MOTO ALLA RAYLEIGH

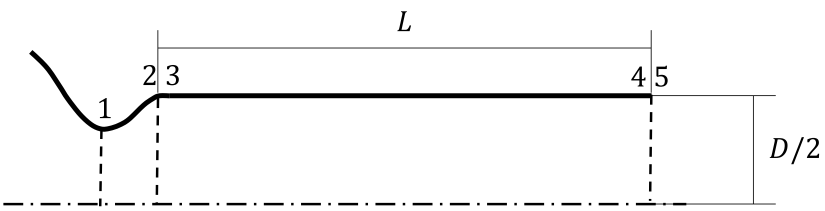
Un ugello convergente divergente è collegato ad un condotto con adduzione di calore, come mostrato in figura. Supponendo che:

$$A_2/A_1 = 2.00 \qquad D = 0.100 \cdot m \qquad\qquad p_0 = 100 \cdot kPa \quad T_0 = 300 \cdot K$$

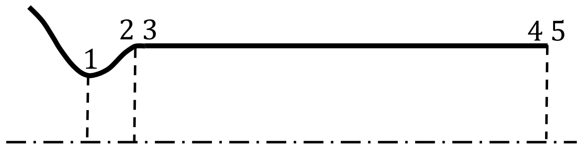
determinare le condizioni termofluidodinamiche all'uscita del condotto e la caduta di pressione di ristagno, nei seguenti casi:

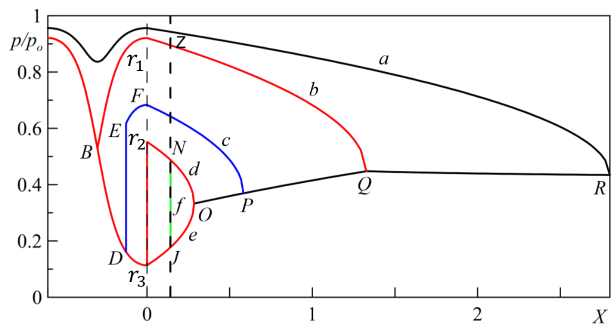
a) $\frac{q}{c_p T_{01}} = 0.150$ per $p_a = 98.0 \cdot kPa, \;\; 60.0 \cdot kPa;$

b) $\frac{q}{c_p T_{01}} = 1.00$ per $p_a = 80.0 \cdot kPa, \;\; 60.0 \cdot kPa.$



Si inizia a determinare i punti caratteristici.





Punto r_1

$$\frac{A_2}{A_1} = 2.0 \xrightarrow[(M<1)]{\text{ISO}} \quad M_{r_1} = 0.306 \qquad \frac{p_{r_1}}{p_0} = 0.937 \qquad \frac{T_{r_1}}{T_0} = 0.982$$

Punto r_3

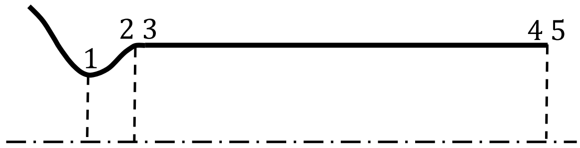
$$\frac{A_2}{A_1} = 2.0 \xrightarrow[(M>1)]{\text{ISO}} \quad M_{r_3} = 2.20 \qquad \frac{p_{r_3}}{p_0} = 0.0939 \qquad \frac{T_{r_3}}{T_0} = 0.509$$

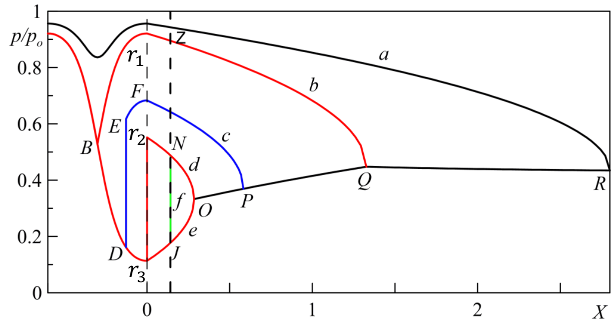
Punto r_2

$$M_{r_3} = 2.20 \xrightarrow{\text{NSW}} \quad M_{r_2} = 0.547$$

$$\frac{p_{r_2}}{p_{r_3}} = 5.47 \quad \Rightarrow \quad \frac{p_{r_2}}{p_0} = \frac{p_{r_2}}{p_{r_3}}\frac{p_{r_3}}{p_0} = 5.47 \cdot 0.0939 = 0.513$$

