$$\gamma(V_1 - V_2) = \frac{a_2^2}{V_2} - \frac{a_1^2}{V_1} = \frac{a_2^2 V_1 - a_1^2 V_2}{V_1 V_2} \qquad (8.6)$$

$$H = h + \frac{V^2}{2} \Rightarrow \frac{a_0^2}{\gamma - 1} = \frac{a^2}{\gamma - 1} + \frac{V^2}{2} \Rightarrow$$

$$\Rightarrow a_0^2 = a^2 + \frac{\gamma - 1}{2} V^2 \Rightarrow a^2 = a_0^2 - \frac{\gamma - 1}{2} V^2$$

Sostituendo in (8.6)

$$\gamma(V_1 - V_2) = \frac{V_1 \cdot \left[a_0^2 - \frac{\gamma-1}{2} V_2^2\right] - V_2 \cdot \left[a_0^2 - \frac{\gamma-1}{2} V_1^2\right]}{V_1 \cdot V_2} =$$

$$= a_0^2 \frac{(V_1 - V_2)}{V_1 V_2} + \frac{\gamma - 1}{2} \frac{V_1^2 V_2 - V_1 V_2^2}{V_1 V_2} \Rightarrow$$

$$\gamma \cancel{(V_1 - V_2)} = \frac{a_0^2 \cancel{(V_1 - V_2)} + \frac{\gamma-1}{2} V_1 V_2 \cancel{(V_1 - V_2)}}{V_1 V_2} \Rightarrow$$

$$\gamma V_1 V_2 = a_0^2 + \frac{\gamma - 1}{2} V_1 V_2 \Rightarrow V_1 V_2 = \frac{2}{\gamma + 1} a_0^2 = a^{*2}$$

$$M_1^{*2} \cdot M_2^{*2} = \frac{M_1^2}{1 + \frac{\gamma - 1}{2} M_1^2} \cdot \frac{\gamma + 1}{2} \cdot \frac{M_2^2}{1 + \frac{\gamma - 1}{2} M_2^2} \cdot \frac{\gamma + 1}{2} \Rightarrow$$

$$M_1^2 M_2^2 \left(\frac{\gamma + 1}{2}\right)^2 = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)\left(1 + \frac{\gamma - 1}{2} M_2^2\right) =$$

$$= 1 + \frac{\gamma - 1}{2} M_1^2 + \frac{\gamma - 1}{2} M_2^2 + \left(\frac{\gamma - 1}{2}\right)^2 M_1^2 M_2^2 \Rightarrow$$

$$M_2^2 \left[M_1^2 \left(\frac{\gamma + 1}{2}\right)^2 - \frac{\gamma - 1}{2} - \left(\frac{\gamma - 1}{2}\right)^2 M_1^2\right] = 1 + \frac{\gamma - 1}{2} M_1^2 \Rightarrow$$

$$M_2^2 = \frac{1 + \frac{\gamma - 1}{2} M_1^2}{\left[M_1^2 \left(\frac{4\gamma}{4}\right) - \frac{\gamma - 1}{2}\right]} = \frac{2 + (\gamma - 1) M_1^2}{2 \gamma M_1^2 - (\gamma - 1)} \qquad (8.11)$$

$$\frac{T_2}{T_1} = \frac{1+\frac{\gamma-1}{2} M_1^2}{1+\frac{\gamma-1}{2} M_2^2} = \frac{1+\frac{\gamma-1}{2} \cdot M_1^2}{1+\frac{\gamma-1}{2}\left[\frac{2+(\gamma-1)M_1^2}{2\gamma M_1^2-(\gamma-1)}\right]} =$$

$$= \frac{\left(1+\frac{\gamma-1}{2} M_1^2\right)\cdot\left[2\gamma M_1^2-(\gamma-1)\right]}{\left[2\gamma M_1^2-(\gamma-1)\right]+\frac{\gamma-1}{2}\cdot\left[2+(\gamma-1)M_1^2\right]} =$$

$$= \frac{\left(1+\frac{\gamma-1}{2} M_1^2\right)\cdot\left[2\gamma M_1^2-(\gamma-1)\right]}{2\gamma M_1^2-\cancel{(\gamma-1)}+\cancel{(\gamma-1)}+\frac{\gamma^2+1-2\gamma}{2} M_1^2} =$$

