Corso di
Complementi di Gasdinamica

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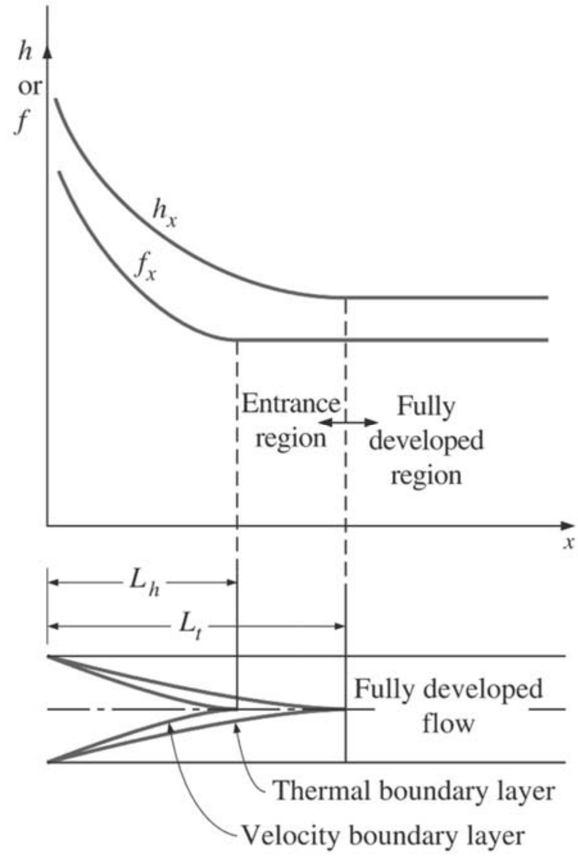
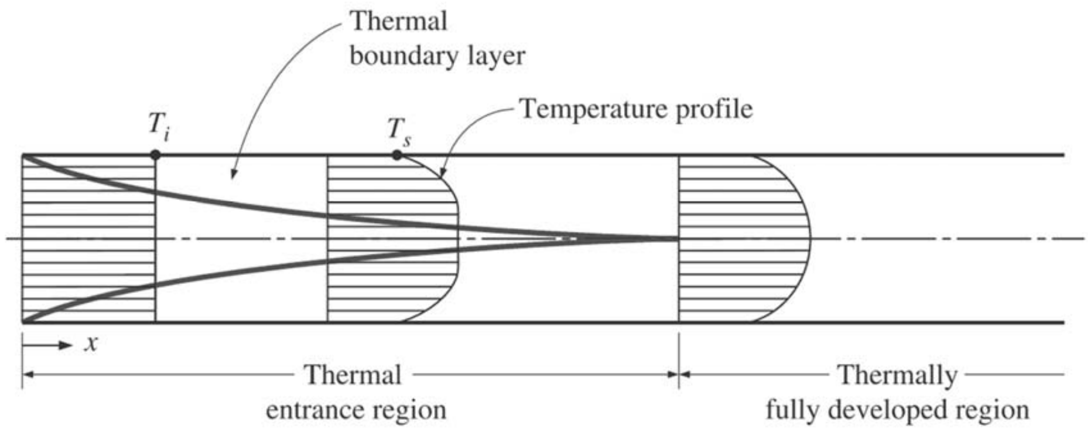
SCAMBIO TERMICO IN CONDOTTI

Variation of the friction factor and the convection heat transfer coefficient in the flow direction for flow in a tube (Pr > 1).



SCAMBIO TERMICO IN CONDOTTI



The concept of a fully developed flow, from the thermal standpoint, is a little more complicated. We must first understand the notion of the *mixing-cup*, or *bulk*, enthalpy and temperature, $\hat{h}_b$ and T_b. The enthalpy is of interest because we use it in writing the First Law of Thermodynamics when calculating the inflow of thermal energy and flow work to open control volumes. The bulk enthalpy is an average enthalpy for the fluid

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flowing through a cross section of the pipe:

$$\dot{m}\,\hat{h}_b \equiv \int_{A_c} \rho u \hat{h}\, dA_c \tag{7.3}$$

If we assume that fluid pressure variations in the pipe are too small to affect the thermodynamic state much (see Sect. 6.3) and if we assume a constant value of c_p, then $\hat{h} = c_p(T - T_{\text{ref}})$ and

$$\dot{m}\,c_p\,(T_b - T_{\text{ref}}) = \int_{A_c} \rho c_p u\,(T - T_{\text{ref}})\; dA_c \tag{7.4}$$

or simply

$$T_b = \frac{\int_{A_c} \rho c_p u T\, dA_c}{\dot{m} c_p} \tag{7.5}$$

In words, then,

$$T_b \equiv \frac{\text{rate of flow of enthalpy through a cross section}}{\text{rate of flow of heat capacity through a cross section}}$$

Thus, if the pipe were broken at any x-station and allowed to discharge into a mixing cup, the enthalpy of the mixed fluid in the cup would equal the average enthalpy of the fluid flowing through the cross section, and the temperature of the fluid in the cup would be T_b. This definition of T_b is perfectly general and applies to either laminar or turbulent flow. For a circular pipe, with $dA_c = 2\pi r\, dr$, eqn. (7.5) becomes

$$T_b = \frac{\int_0^R \rho c_p u T\, 2\pi r\, dr}{\int_0^R \rho c_p u\, 2\pi r\, dr} \tag{7.6}$$

A fully developed flow, from the thermal standpoint, is one for which the relative shape of the temperature profile does not change with x. We state this mathematically as

$$\frac{\partial}{\partial x}\left(\frac{T_w - T}{T_w - T_b}\right) = 0 \tag{7.7}$$

where T generally depends on x and r. This means that the profile can be scaled up or down with $T_w - T_b$. Of course, a flow must be hydrodynamically developed if it is to be thermally developed.

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You will recall that in the absence of any work interactions (such as electric resistance heating), the conservation of energy equation for the steady flow of a fluid in a tube can be expressed as (Fig. 8–10)

$$\dot{Q} = \dot{m} C_p (T_e - T_i) \qquad \text{(W)} \tag{8-15}$$

where T_i and T_e are the mean fluid temperatures at the inlet and exit of the tube, respectively, and $\dot{Q}$ is the rate of heat transfer to or from the fluid. Note that the temperature of a fluid flowing in a tube remains constant in the absence of any energy interactions through the wall of the tube.

The thermal conditions at the surface can usually be approximated with reasonable accuracy to be *constant surface temperature* (T_s = constant) or *constant surface heat flux* ($\dot{q}_s$ = constant). For example, the constant surface temperature condition is realized when a phase change process such as boiling or condensation occurs at the outer surface of a tube. The constant surface heat flux condition is realized when the tube is subjected to radiation or electric resistance heating uniformly from all directions.

