

# A NEW PERSPECTIVE IN SHAPE FROM SHADING FROM SAR IMAGES

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## ABSTRACT

In this paper we propose a new technique for Shape from Shading (SfS) from Synthetic Aperture Radar (SAR) images based on fractals. In particular, the present paper shows that, introducing proper models both for surface and scattering mechanisms, it is possible to estimate with sufficient accuracy the underlying topography also with a very simple and low computational complexity inversion technique. Natural surfaces, which are here of concern, are modeled via fractal geometry, in particular a 2-D fractional Brownian motion (fBm) is used. The scattering mechanisms are described through solutions suitable for fractal surface models; in particular, the Small Perturbation Method (SPM) is used. Considering a simple SAR image model, an appropriate and extremely low-computational inversion technique is used to invert the direct model and estimate the underlying topography. The proposed SAR SfS method is tested and numerically evaluated using an actual SAR image.

## 1. INTRODUCTION

The problem of Shape from Shading is to reconstruct the shape of a surface given a single gray-level image of that surface. The image can be acquired in the optical spectrum (photo) or in the microwave region (radar or SAR image). The main prerequisite to achieve this goal is the knowledge of the reflectivity function of the surface in the spectral region of interest. While very accurate algorithms have been developed for optical images, very little has been obtained from SfS applied to Synthetic Aperture Radar (SAR) images [1], [2], [3]. This is primarily due to the huge number of parameters (like frequency, radar-look angle, resolution, chirp bandwidth, macroscopic and microscopic roughness, local slopes, complex dielectric constant) influencing the surface scattering and then SAR image formation. As a result, SfS is an ill-posed problem: a unique equation dependent on a large number of unknown parameters, of which only local slopes are of interest. In addition, geometrical distortions and speckle cause detrimental effects

on the performance of SAR SfS techniques and have to be properly modeled and faced.

## 2. A NEW PERSPECTIVE FOR SAR SFS

Besides the aforementioned reasons, SAR SfS techniques lay in the widespread use of the Lambertian scattering model [3] (or its generalization [2]), that, although valid in optics, is very inaccurate in describing scattering from natural surfaces in the microwave region of the spectrum, where SAR systems usually operates. In order to overcome this issues and increase performances of SfS applied to SAR images, the direct process – linking the SAR image to the parameters of interest, i.e., the local slopes – needs a proper modelization. To this aim, the forward model is divided in three steps:

- Surface model

Numerous theoretical and experimental studies have assessed that fractal geometry represents the best tool for describing natural surfaces [4], [5]. In this paper, natural surfaces are modeled through a 2-D fBm [6]:

$$\Pr\{z(x, y) - z(x', y') < \bar{\zeta}\} = \frac{1}{\sqrt{2\pi} s \tau^H} \int_{-\infty}^{\bar{\zeta}} \exp\left(-\frac{\zeta^2}{2s^2 \tau^{2H}}\right) d\zeta \quad (1)$$

where Pr stands for “probability”,  $\tau$  is the distance between the two considered points of coordinates  $(x, y)$  and  $(x', y')$ ;  $H$  is the Hurst coefficient ( $0 < H < 1$ );  $s$  is the incremental standard deviation.

- Scattering model

Scattering mechanisms are here described via the SPM suitable for fractal surfaces thanks to its simplicity and validity limits adequate for SAR systems [7]:

$$\sigma_{mn}^0 = 2\pi 8k^4 \cos^4 \theta |\beta_{mn}|^2 \frac{S_0}{(2k \sin \vartheta)^{2+2H}} \quad (2)$$

TABLE I  
PERFORMANCE INDICATORS FOR THE FRACTAL AND LAMBERTIAN MODEL FOR THE SAR IMAGE OF THE VESUVIUS VOLCANO

Error magnitude		Altitude (m)			Range slope (°)			Azimuth slope (°)			
		Median	Mean	Std dev.	Median	Mean	Std dev.	Median	Mean	Std dev.	
Before azimuth filtering	Fractal Model	142.5	166.8	120.8	9.62	11.95	10.73	21.57	27.39	22.27	
	Lambertian Model	267.2	413.6	446.2	25.58	26.39	17.99	69.50	65.51	28.34	
After azimuth filtering	Unknown starting points	Fractal Model	142.4	166.8	120.7	9.60	11.94	10.63	12.21	15.30	13.16
		Lambertian Model	208.5	364.0	444.2	21.15	26.01	21.91	36.18	39.68	27.06
	Known starting points	Fractal Model	98.7	126.3	105.3	9.60	11.94	10.63	9.67	14.14	15.20
		Lambertian Model	155.8	321.1	454.5	21.15	26.01	21.91	33.98	37.87	26.73