$$= \frac{\left(1+\frac{\gamma-1}{2} M_1^2\right)\cdot\left[2\gamma M_1^2-(\gamma-1)\right]}{\frac{(\gamma+1)^2}{2} M_1^2} =$$

$$= \left(1+\frac{\gamma-1}{2} M_1^2\right)\cdot\left[2\gamma M_1^2-(\gamma-1)\right]\cdot\frac{1}{(\gamma+1)^2}\cdot\frac{2}{M_1^2} =$$

$$= \left(\frac{2}{M_1^2}+\gamma-1\right)\cdot\left(2\gamma M_1^2-\gamma+1\right)\cdot\left(\gamma+1\right)^{-2} \quad (8.13)$$

$$\frac{p_2}{p_1} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2} = \frac{1+\gamma M_1^2}{1+\gamma \cdot \left[\frac{2+(\gamma-1) M_1^2}{2\gamma M_1^2 - \gamma + 1}\right]} =$$

$$= \frac{(1+\gamma M_1^2)\cdot(2\gamma M_1^2 - \gamma + 1)}{(2\gamma M_1^2 - \gamma + 1) + 2\gamma + \gamma^2 M_1^2 - \gamma M_1^2} =$$

$$= \frac{(1+\gamma M_1^2)\cdot(2\gamma M_1^2 - \gamma + 1)}{\gamma^2 M_1^2 + \gamma M_1^2 + \gamma + 1} = \frac{\cancel{(1+\gamma M_1^2)}\cdot(2\gamma M_1^2 - \gamma + 1)}{(\gamma+1)\cdot\cancel{(1+\gamma M_1^2)}} =$$

$$= (\gamma+1)^{-1}\cdot(2\gamma M_1^2 - \gamma + 1) \qquad (8.15)$$

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{p_2}{p_1} \cdot \frac{T_1}{T_2} = \frac{\cancel{2\gamma M_1^2 - \gamma + 1}}{\cancel{\gamma + 1}} \cdot \frac{(\gamma+1)^{\cancel{2}}}{\left(\cancel{2\gamma M_1^2 - \gamma + 1}\right)\left(\gamma - 1 + \frac{2}{M_1^2}\right)} =$$

$$= (\gamma+1) M_1^2 \cdot \left[(\gamma-1) M_1^2 + 2\right]^{-1} \qquad (8.14)$$

$$\frac{p_{02}}{p_{01}} = \frac{p_{02}}{p_2} \cdot \frac{p_2}{p_1} \cdot \frac{p_1}{p_{01}} =$$

$$= \frac{\left[1 + \frac{\gamma-1}{2} M_2^2\right]^{\frac{\gamma}{\gamma-1}}}{\left[1 + \frac{\gamma-1}{2} M_1^2\right]^{\frac{\gamma}{\gamma-1}}} \cdot \frac{2\gamma M_1^2 - \gamma + 1}{\gamma + 1} =$$

$$= \left[\frac{2 + (\gamma-1) \cdot \frac{(\gamma-1) M_1^2 + 2}{2\gamma M_1^2 - (\gamma-1)}}{2 + (\gamma-1) M_1^2}\right]^{\frac{\gamma}{\gamma-1}} \cdot \frac{2\gamma M_1^2 - \gamma + 1}{\gamma + 1} =$$

$$= \left[\frac{2\left(2\gamma M_1^2 - \gamma + 1\right) + (\gamma-1)^2 \cdot M_1^2 + 2(\gamma-1)}{2 + (\gamma-1) M_1^2}\right]^{\frac{\gamma}{\gamma-1}} \cdot \frac{\left(2\gamma M_1^2 - \gamma + 1\right)^{\frac{1}{1-\gamma}}}{\gamma + 1} =$$

$$= \left[\frac{(\gamma+1)^2 M_1^2}{2+(\gamma-1)M_1^2}\right]^{\frac{\gamma}{\gamma-1}} \cdot \frac{\left(2\gamma M_1^2-\gamma+1\right)^{\frac{1}{1-\gamma}}}{\gamma+1} =$$

