$$d\delta = -\sqrt{M^2-1}\left(1+\frac{\gamma-1}{2}M^2\right)^{-1}\frac{dM}{M} \qquad (9.11)$$

$$x = M^2 \Rightarrow \frac{dx}{x} = \frac{2M\,dM}{M^2} = 2\frac{dM}{M} \Rightarrow$$

$$\delta = \int \frac{-\sqrt{x-1}}{1+\frac{\gamma-1}{2}x}\,\frac{1}{2x}\,dx = -\frac{1}{2}\int \frac{(x-1)}{\left(1+\frac{\gamma-1}{2}\right)\sqrt{x-1}}\,\frac{dx}{x} =$$

$$= -\frac{1}{2}\int \frac{dx}{\left(1+\frac{\gamma-1}{2}x\right)\sqrt{x-1}} + \frac{1}{2}\int \frac{dx}{\left(1+\frac{\gamma-1}{2}x\right)\sqrt{x-1}\cdot x} =$$

ESSENDO :

$$\frac{1}{\left(1+\frac{\gamma-1}{2}x\right)}\cdot\frac{1}{x} = \frac{A}{x} + \frac{B}{\left(1+\frac{\gamma-1}{2}x\right)} \Rightarrow \begin{matrix} A = 1 \\ B = -\frac{(\gamma-1)}{2} \end{matrix} \Rightarrow$$

$$\delta = -\frac{1}{2}\int \frac{dx}{\left(1+\frac{\gamma-1}{2}x\right)\sqrt{x-1}} + \frac{1}{2}\int \frac{dx}{x\sqrt{x-1}} + \frac{1}{2}\int \frac{-(\gamma-1)/2}{\left(1+\frac{\gamma-1}{2}x\right)\sqrt{x-1}}\,dx =$$

$$= \frac{1}{2}\int \frac{-\frac{(\gamma-1)}{2}-1}{\left(1+\frac{\gamma-1}{2}x\right)\sqrt{x-1}}\,dx + \frac{1}{2}\int \frac{dx}{x\sqrt{x-1}}$$

POICHÉ: $1+\frac{\gamma-1}{2}x = 1+\frac{\gamma-1}{2}(x-1)+\frac{\gamma-1}{2} = \frac{\gamma-1}{2}(x+1)+\frac{\gamma-1}{2} =$

$$= \frac{\gamma+1}{2}\left[1+\frac{\gamma-1}{\gamma+1}(x-1)\right] = \frac{\gamma+1}{2}\left[1+\frac{x-1}{K}\right] \Rightarrow$$

$$\delta = \frac{1}{2}\int \frac{-\cancel{(\gamma+1)}/2}{\cancel{\frac{\gamma+1}{2}}\left[1+\frac{x-1}{K}\right]\sqrt{x-1}}\,dx + \frac{1}{2}\int \frac{dx}{x\sqrt{x-1}} =$$

$$= \sqrt{K}\ \arctan\sqrt{\frac{x-1}{K}} + \arctan\sqrt{x-1} + \text{cost} \overset{x=M^2}{=}$$

$$= \sqrt{K}\ \arctan\sqrt{\frac{M^2-1}{K}} + \arctan\sqrt{M^2-1} + \text{cost} \qquad (9.12)$$

SIA : $y = \sqrt{M^2-1}$

$$\delta(y) = -\sqrt{K}\ \operatorname{arctan}(y/\sqrt{K}) + \operatorname{arctan} y$$

$$\delta \cong \delta(0) + \left.\frac{\partial \delta}{\partial y}\right|_{y=0} \cdot y + \frac{1}{2}\left.\frac{\partial^2 \delta}{\partial y^2}\right|_{y=0} \cdot y^2 + \frac{1}{6}\left.\frac{\partial^3 \delta}{\partial y^3}\right|_{y=0} \cdot y^3$$

