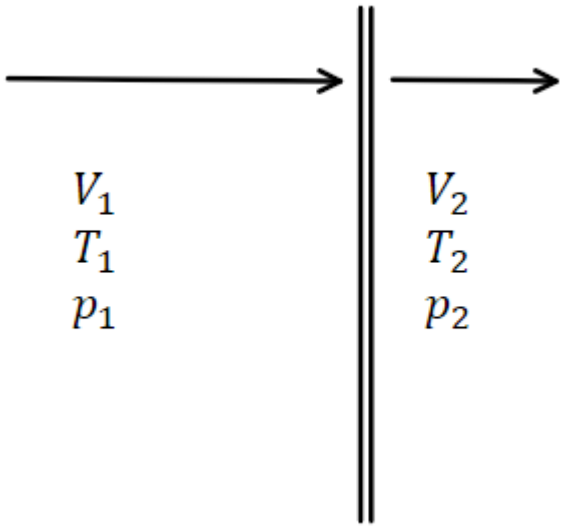
Onde d'urto stazionarie



Esercizio 1

$p_1 = 10 \cdot psi \qquad V_1 = 1500 \cdot \frac{ft}{s} \qquad T_1 = 0F = 459.67R \qquad R = 1716 \cdot \frac{ft^2}{s^2 R} \qquad \gamma = 1.4$

$$M_1 = \frac{V_1}{a_1} = \frac{V_1}{\sqrt{\gamma R T_1}} = \frac{1500}{\sqrt{1.4 \cdot 1716 \cdot 459.7}} = 1.427$$

$$M_1 = 1.43 \xrightarrow{ISO} \frac{p_1}{p_{01}} = 0.301 \rightarrow p_{01} = \frac{p_{01}}{p_1} p_1 = \frac{10.0}{0.301} = 33.2 \cdot psi$$

$$M_1 = 1.43 \xrightarrow{NSW} \begin{cases} M_2 = 0.727 \\ \frac{p_2}{p_1} = 2.22 \\ \frac{T_2}{T_1} = 1.274 \\ \frac{p_{02}}{p_{01}} = 0.950 \end{cases}$$

$$M_1 = 1.43 \xrightarrow{ISO} \frac{p_1}{p_{01}} = 0.301 \rightarrow p_{01} = \frac{p_{01}}{p_1} p_1 = \frac{10.0}{0.301} = 33.2 \cdot psi$$

$$p_2 = \frac{p_2}{p_1} p_1 = 2.22 \cdot 10.0 = 22.2 \cdot psi \qquad T_2 = \frac{T_2}{T_1} T_1 = 1.274 \cdot 459.7 = 586 \cdot R$$

$$p_{02} = \frac{p_{02}}{p_{01}} p_{01} = 0.950 \cdot 33.2 = 31.6 \cdot psi \qquad \Delta p_0 = p_{02} - p_{01} = -(33.2 - 31.6) = -1.6 \cdot psi$$

Esercizio 2

$p_1 = 100 \cdot kPa \qquad p_2 = 400 \cdot kPa \qquad M_1 = ? \qquad M_2 = ? \qquad \Delta p_0 = ?$

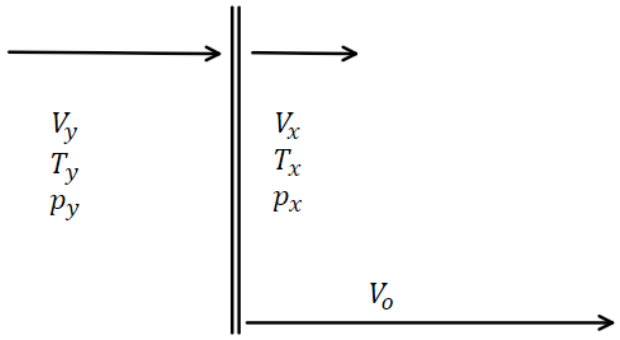
$$\frac{p_2}{p_1} = \frac{400}{100} = 4 \xrightarrow{NSW} \begin{cases} M_1 = 1.89 \\ M_2 = 0.598 \\ \frac{p_{02}}{p_{01}} = 0.772 \end{cases}$$