Noto il numero di Mach dalle tabelle del moto alla Rayleigh (RF) si trova:





Punto r_1

$$M_C = 0.306 \xrightarrow{\text{RF}} \quad \frac{p_{r_1}}{p_{r_1}^*} = 2.12 \qquad \frac{T_{0r_1}}{T_0^*} = 0.358$$

$$p_{r_1}^* = p_Q = \frac{p_{r_1}^*}{p_{r_1}}\frac{p_{r_1}}{p_0}\, p_0 = \frac{1}{2.12} \cdot 0.937 \cdot p_0 = 44.2 \cdot kPa$$

Punto r_2

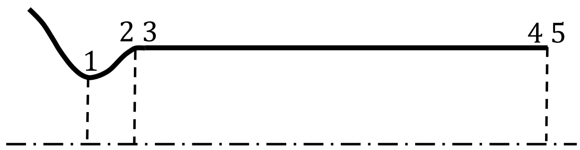
$$M_{r_2} = 0.547 \xrightarrow{\text{RF}} \quad \frac{p_{r_2}}{p_{r_2}^*} = 1.691 \qquad \frac{T_{0r_2}}{T_0^*} = 0.757$$

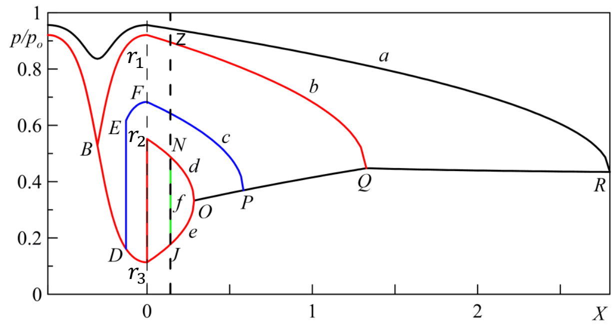
$$p_{r_2}^* = p_O = \frac{p_{r_2}^*}{p_{r_2}}\frac{p_{r_2}}{p_0}\, p_0 = \frac{1}{1.691} \cdot 0.513 \cdot p_0 = 30.4 \cdot kPa$$

Punto r_3

$$M_{r_3} = 2.20 \xrightarrow{\text{RF}} \quad \frac{p_{r_3}}{p_{r_3}^*} = 0.309 \qquad \frac{T_{0r_3}}{T_0^*} = \frac{T_{0r_2}}{T_0^*} = 0.757$$

$p_{r_3}^* = p_O = p_{r_2}^*$ (siccome i punti r_3 ed r_2 hanno stesso G ed I, ovvero sono sulla stessa curva di Rayleigh)





Punto r_1

$$p_{r_1}^* = 44.2 \cdot kPa \qquad \frac{T_{0r_1}}{T_0^*} = 0.358$$

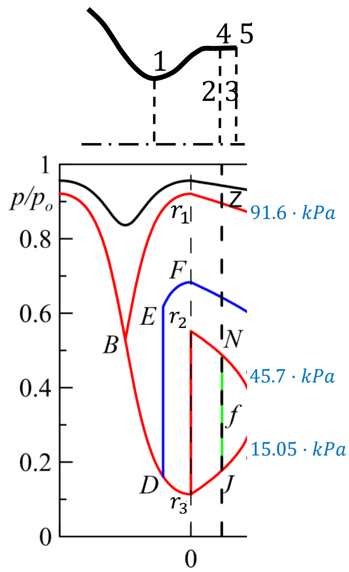
$$\frac{q^*}{c_p T_{01}} = \frac{c_p(T_0^* - T_{01})}{c_p T_{01}} = \frac{T_0^*}{T_{01}} - 1 = \frac{1}{0.358} - 1 = 1.796$$

Punto r_2, r_3

$$p_{r_2}^* = p_{r_3}^* = 30.4 \cdot kPa \qquad \frac{T_{0r_2}}{T_0^*} = \frac{T_{0r_3}}{T_0^*} = 0.757$$

$$\frac{q^*}{c_p T_{01}} = \frac{c_p(T_0^* - T_{01})}{c_p T_{01}} = \frac{T_0^*}{T_{01}} - 1 = \frac{1}{0.757} - 1 = 0.322$$