$$= \frac{(\gamma+1)^{\frac{2\gamma}{\gamma-1}}}{\left[\frac{2}{M_1^2}+\gamma-1\right]^{\frac{\gamma}{\gamma-1}}} \cdot \frac{\left(2\gamma M_1^2-\gamma+1\right)^{\frac{1}{1-\gamma}}}{\gamma+1} =$$

$$= (\gamma+1)^{\frac{\gamma+1}{\gamma-1}} \cdot \left(\frac{2}{M_1^2}+\gamma-1\right)^{-\frac{\gamma}{\gamma-1}} \cdot \left(2\gamma M_1^2-\gamma+1\right)^{\frac{1}{1-\gamma}} =$$

$$= \left[(\gamma+1)^{-(\gamma+1)} \cdot \left(\frac{2}{M_1^2}+\gamma-1\right)^{\gamma} \cdot \left(2\gamma M_1^2-\gamma+1\right)\right]^{\frac{1}{1-\gamma}} \quad (8.17)$$

$$\frac{\Delta s}{R} = -\ln\frac{p_{02}}{p_{01}} =$$

$$= -\ln\left[(\gamma+1)^{-(\gamma+1)} \cdot \left(\gamma-1+\frac{2}{M_1^2}\right)^{\gamma} \cdot \left(2\gamma M_1^2-\gamma+1\right)\right]^{\frac{1}{1-\gamma}} =$$

$$= \frac{1}{\gamma-1} \cdot \ln\left[(\gamma+1)^{-(\gamma+1)} \cdot \left(\gamma-1+\frac{2}{M_1^2}\right)^{\gamma} \cdot \left(2\gamma M_1^2-\gamma+1\right)\right] =$$

$$= \frac{1}{\gamma-1} \cdot \left[\ln(\gamma+1)^{-(\gamma+1)} + \ln\left(\gamma-1+\frac{2}{M_1^2}\right)^{\gamma} + \ln\left(2\gamma M_1^2-\gamma+1\right)\right] =$$

$$= \frac{1}{\gamma-1} \cdot \left[-(\gamma+1) \cdot \ln(\gamma+1) + \gamma \cdot \ln\left(\gamma-1+\frac{2}{M_1^2}\right) + \ln\left(2\gamma M_1^2-\gamma+1\right)\right] \quad (3.20)$$

$$s(Mq) := \frac{\left[-(\gamma+1)\cdot\ln(\gamma+1) + \ln[2\gamma\cdot Mq - (\gamma-1)] + \gamma\cdot\ln\left(\gamma - 1 + \frac{2}{Mq}\right)\right]}{\gamma - 1}$$

$$D1s(M) := \frac{d}{dM}s(M) \qquad D2s(M) := \frac{d}{dM}D1s(M) \qquad D3s(M) := \frac{d}{dM}D2s(M)$$

$$D1s(M) \rightarrow \frac{2\cdot\frac{\gamma}{2\cdot\gamma\cdot M - \gamma + 1} - 2\cdot\frac{\gamma}{M^2\cdot\left(\gamma - 1 + \frac{2}{M}\right)}}{\gamma - 1} \qquad D1s(1) \rightarrow 0$$

$$D2s(M) \rightarrow \frac{-4\cdot\frac{\gamma^2}{(2\cdot\gamma\cdot M - \gamma + 1)^2} + 4\cdot\frac{\gamma}{M^3\cdot\left(\gamma - 1 + \frac{2}{M}\right)} - 4\cdot\frac{\gamma}{M^4\cdot\left(\gamma - 1 + \frac{2}{M}\right)^2}}{\gamma - 1}$$

$$D2s(1) \text{ simplify } \rightarrow 0$$

$$D3s(M) \rightarrow \frac{16 \cdot \frac{\gamma^3}{(2 \cdot \gamma \cdot M - \gamma + 1)^3} - 12 \cdot \frac{\gamma}{M^4 \cdot \left(\gamma - 1 + \frac{2}{M}\right)} + 24 \cdot \frac{\gamma}{M^5 \cdot \left(\gamma - 1 + \frac{2}{M}\right)^2} - 16 \cdot \frac{\gamma}{M^6 \cdot \left(\gamma - 1 + \frac{2}{M}\right)^3}}{\gamma - 1}$$