SCAMBIO TERMICO IN CONDOTTI

Surface heat flux is expressed as

$$\dot{q}_s = h_x(T_s - T_m) \qquad (\mathrm{W/m^2}) \tag{8-16}$$

where h_x is the *local* heat transfer coefficient and T_s and T_m are the surface and the mean fluid temperatures at that location. Note that the mean fluid temperature T_m of a fluid flowing in a tube must change during heating or cooling. Therefore, when $h_x = h =$ constant, the surface temperature T_s must change when $\dot{q}_s =$ constant, and the surface heat flux $\dot{q}_s$ must change when $T_s =$ constant. Thus we may have either $T_s =$ constant or $\dot{q}_s =$ constant at the surface of a tube, but not both. Next we consider convection heat transfer for these two common cases.

FLUSSO TERMICO COSTANTE

In the case of $\dot{q}_s =$ constant, the rate of heat transfer can also be expressed as

$$\dot{Q} = \dot{q}_s A_s = \dot{m} C_p (T_e - T_i) \qquad (\mathrm{W}) \tag{8-17}$$

Then the mean fluid temperature at the tube exit becomes

$$T_e = T_i + \frac{\dot{q}_s A_s}{\dot{m} C_p} \tag{8-18}$$

Note that the mean fluid temperature increases *linearly* in the flow direction in the case of constant surface heat flux, since the surface area increases linearly in the flow direction (A_s is equal to the perimeter, which is constant, times the tube length).

The surface temperature in the case of constant surface heat flux $\dot{q}_s$ can be determined from

$$\dot{q}_s = h(T_s - T_m) \longrightarrow T_s = T_m + \frac{\dot{q}_s}{h} \tag{8-19}$$

In the fully developed region, the surface temperature T_s will also increase linearly in the flow direction since h is constant and thus $T_s - T_m =$ constant (Fig. 8–11). Of course this is true when the fluid properties remain constant during flow.

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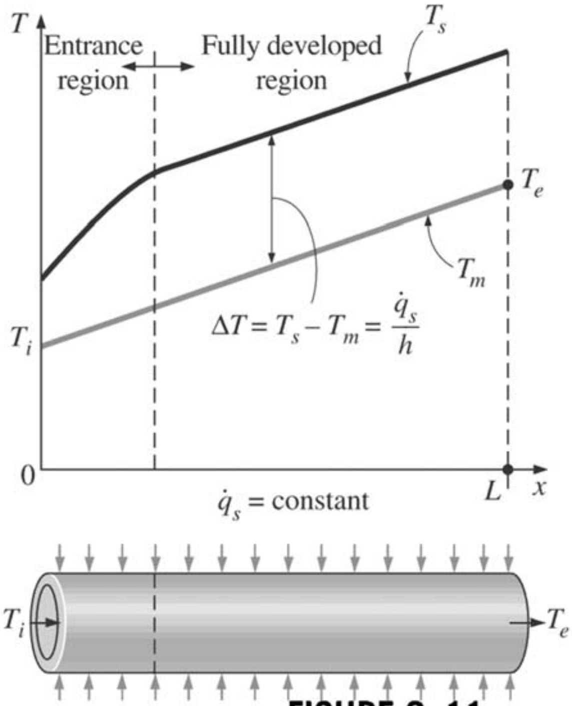


FIGURE 8–11
Variation of the *tube surface* and the *mean fluid* temperatures along the tube for the case of constant surface heat flux.

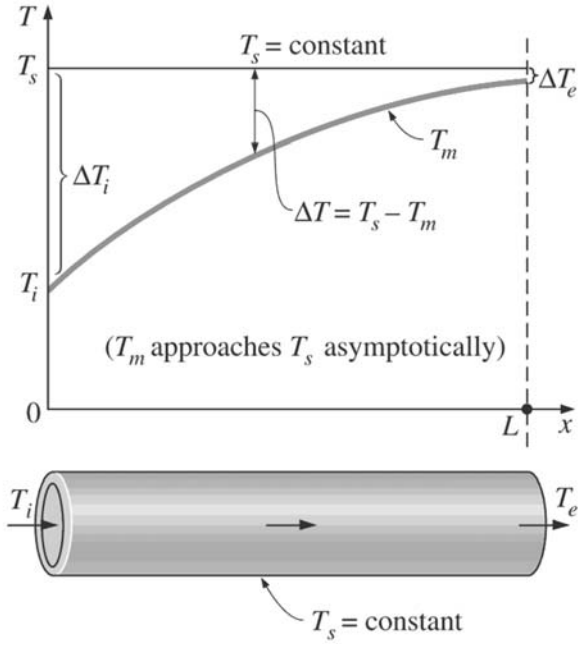


FIGURE 8–14
The variation of the *mean fluid* temperature along the tube for the case of constant temperature.

FLUSSO TERMICO COSTANTE

The slope of the mean fluid temperature T_m on a T-x diagram can be determined by applying the steady-flow energy balance to a tube slice of thickness dx shown in Figure 8–12. It gives

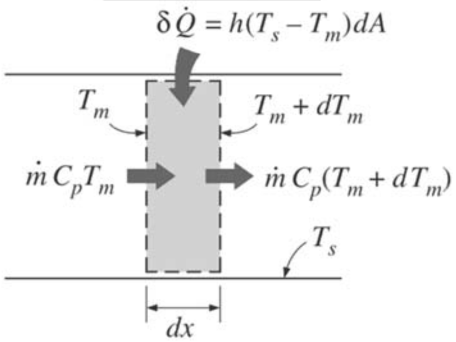
$$\dot{m} C_p \, dT_m = \dot{q}_s (p dx) \longrightarrow \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m} C_p} = \text{constant} \qquad \textbf{(8-20)}$$

where p is the perimeter of the tube.

Noting that both $\dot{q}_s$ and h are constants, the differentiation of Eq. 8–19 with respect to x gives

$$T_s = T_m + \frac{\dot{q}_s}{h}$$

$$\frac{dT_m}{dx} = \frac{dT_s}{dx} \qquad \textbf{(8-21)}$$



$$\frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m} C_p}$$

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$$\frac{dT_m}{dx} = \frac{dT_s}{dx}$$

Also, the requirement that the dimensionless temperature profile remains unchanged in the fully developed region gives

$$\frac{\partial}{\partial x}\left(\frac{T_s - T}{T_s - T_m}\right) = 0 \longrightarrow \frac{1}{T_s - T_m}\left(\frac{\partial T_s}{\partial x} - \frac{\partial T}{\partial x}\right) = 0 \longrightarrow \frac{\partial T}{\partial x} = \frac{dT_s}{dx} \quad \textbf{(8-22)}$$

since $T_s - T_m$ = constant. Combining Eqs. 8–20, 8–21, and 8–22 gives

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m} C_p} = \text{constant} \quad \textbf{(8-23)}$$

Then we conclude that *in fully developed flow in a tube subjected to constant surface heat flux, the temperature gradient is independent of x and thus the shape of the temperature profile does not change along the tube* (Fig. 8–13).

