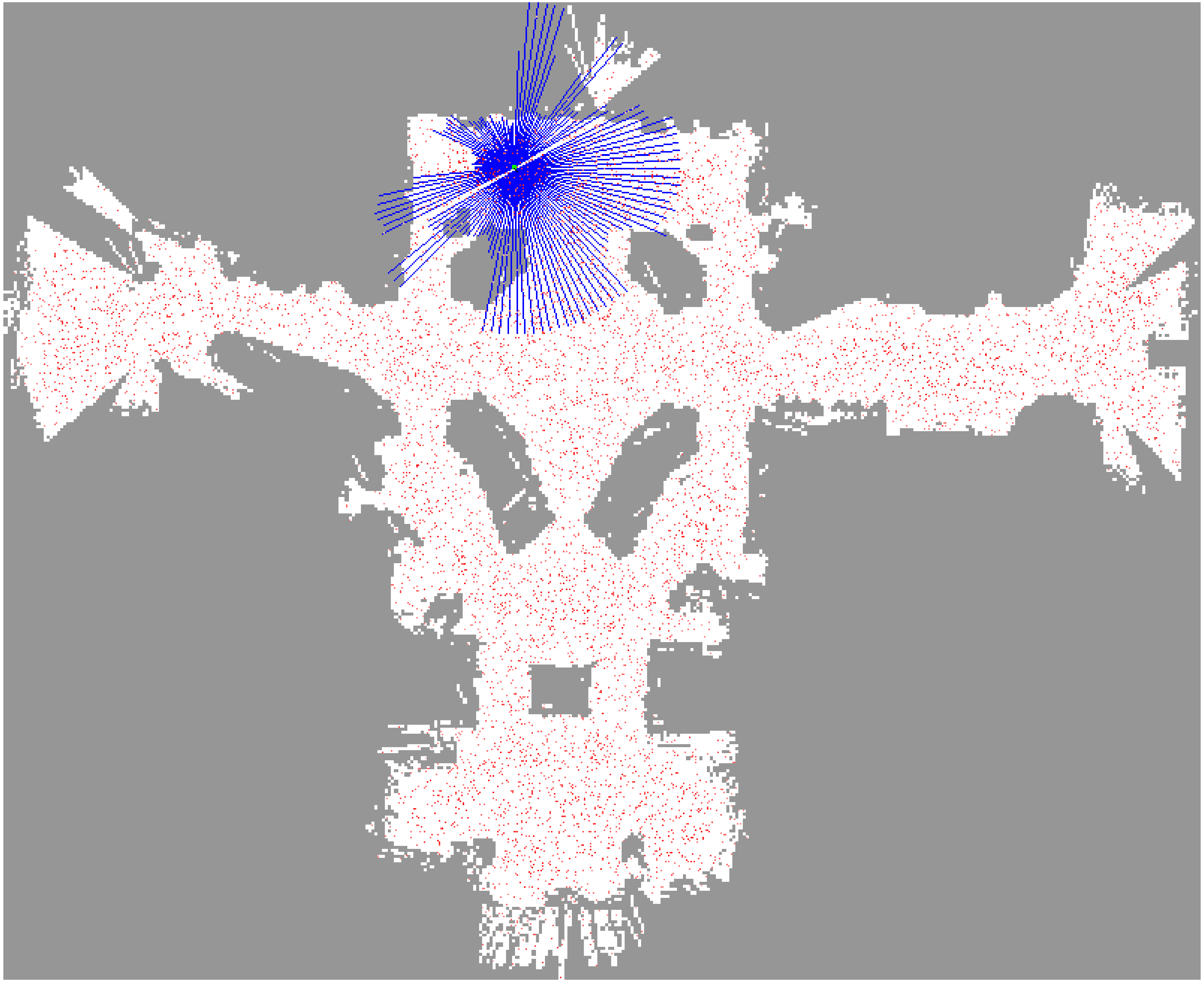


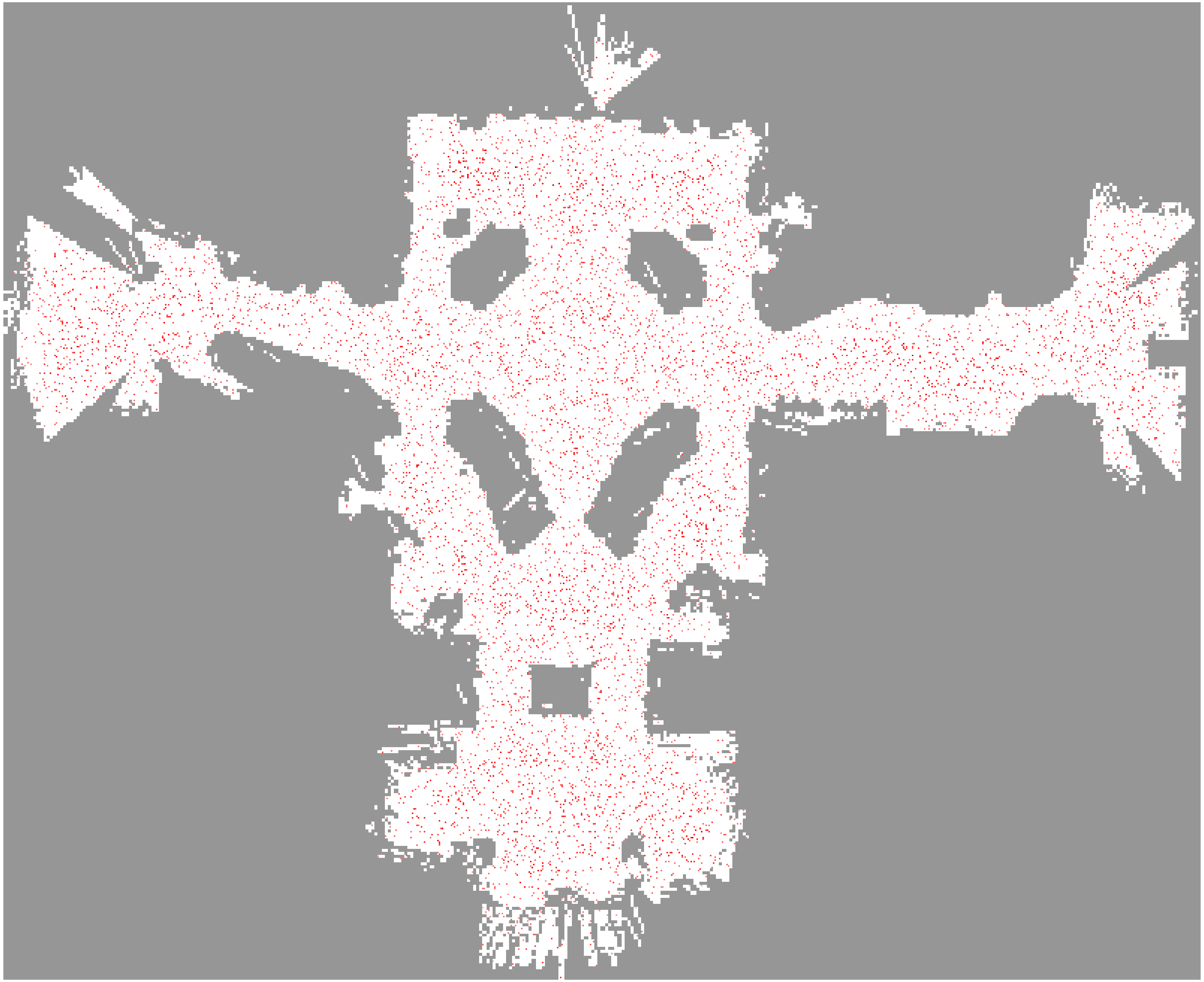
Laser sensor

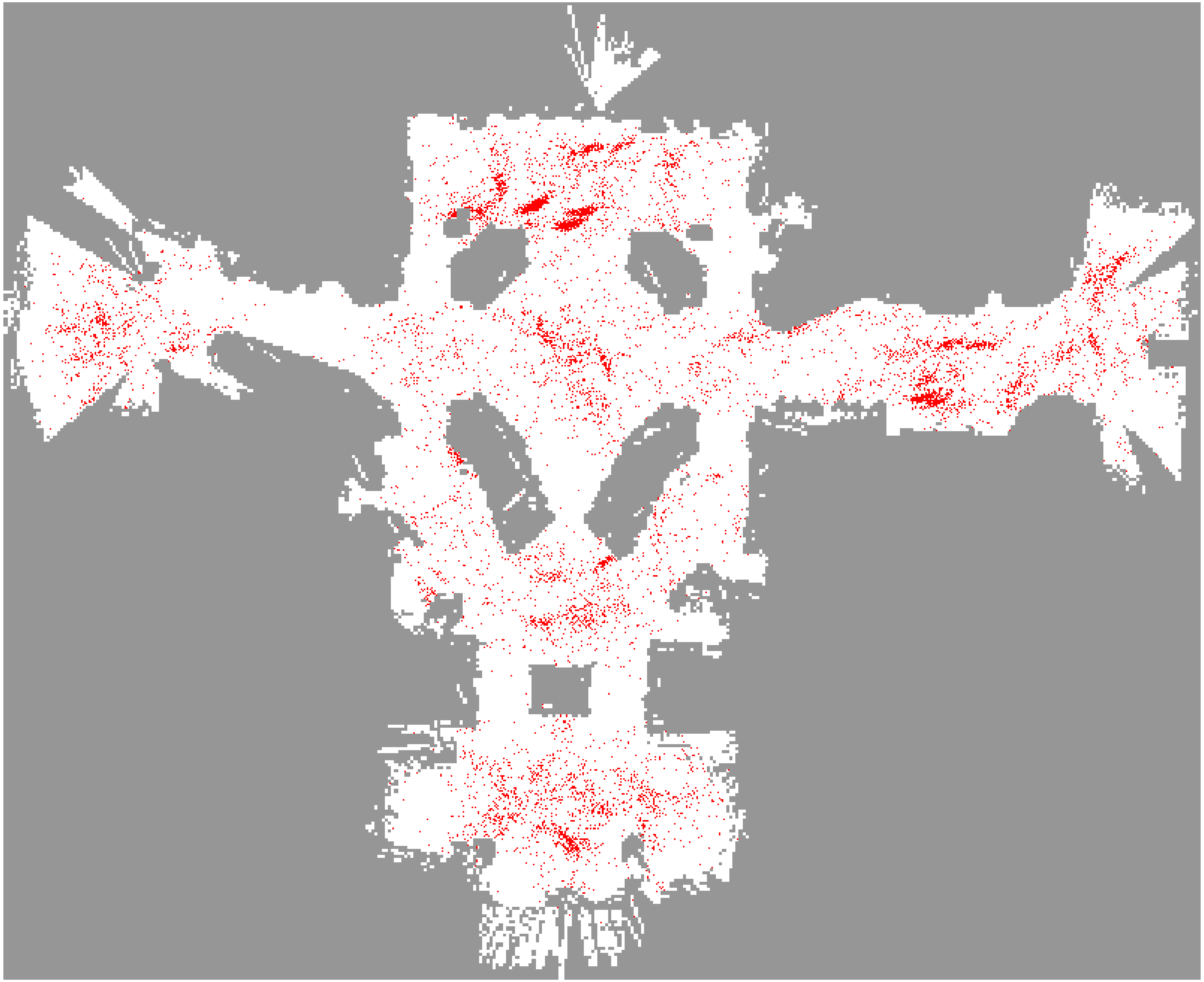