$$M_1 = 1.89 \xrightarrow{ISO} \frac{p_1}{p_{01}} = 0.1516 \quad \rightarrow \quad p_{01} = \frac{p_{01}}{p_1} p_1 = \frac{100}{0.152} = 660 \cdot kPa$$

$$p_{02} = \frac{p_{02}}{p_{01}} p_{01} = 0.772 \cdot 660 = 509 \cdot kPa$$

$$\Delta p_0 = \left(\frac{p_{02}}{p_{01}} - 1\right) p_{01} = p_{02} - p_{01} = 509 - 660 = 151 \cdot kPa$$

Onde d'urto non stazionarie



$$V_2 = V_0 - V_y \qquad V_1 = V_0 - V_x$$
$$T_2 = T_y \qquad T_1 = T_x$$
$$p_2 = p_y \qquad p_1 = p_x$$

Esercizio 1

$$V_0 = 600 \cdot \frac{m}{s} \qquad T_x = 300K \qquad p_x = 101 \cdot kPa \qquad V_x = 0 \cdot \frac{m}{s}$$

$$V_1 = V_o - V_x = V_o = 600 \cdot \frac{m}{s} \qquad V_2 = V_o - V_y = ?$$

$$a_1 = \sqrt{\gamma R T_1} = \sqrt{1.4 \cdot 287 \cdot 300} = 347 \cdot \frac{m}{s} \qquad M_1 = \frac{V_1}{a_1} = \frac{600}{347} = 1.728$$

$$M_1 = 1.728 \xrightarrow{NSW} \begin{cases} \frac{p_2}{p_1} = 3.32 \\ \frac{T_2}{T_1} = 1.478 \\ \frac{p_{02}}{p_{01}} = 0.843 \\ \frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = 2.25 \end{cases} \qquad M_1 = 1.728 \xrightarrow{ISO} \begin{cases} \frac{p_1}{p_{01}} = 0.194 \\ \frac{T_1}{T_{01}} = 0.626 \end{cases}$$

$$p_{01} = \frac{p_1}{\frac{p_1}{p_{01}}} = \frac{p_{01}}{p_1} p_1 = \frac{101}{0.194} = 522 \cdot kPa \qquad T_{01} = \frac{T_{01}}{T_1} T_1 = \frac{300}{0.626} = 480 \cdot K$$

$$p_2 = \frac{p_2}{p_1} p_1 = 3.32 \cdot 101 = 336 \cdot kPa \qquad T_2 = \frac{T_2}{T_1} T_1 = 1.478 \cdot 300 = 443 \cdot K$$

$$p_{02} = \frac{p_{02}}{p_{01}} p_{01} = 0.843 \cdot 522 = 440 \cdot kPa \qquad T_{02} = \frac{T_{02}}{T_{01}} T_{01} = 1 \cdot 480 = 480 \cdot K$$

$$V_2 = \frac{V_2}{V_1} V_1 = \frac{600}{2.25} = 267 \cdot \frac{m}{s}$$

Sistema di riferimento in cui l'onda è non stazionaria

$$V_y = V_o - V_2 = 600 - 267 = 333 \cdot \frac{m}{s} \qquad M_y = \frac{V_y}{a_y} = \frac{333}{\sqrt{1.4 \cdot 287 \cdot 443}} = \frac{333}{422} = 0.788$$

$$M_y = 0.788 \xrightarrow{ISO} \frac{p_y}{p_{0y}} = 0.664 \qquad \frac{T_y}{T_{0y}} = 0.890$$

$$p_{oy} = \frac{p_{oy}}{p_y} p_y = \frac{336}{0.664} = 506 \cdot kPa \qquad T_{oy} = \frac{T_{oy}}{T_y} T_y = \frac{443}{0.890} = 498 \cdot K$$