For a circular tube, $p = 2\pi R$ and $\dot{m} = \rho \mathcal{V}_m A_c = \rho \mathcal{V}_m (\pi R^2)$, and Eq. 8–23 becomes

Circular tube:
$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{2\dot{q}_s}{\rho \mathcal{V}_m C_p R} = \text{constant} \quad \textbf{(8-24)}$$

where $\mathcal{V}_m$ is the mean velocity of the fluid.

TEMPERATURA COSTANTE

Consider the heating of a fluid in a tube of constant cross section whose inner surface is maintained at a constant temperature of T_s. We know that the mean temperature of the fluid T_m will increase in the flow direction as a result of heat transfer. The energy balance on a differential control volume shown in Figure 8–12 gives

$$\dot{m} C_p dT_m = h(T_s - T_m) dA_s \quad \textbf{(8-27)}$$

That is, the increase in the energy of the fluid (represented by an increase in its mean temperature by dT_m) is equal to the heat transferred to the fluid from the tube surface by convection. Noting that the differential surface area is $dA_s = pdx$, where p is the perimeter of the tube, and that $dT_m = -d(T_s - T_m)$, since T_s is constant, the relation above can be rearranged as

$$\frac{d(T_s - T_m)}{T_s - T_m} = -\frac{hp}{\dot{m} C_p} dx \quad \textbf{(8-28)}$$

Integrating from $x = 0$ (tube inlet where $T_m = T_i$) to $x = L$ (tube exit where $T_m = T_e$) gives

$$\ln \frac{T_s - T_e}{T_s - T_i} = -\frac{hA_s}{\dot{m} C_p} \quad \textbf{(8-29)}$$

$$ln\frac{T_s - T_e}{T_s - T_i} = -\frac{hA_s}{\dot{m}C_p}$$

TEMPERATURA COSTANTE

where $A_s = pL$ is the surface area of the tube and h is the constant *average* convection heat transfer coefficient. Taking the exponential of both sides and solving for T_e gives the following relation which is very useful for the determination of the *mean fluid temperature at the tube exit:*

$$T_e = T_s - (T_s - T_i)\exp(-hA_s/\dot{m}C_p) \qquad \textbf{(8-30)}$$

This relation can also be used to determine the mean fluid temperature $T_m(x)$ at any x by replacing $A_s = pL$ by px.

Solving Eq. 8–29 for $\dot{m}C_p$ gives

$$\dot{m}C_p = -\frac{hA_s}{\ln[(T_s - T_e)/(T_s - T_i)]} \qquad \textbf{(8-31)}$$

Substituting this into Eq. 8–17, we obtain

$$\dot{Q} = \dot{q}_s A_s = \dot{m}C_p(T_e - T_i) \qquad \textbf{(8-17)}$$

$$\dot{Q} = hA_s\Delta T_{\ln} \qquad \textbf{(8-32)}$$

where

$$\Delta T_{\ln} = \frac{T_i - T_e}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)} \qquad \textbf{(8-33)}$$

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is the **logarithmic mean temperature difference.** Note that $\Delta T_i = T_s - T_i$ and $\Delta T_e = T_s - T_e$ are the temperature differences between the surface and the fluid at the inlet and the exit of the tube, respectively. This $\Delta T_{\ln}$ relation appears to be prone to misuse, but it is practically fail-safe, since using T_i in place of T_e and vice versa in the numerator and/or the denominator will, at most, affect the sign, not the magnitude. Also, it can be used for both heating ($T_s > T_i$ and T_e) and cooling ($T_s < T_i$ and T_e) of a fluid in a tube.

The logarithmic mean temperature difference $\Delta T_{\ln}$ is obtained by tracing the actual temperature profile of the fluid along the tube, and is an *exact* representation of the *average temperature difference* between the fluid and the surface. It truly reflects the exponential decay of the local temperature difference. When ΔT_e differs from ΔT_i by no more than 40 percent, the error in using the arithmetic mean temperature difference is less than 1 percent. But the error increases to undesirable levels when ΔT_e differs from ΔT_i by greater amounts. Therefore, we should always use the logarithmic mean temperature difference when determining the convection heat transfer in a tube whose surface is maintained at a constant temperature T_s.

SCAMBIO TERMICO IN CONDOTTI MOTO Laminare

Circular tube, laminar ($\dot{q}_x$ = constant): $$\text{Nu} = \frac{hD}{k} = 4.36 \quad \textbf{(8-60)}$$

Circular tube, laminar (T_s = constant): $$\text{Nu} = \frac{hD}{k} = 3.66 \quad \textbf{(8-61)}$$

SCAMBIO TERMICO IN CONDOTTI MOTO TURBOLENTO

$$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{n} \quad \textbf{(8-68)}$$

where $n = 0.4$ for *heating* and 0.3 for *cooling* of the fluid flowing through the tube. This equation is known as the *Dittus–Boelter equation* [Dittus and Boelter (1930), Ref. 6] and it is preferred to the Colburn equation.

The Nusselt number relations above are fairly simple, but they may give errors as large as 25 percent. This error can be reduced considerably to less than 10 percent by using more complex but accurate relations such as the *second Petukhov equation* expressed as

$$\text{Nu} = \frac{(f/8)\,\text{Re}\,\text{Pr}}{1.07 + 12.7(f/8)^{0.5}\,(\text{Pr}^{2/3} - 1)} \qquad \begin{pmatrix} 0.5 \le \text{Pr} \le 2000 \\ 10^4 < \text{Re} < 5 \times 10^6 \end{pmatrix} \quad \textbf{(8-69)}$$

The accuracy of this relation at lower Reynolds numbers is improved by modifying it as [Gnielinski (1976), Ref. 8]

$$\text{Nu} = \frac{(f/8)(\text{Re} - 1000)\,\text{Pr}}{1 + 12.7(f/8)^{0.5}\,(\text{Pr}^{2/3} - 1)} \qquad \begin{pmatrix} 0.5 \le \text{Pr} \le 2000 \\ 3 \times 10^3 < \text{Re} < 5 \times 10^6 \end{pmatrix} \quad \textbf{(8-70)}$$