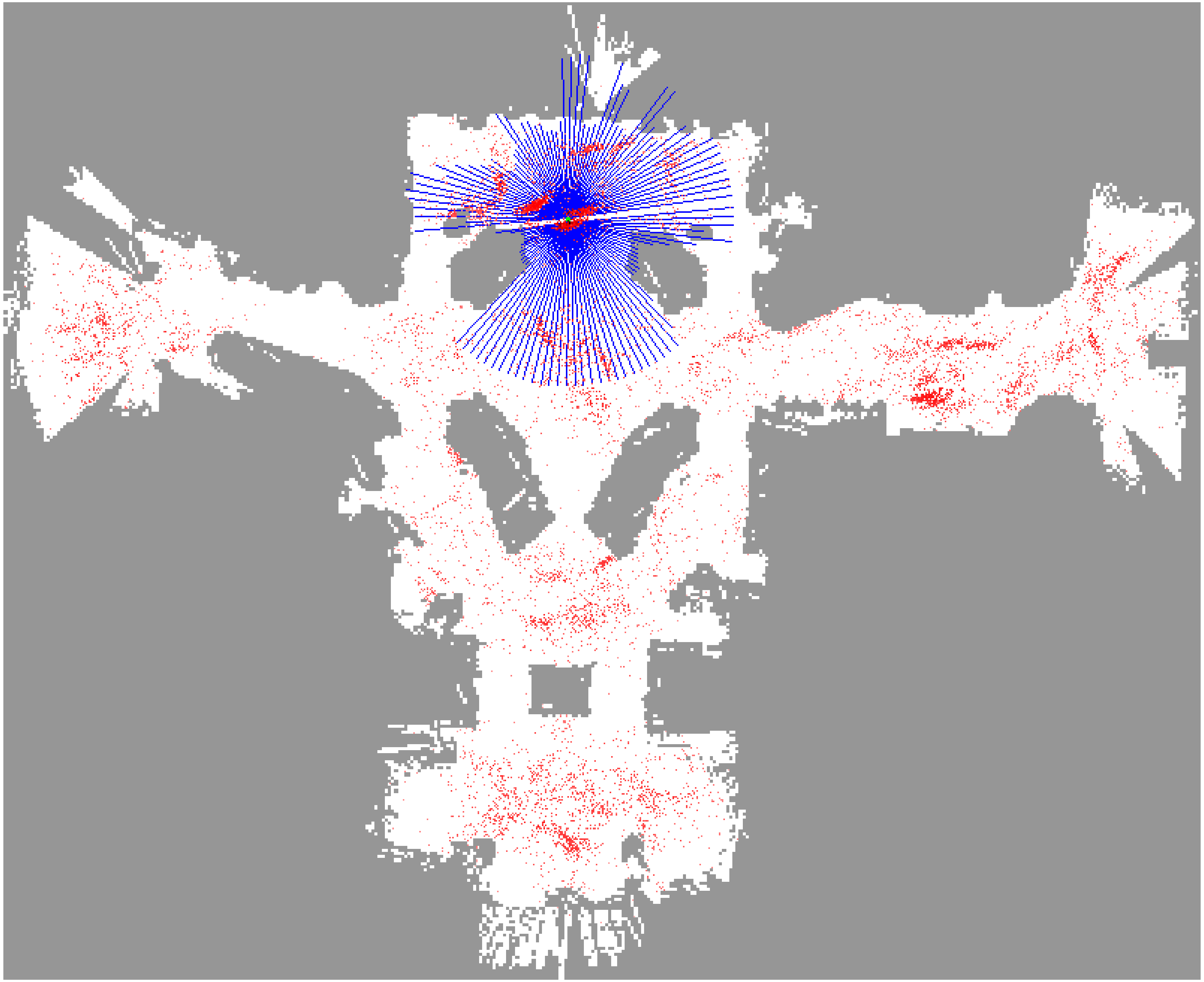
Sonar sensor

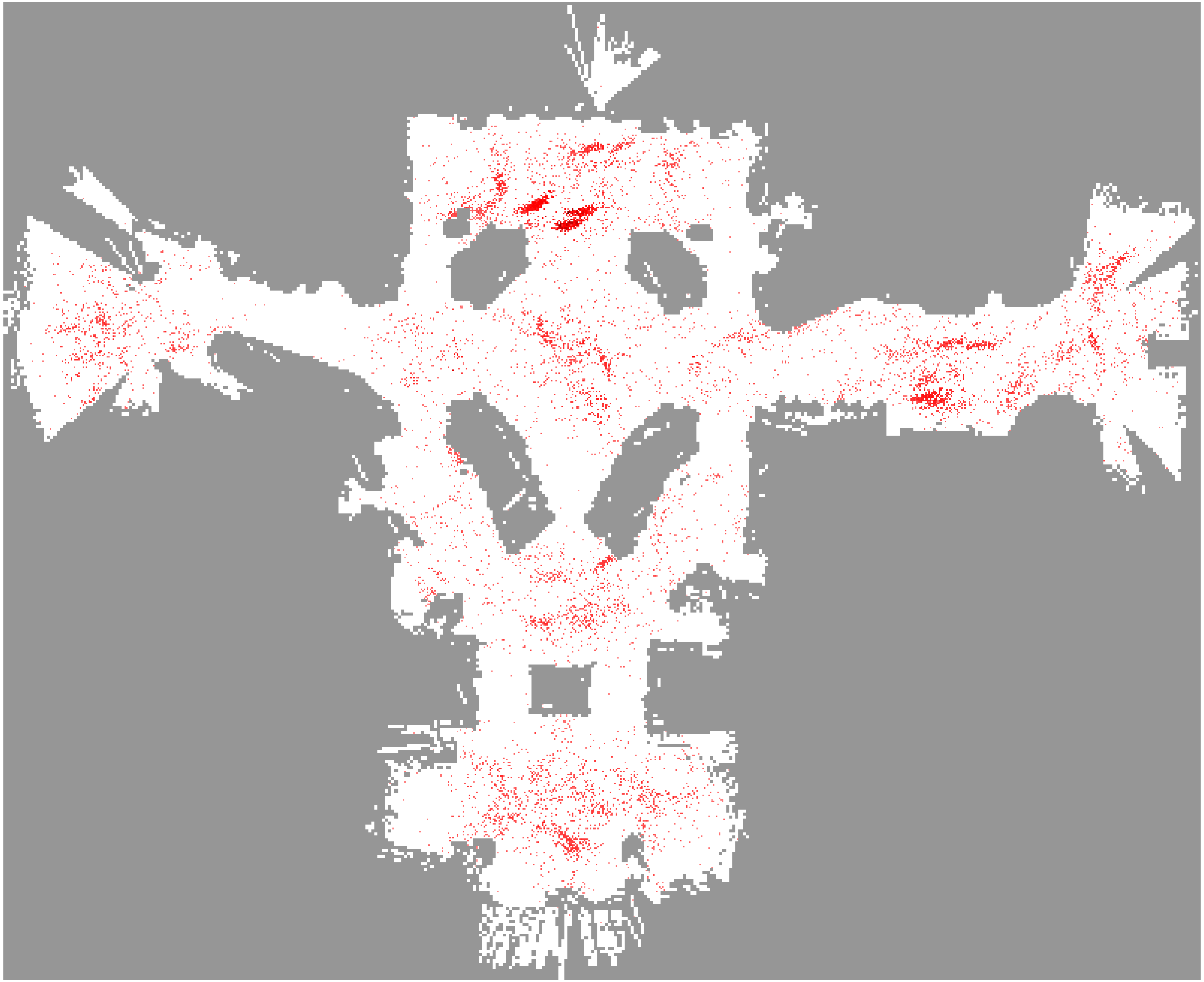


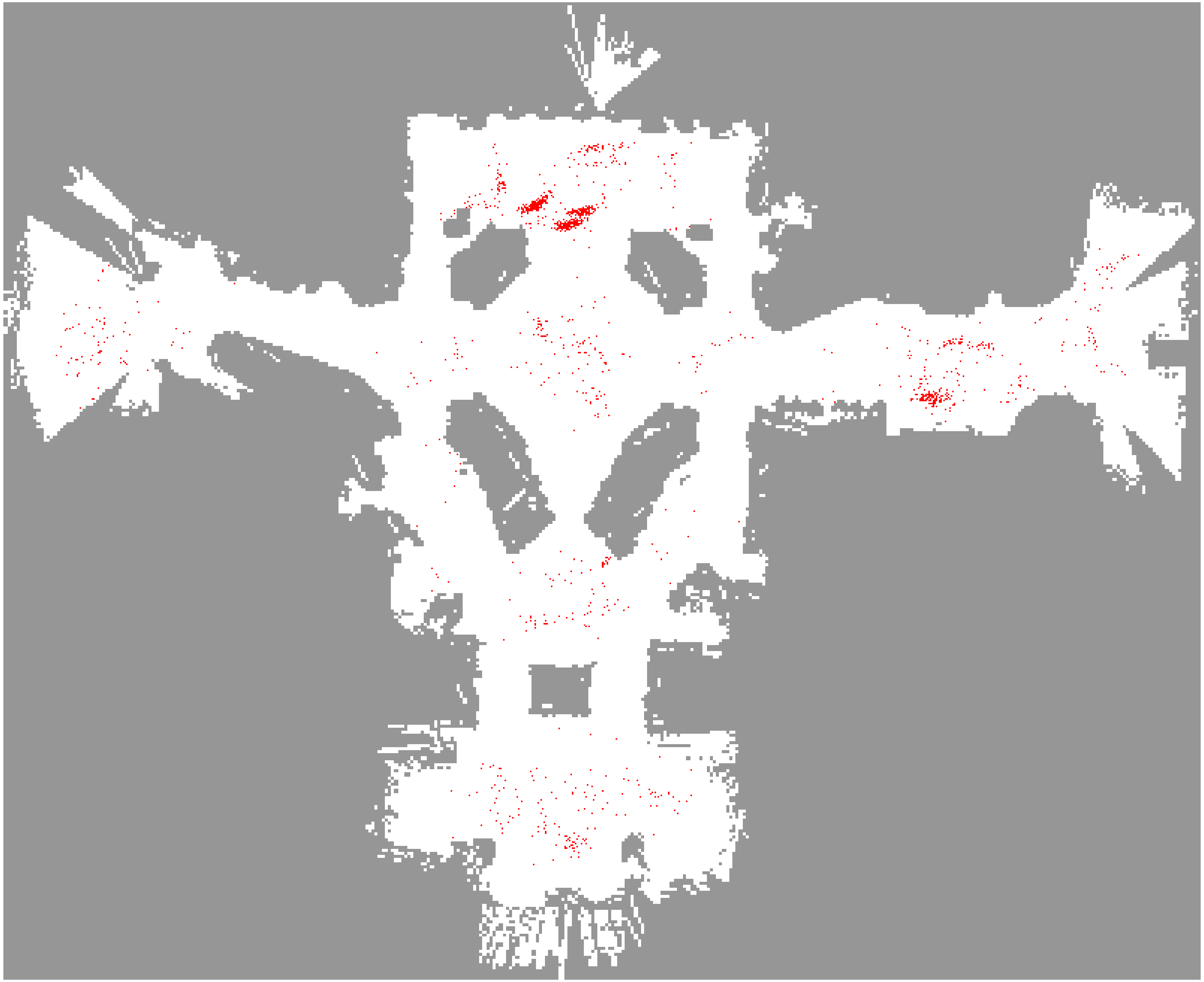