$$V_2 = 267\frac{m}{s} \qquad V_1 = 600\frac{m}{s}$$
$$T_2 = T_y = 443K \qquad T_1 = T_x = 300K$$
$$p_2 = p_y = 336 \cdot kPa \qquad p_1 = p_x = 101kPa$$
$$T_{02} = 480 \cdot K \qquad T_{01} = 480K$$
$$p_{02} = 440 \cdot kPa \qquad p_{01} = 522 \cdot kPa$$

$$V_y = 333\frac{m}{s} \qquad V_x = 0\frac{m}{s}$$
$$T_y = 443K \qquad T_x = 300K$$
$$p_y = 336kPa \qquad p_x = 101kPa$$
$$T_{0y} = 498K \qquad T_{0x} = 300K$$
$$p_{0y} = 506 \cdot kPa \qquad p_{0x} = 101 \cdot kPa$$

$$V_o = 600\frac{m}{s}$$

Esercizio 2

$V_0 = ?$ $\quad T_x = 300K$ $\quad V_x = 0 \cdot \frac{m}{s}$ $\quad V_y = 100 \cdot \frac{m}{s}$

$a_x = \sqrt{\gamma R T_x} = \sqrt{1.4 \cdot 287 \cdot 300} = 347 \cdot \frac{m}{s}$

Primo tentativo

$M_0^0 = 1.4 \quad \rightarrow \quad V_o = M_0 a_x = 1.4 \cdot 347 = 486.1 \cdot \frac{m}{s} \quad V_x = 0 \quad \rightarrow \quad M_o = M_1 \quad V_1 = V_o$

$M_1 = 1.4 \xrightarrow{NSW} \frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = 1.69 \quad \rightarrow \quad V_2 = \frac{V_2}{V_1} V_1 = \frac{486.1}{1.69} = 288 \cdot \frac{m}{s}$

$V_y' = V_o - V_2 = 486.1 - 288 = 198 \cdot \frac{m}{s} \qquad e = \frac{V_y' - V_y}{V_y} = \frac{198 - 100}{100} = .98$

Secondo tentativo

$M_0^1 = 1.2 \quad \rightarrow \quad V_o = M_0 a_x = 1.2 \cdot 347 = 416.6 \cdot \frac{m}{s}$

$V_x = 0 \quad \rightarrow \quad M_o = M_1 \quad V_1 = V_o$

$M_1 = 1.2 \xrightarrow{NSW} \frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = 1.342 \quad \rightarrow \quad V_2 = \frac{V_2}{V_1} V_1 = \frac{416.6}{1.342} = 311 \cdot \frac{m}{s}$

$V_y' = V_o - V_2 = 417 - 311 = 106 \cdot \frac{m}{s} \qquad e = \frac{V_y' - V_y}{V_y} = \frac{106 - 100}{100} = .06$

$$M_o^2 = \frac{M_o^0 e^1 - M_o^1 e^0}{e^1 - e^0} = \frac{1.4 \cdot 0.06 - 1.2 \cdot 0.98}{0.06 - 0.98} = 1.19$$

Tx	300	K	ax	347.2	m/s	Vy	100
M0	V0=V1	ρ2/ρ1	V2	Vy'	e%		
1.4	486.1	1.690	287.6706	198.4	98.39355	1.2	
1.2	416.6	1.342	310.541	106.1	6.085	1.187	
1.187	412.0	1.319	312.4566	99.6	-0.408	1.188	
1.188	412.3	1.320	312.3345	100.0	0.002	1.188	
1.188	412.3	1.320	312.3351	100.0	0.000	1.188	

Esercizio 3

$V_0 = ?$ $\quad T_x = 280 \cdot K$ $\quad V_x = 300 \cdot \frac{m}{s}$ $\quad V_y = 500 \cdot \frac{m}{s}$

$$a_x = \sqrt{\gamma R T_x} = \sqrt{1.4 \cdot 287 \cdot 280} = 335 \cdot \frac{m}{s}$$