ESSENDO:

$$\frac{\partial \delta}{\partial y} = \frac{\cancel{\sqrt{K}} \cdot 1/\cancel{\sqrt{K}}}{1+\frac{y^2}{K}} - \frac{1}{1+y^2} = \frac{(1+y^2)-(1+y^2/K)}{\left(1+\frac{y^2}{K}\right)(1+y^2)} = \left(1-\frac{1}{K}\right)\frac{y^2}{\left(1+\frac{y^2}{K}\right)(1+y^2)}$$

$$\frac{\partial^2 \delta}{\partial y^2} = \frac{K-1}{K} \cdot \frac{2y(1+y^2+y^2/K+y^4/K) - y^2(2y+2y/K+4y^3/K)}{\left[\left(1+\frac{y^2}{K}\right)(1+y^2)\right]^2} =$$

$$= \frac{K-1}{K} \cdot \frac{2y+\cancel{2y^3}+\cancel{\frac{2}{K}y^3}+\frac{2}{K}y^5-\cancel{2y^3}-\cancel{\frac{2}{K}y^3}-\frac{4}{K}y^5}{\left[\left(1+\frac{y^2}{K}\right)(1+y^2)\right]^2} =$$

$$= \frac{K-1}{K} \frac{2y - \frac{2}{K} y^5}{\left[\left(1 + \frac{y^2}{K}\right)\left(1 + y^2\right) \right]^2}$$

$$\frac{\partial^3 \delta}{\partial y^3} = \frac{K-1}{K} \frac{\left(2 + \frac{10}{K} y^4\right)\left[\left(1 + \frac{y^2}{K}\right)\left(1+y^2\right)\right]^2 - \left(2y - \frac{2}{K} y^5\right) \cdot 2\left[1 + y^2 + \frac{y^2}{K} + \frac{y^4}{K}\right] \cdot \left(2y + \frac{2}{K} y + \frac{4}{K} y^3\right)}{\left[\left(1 + \frac{y^2}{K}\right)\left(1+y^2\right)\right]^4}$$

$$\left.\frac{\partial^3 \delta}{\partial y^3}\right|_{y=0} = \frac{K-1}{K} \cdot \frac{2}{1}$$

$$\delta = 0 + 0 \cdot y + 0 \cdot y^2 + \frac{1}{6} \cdot 2 \cdot \frac{K-1}{K} y^3 =$$

$$= \frac{1}{3} \frac{K-1}{K} y^3 = \frac{K-1}{3K} \left(M^2 - 1\right)^{\frac{3}{2}} \qquad (4.13)$$

$$\rho V A = \rho^* V^* A^* \Rightarrow$$

$$\frac{A}{A^*} = \frac{\rho^*}{\rho} \cdot \frac{V^*}{V} = \frac{\rho^*}{\rho_0} \cdot \frac{\rho_0}{\rho} \cdot \frac{V^*}{a_0} \cdot \frac{a_0}{a} \cdot \frac{a}{V} =$$

$$= \frac{\rho^*}{\rho_0} \cdot \frac{\rho_0}{\rho} \cdot \frac{V^*}{a_0} \cdot \sqrt{\frac{T_0}{T}} \cdot \frac{a}{V}$$

SOSTITUENDO LE (7.34), (7.36), (7.44) E (10.2) $\Rightarrow$

$$\frac{A}{A^*} = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \cdot \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{1}{\gamma-1}} \cdot \left(\frac{2}{\gamma+1}\right)^{\frac{1}{2}} \cdot \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{1}{2}} \cdot \frac{1}{M} =$$

$$= \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}+\frac{1}{2}} \cdot \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{1}{\gamma-1}+\frac{1}{2}} \cdot \frac{1}{M} =$$

$$= \frac{1}{M} \cdot \left[\frac{2}{\gamma+1} \cdot \left(1 + \frac{\gamma-1}{2} M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}} \qquad (7.56)$$

$$\Psi = K\,\eta^{\frac{1}{\gamma}} \cdot \left[1 - \eta^{\frac{(\gamma-1)}{\gamma}}\right]^{\frac{1}{2}} \Longrightarrow$$

$$\left.\frac{d\Psi}{d\eta}\right|_{\eta=\bar{\eta}} = K \cdot \frac{1}{\gamma} \cdot \bar{\eta}^{\left(\frac{1}{\gamma}-1\right)} \cdot \left[1 - \bar{\eta}^{\frac{(\gamma-1)}{\gamma}}\right]^{\frac{1}{2}} - K\,\bar{\eta}^{\frac{1}{\gamma}} \cdot \frac{1}{2} \cdot \frac{\frac{\gamma-1}{\gamma} \cdot \bar{\eta}^{\left(\frac{\gamma-1}{\gamma}-1\right)}}{\left[1 - \bar{\eta}^{\frac{(\gamma-1)}{\gamma}}\right]^{\frac{1}{2}}} =$$