SCAMBIO TERMICO IN CONDOTTI FLUSSO TERMICO COSTANTE

V	5	m/s				
D	0.1	m				
L	10	m				
ΔT	10	K				
	Aria	Acqua			Rapporto	
ρ	1.2	1000	kg/m³		833	
cp	1.004	4.178	kJ/kgK		4.16	
m	0.047124	39.3	kg/s		833	
q	0.15	522	kW/m²		**3468**	
k	0.025	0.6	W/mK			
μ	1.81E-05	1.00E-03	Pas		55.3	
Re	3.31E+04	5.00E+05			15.1	
Pr	0.73	6.97			9.6	
Nu	83.7	1811			21.6	Heating
h	20.9	10864			**519**	
Nu	86.4	1491			17.26	Cooling
h	21.6	8947			**414**	

$$\dot{m}c_p\left(T_e - T_i\right) = \dot{q}A_s \rightarrow \dot{q} = \frac{\dot{m}c_p\left(T_e - T_i\right)}{\pi DL}$$

$$Nu = 0.023\left(\frac{VD}{\nu}\right)^{0.8}\left(\frac{c_p\mu}{k}\right)^n$$

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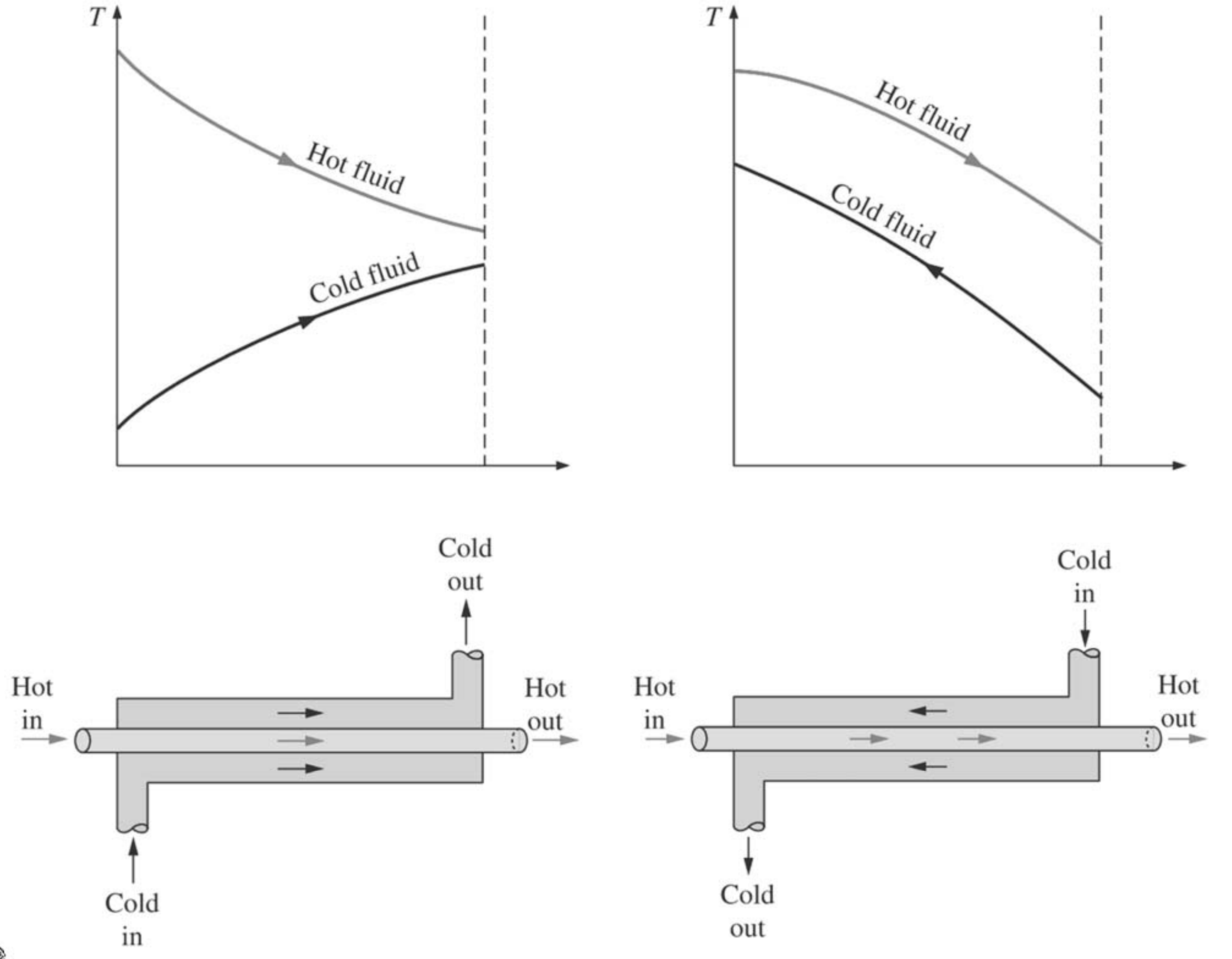
Equicorrente

The simplest type of heat exchanger consists of two concentric pipes of different diameters, as shown in Figure 13–1, called the **double-pipe** heat exchanger. One fluid in a double-pipe heat exchanger flows through the smaller pipe while the other fluid flows through the annular space between the two pipes. Two types of flow arrangement are possible in a double-pipe heat exchanger: in **parallel flow,** both the hot and cold fluids enter the heat exchanger at the same end and move in the *same* direction. In **counter flow,** on the other hand, the hot and cold fluids enter the heat exchanger at opposite ends and flow in *opposite* directions.

Another type of heat exchanger, which is specifically designed to realize a large heat transfer surface area per unit volume, is the **compact** heat exchanger. The ratio of the heat transfer surface area of a heat exchanger to its volume is called the *area density* β. A heat exchanger with $\beta > 700\ m^2/m^3$ (or 200 ft^2/ft^3) is classified as being compact. Examples of compact heat exchangers are car radiators ($\beta \approx 1000\ m^2/m^3$), glass ceramic gas turbine heat exchangers ($\beta \approx 6000\ m^2/m^3$), the regenerator of a Stirling engine ($\beta \approx 15{,}000\ m^2/m^3$), and the human lung ($\beta \approx 20{,}000\ m^2/m^3$). Compact heat exchangers enable us to achieve high heat transfer rates between two fluids in a small volume, and they are commonly used in applications with strict limitations on the weight and volume of heat exchangers (Fig. 13–2).