Caso a): $\frac{q}{c_p T_{01}} = 0.150 \quad \rightarrow \quad \frac{T_{05}}{T_{02}} = 1 + 0.150 = 1.150$



Punto Z

$$\frac{T_{05}}{T_0^*} = \frac{T_{05}}{T_{02}}\frac{T_{02}}{T_0^*} = 1.150 \cdot 0.358 = 0.411$$

$$\frac{T_{05}}{T_0^*} = 0.411 \quad \xrightarrow[(M<1)]{\text{RF}} \quad M_5 = 0.335 \qquad \frac{p_5}{p^*} = 2.07$$

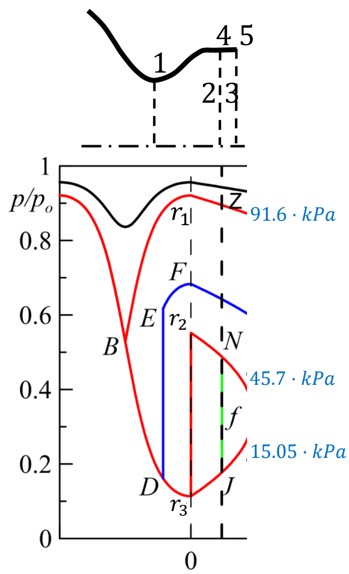
$$p_Z = p_5 = \frac{p_5}{p^*}p^* = 2.07 \cdot 44.2 = 91.6 \cdot kPa$$

Analogamente:

Punto N $\quad \frac{T_{05}}{T_0^*} = \frac{T_{05}}{T_{02}}\frac{T_{02}}{T_0^*} = 1.150 \cdot 0.757 = 0.870 \quad \xrightarrow[(M<1)]{\text{RF}} \quad M_5 = 0.652 \quad \frac{p_5}{p^*} = 1.505 \Rightarrow p_5 = 45.7 \cdot kPa$

Punto J $\quad \frac{T_{05}}{T_0^*} = \frac{T_{05}}{T_{02}}\frac{T_{02}}{T_0^*} = 1.150 \cdot 0.757 = 0.870 \quad \xrightarrow[(M>1)]{\text{RF}} \quad M_5 = 1.657 \quad \frac{p_5}{p^*} = 0.495 \Rightarrow p_5 = 15.05 \cdot kPa$

Caso a): $\frac{q}{c_p T_{01}} = 0.150 \quad \rightarrow \quad \frac{T_{05}}{T_{02}} = 1 + 0.150 = 1.150$



Analizziamo i 2 sottocasi proposti nella traccia:

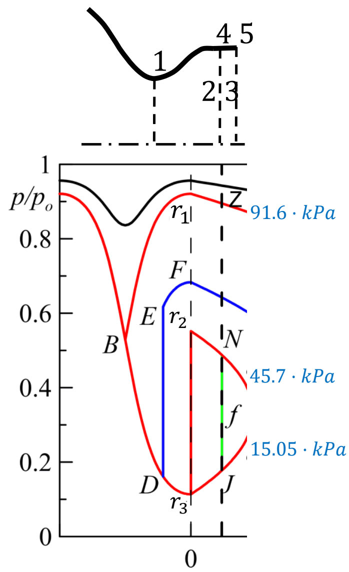
i. $p_a = 95.0 \cdot kPa$

ii. $p_a = 60.0 \cdot kPa$

i. $p_a > pz$: il moto è completamente subsonico (funzionamento alla Venturi);

ii. $p_V < p_a < p_Z$: è presente un'onda d'urto nel divergente 1-2;

Caso a) i.: $\frac{q}{c_p T_{01}} = 0.15 \quad \rightarrow \quad \frac{T_{05}}{T_{02}} = 1 + 0.15 = 1.15,\ p_a = 95 \cdot kPa$



Il moto è completamente subsonico (funzionamento alla Venturi),

Si procede iterativamente assegnando il numero di Mach all'uscita del condotto.