Primo tentativo

$$M_1^0 = 2 \quad \rightarrow \quad V_1 = M_1 a_x = 2 \cdot 335 = 670 \cdot \frac{m}{s} \qquad V_o = V_1 + V_x = 670 + 300 = 970 \cdot \frac{m}{s}$$

$$M_1^0 = 2 \quad \xrightarrow{NSW} \quad \frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = 2.67 \quad \rightarrow \quad V_2 = V_1 \frac{V_2}{V_1} = \frac{670}{2.67} = 251 \cdot \frac{m}{s}$$

$$V_y' = V_o - V_2 = 970 - 251 = 719 \cdot \frac{m}{s} \qquad e = \frac{V_y' - V_y}{V_y} = \frac{719 - 500}{500} = 0.438$$

Secondo tentativo

$$M_1^1 = 1.6 \quad \rightarrow \quad V_1 = M_1 a_x = 1.6 \cdot 335 = 537 \cdot \frac{m}{s} \qquad V_o = V_1 + V_x = 537 + 300 = 837 \cdot \frac{m}{s}$$

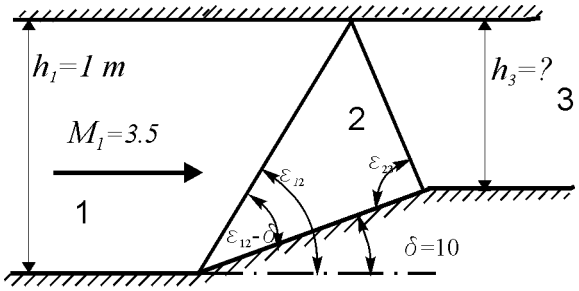
$$M_1^0 = 1.6 \quad \xrightarrow{NSW} \quad \frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = 2.032 \quad \rightarrow \quad V_2 = V_1 \frac{V_2}{V_1} = \frac{537}{2.032} = 264 \cdot \frac{m}{s}$$

$$V_y' = V_o - V_2 = 837 - 264 = 572 \cdot \frac{m}{s} \qquad e = \frac{V_y' - V_y}{V_y} = \frac{572 - 500}{500} = 0.145$$

$$M_1^2 = \frac{M_1^0 e^1 - M_1^1 e^0}{e^1 - e^0} = \frac{2 \cdot 0.145 - 1.6 \cdot 0.438}{0.145 - 0.438} = 1.40$$

Tx	280	K	ax	335.4	m/s		
Vx	300	Vy	500				
M1	V1	Vo	ρ2/ρ1	V2	Vy'	e%	
2	670.8	970.8	2.667	251.5621	719.3	43.85404	1.6
1.6	536.7	836.7	2.032	264.1402	572.5	14.50513	1.402
1.402	470.4	770.4	1.694	277.7167	492.6	-1.472	1.421
1.421	476.5	776.5	1.725	276.1792	500.3	0.057337	1.420
1.420	476.2	776.2	1.724	276.2356	500.0	0.000237	1.420

Onde oblique 1



$M_1 = 3.5 \quad \delta = 10° \quad \xrightarrow{\delta\epsilon M} \quad \epsilon_{12} = 24.5°$

$M_{n1} = M_1 \sin(\epsilon_{12}) = 3.5 \cdot \sin(24.5) = 1.45$

$M_{t1} = M_1 \cos(\epsilon_{12}) = 3.5 \cdot \cos(24.5) = 3.18$ M_t1=M_1 cos(ϵ_12)=3.5·cos(24.5)=3.18

$M_{n1} = 1.45 \quad \xrightarrow{NSW} \quad \frac{\rho_2}{\rho_1} = 1.776 \quad \frac{T_2}{T_1} = 1.287 \quad M_{n2} = 0.719 \quad \frac{p_{02}}{p_{01}} = 0.945$

$$V_{t2} = V_{t1} \quad \rightarrow \quad M_{t2} = M_{t1}\frac{a_1}{a_2} = M_{t1}\sqrt{\frac{T_1}{T_2}} = 3.18\sqrt{\frac{1}{1.287}} = 2.8$$

$$M_2 = \sqrt{M_{t2}^2 + M_{n2}^2} = \sqrt{2.8^2 + 0.719^2} = 2.89$$

$M_2 = 2.89 \quad \delta = 10° \quad \xrightarrow{\delta\epsilon M} \quad \epsilon_{23} = 28°$