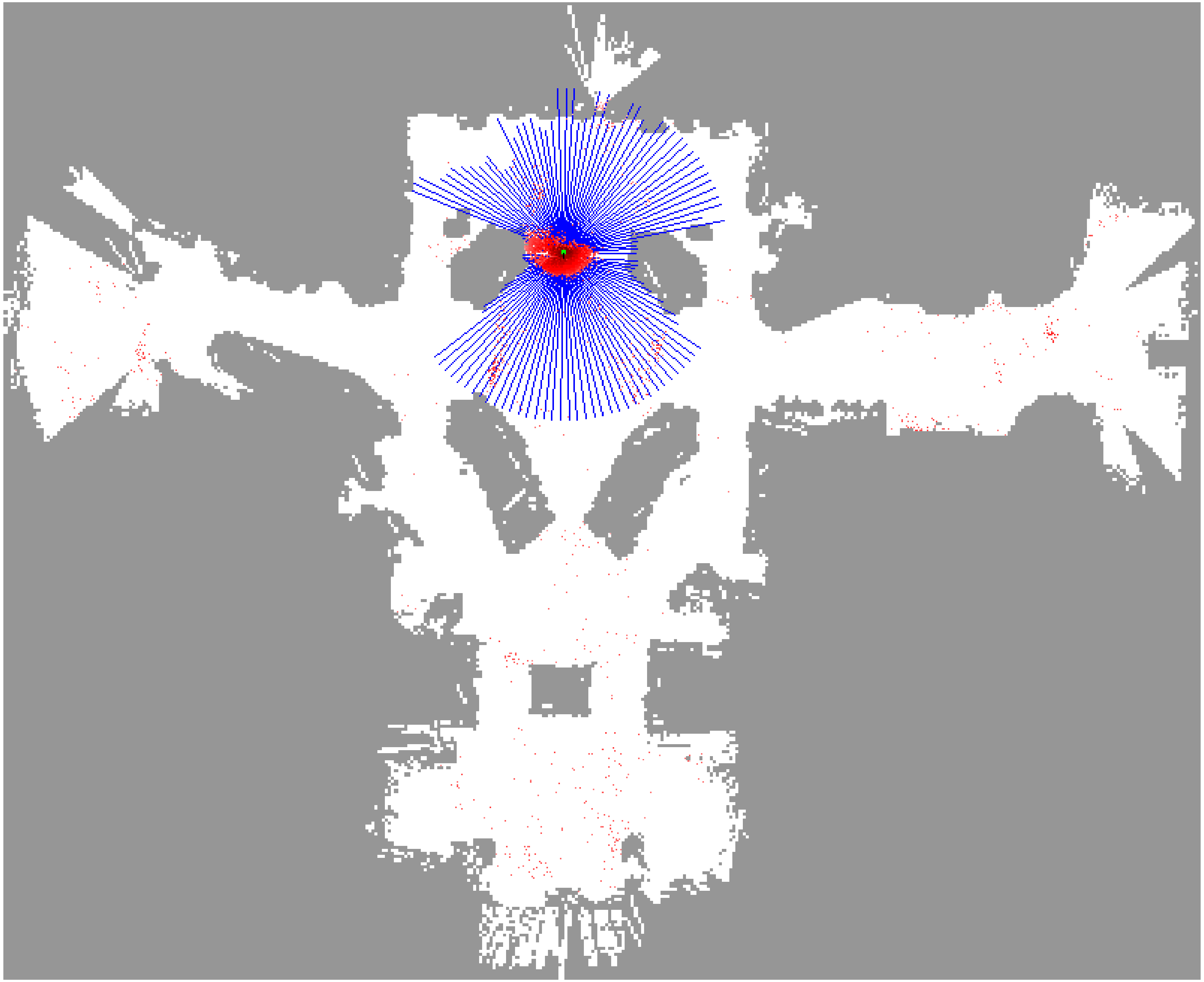


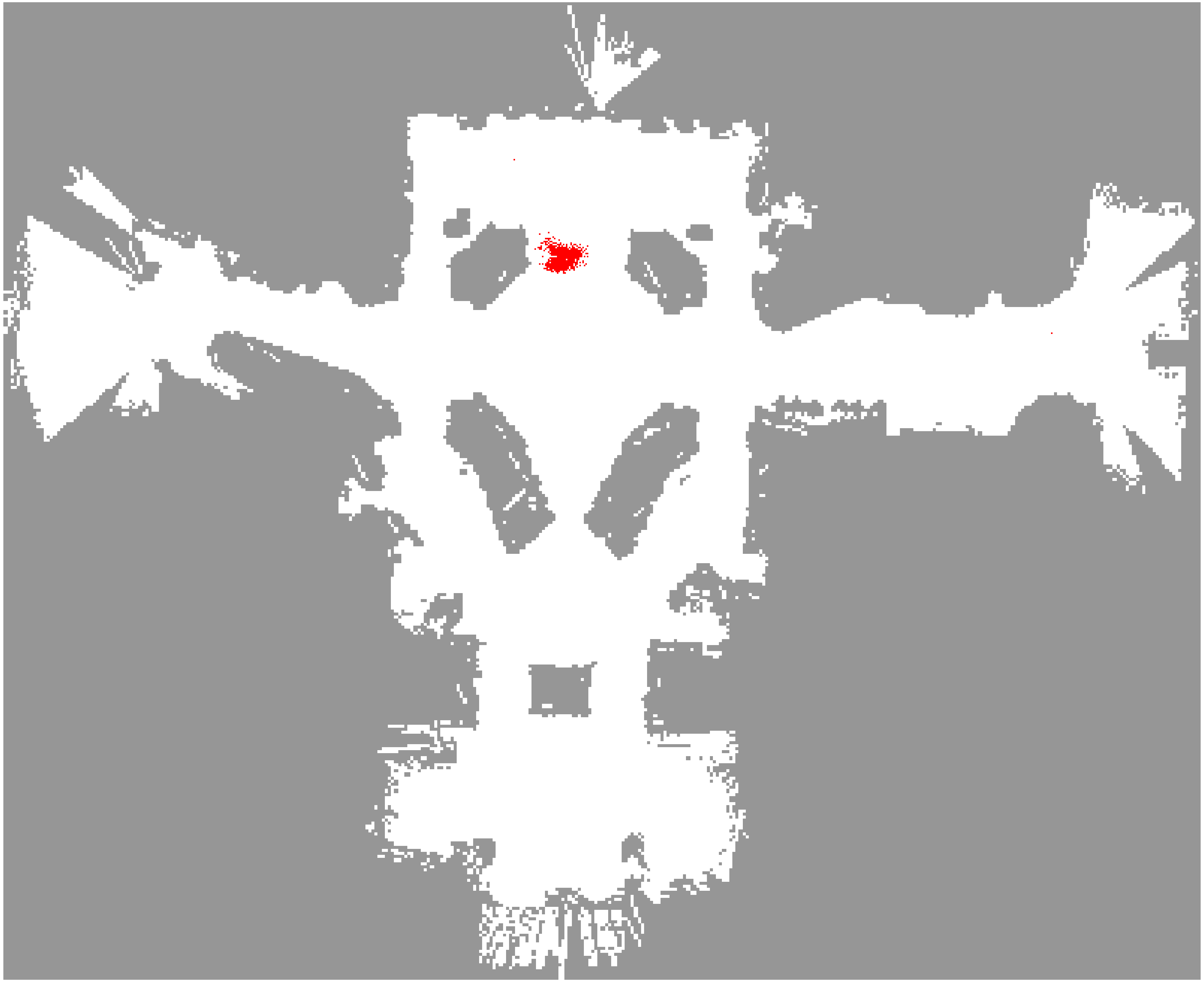


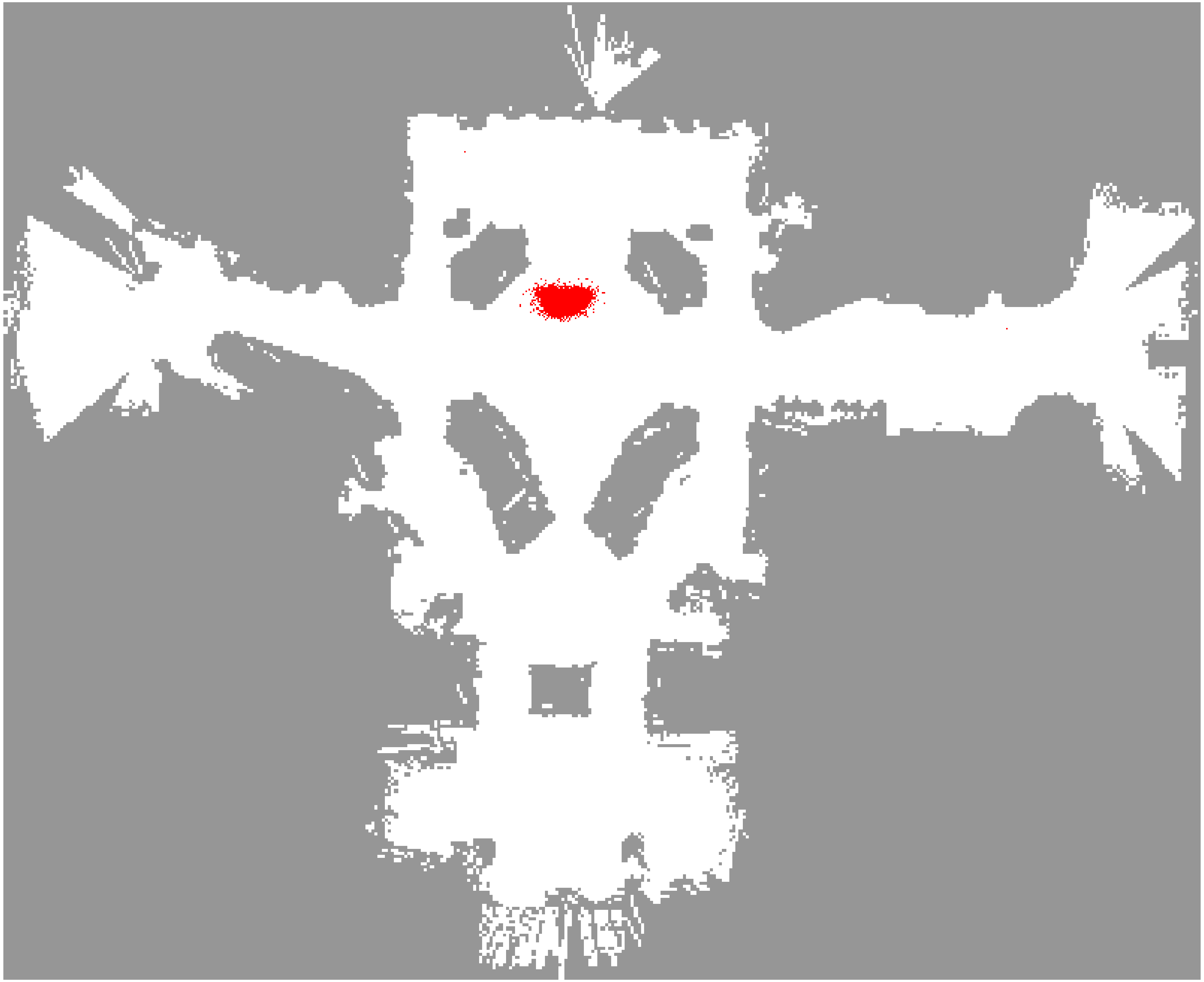