Supponendo che $M_5 = 0.300$ ($< M_Z = 0.335$).

$$M_5 = 0.300 \xrightarrow{\text{RF}} \frac{T_{05}}{T_0^*} = 0.347 \qquad \frac{p_5}{p^*} = 2.13$$

$$\frac{T_{02}}{T_0^*} = \frac{T_{02}}{T_{05}}\frac{T_{05}}{T_0^*} = \frac{0.347}{1.150} = 0.302 \xrightarrow[(M<1)]{\text{RF}} \frac{p_2}{p^*} = 2.17 \qquad M_2 = 0.275 \xrightarrow{\text{ISO}} \frac{p_2}{p_{0_2}} = 0.949$$

Infine:

$$p_5 = \frac{p_5}{p^*}\frac{p^*}{p_2}\frac{p_2}{p_{0_2}}\,p_{0_2} = \frac{2.13}{2.17} \cdot 0.949 \cdot 100 = 93.2 \cdot kPa \quad \Rightarrow \quad e = \frac{p_5 - p_a}{p_a} = -1.904\%$$

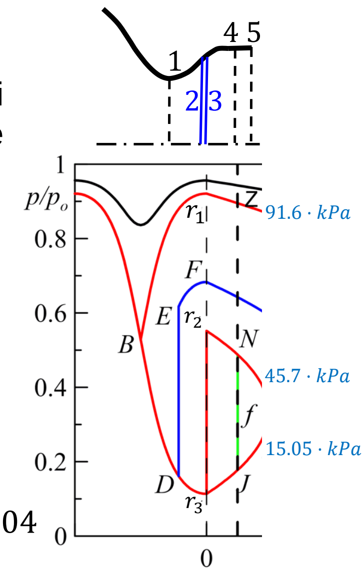
Si procede con il metodo di falsa posizione.

$$\frac{q}{c_p T_{01}} = 0.150 \qquad p_0 = 100 \cdot kPa \quad p_a = 95.0 \cdot kPa$$

		dalle tabelle (RF) entrando con M_5		$= \frac{T_{02}}{T_{05}}\frac{T_{05}}{T_0^*}$	dalle tabelle (RF) entrando con $\frac{T_{02}}{T_0^*}$		da (ISO) con M_2	$p_5 = p_5/p^* \cdot p^*/p_2 \cdot p_2/p_{0_2} \cdot p_{0_2}$	$= \frac{p_5 - p_a}{p_a}$
tenta-tivo #	M_5	$\frac{p_5}{p^*}$	$\frac{T_{05}}{T_0^*}$	$\frac{T_{02}}{T_0^*}$	M_2	$\frac{p_2}{p^*}$	$\frac{p_2}{p_{0_2}}$	p_5 (kPa)	**errore (%)**
(1)	0.335	0.4114	2.0741	0.3577	0.3059	2.122	0.9372	91.601	-3.578
(2)	0.300	0.3469	2.1314	0.3016	0.2752	2.17	0.9488	93.191	-1.904
(3)	0.2602	0.2747	2.1923	0.2389	0.2397	2.2214	0.9608	94.823	-0.187
(4)	0.2558	0.2671	2.1986	0.2322	0.2358	2.2267	0.962	94.988	-0.013
(5)	0.2555	0.2665	2.199	0.2317	0.2355	2.2271	0.9621	95.000	-1E-04
(6)	0.2555	0.2665	2.199	0.2317	0.2355	2.2271	0.9621	95.000	-5E-08

$$M_5^{(n)} = \frac{M_5^{(n-1)} e^{(n-2)} - M_5^{(n-2)} e^{(n-1)}}{e^{(n-2)} - e^{(n-1)}}$$

Caso a) ii.: $\frac{q}{c_p T_{01}} = 0.15 \quad \rightarrow \quad \frac{T_{05}}{T_{02}} = 1 + 0.150 = 1.150,\ p_a = 60.0 \cdot kPa$



È presente un'onda d'urto nel divergente 1-2. In questo caso al contrario di quello che succedeva nel moto alla Fanno e per ugello convergente divergente si deve procedere per tentativi.