$$= K \cdot \frac{1}{\gamma}\,\bar{\eta}^{\left(\frac{1-\gamma}{\gamma}\right)} \cdot \left[1 - \bar{\eta}^{\left(\frac{\gamma-1}{\gamma}\right)}\right]^{\frac{1}{2}} - K\,\frac{\bar{\eta}^{\frac{1}{\gamma}}}{2} \cdot \frac{(\gamma-1)\,\bar{\eta}^{-\frac{1}{\gamma}}}{\left[1 - \bar{\eta}^{\left(\frac{\gamma-1}{\gamma}\right)}\right]^{\frac{1}{2}}} = 0$$

$$\Longrightarrow \quad \bar{\eta} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\Psi^* = \Psi\left[\bar{\eta} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}\right] =$$

$$= K \cdot \left[\left(\frac{2}{\gamma+1} \right)^{\frac{\cancel{\gamma}}{\gamma-1}} \right]^{\frac{1}{\cancel{\gamma}}} \cdot \left[1 - \left(\frac{2}{\gamma+1} \right)^{\cancel{\frac{\gamma}{\gamma-1}} \cdot \cancel{\frac{\gamma-1}{\gamma}}} \right]^{\frac{1}{2}} =$$

$$= K \cdot \left[\left(\frac{2}{\gamma+1} \right)^{\frac{1}{\gamma-1}} \right] \cdot \left[1 - \frac{2}{\gamma+1} \right]^{\frac{1}{2}}$$

ESSENDO : $K = \left[\frac{2\gamma^2}{\gamma-1} \right]^{\frac{1}{2}} \Longrightarrow$

$$\psi^* = \left(\frac{2\gamma^2}{\cancel{\gamma-1}} \right)^{\frac{1}{2}} \cdot \left(\frac{2}{\gamma+1} \right)^{\frac{1}{\gamma-1}} \cdot \left(\frac{\cancel{\gamma-1}}{\gamma+1} \right)^{\frac{1}{2}} =$$

$$= \gamma \cdot \left(\frac{2}{\gamma+1} \right)^{\frac{1}{2}} \cdot \left(\frac{2}{\gamma+1} \right)^{\frac{1}{\gamma-1}} = \gamma \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \quad (10.9)$$

$$\frac{p^*}{\cancel{R}T^*}\sqrt{\cancel{\gamma R}T^*}\cdot A^* = \frac{p_u}{\cancel{R}T_u} M_u \sqrt{\cancel{\gamma R}T_u}\cdot A_u \Rightarrow$$

$$\frac{M_u}{\sqrt{T_u}} = \frac{p^*}{p_u}\cdot\frac{A^*}{A_u}\cdot\frac{1}{\sqrt{T^*}} = \frac{p^*}{p_u}\cdot\frac{A^*}{A_u}\cdot\sqrt{\frac{T_0}{T^*}}\cdot\frac{1}{\sqrt{T_0}}$$

POICHÉ:

$$\frac{p^*}{p_u} = \frac{p^*}{p_0}\cdot\frac{p_0}{p_u} \overset{(10.3)}{=} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}\cdot\frac{p_0}{p_u}$$

$$\frac{T_0}{T^*} \overset{(7.34)}{=} 1+\frac{\gamma-1}{2}M^{*2} = 1+\frac{\gamma-1}{2} = \frac{\gamma+1}{2} \Rightarrow$$

$$\frac{M_u}{\sqrt{\frac{T_u}{T_0}}} = \frac{p^*}{p_u}\cdot\frac{A^*}{A_u}\cdot\sqrt{\frac{T_0}{T^*}} = \frac{p_0}{p_u}\cdot\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}\frac{A^*}{A_u}\cdot\sqrt{\frac{\gamma+1}{2}} =$$

$$= \frac{p_0}{p_u} \cdot \frac{A^*}{A_u} \cdot \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \cdot \left(\frac{2}{\gamma+1}\right)^{-\frac{1}{2}} =$$

$$= \frac{p_0}{p_u} \cdot \frac{A^*}{A_u} \cdot \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} = B \qquad (10.14)$$