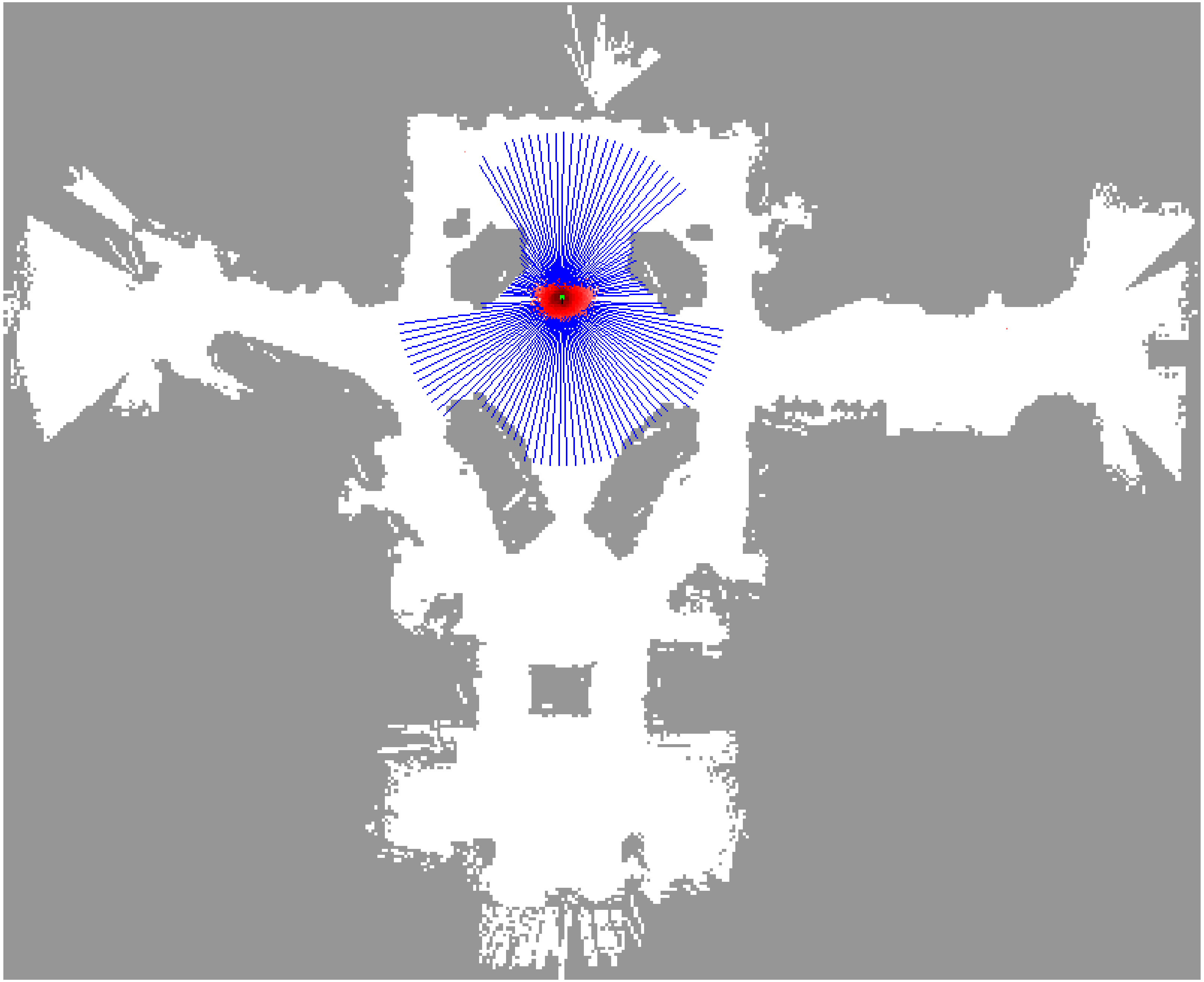


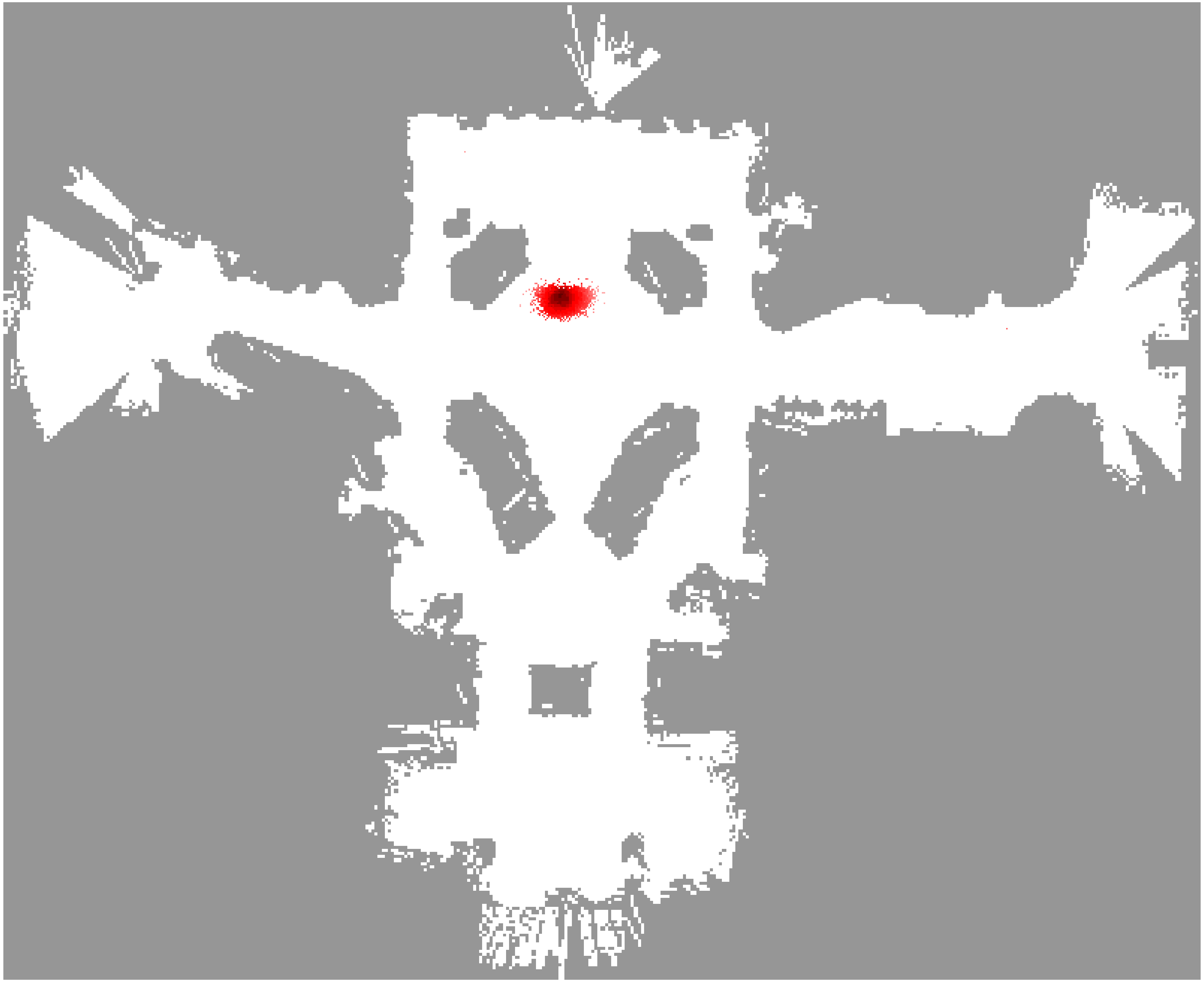


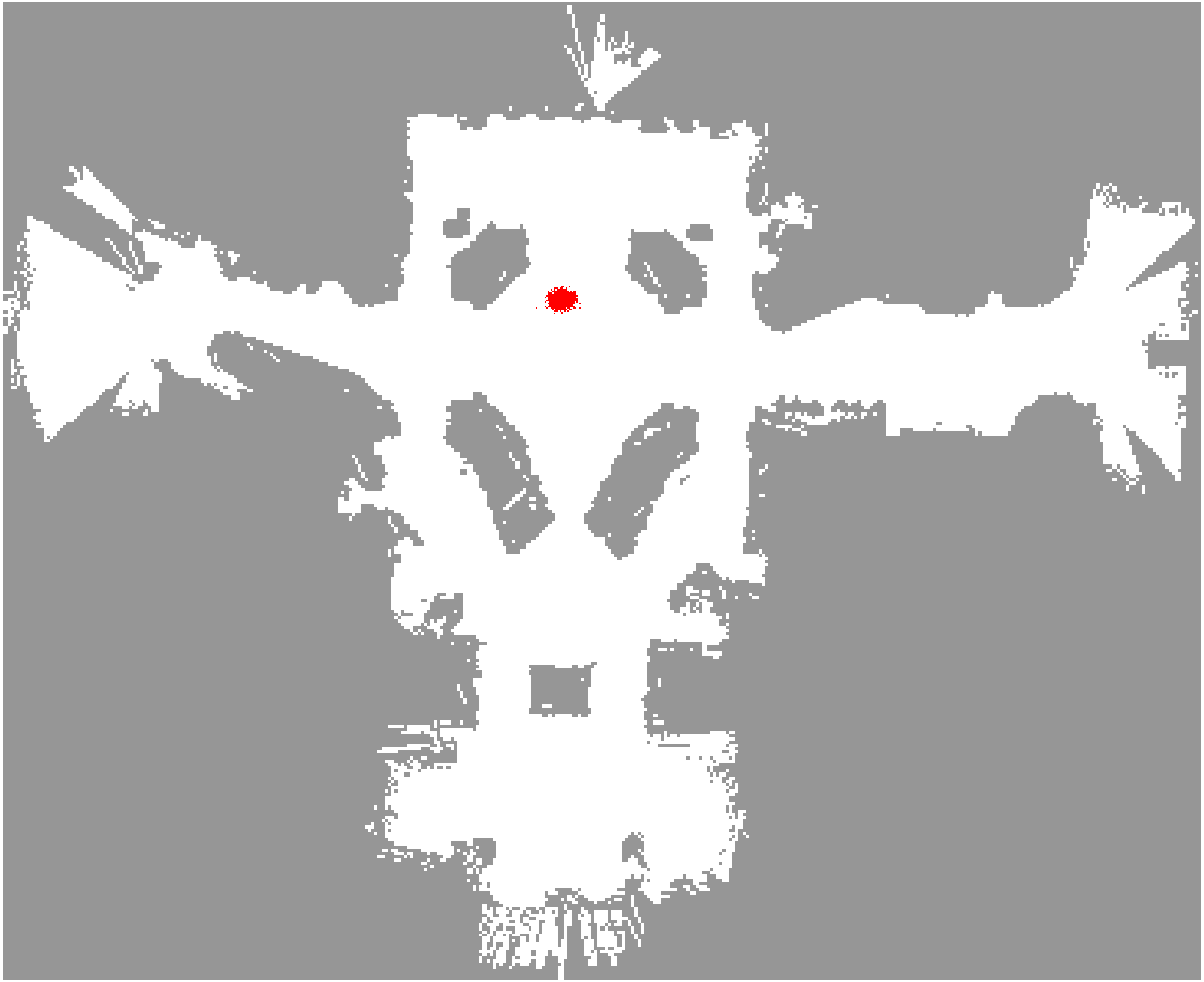


