The sequence of open and closed prefixes of a Sturmian word

Alessandro De Luca[1] Gabriele Fici[2] Luca Q. Zamboni[3]

[1]Università di Napoli Federico II
[2]Università di Palermo
[3]Université Claude Bernard Lyon 1

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Outline

1. Open and closed words
   - Basic definitions
   - Open and closed prefixes: oc-sequences
2. Main results
   - General properties
   - The case of Sturmian words
   - Algorithms
3. Conclusions

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Closed Words

A finite word is closed if it has a factor which occurs exactly twice, as a prefix and a suffix.

Example

- The words $abaab$ and aaa are closed.
- Single letters are closed too (ε being the relevant factor).

Also known as periodic-like words, or complete (first) returns to a factor.

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Open Words and Characterizations

A word $w \in \Sigma^*$ is open if it is not closed.

Example

The words ab and $ababba$ are both open.

Open/closed words admit several characterizations, e.g.:

Proposition

A word is open iff its longest repeated prefix is right special.

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The oc-sequence

Let w be a finite or infinite word.
The n-th term of the sequence $\mathrm{oc}(w)$ is

- 1 if the prefix $w[1 \ldots n]$ is closed,
- 0 otherwise.

Example

Let $w = abbababbb$. Then
$\mathrm{oc}(w) = 100110010$.

Example (Bucci, De Luca, Fici 2013)

Let $f = abaababaabaababaabab\cdots$ be the Fibonacci word. Then

$$\mathrm{oc}(f) = 10101100\cdots 1^{F_n}0^{F_n}1^{F_{n+1}}0^{F_{n+1}}\cdots$$

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Reformulation

- By definition, the position of the n-th 1 in $\mathrm{oc}(w)$ corresponds to the end of the second occurrence of $w[1 \dots n-1]$, for all $n > 0$.
- Hence, if w has a border of length ℓ, then $|\mathrm{oc}(w)|_1 > \ell$.
- In particular, the period of a closed w equals $1 + |\mathrm{oc}(w)|_0$.

Example

- In $w = abbababbb$, the 3rd 1 in $\mathrm{oc}(w) = 100110010$ is at position 5.
- Correspondingly, the 2nd occurrence of the prefix ab in w ends in the same position.

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Recurrence and Periodicity

Let $w \in \Sigma^\omega$ be an infinite word.

Proposition

$\mathrm{oc}(w)$ *is recurrent* $\iff$ $w = x^\omega$ *for some letter* $x \in \Sigma$
$(\iff \mathrm{oc}(w) = 1^\omega)$.

Proposition

$\mathrm{oc}(w)$ *is ultimately periodic* $\iff$ w *is either periodic or non-recurrent.*

- *If* w *is periodic,* $\mathrm{oc}(w)$ *ends in* 1^ω*.*
- *If* w *is not recurrent,* $\mathrm{oc}(w)$ *ends in* 0^ω*.*

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Run Structure in oc-sequences

Lemma

Let $t, s > 0$ *be such that* $1^t 0^s 1$ *occurs in* $\mathrm{oc}(w)$. *Then* $t \leq s$.

- For example, if a word has 6 consecutive closed prefixes and the next one is open, then so are the next 5.
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In general, unrelated words may share the same oc-sequence:

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And This Makes St Special

- Let C be the family of factorial languages whose elements are determined by their oc-sequences up to isomorphism:

$$C = \{X \subseteq \Sigma^* \mid \mathrm{Fact}(X) \subseteq X \text{ and } \forall u, v \in X, \mathrm{oc}(u) = \mathrm{oc}(v) \Rightarrow u \sim v\}.$$

- C is nonempty (e.g. $\{\varepsilon, a\} \in C$), and every ascending chain $X_1 \subseteq X_2 \subseteq \cdots \subseteq X_n \subseteq \cdots$ of languages in C has the upper bound $\bigcup_{n>0} X_n \in C$.
- By Zorn's lemma, C has a maximal element.

Theorem

The set St of finite Sturmian words is a maximal element of C.

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- Let $\mathcal{C}$ be the family of factorial languages whose elements are determined by their oc-sequences up to isomorphism:

$$\mathcal{C} = \{X \subseteq \Sigma^* \mid \mathrm{Fact}(X) \subseteq X \text{ and } \forall u, v \in X, \mathrm{oc}(u) = \mathrm{oc}(v) \Rightarrow u \sim v\}.$$

- $\mathcal{C}$ is nonempty (e.g. $\{\varepsilon, a\} \in \mathcal{C}$), and every ascending chain $X_1 \subseteq X_2 \subseteq \cdots \subseteq X_n \subseteq \cdots$ of languages in $\mathcal{C}$ has the upper bound $\bigcup_{n>0} X_n \in \mathcal{C}$.
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Theorem

The set St of finite Sturmian words is a maximal element of $\mathcal{C}$.

Characteristic Sturmian Words

Recall that the characteristic Sturmian word of slope $\alpha \in (0,1)$ can be defined as the limit of a standard sequence $(s_n)_{n \geq -1}$, where

- $s_{-1} = b, \quad s_0 = a,$
- $s_{n+1} = s_n^{d_n} s_{n-1}$ for $n \geq 0$,

and $[0; d_0 + 1, d_1, \ldots, d_n, \ldots]$ is the continued fraction expansion of α.

Example

The Fibonacci word has slope $1/\varphi^2 = [0; 2, 1, 1, \ldots]$, i.e., $d_n = 1$ for $n \geq 0$, and the corresponding standard sequence is made of finite Fibonacci words.

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Characterized by Their oc-sequences

Theorem

Let $\mathrm{oc}(w) = 1^{k_0}0^{k'_0}1^{k_1}0^{k'_1}\cdots 1^{k_n}0^{k'_n}1.$
Then w *is prefix of a characteristic Sturmian word* $\Leftrightarrow k_i = k'_i$ *for* $0 \leq i \leq n$.

- Moreover, in such a case k_i is the continuant $K[1, d_0, \ldots, d_{i-1}, d_i - 1]$, for all i.
- Proof uses characterizations of "maximal" open & closed prefixes, in terms of the standard sequence.

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Byproduct: a New Factorization

Theorem

Let w be a characteristic Sturmian word of slope $\alpha < 1/2$ (i.e., starting with a). Then

$$ba^{-1}w = \prod_{n \geq 0} (\widetilde{s_n^{-1} s_{n+1}})^2.$$

That is, the word $ba^{-1}w$, obtained from w by changing its first letter, can be written by concatenating squares of reversed standard words.

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Linear Algorithms for oc-sequences

- Let $B(w)$ be the border array of w,
  whose i-th entry is the length of the longest border of $w[1\ldots i]$.
- Define $B'(w)$ by $B'(w)[i] = \max_{j\leq i} B(w)[j]$.

Proposition

For $i > 1$, $\mathrm{oc}(w)[i] = B'(w)[i] - B'(w)[i-1]$.

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Summary

- We have considered open and closed prefixes of words, and studied the corresponding oc-sequence in general;
- Sturmian words are determined by their oc-sequences up to isomorphism, and make up a maximal factorial language with this property;
- Characteristic Sturmian words are characterized by their simple oc-sequence;
- Linear-time algorithms for calculating oc-sequences, or finite Sturmian word from given oc-sequence.
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