wherein  $k$  is the electromagnetic wavenumber of the incident field;  $S_0$  is the spectral amplitude of the fBM surface;  $\beta_{mn}$  is a coefficient depending on transmitted and received signal polarization and the radar look angle  $\theta$ .

- Imaging model

In order to link the SAR intensity map to the backscattering coefficient and then to the local slopes, a SAR imaging model is needed. To this aim, the intensity of a SAR image can be assumed to be equal to the electromagnetic energy backscattered from the resolution cell.

In order to simplify the inversion technique, a small-slopes regime for the surface is assumed and then a first-order approximation of the image intensity can be evaluated with regard to the local range slopes of the observed surface  $p$  (in a first order approximation no dependence on the azimuth slopes is present):

$$I = G(a_0 + a_1 p) \quad (3)$$

where  $a_0$  and  $a_1$  are the coefficients of the expansion depending on the forward model and the geometric and electromagnetic parameters of both the surface and the sensor. Absolute calibration constant  $G$  estimation needs great care because it greatly affects the accuracy of the surface reconstruction. To this aim, we propose a simple estimation method from data:

$$G = \frac{\langle I \rangle}{a_0} \quad (4)$$

where  $\langle I \rangle$  represents the mean value of the intensity map and is assumed to be equal to the intensity linked to a flat surface.

To take into account the azimuth slopes (which are usually neglected in other techniques [1], [2] or introduced using polarimetric concepts [3]) we propose the use of a regularization procedure based on Bayesian Minimum Mean Squared Error (MMSE) estimation.

The result obtained applying the proposed technique on a multilook Cosmo/SkyMed stripmap SAR image of the Somma-Vesuvius volcanic complex, close to Naples, Italy, are presented in Fig. 1 and Fig. 2. In particular in Fig. 1

(c)-(d) the estimated DEMs before and after the azimuth regularization procedure are shown assuming unknown starting points. As can be seen, the azimuth filtering greatly reduces the linage effects clearly visible in Fig. 1 (c). In Fig. 2 range and azimuth profiles of the estimated DEM are reported and compared with those obtained using a Lambertian model. A quantitative assessment of the proposed method is reported in Table I, where the benefits provided by the fractal model can be clearly appreciated.