Controcorrente



(*a*) Parallel flow (*b*) Counter flow

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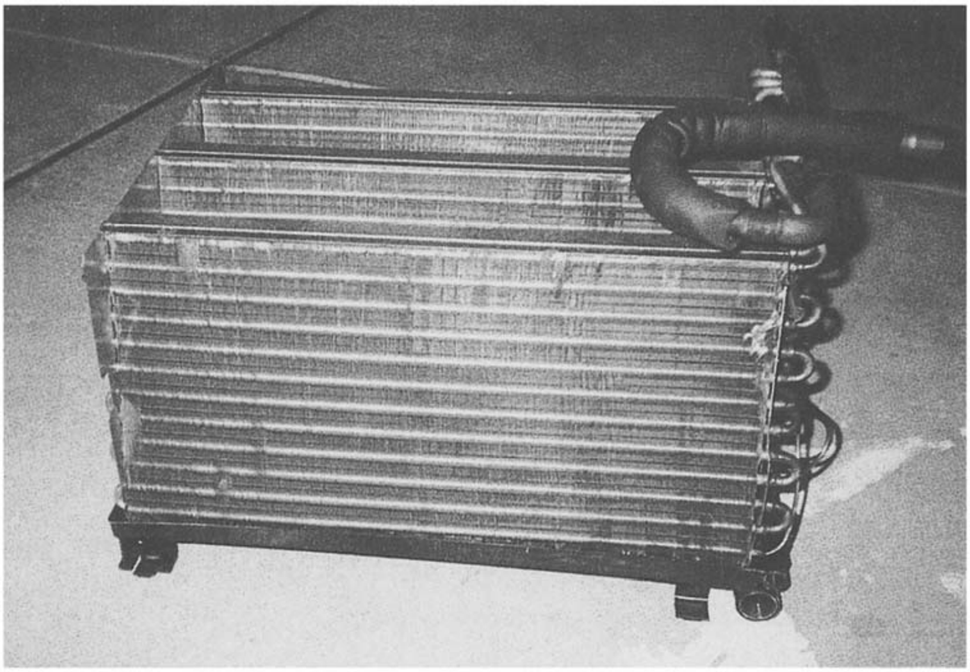

FIGURE 13–2
A gas-to-liquid compact heat exchanger for a residential air-conditioning system.

Scambiatore a pacco alettato

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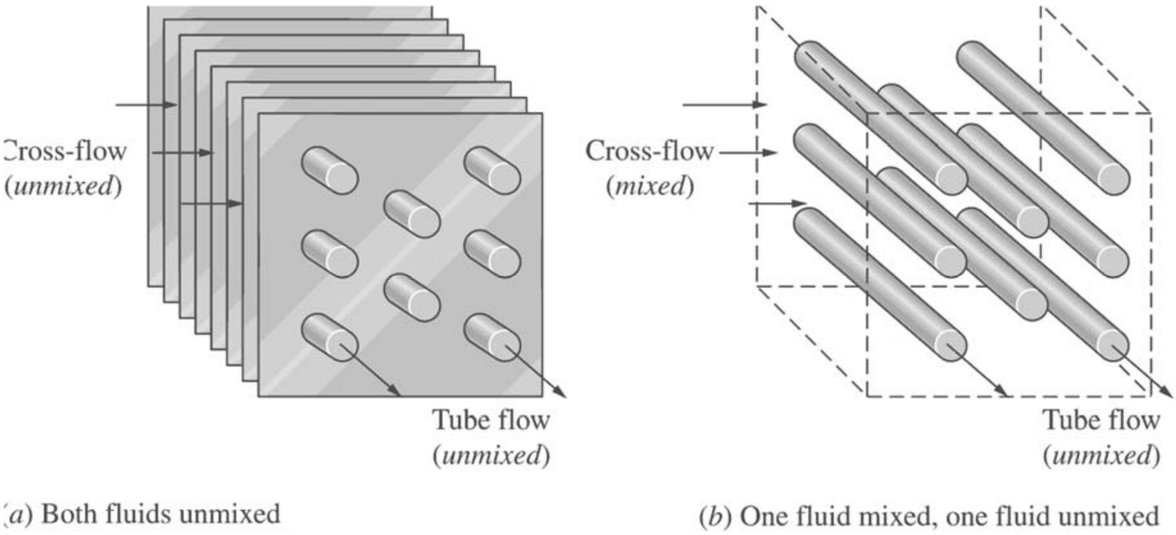

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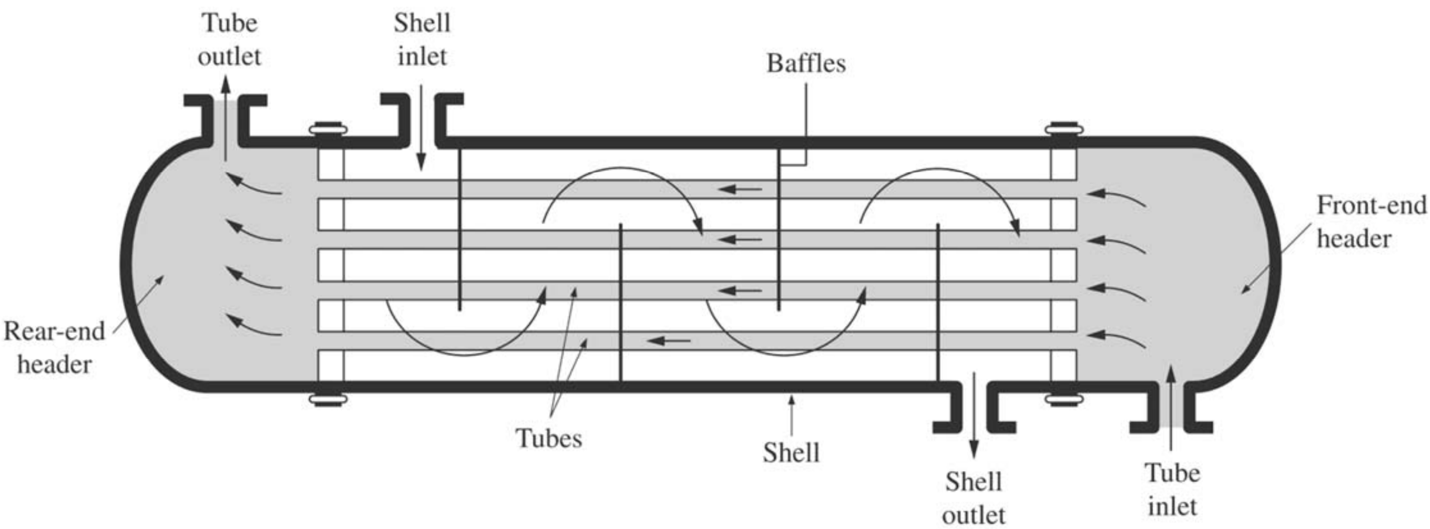

Scambiatore a fascio tubiero e mantello (shell and tube)

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A heat exchanger typically involves two flowing fluids separated by a solid wall. Heat is first transferred from the hot fluid to the wall by *convection,* through the wall by *conduction,* and from the wall to the cold fluid again by *convection.* Any radiation effects are usually included in the convection heat transfer coefficients.