$$D3s(1) \text{ simplify } \rightarrow 4 \cdot \frac{\gamma}{(\gamma + 1)^2}$$

$$\frac{p_{02}}{p_1} = \frac{p_{02}}{p_2} \cdot \frac{p_2}{p_1} =$$

$$= \left(1 + \frac{\gamma-1}{2} M_2^2\right)^{\frac{\gamma}{\gamma-1}} \cdot \frac{1}{\gamma+1} \cdot \left(2\gamma M_1^2 - \gamma + 1\right) =$$

$$= \left[1 + \frac{\gamma-1}{2} \cdot \frac{(\gamma-1) M_1^2 + 2}{2\gamma M_1^2 - (\gamma-1)}\right]^{\frac{\gamma}{\gamma-1}} \cdot \frac{1}{\gamma+1} \cdot \left(2\gamma M_1^2 - \gamma + 1\right) =$$

$$= \left[\frac{2\gamma M_1^2 - \cancel{(\gamma-1)} + \frac{(\gamma-1)^2}{2} M_1^2 + \cancel{2}\frac{\cancel{(\gamma-1)}}{\cancel{2}}}{2\gamma M_1^2 - (\gamma-1)}\right]^{\frac{\gamma}{\gamma-1}} \cdot \frac{1}{\gamma+1} \cdot \left(2\gamma M_1^2 - \gamma + 1\right) =$$

$$= \left[2\gamma M_1^2 + \frac{\gamma^2}{2} M_1^2 + \frac{1}{2} M_1^2 - \gamma M_1^2 \right]^{\frac{\gamma}{\gamma-1}} \cdot \frac{1}{\gamma+1} \cdot \left(2\gamma M_1^2 - \gamma + 1 \right)^{1-\frac{\gamma}{\gamma-1}} =$$

$$= \left[\frac{(\gamma+1)^2}{2} M_1^2 \right]^{\frac{\gamma}{\gamma-1}} \cdot \frac{1}{\gamma+1} \cdot \left(2\gamma M_1^2 - \gamma + 1 \right)^{\frac{1}{1-\gamma}} =$$

$$= \left(\frac{\gamma+1}{2} \right)^{\frac{2\gamma}{\gamma-1}} \cdot \frac{2}{\gamma+1} \cdot \frac{1}{2} \left(2 M_1 \right)^{\frac{2\gamma}{\gamma-1}} \cdot \left(2\gamma M_1^2 - \gamma + 1 \right)^{\frac{1}{1-\gamma}} =$$

$$= \left(\frac{\gamma+1}{2} \right)^{\frac{\gamma+1}{\gamma-1}} \cdot \left(2 M_1^{2\gamma} \right)^{\frac{1}{\gamma-1}} \cdot \left(2\gamma M_1^2 - \gamma + 1 \right)^{\frac{1}{1-\gamma}} =$$

$$= \left(\frac{\gamma+1}{2} \right)^{\frac{\gamma+1}{\gamma-1}} \cdot \left[\frac{2 M_1^{2\gamma}}{2\gamma M_1^2 - \gamma + 1} \right]^{\frac{1}{\gamma-1}} \qquad (8.22)$$

$$\frac{V_1}{V_2} = (\gamma+1)\, M_1^2 \left(\frac{1}{2+(\gamma-1) M_1^2} \right) \xrightarrow{M_1 = \frac{v_1^2}{a_1^2}}$$

$$\cancel{V_1} \left(2 + (\gamma-1) \frac{v_1^2}{a_1^2} \right) = (\gamma+1) \frac{v_1^{\cancel{2}}}{a_1^2} V_2 \xrightarrow[V_2 = V_0 - V_y]{V_1 = V_0 - V_x}$$

$$2 a_1^2 + (\gamma-1)\left(V_0^2 - 2 V_0 V_x + V_x^2\right) = (\gamma+1)\left(V_0^2 - V_0 V_y - V_0 V_x + V_x V_y\right)$$