$M_{n2} = M_2 \sin(\epsilon_{23}) = 2.89 \cdot \sin(28) = 1.36$

$M_{t2} = M_2 \cos(\epsilon_{23}) = 2.89 \cdot \cos(28) = 2.55$

$M_{n2} = 1.36 \quad \xrightarrow{NSW} \quad \frac{\rho_3}{\rho_2} = 1.620 \quad \frac{T_3}{T_2} = 1.229 \quad M_{n3} = 0.757 \quad \frac{p_{03}}{p_{02}} = 0.968$

$$V_{t3} = V_{t2} \quad \rightarrow \quad M_{t3} = M_{t2}\frac{a_2}{a_1} = M_{t2}\sqrt{\frac{T_2}{T_3}} = 2.55\sqrt{\frac{1}{1.229}} = 2.30$$

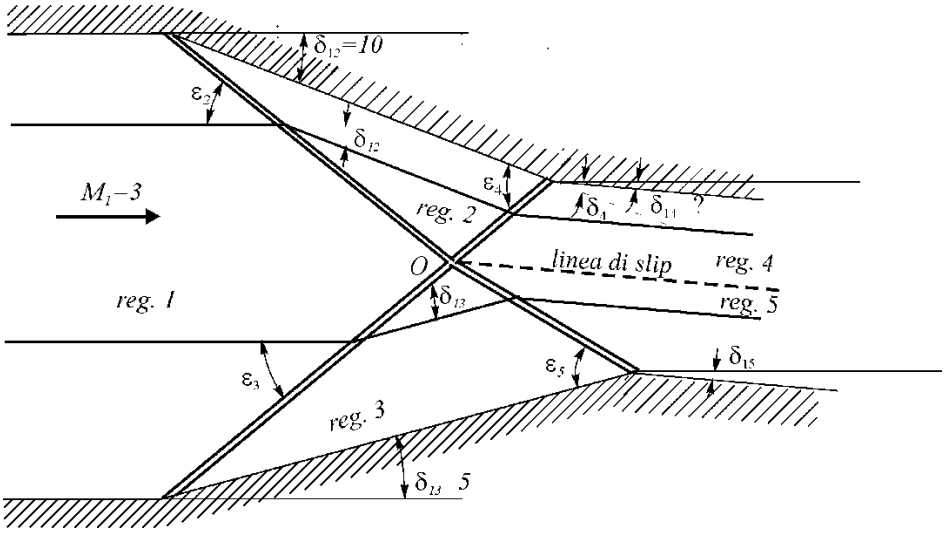
$$M_3 = \sqrt{M_{t3}^2 + M_{n3}^2} = \sqrt{2.30^2 + 0.757^2} = 2.42$$

$$\rho_1 V_1 A_1 = \rho_3 V_3 A_3 \quad \rightarrow \quad \frac{A_3}{A_1} = \frac{\mathrm{h}_3}{\mathrm{h}_1} = \frac{\rho_1 V_1}{\rho_3 V_3} = \frac{\rho_1}{\rho_2}\frac{\rho_2}{\rho_3}\frac{M_1 a_1}{M_3 a_3} = \frac{\rho_1}{\rho_2}\frac{\rho_2}{\rho_3}\frac{M_1}{M_3}\sqrt{\frac{T_1}{T_2}\frac{T_2}{T_3}}$$

$$\frac{h_3}{h_1} = \frac{\rho_1}{\rho_2}\frac{\rho_2}{\rho_3}\frac{M_1}{M_3}\sqrt{\frac{T_1}{T_2}\frac{T_2}{T_3}} = \frac{1}{1.776}\frac{1}{1.620}\frac{3.5}{2.42}\sqrt{\frac{1}{1.287}\frac{1}{1.229}} = 0.397m$$

$$\frac{p_{03}}{p_{01}} = \frac{p_{03}}{p_{02}}\frac{p_{02}}{p_{01}} = 0.968 \cdot 0.945 = 0.915$$