Avendo già trovato i punti N e Z si può immediatamente applicare il metodo di falsa posizione.

$$M_Z = 0.335, p_Z = 91.6 \cdot kPa \qquad M_2^{(n-2)} = M_B = 1, e^{(n-2)} = 52.6\%$$

$$M_N = 0.652, p_Z = 45.7 \cdot kPa \quad \rightarrow \quad M_2^{(n-1)} = M_{r_3} = 2.20, e^{(n-1)} = -23.9\%$$

da cui:

$$M_2^{(n)} = \frac{M_2^{(n-1)} e^{(n-2)} - M_2^{(n-2)} e^{(n-1)}}{e^{(n-2)} - e^{(n-1)}} = 1.82$$

$$M_2 = 1.82 \xrightarrow{\text{NSW}} \frac{p_{03}}{p_{02}} = 0.802 \qquad \frac{A_4}{A_4^*} = \frac{A_4}{A_1}\frac{A_1}{A_4^*} = \frac{A_4}{A_1}\frac{p_{03}}{p_{02}} = 2 \cdot 0.802 = 1.604$$

$$\frac{A_4}{A_4^*} = 1.604 \xrightarrow[(M<1)]{\text{ISO}} M_4 = 0.396 \qquad \frac{p_4}{p_{04}} = 0.898 \qquad \xrightarrow{\text{RF}} \frac{p_4}{p^*} = 1.968 \qquad \frac{T_{04}}{T_0^*} = 0.522$$

$$\frac{T_{05}}{T_0^*} = \frac{T_{05}}{T_{04}}\frac{T_{04}}{T_0^*} = 0.600 \xrightarrow{\text{RF}} \begin{matrix} M_5 = 0.441 \\ \frac{p_5}{p^*} = 1.886 \end{matrix} \quad p_5 = \frac{p_5}{p^*}\frac{p^*}{p_4}\frac{p_4}{p_{04}}\frac{p_{04}}{p_{02}} p_0 = \frac{1.886}{1.968} 0.898 \cdot 0.802 \cdot 100 = 69.0 \cdot kPa$$

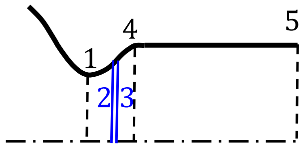
Si procede con il metodo di falsa posizione.

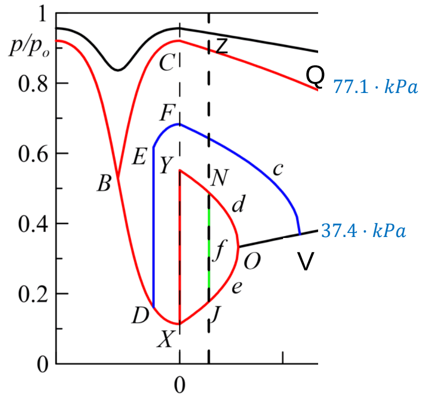
$\frac{q}{c_p T_{01}} = 0.150 \qquad \frac{T_{05}}{T_{04}} = 1.150 \qquad p_a = 60.0 \cdot kPa$

		(NSW)	$= \frac{A_4}{A_1}\frac{p_{03}}{p_{02}}$	(ISO)	(ISO)	(RF) M_4	(RF) M_4	$= \frac{T_{05}}{T_{04}}\frac{T_{04}}{T_0^*}$	(RF)	(RF)	$\frac{p_5}{p^*}\frac{p^*}{p_4}\frac{p_4}{p_{04}}\frac{p_{04}}{p_{02}} p_0$	$= \frac{p_5 - p_a}{p_a}$
#	$\boldsymbol{M_2}$	$\boldsymbol{\frac{p_{03}}{p_{02}}}$	$\boldsymbol{\frac{A_4}{A_4^*}}$	$\boldsymbol{M_4}$	$\boldsymbol{\frac{p_4}{p_{04}}}$	$\boldsymbol{\frac{p_4}{p^*}}$	$\boldsymbol{\frac{T_{04}}{T_0^*}}$	$\boldsymbol{\frac{T_{05}}{T_0^*}}$	$\boldsymbol{M_5}$	$\boldsymbol{\frac{p_5}{p^*}}$	$\boldsymbol{p_5}$ **(kPa)**	**errore (%)**
(1)	1	1	2	0.3059	0.9372	2.122	0.3577	0.4114	0.335	2.0741	91.601	52.669
(2)	2.1972	0.6294	1.2588	0.5474	0.8157	1.691	0.7566	0.8701	0.652	1.5046	45.689	-23.85
(3)	1.824	0.802	1.6039	0.3958	0.8977	1.968	0.5216	0.5998	0.4414	1.8856	68.963	14.938
(4)	1.9677	0.7359	1.4719	0.4408	0.8751	1.887	0.5988	0.6887	0.4982	1.7812	60.797	1.3279
(5)	1.9818	0.7294	1.4588	0.4459	0.8724	1.877	0.6073	0.6984	0.5048	1.7689	59.956	-0.073
(6)	1.981	0.7297	1.4595	0.4457	0.8725	1.878	0.6068	0.6979	0.5045	1.7695	60	0.0003