$$V_0^2 \overbrace{\left(\gamma-1-(\gamma+1)\right)}^{-2} + V_0 \overbrace{\left((\gamma+1)(V_x+V_y) - 2(\gamma-1) V_x\right)}^{(\gamma+1) V_y + V_x(-\gamma+3)} + 2 a_1^2 +$$

$$+ (\gamma-1) V_x^2 - (\gamma+1) V_x V_y = 0$$

$$2 V_0^2 + \left(V_x(\gamma-3) - V_y(\gamma+1)\right) V_0 + (\gamma+1) V_x V_y - 2 a_1^2 - (\gamma-1) V_x^2 = 0$$

$$V_0 = \frac{V_y(\gamma+1) - V_x(\gamma-3)}{4} \pm \sqrt{\underbrace{\frac{\left[V_x(\gamma-3) - V_y(\gamma+1)\right]^2 - 8\left[(\gamma+1)V_xV_y - 2a_1^2 - (\gamma-1)V_x^2\right]}{16}}_{}}$$

$$\frac{V_x^2(\gamma^2-6\gamma+9) - 2V_xV_y(\gamma+1)(\gamma-3) + V_y(\gamma+1)^2}{16} - \frac{1}{2}V_xV_y(\gamma+1) + a_1^2 + \frac{\gamma-1}{2}V_x^2 =$$

$$= \frac{V_x^2}{16}\underbrace{(\gamma^2-6\gamma+9+8\gamma-8)}_{(\gamma+1)^2} + \frac{V_y^2}{16}(\gamma+1)^2 - 2V_xV_y\frac{(\gamma+1)}{16}\underbrace{\left[\gamma-3+4\right]}_{(\gamma+1)} + a_1^2 =$$

$$= \left[\frac{(\gamma+1)}{4}\right]^2 \left[V_x^2 - 2V_xV_y + V_y^2\right] + a_1^2$$

$$V_0 = \frac{V_y(\gamma+1) - V_x(\gamma-3)}{4} \pm \sqrt{a_1^2 + \left(\frac{\gamma+1}{4}\right)^2 (V_y - V_x)^2} \quad (8.29)$$

$$M_p = \frac{V_p}{a_1} = M_1 - \sqrt{\frac{T_2}{T_1}}\, M_2 \qquad (8.30)$$

POICHÉ :

$$\frac{T_2}{T_1} M_2^2 = (\gamma+1)^{-2} \cdot \cancel{\left(2\gamma M_1^2 - \gamma+1\right)} \cdot \left(\gamma - 1 + \frac{2}{M_1^2}\right) \cdot \frac{(\gamma-1)M_1^2+2}{\cancel{\left(2\gamma M_1^2-\gamma+1\right)}} =$$

$$= \frac{\left[(\gamma-1)M_1^2+2\right]^2}{(\gamma+1)^2 \cdot M_1^2} \Longrightarrow M_p = M_1 - \frac{(\gamma-1)M_1^2+2}{(\gamma+1)\,M_1} =$$

$$= \frac{(\gamma+1)M_1^2 - (\gamma-1)M_1^2 - 2}{(\gamma+1)\,M_1} = \frac{2M_1^2-2}{(\gamma+1)\,M_1} =$$

$$= \frac{2}{\gamma+1} \cdot \frac{M_1^2-1}{M_1}$$

$$\frac{(\gamma+1)M_1^2 \operatorname{sen}^2\varepsilon}{2+(\gamma-1)M_1^2 \operatorname{sen}^2\varepsilon} = \frac{\operatorname{Tan}\varepsilon}{\operatorname{Tan}(\varepsilon-\delta)}$$

PONIAMO : $A = \dfrac{(\gamma+1)M_1^2 \operatorname{sen}^2\varepsilon}{2+(\gamma-1)M_1^2 \operatorname{sen}^2\varepsilon}$