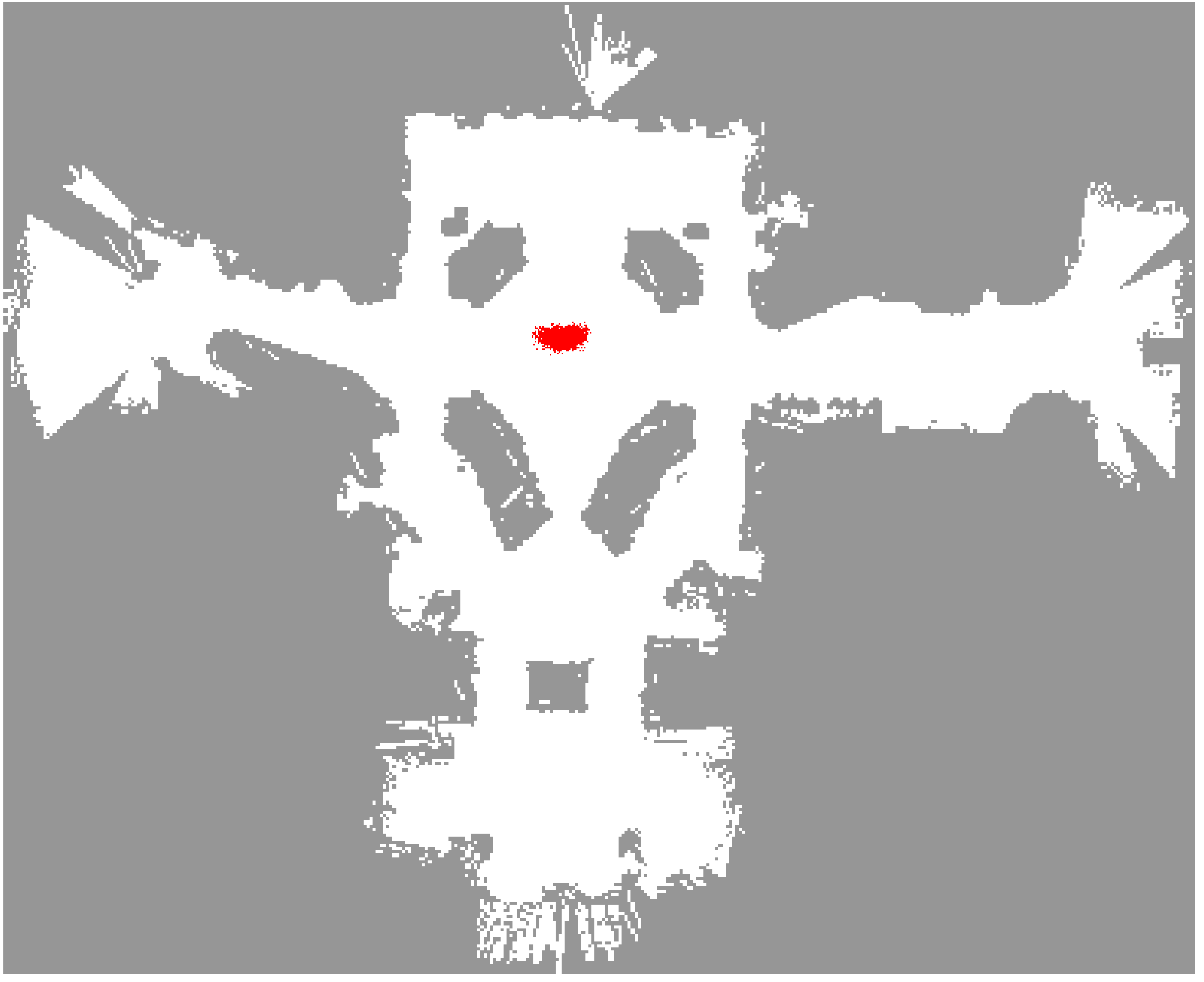


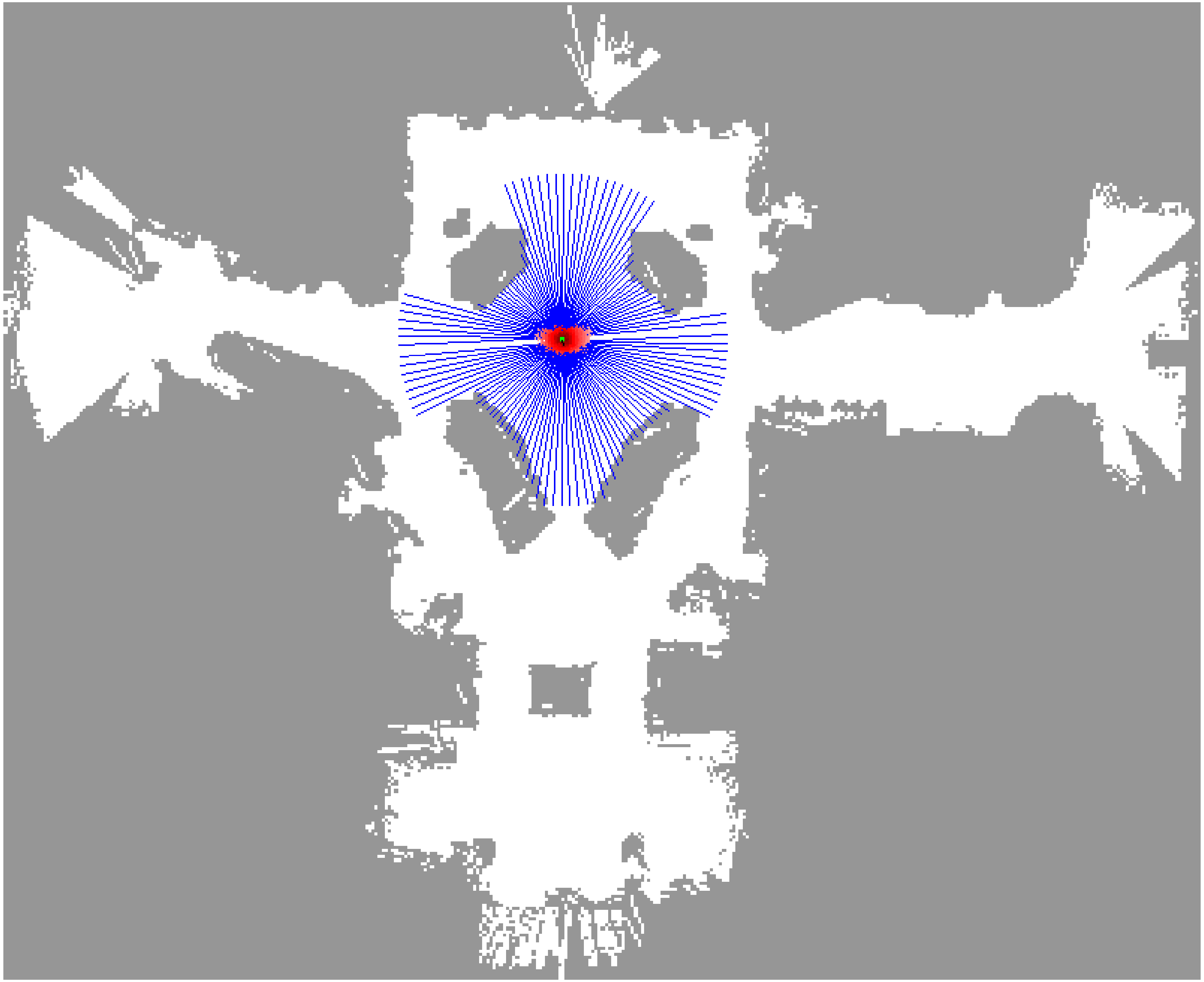


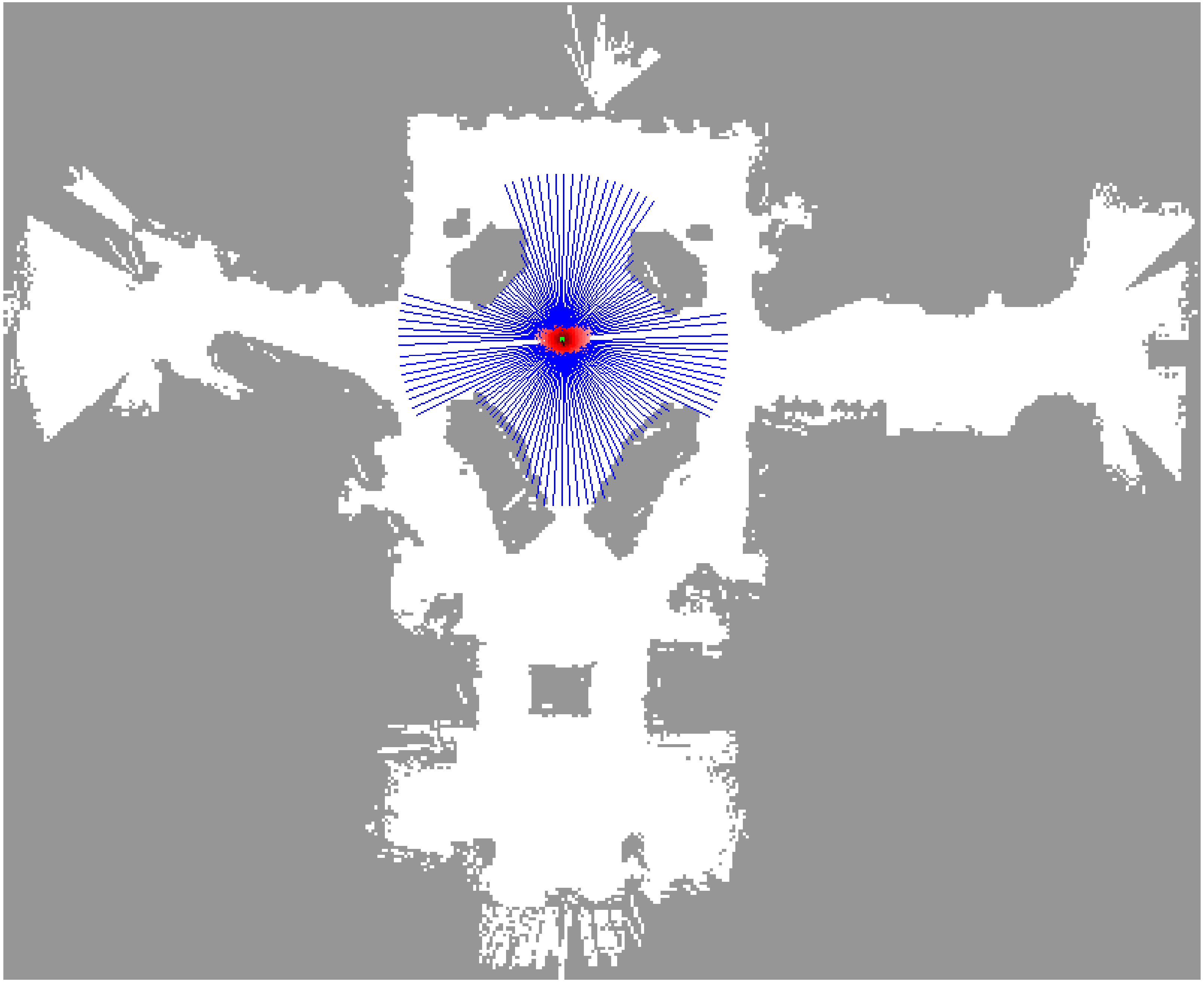




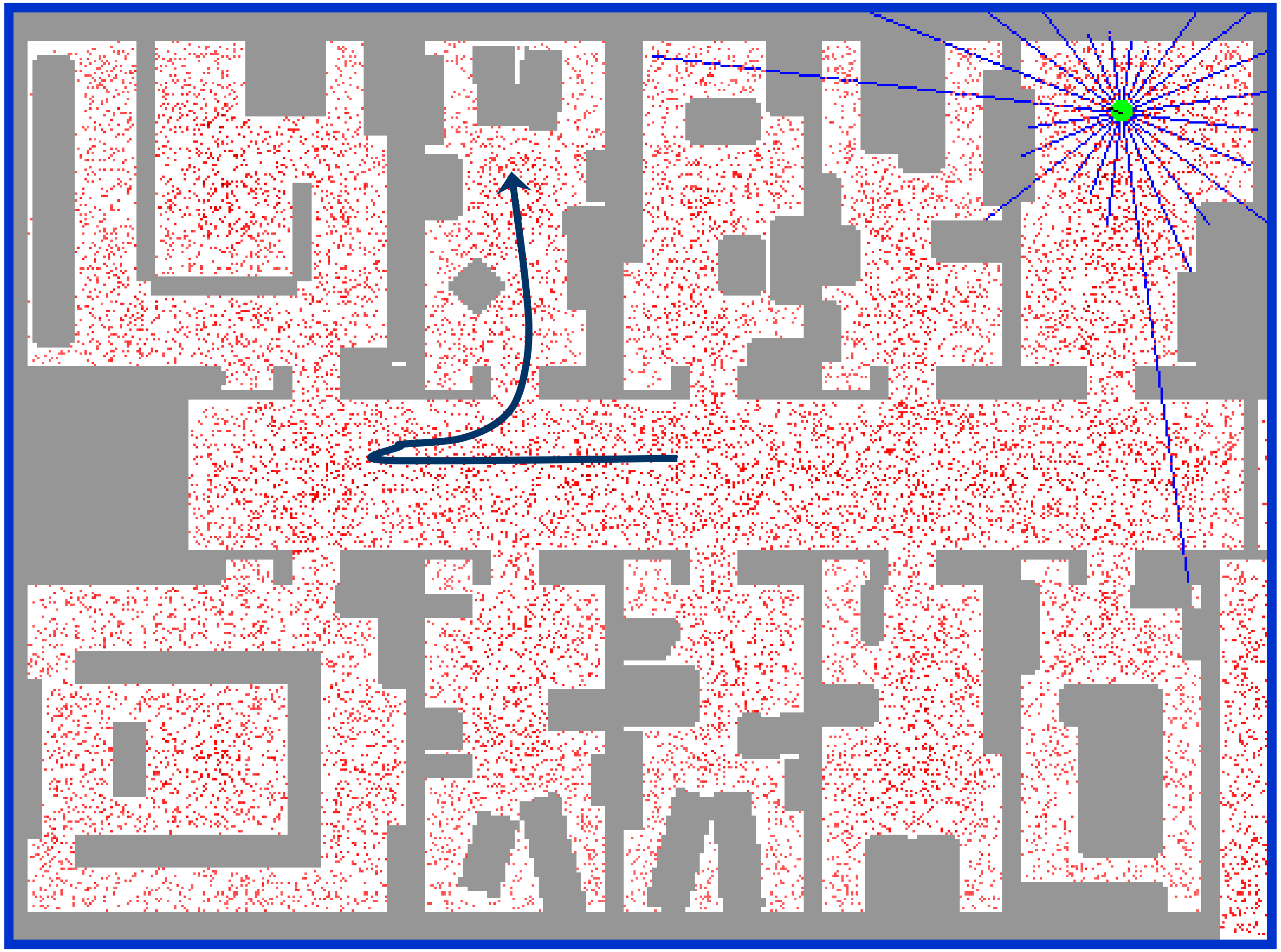