$$M_2^{(n)} = \frac{M_2^{(n-1)} e^{(n-2)} - M_2^{(n-2)} e^{(n-1)}}{e^{(n-2)} - e^{(n-1)}}$$

Caso b): $\frac{q}{c_p T_{01}} = 1.00 \quad \rightarrow \quad \frac{T_{05}}{T_{02}} = 1 + 1.00 = 2.00$





<u>Punto</u> Q

$$\frac{T_{05}}{T_0^*} = \frac{T_{05}}{T_{02}}\frac{T_{02}}{T_0^*} = 2 \cdot 0.358 = \quad 0.715$$

$$\frac{T_{05}}{T_0^*} = 0.715 \quad \xrightarrow[(M<1)]{\text{RF}} \quad M_5 = \quad 0.517 \qquad \frac{p_5}{p^*} = \quad 1.747$$

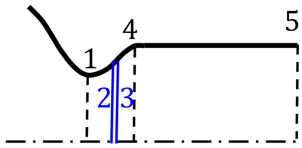
$$p_Q = p_5 = \frac{p_5}{p^*} p^* = 1.747 \cdot 44.2 = \quad 77.1 \cdot kPa$$

<u>Punto</u> V

$$\frac{T_{04}}{T_0^*} = \frac{T_{04}}{T_{05}} = 0.500 \quad \xrightarrow[(M<1)]{\text{RF}} \quad M_4 = \quad 0.384 \qquad \frac{p_4}{p^*} = \quad 1.990 \quad \xrightarrow{\text{ISO}} \quad \frac{p_4}{p_{04}} = \quad 0.903 \qquad \frac{A_4}{A^*} = \quad 1.646$$

$$\frac{p_{03}}{p_{02}} = \frac{A_2^*}{A_3^*} = \frac{A_1}{A_4}\frac{A_4}{A_4^*} = \frac{1.644}{2} = 0.823 \quad \xrightarrow{\text{NSW}} \quad M_2 = \quad 1.777 \qquad p_5 = p^* = \frac{p^*}{p_4}\frac{p_4}{p_{04}}\frac{p_{04}}{p_{02}} p_0 = 37.4 \cdot kpa$$

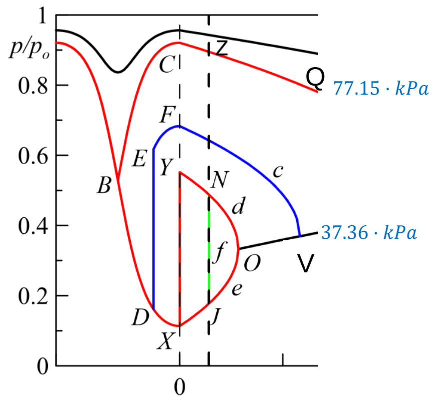
Caso b): $\frac{q}{c_p T_{01}} = 1.00 \quad \rightarrow \quad \frac{T_{05}}{T_{02}} = 1 + 1.00 = 2.00$



Analizziamo i 2 sottocasi proposti nella traccia:

i. $p_a = 80.0 \cdot kPa$

ii. $p_a = 60.0 \cdot kPa$



i. $p_a > pz$: il moto è completamente subsonico (funzionamento alla Venturi);

ii. $p_V < p_a < p_Z$: è presente un'onda d'urto nel divergente 1-2;

Si procede con il metodo di falsa posizione.