ESSENDO : $\operatorname{Tan}(\varepsilon-\delta) = \dfrac{\operatorname{Tan}\varepsilon - \operatorname{Tan}\delta}{1+\operatorname{Tan}\varepsilon\cdot\operatorname{Tan}\delta} \Rightarrow$

$$(\operatorname{Tan}\varepsilon - \operatorname{Tan}\delta)\cdot A = \operatorname{Tan}\varepsilon + \operatorname{Tan}^2\varepsilon\cdot\operatorname{Tan}\delta \Rightarrow$$

$$\operatorname{Tan}\delta\cdot(\operatorname{Tan}^2\varepsilon + A) = \operatorname{Tan}\varepsilon\,(A-1) \Rightarrow$$

$$\operatorname{Tan}\delta = \operatorname{Tan}\varepsilon\cdot\frac{(A-1)}{\operatorname{Tan}^2\varepsilon + A}\cdot\frac{\operatorname{Tan}^2\varepsilon}{\operatorname{Tan}^2\varepsilon} = \frac{1}{\operatorname{Tan}\varepsilon}\cdot\frac{(A-1)}{1+\dfrac{A}{\operatorname{Tan}^2\varepsilon}} =$$

$$= \operatorname{cot}\varepsilon\cdot\frac{1}{\dfrac{1+A/\operatorname{Tan}^2\varepsilon}{(A-1)}}$$

ESSENDO :

$$A-1 = \frac{(\gamma+1) M_1^2 \operatorname{sen}^2 \varepsilon - 2 - (\gamma-1) M_1^2 \operatorname{sen}^2 \varepsilon}{2+(\gamma-1) M_1^2 \operatorname{sen}^2 \varepsilon} = \frac{2\left(M_1^2 \operatorname{sen}^2 \varepsilon - 1\right)}{2+(\gamma-1) M_1^2 \operatorname{sen}^2 \varepsilon}$$

E:

$$1 + A/\tan^2 \varepsilon = 1 + \frac{\cos^2 \varepsilon}{\operatorname{sen}^2 \varepsilon} \cdot A = 1 + \frac{(\gamma+1) M_1^2 \cos^2 \varepsilon}{2+(\gamma-1) M_1^2 \operatorname{sen}^2 \varepsilon} =$$

$$= \frac{2+(\gamma-1) M_1^2 \operatorname{sen}^2 \varepsilon + (\gamma+1) M_1^2 (1-\operatorname{sen}^2 \varepsilon)}{2+(\gamma-1) M_1^2 \operatorname{sen}^2 \varepsilon} =$$

$$= \frac{2 - 2 M_1^2 \operatorname{sen}^2 \varepsilon + (\gamma+1) M_1^2}{2+(\gamma-1) M_1^2 \operatorname{sen}^2 \varepsilon} \quad \Rightarrow$$

$$\frac{1 + A/\tan^2\varepsilon}{A-1} = \frac{2 - 2M_1^2 \operatorname{sen}^2\varepsilon + (\gamma+1)M_1^2}{2 + \cancel{(\gamma-1)M_1^2 \operatorname{sen}^2\varepsilon}} \cdot \frac{2 + \cancel{(\gamma-1)M_1^2 \operatorname{sen}^2\varepsilon}}{2(M_1^2 \operatorname{sen}^2\varepsilon - 1)} =$$

$$= \frac{2(1 - M_1^2 \operatorname{sen}^2\varepsilon) + (\gamma+1)M_1^2}{2(M_1^2 \operatorname{sen}^2\varepsilon - 1)} \cdot \frac{1/M_1^2}{1/M_1^2} =$$

$$= \frac{2\left(\frac{1}{M_1^2} - \operatorname{sen}^2\varepsilon\right) + (\gamma+1)}{2\left(\operatorname{sen}^2\varepsilon - \frac{1}{M_1^2}\right)} =$$

$$= \frac{\gamma+1}{2\left(\operatorname{sen}^2\varepsilon - \frac{1}{M_1^2}\right)} - 1$$