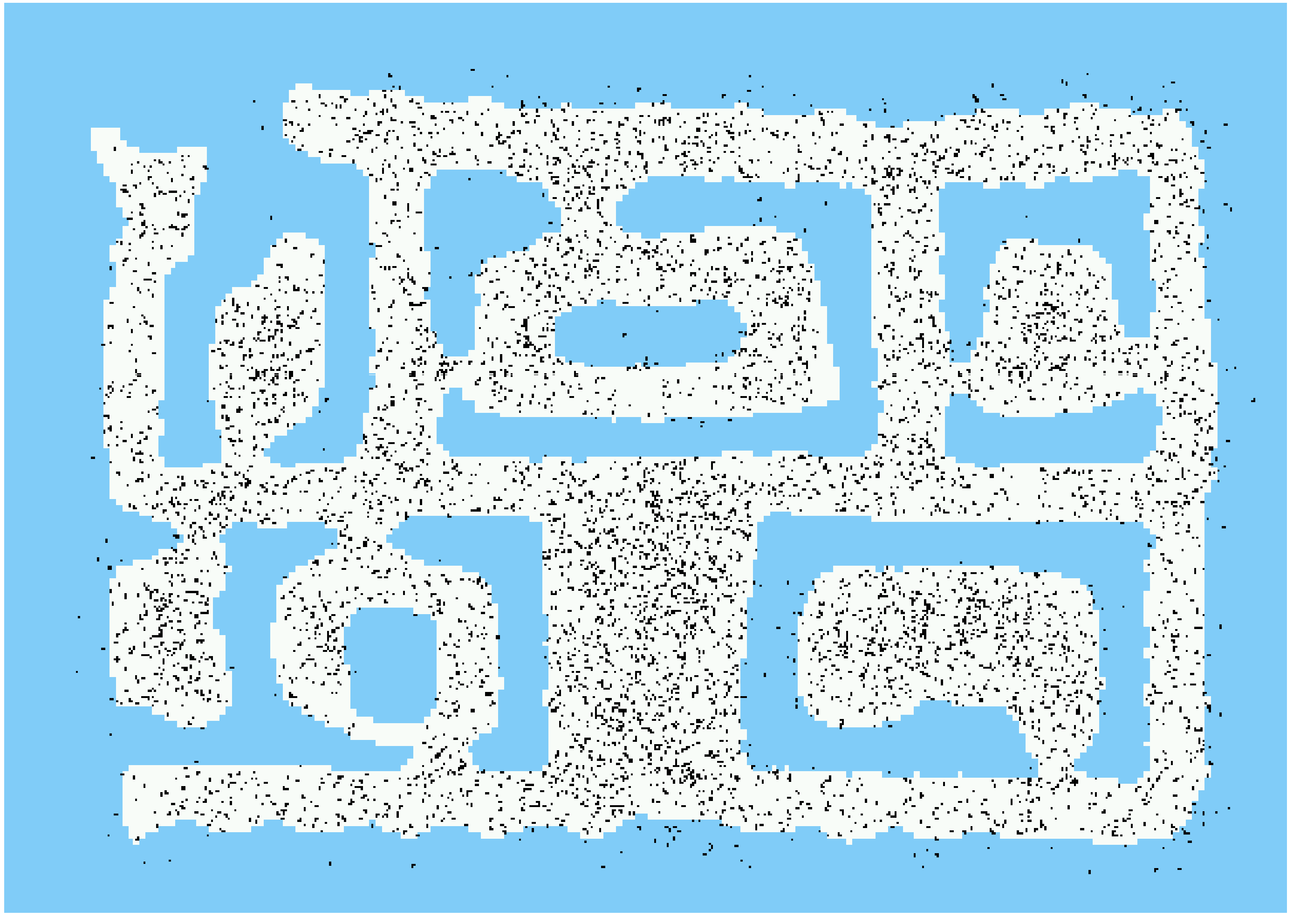




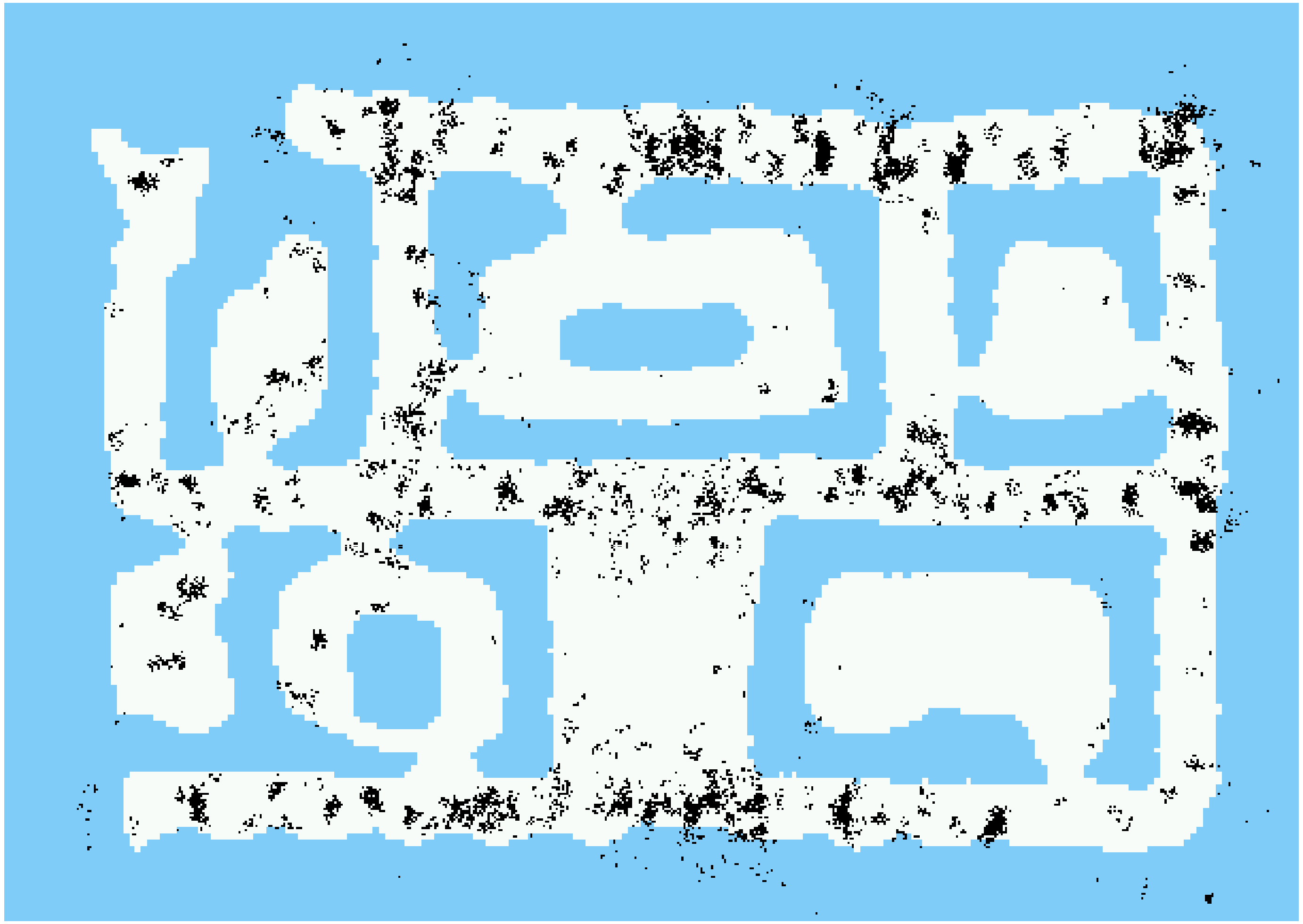
Localizzazione Sample-based (sonar)