$$\frac{q}{c_p T_{01}} = 1.00$$

$p_0 = 100 \cdot kPa$ $p_a = 80.0 \cdot kPa$

		dalle tabelle (RF) entrando con M_5		$= \frac{T_{02}}{T_{05}}\frac{T_{05}}{T_0^*}$	dalle tabelle (RF) entrando con $\frac{T_{02}}{T_0^*}$		da (ISO) con M_2	$p_5 = p_5/p^*\ p^*/p_2\ p_2/p_{0_2}\ p_{0_2}$	$= \frac{p_5 - p_a}{p_a}$
tenta-tivo #	$\boldsymbol{M_5}$	$\boldsymbol{\frac{p_5}{p^*}}$	$\frac{T_{05}}{T_0^*}$	$\frac{T_{02}}{T_0^*}$	$\boldsymbol{M_2}$	$\boldsymbol{\frac{p_2}{p^*}}$	$\boldsymbol{\frac{p_2}{p_{0_2}}}$	**p_5 (kPa)**	**errore (%)**
(1)	0.5168	0.7154	1.7468	0.3577	0.3059	2.122	0.9372	77.148	-3.565
(2)	0.3	0.3469	2.1314	0.1734	0.1999	2.2728	0.9725	91.202	14.003
(3)	0.4728	0.6503	1.8279	0.3251	0.2881	2.1501	0.944	80.255	0.3187
(4)	0.4768	0.6565	1.8205	0.3283	0.2898	2.1474	0.9434	79.974	-0.033
(5)	0.4764	0.6559	1.8212	0.328	0.2897	2.1477	0.9434	80.000	4E-05
(6)	0.4764	0.6559	1.8212	0.328	0.2897	2.1477	0.9434	80.000	5E-09

$$M_5^{(n)} = \frac{M_5^{(n-1)} e^{(n-2)} - M_5^{(n-2)} e^{(n-1)}}{e^{(n-2)} - e^{(n-1)}}$$

Si procede con il metodo di falsa posizione.

$$\frac{q}{c_p T_{01}} = 1.00 \qquad \frac{T_{05}}{T_{04}} = 2.00$$

$p_a = 60.0 \cdot kPa$

		(NSW)	$= \frac{A_4}{A_1}\frac{p_{03}}{p_{02}}$	(ISO)	(ISO)	(RF) M_4	(RF) M_4	$= \frac{T_{05}}{T_{04}}\frac{T_{04}}{T_0^*}$	(RF)	(RF)	$\frac{p_5}{p^*}\frac{p^*}{p_4}\frac{p_4}{p_{04}}\frac{p_{04}}{p_{02}} p_0$	$= \frac{p_5 - p_a}{p_a}$
#	$\boldsymbol{M_2}$	$\boldsymbol{\frac{p_{03}}{p_{02}}}$	$\boldsymbol{\frac{A_4}{A_4^*}}$	$\boldsymbol{M_4}$	$\boldsymbol{\frac{p_4}{p_{04}}}$	$\boldsymbol{\frac{p_4}{p^*}}$	$\boldsymbol{\frac{T_{04}}{T_0^*}}$	$\boldsymbol{\frac{T_{05}}{T_0^*}}$	$\boldsymbol{M_5}$	$\boldsymbol{\frac{p_5}{p^*}}$	**p_5 (kPa)**	**errore (%)**
(1)	1	1	2	0.3059	0.9372	2.122	0.3577	0.7154	0.5168	1.7468	77.148	28.579
(2)	1.7771	0.8228	1.6456	0.3836	0.9034	1.9899	0.5	1	1	1	37.355	-37.74
(3)	1.3349	0.9729	1.9457	0.3155	0.9333	2.1064	0.3755	0.751	0.5431	1.6987	73.223	22.038
(4)	1.4979	0.9305	1.8609	0.332	0.9265	2.0792	0.4058	0.8117	0.5934	1.6076	66.655	11.092
(5)	1.6631	0.8708	1.7416	0.3586	0.915	2.0339	0.4547	0.9093	0.7012	1.4216	55.687	-7.188
(6)	1.5981	0.8959	1.7918	0.3469	0.9202	2.054	0.4332	0.8663	0.6478	1.5118	60.676	1.1266

$$M_2^{(n)} = \frac{M_2^{(n-1)} e^{(n-2)} - M_2^{(n-2)} e^{(n-1)}}{e^{(n-2)} - e^{(n-1)}}$$