### 3. ACKNOWLEDGEMENT

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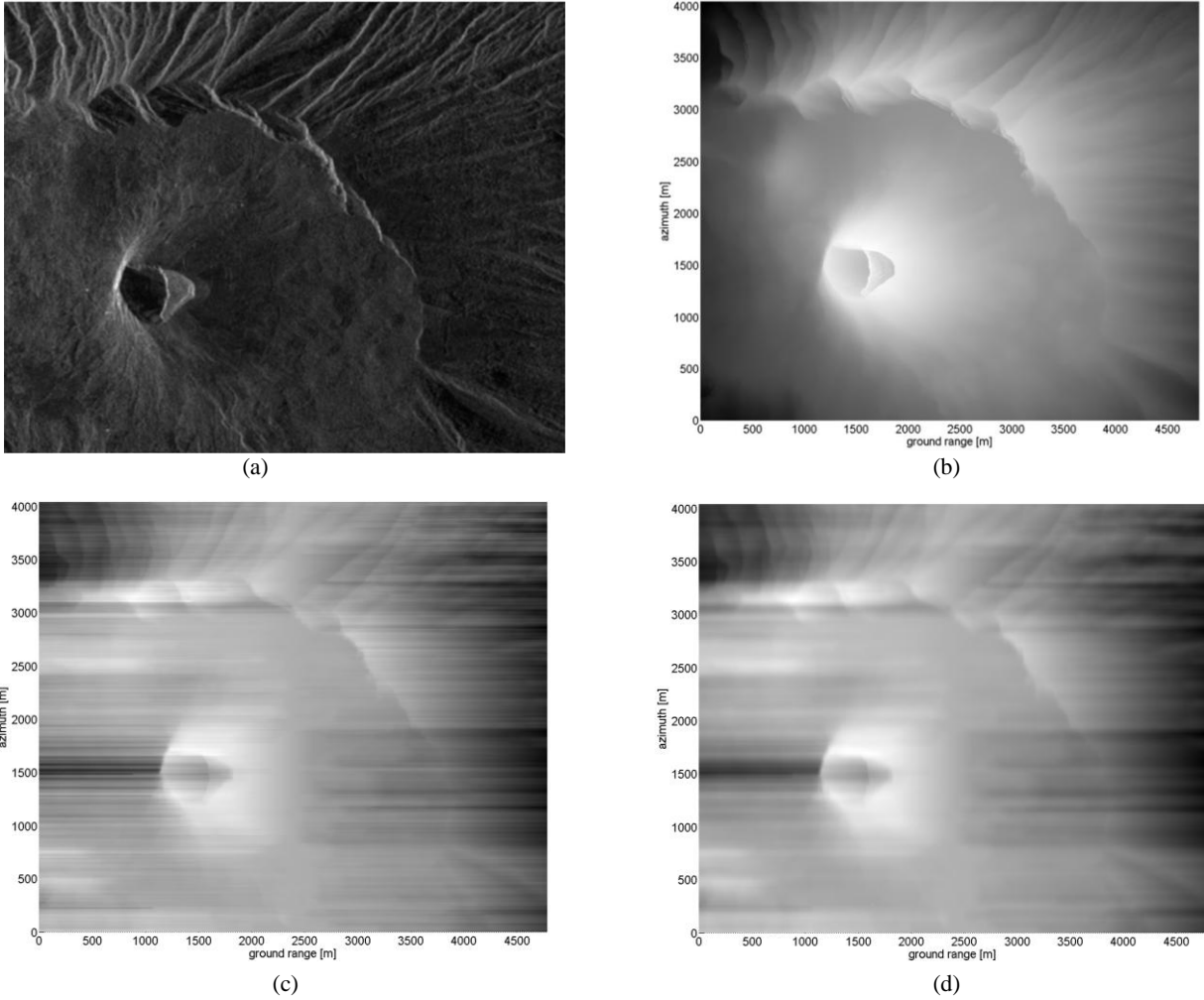


Fig. 1 Cosmo/SkyMed SAR image of the Vesuvius complex (a); ground-truth DEM in azimuth-slant range coordinates (b) with lines identifying range and azimuth cuts; obtained SfS DEM before (c) and after (d) azimuth regularization procedure assuming unknown starting points in the range integration step.

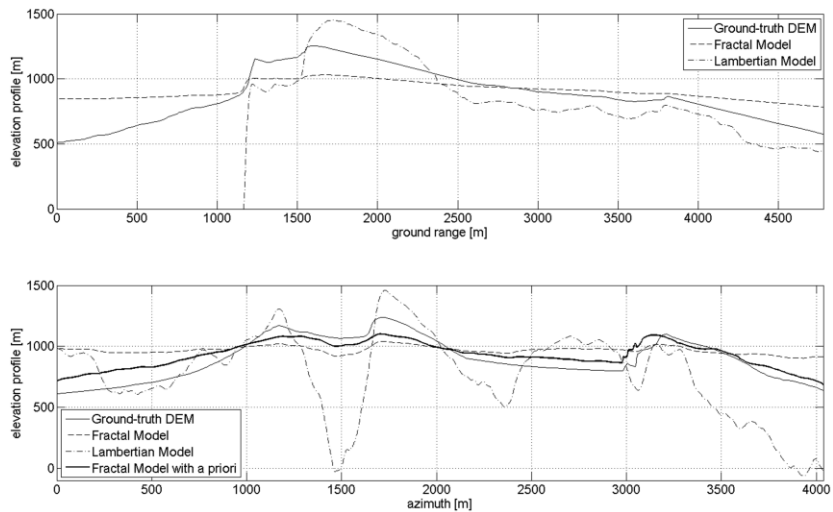


Fig. 2 Range (*top*) and azimuth (*bottom*) cuts of the obtained SfS DEM. Better results provided by the fractal model are visible especially in the azimuth cut, where a priori knowledge about starting points in the range integration step can improve performances.