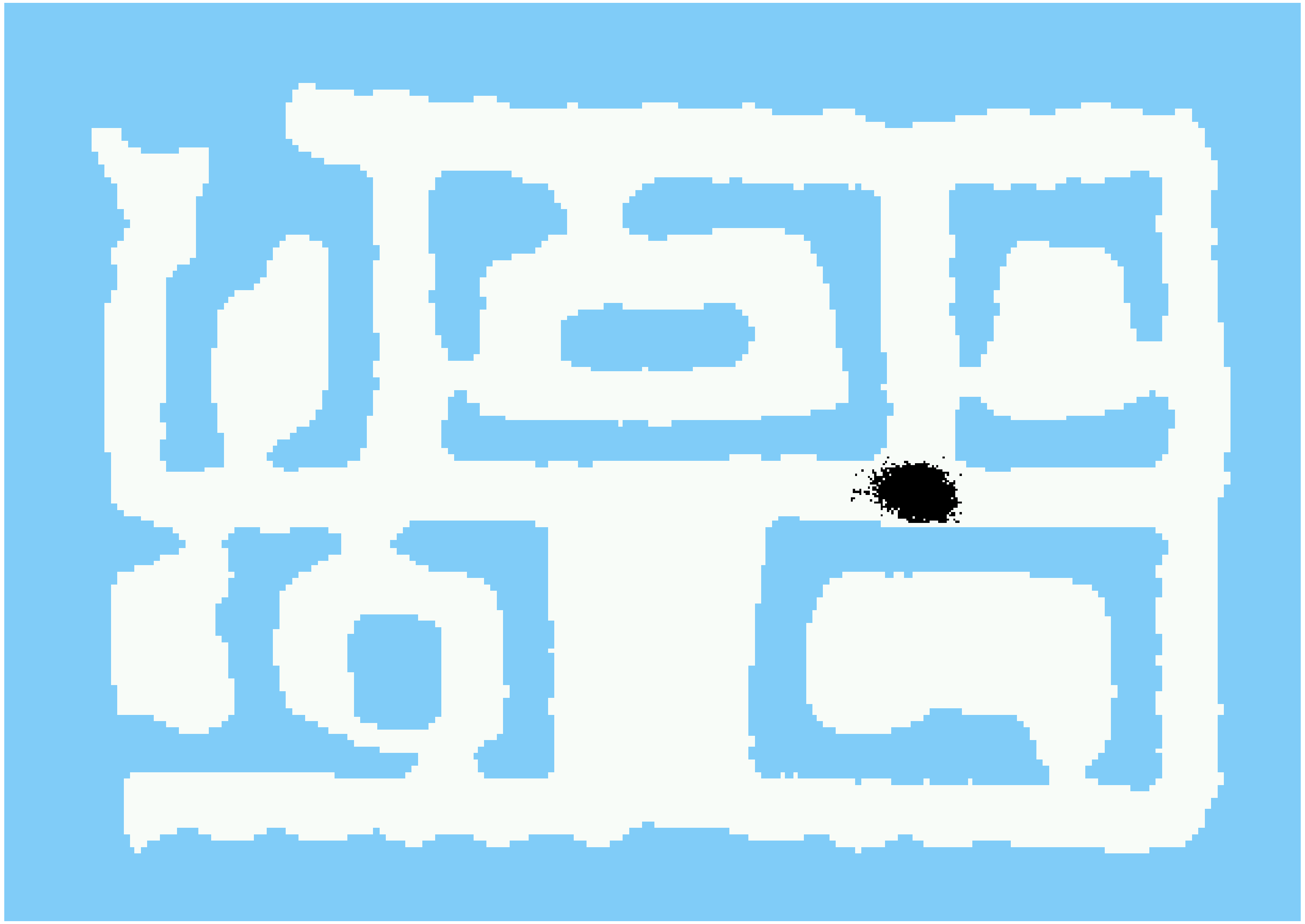
Initial Distribution



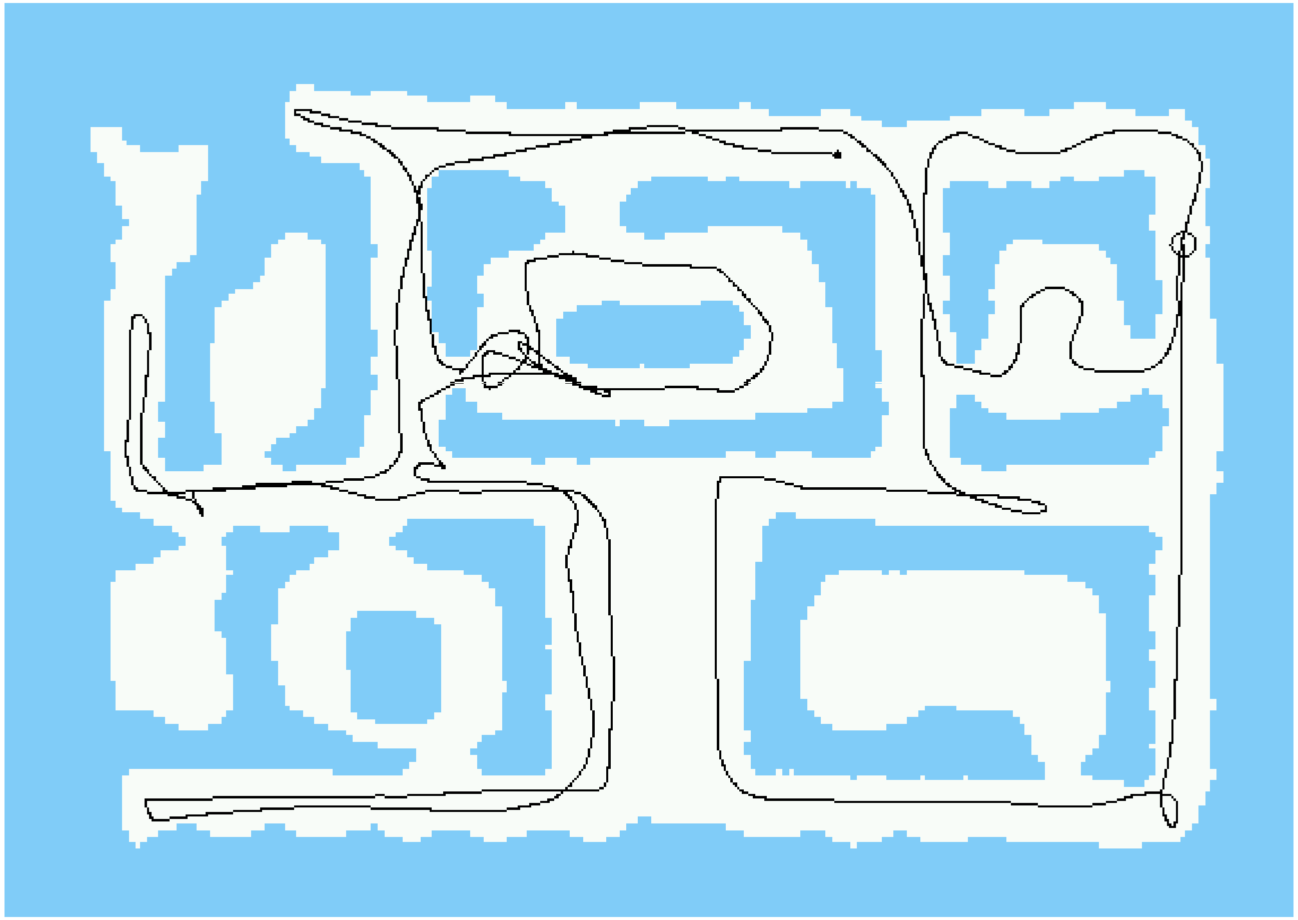
After Incorporating Ten Ultrasound Scans



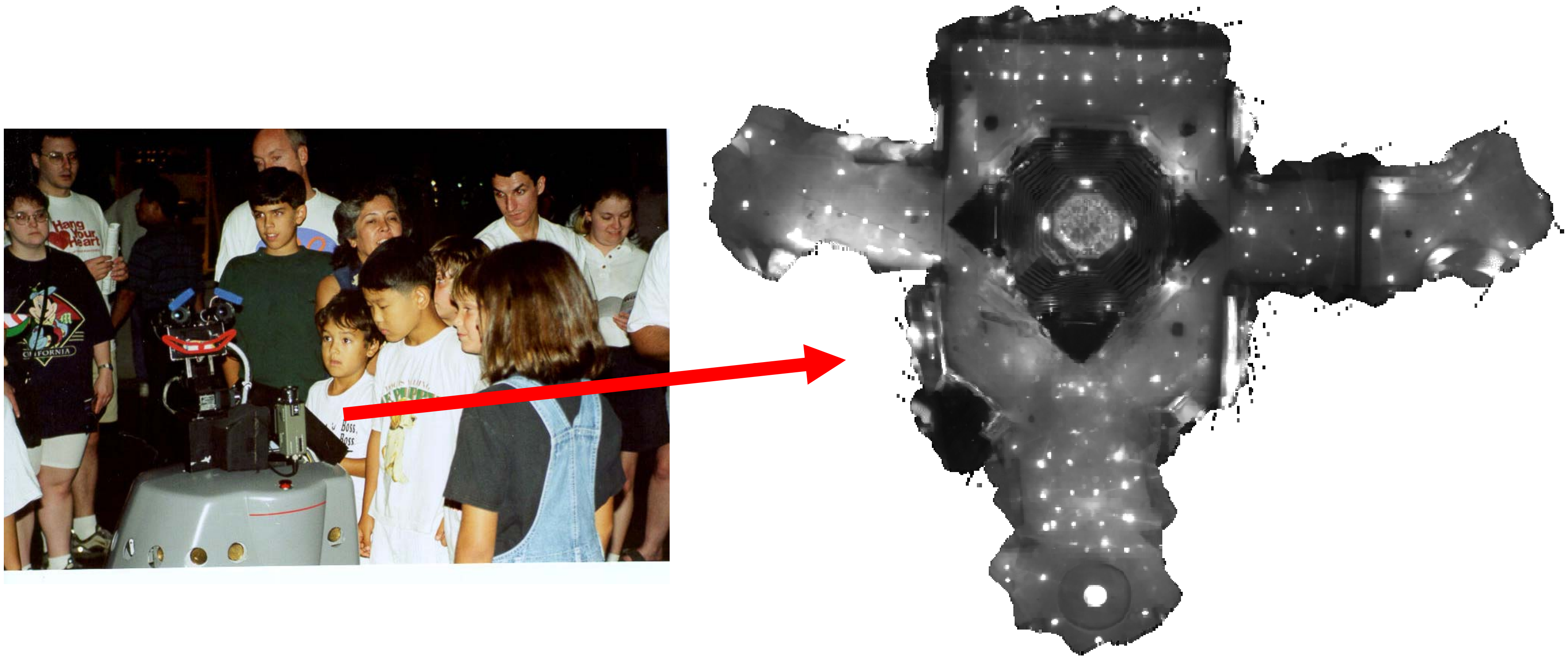
After Incorporating 65 Ultrasound Scans



Percorso stimato



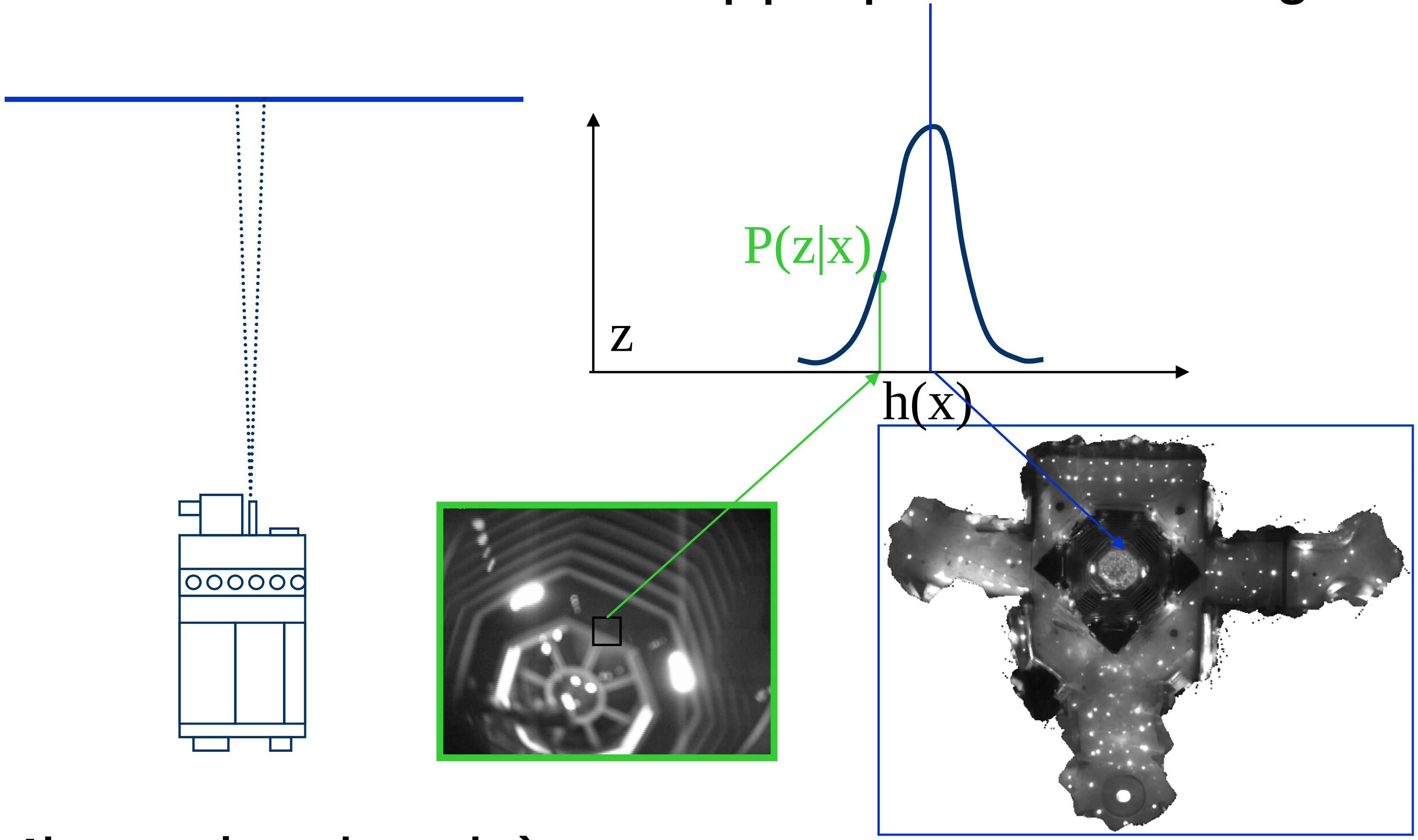
Mappe dei Soffitti per la Localizzazione Vision-based



Mappa di likelihood

Localizzazione Vision-based

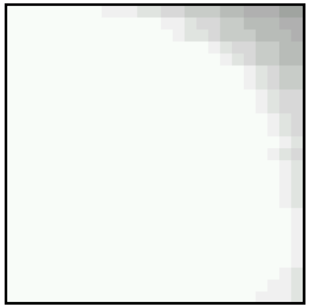
Modello del sensore mappa posa a immagine



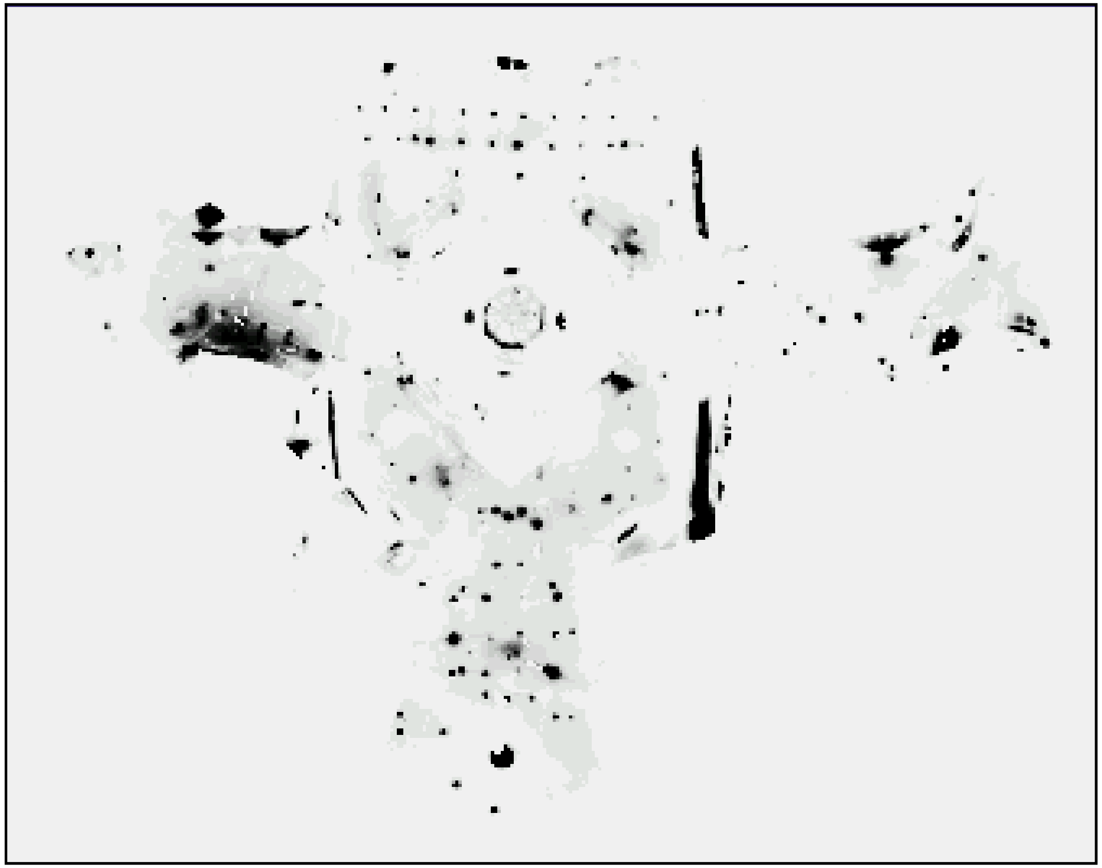
Misura luminosità

Sotto una luce

Measurement z:

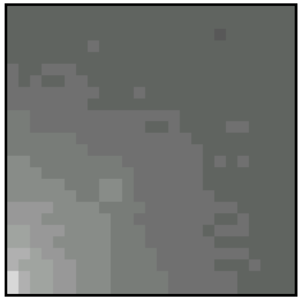


***P(z|x)*:** Mappa di likelihood per luminosità intensa

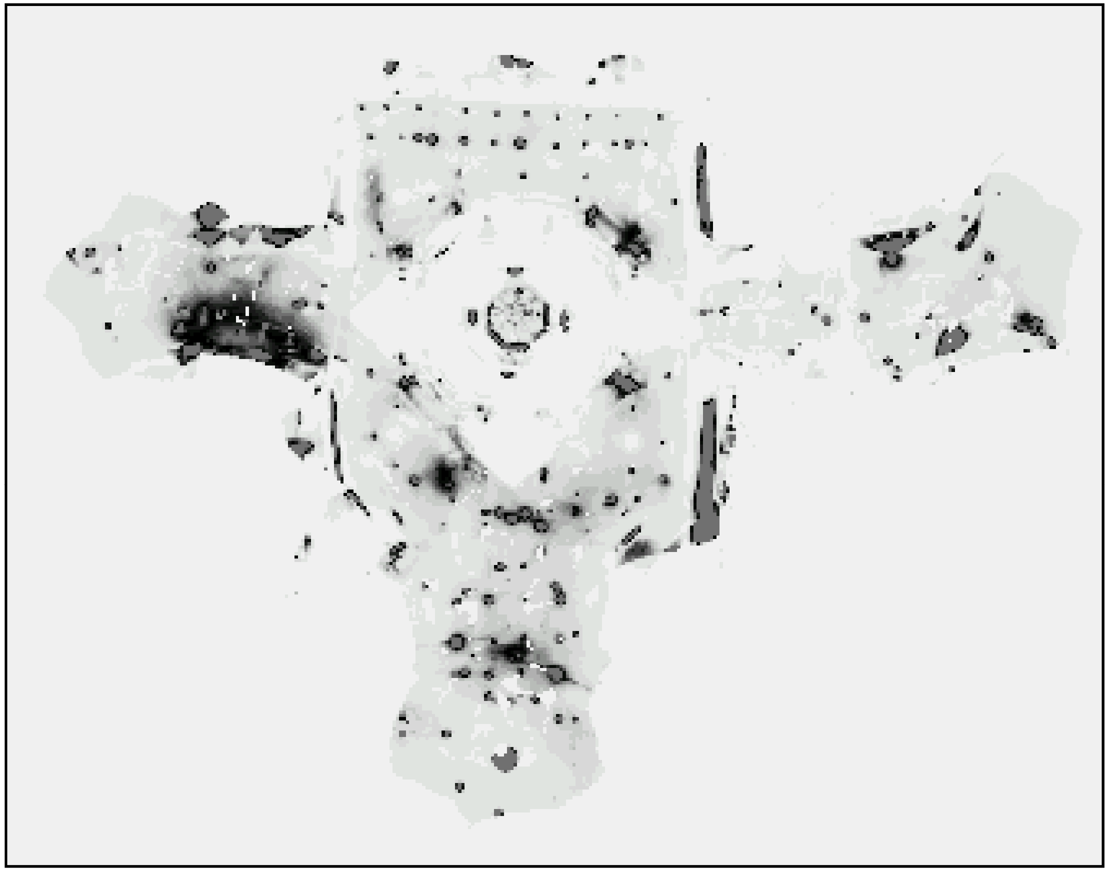


Vicino ad una luce

Measurement z:



***P(z|x)*:** Mappa di likelihood per luminosità meno intensa

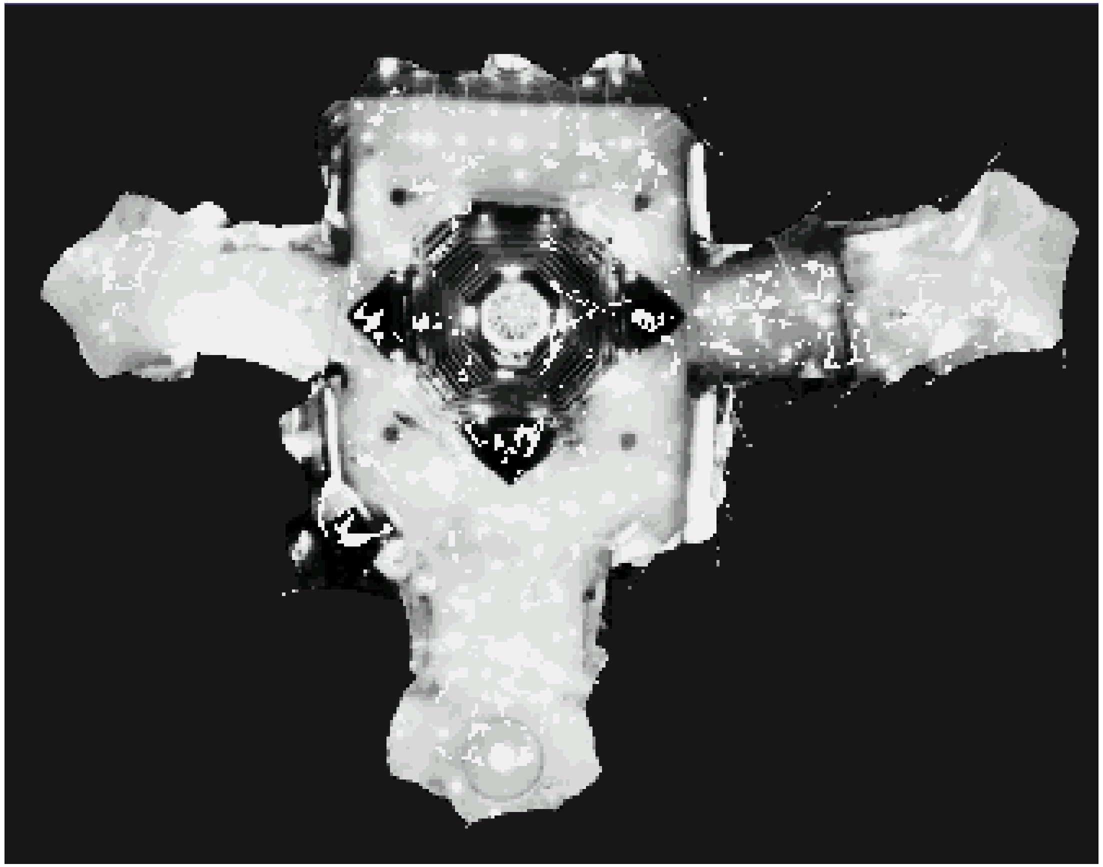


Non sotto una luce

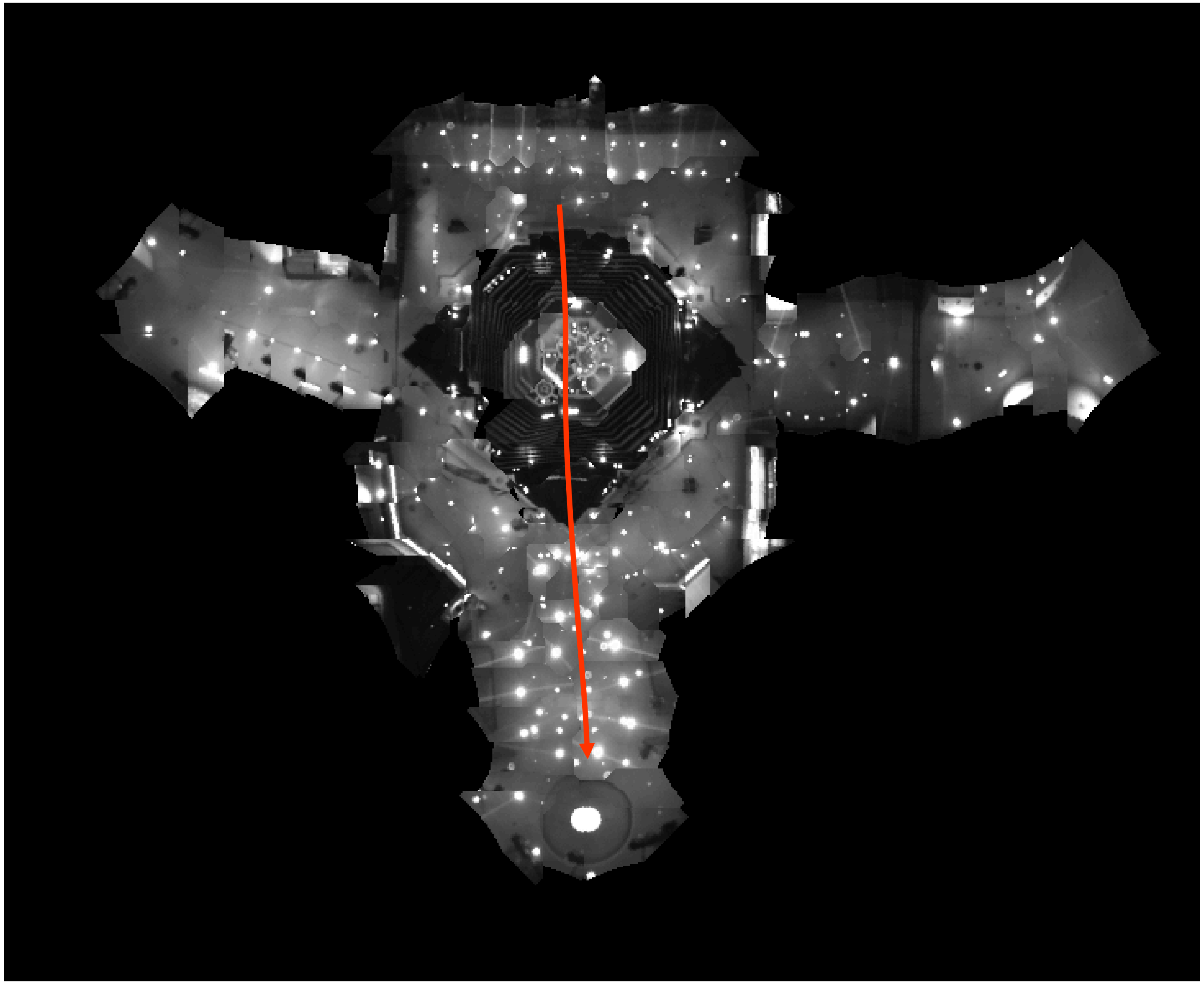
Measurement z:

***P(z|x)*:**

Mappa di likelihood per
poca luminosità



Localizzazione Globale



Robots in Action: Albert

Application: Rhino and Albert Synchronized in Munich and Bonn