The thermal resistance network associated with this heat transfer process involves two convection and one conduction resistances, as shown in Figure 13–7. Here the subscripts i and o represent the inner and outer surfaces of the inner tube. For a double-pipe heat exchanger, we have $A_i = \pi D_i L$ and $A_o = \pi D_o L$, and the *thermal resistance* of the tube wall in this case is

$$R_{\text{wall}} = \frac{\ln (D_o/D_i)}{2\pi k L} \quad \textbf{(13-1)}$$

where k is the thermal conductivity of the wall material and L is the length of the tube. Then the *total thermal resistance* becomes

$$R = R_{\text{total}} = R_i + R_{\text{wall}} + R_o = \frac{1}{h_i A_i} + \frac{\ln (D_o/D_i)}{2\pi k L} + \frac{1}{h_o A_o} \quad \textbf{(13-2)}$$

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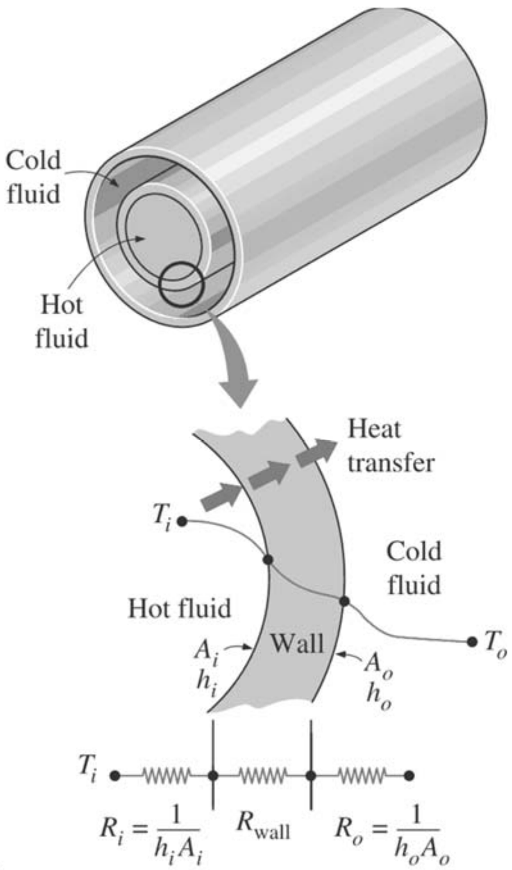

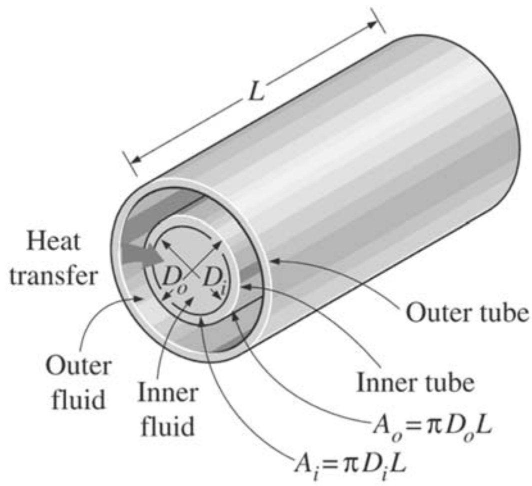

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In the analysis of heat exchangers, it is convenient to combine all the thermal resistances in the path of heat flow from the hot fluid to the cold one into a single resistance R, and to express the rate of heat transfer between the two fluids as

$$\dot{Q} = \frac{\Delta T}{R} = UA\,\Delta T = U_i A_i\,\Delta T = U_o A_o\,\Delta T \qquad \textbf{(13-3)}$$

where U is the **overall heat transfer coefficient,** whose unit is $W/m^2 \cdot °C$, which is identical to the unit of the ordinary convection coefficient h. Canceling ΔT, Eq. 13-3 reduces to

$$\frac{1}{UA_s} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = R = \frac{1}{h_i A_i} + R_{\text{wall}} + \frac{1}{h_o A_o} \qquad \textbf{(13-4)}$$

Perhaps you are wondering why we have two overall heat transfer coefficients U_i and U_o for a heat exchanger. The reason is that every heat exchanger has two heat transfer surface areas A_i and A_o, which, in general, are not equal to each other.

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Representative values of the overall heat transfer coefficients in heat exchangers

Type of heat exchanger	U, $W/m^2 \cdot °C$*
Water-to-water	850–1700
Water-to-oil	100–350
Water-to-gasoline or kerosene	300–1000
Feedwater heaters	1000–8500
Steam-to-light fuel oil	200–400
Steam-to-heavy fuel oil	50–200
Steam condenser	1000–6000
Freon condenser (water cooled)	300–1000
Ammonia condenser (water cooled)	800–1400
Alcohol condensers (water cooled)	250–700
Gas-to-gas	10–40
Water-to-air in finned tubes (water in tubes)	30–60†
	400–850†
Steam-to-air in finned tubes (steam in tubes)	30–300†
	400–4000‡

Tipicamente limitato da uno dei due scambi convettivi

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To begin with, we take U to be a constant value. This is fairly reasonable in compact single-phase heat exchangers. In larger exchangers, particularly in shell-and-tube configurations and large condensers, U is apt to vary with position in the exchanger and/or with local temperature. But in situations in which U is fairly constant, we can deal with the varying temperatures of the fluid streams by writing the overall heat transfer in terms of a mean temperature difference between the two fluid streams:

$$Q = UA\,\Delta T_{\text{mean}} \tag{3.1}$$

Our problem then reduces to finding the appropriate mean temperature difference that will make this equation true. Let us do this for the simple parallel and counterflow configurations, as sketched in Fig. 3.8.

The temperature of both streams is plotted in Fig. 3.8 for both single-pass arrangements—the parallel and counterflow configurations—as a function of the length of travel (or area passed over). Notice that, in the parallel-flow configuration, temperatures tend to change more rapidly with position and less length is required. But the counterflow arrangement achieves generally more complete heat exchange from one flow to the other.