Onde oblique 2



$M_1 = 3 \quad \delta_{12} = 10° \xrightarrow{\delta\epsilon M} \epsilon_2 = 27.5°$

$M_{n1} = M_1 \sin(\epsilon_2) = 3.0 \cdot \sin(27.5) = 1.385 \quad M_{t1} = M_1 \cos(\epsilon_2) = 3.0 \cdot \cos(27.5) = 2.66$

$M_{n1} = 1.385 \xrightarrow{NSW} \frac{p_2}{p_1} = 2.09 \quad \frac{T_2}{T_1} = 1.248 \quad M_{n2} = 0.744$

$M_{t2} = M_{t1}\frac{a_1}{a_2} = M_{t1}\sqrt{\frac{T_1}{T_2}} = 2.66\sqrt{\frac{1}{1.218}} = 2.38 \quad M_2 = \sqrt{M_{t2}^2 + M_{n2}^2} = \sqrt{2.38^2 + 0.744^2} = 2.49$

$M_1 = 3 \quad \delta_{13} = 5° \xrightarrow{\delta\epsilon M} \epsilon_3 = 23°$

$M_{n1} = M_1 \sin(\epsilon_2) = 3.0 \cdot \sin(23.0) = 1.172 \quad M_{t1} = M_1 \cos(\epsilon_2) = 3.0 \cdot \cos(23.0) = 2.76$

$M_{n1} = 1.172 \xrightarrow{NSW} \frac{p_3}{p_1} = 1.43 \quad \frac{T_3}{T_1} = 1.109 \quad M_{n3} = 0.861$

$M_{t3} = M_{t1}\frac{a_1}{a_3} = M_{t1}\sqrt{\frac{T_1}{T_3}} = 2.76\sqrt{\frac{1}{1.109}} = 2.62 \quad M_3 = \sqrt{M_{t3}^2 + M_{n3}^2} = \sqrt{2.62^2 + 0.861^2} = 2.75$

Deviazioni positive verso il basso

$\delta_{14} = \delta_{12} + \delta_{24} = \delta_{15} = \delta_{13} + \delta_{35} \qquad \delta_{24} = \delta_{14} - \delta_{12} \qquad \delta_{35} = \delta_{14} - \delta_{13}$

Primo tentativo

$\delta_{14} = 10° \qquad \delta_{24} = \delta_{14} - \delta_{12} = 0 - 10 = -10$

$M_2 = 2.49 \quad |\delta_{24}| = 10° \xrightarrow{\delta\epsilon M} \epsilon_4 = 32.0°$

$M_{n2} = M_2 \sin(\epsilon_4) = 2.49 \cdot \sin(32) = 1.319 \quad M_{n2} = 1.319 \xrightarrow{NSW} \frac{p_4}{p_2} = 1.866$

$\delta_{14} = 10° \qquad \delta_{35} = \delta_{14} - \delta_{13} = 0 - (-5) = 5$

$M_3 = 2.75 \quad |\delta_{24}| = 5° \xrightarrow{\delta\epsilon M} \epsilon_4 = 25.0°$

$M_{n3} = M_3 \sin(\epsilon_4) = 2.75 \cdot \sin(25) = 1.165 \quad M_{n3} = 1.165 \xrightarrow{NSW} \frac{p_5}{p_3} = 1.403$

$$\frac{p_5}{p_4} = \frac{p_5}{p_3}\frac{p_3}{p_1}\frac{p_1}{p_2}\frac{p_2}{p_4} = 1.403 \cdot \left(\frac{1.43}{2.09}\right)\frac{1}{1.866} = 0.537$$