QUADRANDO :

$$M_u^2 \cdot \frac{T_0}{T_u} \overset{(7.34)}{=} M_u^2 \cdot \left[1 + \frac{\gamma-1}{2} M_u^2\right] = M_u^2 + \frac{\gamma-1}{2} M_u^4 = B^2 \Longrightarrow$$

$$M_u^2 = \frac{-1 \pm \sqrt{1+2(\gamma-1)\,B^2}}{\gamma-1} \qquad (10.15)$$

$$T_0 = T_{0i}\left(\frac{p_0}{p_{0i}}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow$$

$$\frac{p_0}{T_0} = \frac{p_0^{\left(1-\frac{\gamma-1}{\gamma}\right)}}{T_{0i}} \cdot p_{0i}^{\frac{\gamma-1}{\gamma}} = \frac{p_0^{\frac{1}{\gamma}}}{T_{0i}} \cdot p_{0i}^{\frac{\gamma-1}{\gamma}} \Rightarrow$$

$$\frac{d}{dt}\left(\frac{p_0}{T_0}\right) = \frac{p_{0i}^{\frac{\gamma-1}{\gamma}}}{T_{0i}} \cdot \frac{d}{dt}\left(p_0^{\frac{1}{\gamma}}\right) = \frac{p_{0i}^{\frac{\gamma-1}{\gamma}}}{T_{0i}} \cdot \frac{1}{\gamma} p_0^{\frac{1}{\gamma}-1} \cdot \frac{dp_0}{dt} =$$

$$= \frac{p_{0i}^{\frac{\gamma-1}{\gamma}}}{\gamma T_{0i}} \cdot p_0^{\frac{1-\gamma}{\gamma}} \frac{dp_0}{dt} \qquad (11.9)$$

$$T_0 = T_{0i} \left(\frac{p_0}{p_{0i}} \right)^{\frac{\gamma-1}{\gamma}} \Rightarrow$$

$$\frac{p_0}{\sqrt{T_0}} = \frac{p_0}{\sqrt{T_{0i} \left(\frac{p_0}{p_{0i}} \right)^{\frac{\gamma-1}{\gamma}}}} = \frac{p_0 \cdot p_{0i}^{\frac{\gamma-1}{2\gamma}}}{\sqrt{T_{0i}} \cdot p_0^{\frac{\gamma-1}{2\gamma}}} =$$

$$= \frac{p_{0i}^{\frac{\gamma-1}{2\gamma}}}{\sqrt{T_{0i}}} \cdot p_0^{1-\frac{\gamma-1}{2\gamma}} = \frac{p_{0i}^{\frac{\gamma-1}{2\gamma}}}{\sqrt{T_{0i}}} \cdot p_0^{\frac{\gamma+1}{2\gamma}} \quad (11.10)$$

$$\frac{V_s}{\gamma R T_{oi}} \, p_{oi}^{\frac{\gamma-1}{\gamma}} \cdot p_o^{\frac{1-\gamma}{\gamma}} \, \frac{dp_o}{dt} + \frac{A_u \psi^*}{\sqrt{\gamma R T_{oi}}} \cdot p_{oi}^{\frac{\gamma-1}{2\gamma}} \cdot p_o^{\frac{\gamma+1}{2\gamma}} = 0$$

POICHÉ: $a_{oi} = \sqrt{\gamma R T_{oi}}$ E $\vartheta = \dfrac{\gamma V_s}{A_u a_o \psi^*} \Longrightarrow$

$$\frac{V_s}{a_{oi}^2} \, p_{oi}^{\frac{\gamma-1}{\gamma}} \cdot p_o^{\frac{1-\gamma}{\gamma}} \, \frac{dp_o}{dt} + \frac{A_u \psi^*}{a_{oi}} \, p_{oi}^{\frac{\gamma-1}{2\gamma}} \cdot p_o^{\frac{\gamma+1}{2\gamma}} = 0$$

$$p_{oi}^{\frac{\gamma-1}{\gamma}} \cdot p_{oi}^{\frac{1-\gamma}{2\gamma}} \cdot p_o^{\frac{1-\gamma}{\gamma}} \cdot p_o^{-\frac{\gamma+1}{2\gamma}} \, \frac{dp_o}{dt} =$$

$$= -\frac{A_u \psi^*}{a_{oi}} \cdot \frac{a_{oi}^2}{V_s} \Longrightarrow$$

$$p_{oi}^{\frac{\gamma-1}{2\gamma}} \cdot p_o^{\frac{1-3\gamma}{2\gamma}} \, dp_o = -\frac{\gamma}{\vartheta_i} \, dt \qquad (11.11)$$