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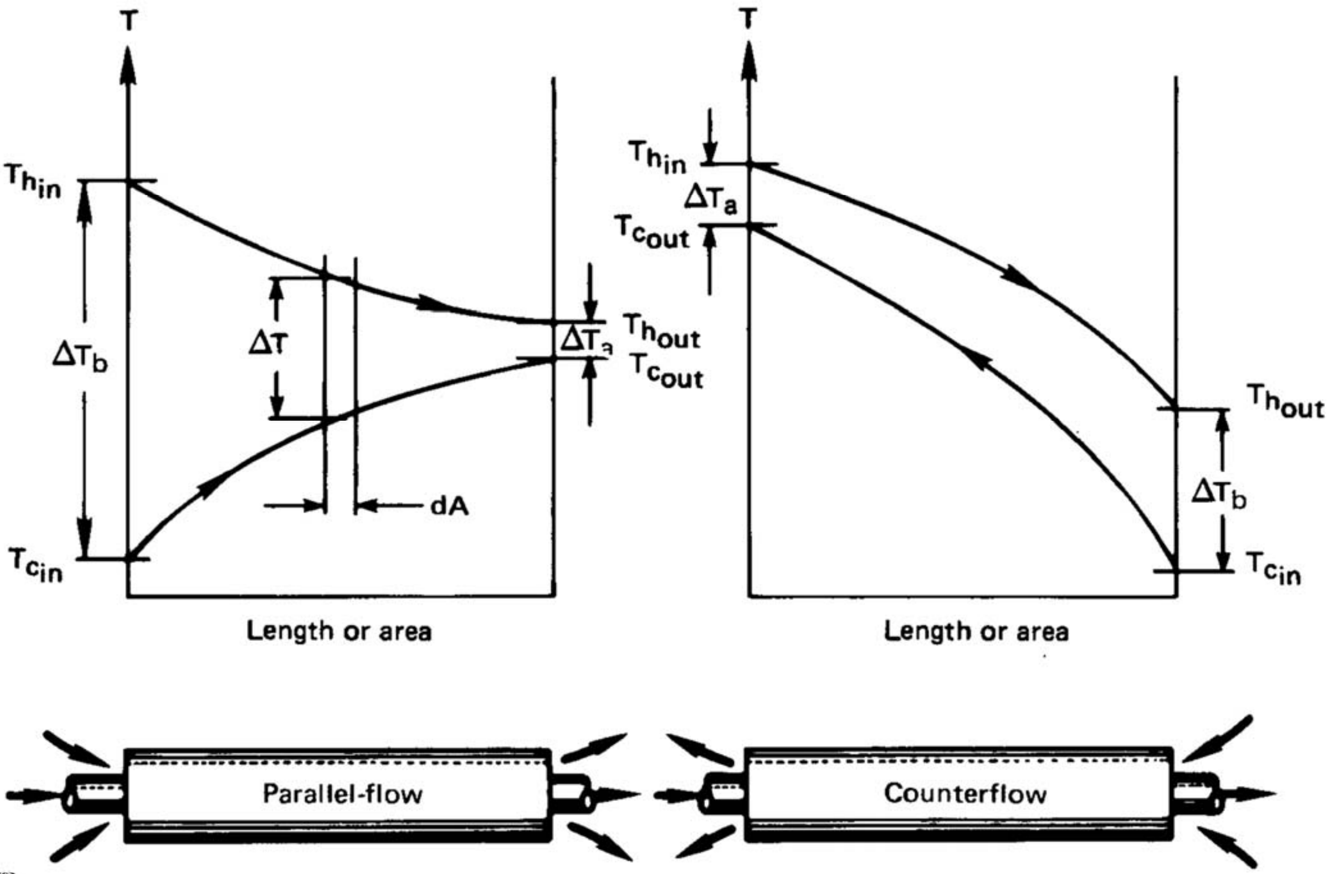

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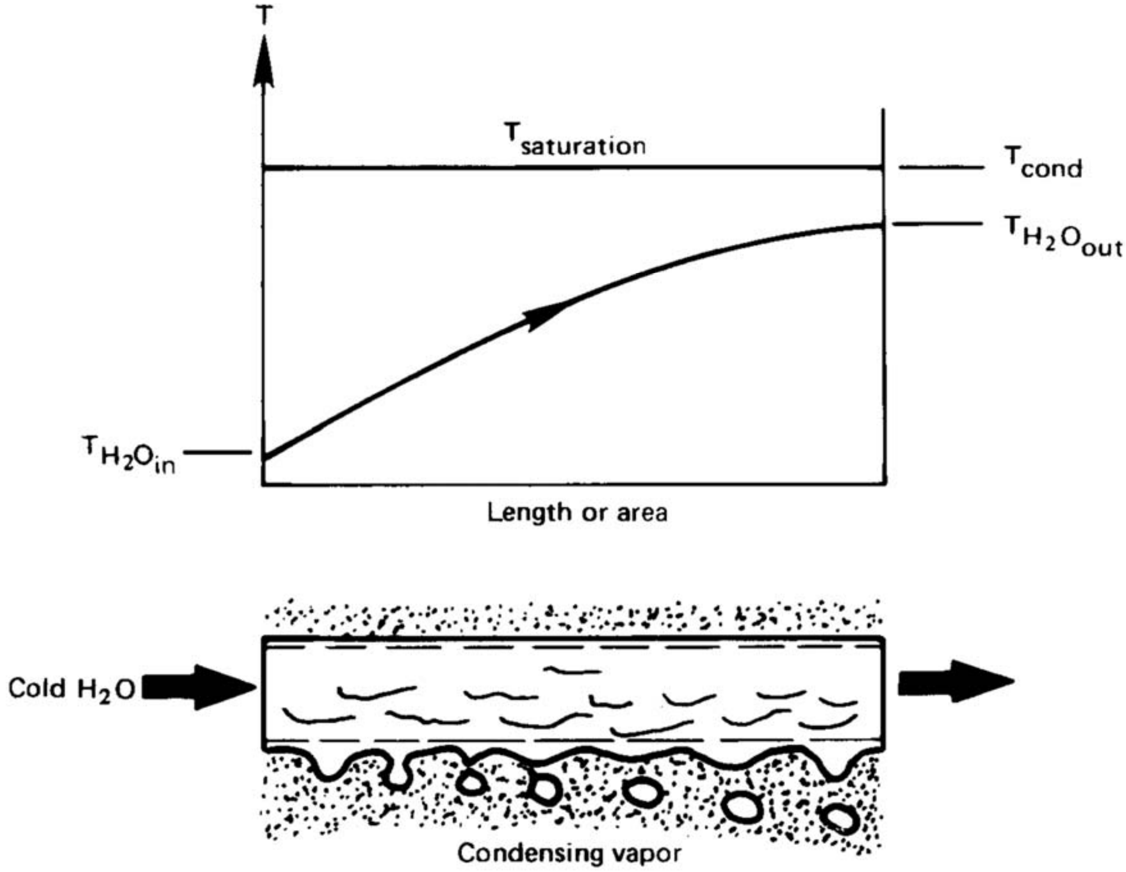

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Figure 3.9 shows another variation on the single-pass configuration. This is a condenser in which one stream flows through with its temperature changing, but the other simply condenses at uniform temperature. This arrangement has some special characteristics, which we point out shortly.