SOSTITUENDO:

$$\tan\delta = \cot\varepsilon \cdot \frac{1}{\dfrac{1 + A/\tan^2\varepsilon}{(A-1)}} =$$

$$= \cot\varepsilon \cdot \left[\frac{\gamma+1}{2\left(\operatorname{sen}^2\varepsilon - \dfrac{1}{M_1^2}\right)} - 1 \right]^{-1} \qquad (8.45)$$

$$\frac{V_{n1}}{V_{n2}} = \frac{(\gamma+1)\, M_1^2 \operatorname{sen}^2 \varepsilon}{2+(\gamma-1)\, M_1^2 \operatorname{sen}^2 \varepsilon} = \frac{(\gamma+1)\, M_1^2 \frac{V_{n1}^2}{V_1^2}}{2+(\gamma-1)\, M_1^2 \frac{V_{n1}^2}{V_1^2}} =$$

$$= \frac{(\gamma+1)\, \frac{V_{n1}^2}{a_1^2}}{2+(\gamma-1)\, \frac{V_{n1}^2}{a_1^2}} = \frac{(\gamma+1)\, V_{n1}^2}{2\, a_1^2 + (\gamma-1) V_{n1}^2} \Longrightarrow$$

$$\left[2\, a_1^2 + (\gamma-1)\, V_{n1}^2 \right] \cdot \cancel{V_{n1}} = (\gamma+1) \cdot V_{n1}^{\cancel{2}} \cdot V_{n2} \Longrightarrow$$

$$V_{n1} \cdot V_{n2} = \frac{2}{\gamma+1}\, a_1^2 + \frac{\gamma-1}{\gamma+1}\, V_{n1}^2 \qquad (8.47)$$

POICHÉ: $V_1^2 = V_{n_1}^2 + V_t^2 \Longrightarrow$

$$V_{n_1} \cdot V_{n_2} = \frac{2}{\gamma+1} a_1^2 + \frac{\gamma-1}{\gamma+1} V_1^2 - \frac{\gamma-1}{\gamma+1} V_t^2 =$$

$$= \frac{2(\gamma-1)}{\gamma+1} \cdot \left(\frac{V_1^2}{2} + \frac{a_1^2}{\gamma-1} \right) - \frac{\gamma-1}{\gamma+1} V_t^2$$

POICHÉ: $H = \frac{V^2}{2} + C_p T = \frac{V^2}{2} + \frac{a^2}{\gamma-1}$

E: $\sqrt{\frac{2H(\gamma-1)}{(\gamma+1)}} = a^*$ (7.44)

ABBIAMO:

$$V_{n_1} \cdot V_{n_2} = a^{*2} - \frac{\gamma-1}{\gamma+1} V_t^2 \qquad (8.48)$$

$$V_1^2 \operatorname{sen}^2\varepsilon - \nu V_1 \tan\varepsilon = a^{*2} - \frac{\gamma-1}{\gamma+1} V_1^2 \cos^2\varepsilon \quad (8.50)$$

SOSTITUENDO LE (8.49), (8.51) ED (8.52):

$$V_1^2 \cdot \frac{(V_1-U)^2}{\nu^2+(V_1-U)^2} - \cancel{\nu} V_1 \cdot \frac{V_1-U}{\cancel{\nu}} = a^{*2} - \frac{\gamma-1}{\gamma+1} V_1^2 \cdot \frac{\nu^2}{\nu^2+(V_1-U)^2}$$

MOLTIPLICANDO ENTRAMBI I MEMBRI PER $\left[\nu^2+(V_1-U)^2\right]$:

$$V_1^2 (V_1-U)^2 - V_1(V_1-U) \cdot \left[\nu^2+(V_1-U)^2\right] =$$

$$= a^{*2}\left[\nu^2+(V_1-U)^2\right] - \frac{\gamma-1}{\gamma+1} V_1^2 \nu^2$$

EVIDENZIANDO I TERMINI IN U^2:

$$U^2\left[a^{*2}-\frac{\gamma-1}{\gamma+1}V_1^2+V_1(V_1-U)\right]=V_1^2(V_1-U)^2-V_1(V_1-U)^3-a^*(V_1-U)^2$$

$$\Downarrow$$

$$U^2=\frac{\left[V_1^2-V_1(V_1-U)-a^*\right]\cdot(V_1-U)^2}{a^*+2\frac{V_1^2}{\gamma+1}-V_1\cdot U}=$$

$$=\frac{(V_1\cdot U-a^*)\cdot(V_1-U)^2}{a^*+2\frac{V_1^2}{\gamma+1}-V_1\cdot U}\qquad(8.54)$$