The determination of ΔT_{mean} for such arrangements proceeds as follows: the differential heat transfer within either arrangement (see Fig. 3.8) is

$$dQ = U\Delta T\, dA = -(\dot{m}c_p)_h\, dT_h = \pm(\dot{m}c_p)_c\, dT_c \qquad (3.2)$$

where the subscripts h and c denote the hot and cold streams, respectively; the upper and lower signs are for the parallel and counterflow cases, respectively; and dT denotes a change from left to right in the exchanger. We give symbols to the total heat capacities of the hot and cold streams:

$$C_h \equiv (\dot{m}c_p)_h \,\text{W/K} \qquad \text{and} \qquad C_c \equiv (\dot{m}c_p)_c \,\text{W/K} \qquad (3.3)$$

Thus, for either heat exchanger, $\mp C_h dT_h = C_c dT_c$. This equation can be integrated from the lefthand side, where $T_h = T_{h_{\text{in}}}$ and $T_c = T_{c_{\text{in}}}$ for

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parallel flow or $T_h = T_{h_{in}}$ and $T_c = T_{c_{out}}$ for counterflow, to some arbitrary point inside the exchanger. The temperatures inside are thus:

$$\begin{aligned} \text{parallel flow:} \quad & T_h = T_{h_{in}} - \frac{C_c}{C_h}(T_c - T_{c_{in}}) \quad = T_{h_{in}} - \frac{Q}{C_h} \\ \text{counterflow:} \quad & T_h = T_{h_{in}} - \frac{C_c}{C_h}(T_{c_{out}} - T_c) \quad = T_{h_{in}} - \frac{Q}{C_h} \end{aligned} \tag{3.4}$$

where Q is the total heat transfer from the entrance to the point of interest. Equations (3.4) can be solved for the local temperature differences:

$$\begin{aligned} \Delta T_{\text{parallel}} &= T_h - T_c = T_{h_{in}} - \left(1 + \frac{C_c}{C_h}\right) T_c + \frac{C_c}{C_h} T_{c_{in}} \\ \Delta T_{\text{counter}} &= T_h - T_c = T_{h_{in}} - \left(1 - \frac{C_c}{C_h}\right) T_c - \frac{C_c}{C_h} T_{c_{out}} \end{aligned} \tag{3.5}$$

Substitution of these in $dQ = C_c dT_c = U\Delta T\, dA$ yields

$$\begin{aligned} \left.\frac{UdA}{C_c}\right|_{\text{parallel}} &= \frac{dT_c}{\left[-\left(1 + \frac{C_c}{C_h}\right) T_c + \frac{C_c}{C_h} T_{c_{in}} + T_{h_{in}}\right]} \\ \left.\frac{UdA}{C_c}\right|_{\text{counter}} &= \frac{dT_c}{\left[-\left(1 - \frac{C_c}{C_h}\right) T_c - \frac{C_c}{C_h} T_{c_{out}} + T_{h_{in}}\right]} \end{aligned} \tag{3.6}$$

Equations (3.6) can be integrated across the exchanger:

$$\int_0^A \frac{U}{C_c} dA = \int_{T_{c\,in}}^{T_{c\,out}} \frac{dT_c}{[---]} \tag{3.7}$$

If U and C_c can be treated as constant, this integration gives

$$\begin{aligned} \text{parallel:} \quad & \ln\left[\frac{-\left(1 + \frac{C_c}{C_h}\right) T_{c_{out}} + \frac{C_c}{C_h} T_{c_{in}} + T_{h_{in}}}{-\left(1 + \frac{C_c}{C_h}\right) T_{c_{in}} + \frac{C_c}{C_h} T_{c_{in}} + T_{h_{in}}}\right] = -\frac{UA}{C_c}\left(1 + \frac{C_c}{C_h}\right) \\ \text{counter:} \quad & \ln\left[\frac{-\left(1 - \frac{C_c}{C_h}\right) T_{c_{out}} - \frac{C_c}{C_h} T_{c_{out}} + T_{h_{in}}}{-\left(1 - \frac{C_c}{C_h}\right) T_{c_{in}} - \frac{C_c}{C_h} T_{c_{out}} + T_{h_{in}}}\right] = -\frac{UA}{C_c}\left(1 - \frac{C_c}{C_h}\right) \end{aligned} \tag{3.8}$$

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If U were variable, the integration leading from eqn. (3.7) to eqns. (3.8) is where its variability would have to be considered. Any such variability of U can complicate eqns. (3.8) terribly. Presuming that eqns. (3.8) are valid, we can simplify them with the help of the definitions of ΔT_a and ΔT_b, given in Fig. 3.8:

$$\begin{aligned} \text{parallel:} \quad & \ln\left[\frac{(1 + C_c/C_h)(T_{c_{\text{in}}} - T_{c_{\text{out}}}) + \Delta T_b}{\Delta T_b}\right] = -UA\left(\frac{1}{C_c} + \frac{1}{C_h}\right) \\ \text{counter:} \quad & \ln\frac{\Delta T_a}{(-1 + C_c/C_h)(T_{c_{\text{in}}} - T_{c_{\text{out}}}) + \Delta T_a} = -UA\left(\frac{1}{C_c} - \frac{1}{C_h}\right) \end{aligned} \qquad (3.9)$$

Conservation of energy ($Q_c = Q_h$) requires that

$$\boxed{\frac{C_c}{C_h} = -\frac{T_{h_{\text{out}}} - T_{h_{\text{in}}}}{T_{c_{\text{out}}} - T_{c_{\text{in}}}}} \qquad (3.10)$$

Then eqn. (3.9) and eqn. (3.10) give

$$\begin{aligned} \text{parallel:} \quad & \ln\left[\frac{\overbrace{(T_{c_{\text{in}}} - T_{c_{\text{out}}}) + (T_{h_{\text{out}}} - T_{h_{\text{in}}})}^{\Delta T_a - \Delta T_b} + \Delta T_b}{\Delta T_b}\right] \\ & = \ln\left(\frac{\Delta T_a}{\Delta T_b}\right) = -UA\left(\frac{1}{C_c} + \frac{1}{C_h}\right) \\ \text{counter:} \quad & \ln\left(\frac{\Delta T_a}{\Delta T_b - \Delta T_a + \Delta T_a}\right) = \ln\left(\frac{\Delta T_a}{\Delta T_b}\right) = -UA\left(\frac{1}{C_c} - \frac{1}{C_h}\right) \end{aligned} \qquad (3.11)$$

Finally, we write $1/C_c = (T_{c_{\text{out}}} - T_{c_{\text{in}}})/Q$ and $1/C_h = (T_{h_{\text{in}}} - T_{h_{\text{out}}})/Q$ on the right-hand side of either of eqns. (3.11) and get for either parallel or counterflow,

$$\boxed{Q = UA\left(\frac{\Delta T_a - \Delta T_b}{\ln(\Delta T_a/\Delta T_b)}\right)} \qquad (3.12)$$

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The appropriate ΔT_{mean} for use in eqn. (3.11) is thus the *logarithmic mean temperature difference* (LMTD):

$$\Delta T_{\text{mean}} = \text{LMTD} \equiv \frac{\Delta T_a - \Delta T_b}{\ln\left(\dfrac{\Delta T_a}{\Delta T_b}\right)} \qquad (3.13)$$