MOTO ALLA RAYLEIGH

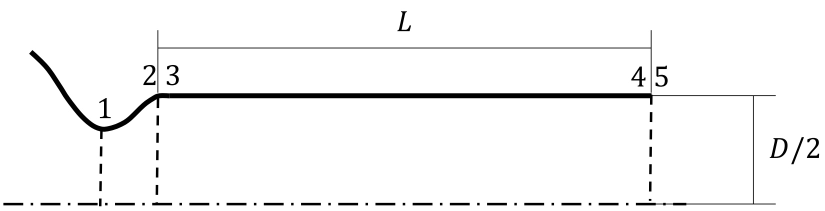
Un ugello convergente divergente è collegato ad un condotto con adduzione di calore, come mostrato in figura. Supponendo che:

$$A_2/A_1 = 2.00 \qquad D = 0.100 \cdot m \qquad p_0 = 100 \cdot kPa \quad T_0 = 300 \cdot K$$

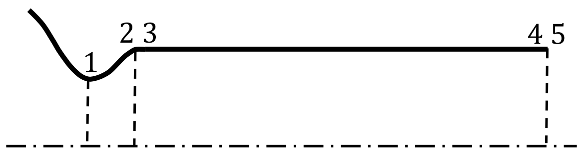
determinare le condizioni termofluidodinamiche all'uscita del condotto e la caduta di pressione di ristagno, nei seguenti casi:

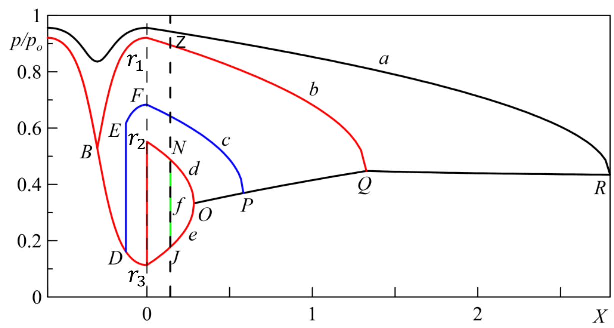
a) $\frac{q}{c_p T_{01}} = 0.150$ per $p_a = 98.0 \cdot kPa, \; 60.0 \cdot kPa;$

b) $\frac{q}{c_p T_{01}} = 1.00$ per $p_a = 80.0 \cdot kPa, \; 60.0 \cdot kPa.$



Si inizia a determinare i punti caratteristici.





Punto r_1

$$\frac{A_2}{A_1} = 2.0 \xrightarrow[(M<1)]{\text{ISO}} \quad M_{r_1} = 0.306 \quad \frac{p_{r_1}}{p_0} = 0.937 \quad \frac{T_{r_1}}{T_0} = 0.982$$

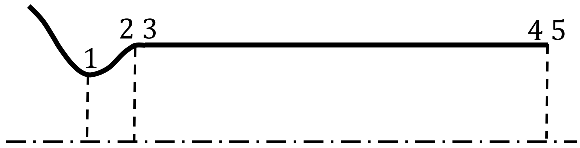
Punto r_3

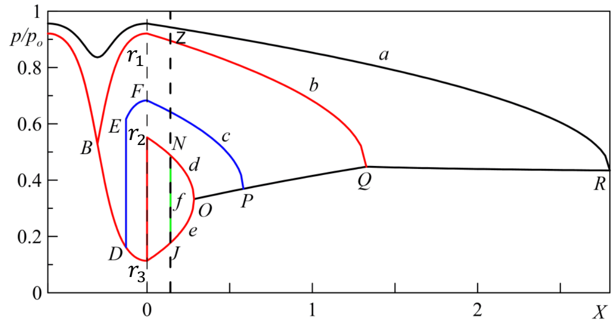
$$\frac{A_2}{A_1} = 2.0 \xrightarrow[(M>1)]{\text{ISO}} \quad M_{r_3} = 2.20 \quad \frac{p_{r_3}}{p_0} = 0.0935 \quad \frac{T_{r_3}}{T_0} = 0.508$$

Punto r_2

$$M_{r_3} = 2.20 \xrightarrow{\text{NSW}} \quad M_{r_2} = 0.547 \quad \frac{p_{r_2}}{p_{r_3}} = 5.48 \quad \Rightarrow \quad \frac{p_{r_2}}{p_0} = \frac{p_{r_2}}{p_{r_3}}\frac{p_{r_3}}{p_0} = 5.48 \cdot 0.0935 = 0.512$$

Noto il numero di Mach dalle tabelle del moto alla Rayleigh (RF) si trova:





Punto r_1

$$M_C = 0.306 \xrightarrow{\text{RF}} \quad \frac{p_{r_1}}{p^*_{r_1}} = 2.12 \qquad \frac{T_{0r_1}}{T^*_0} = 0.358$$

$$p^*_{r_1} = p_Q = \frac{p^*_{r_1}}{p_{r_1}}\frac{p_{r_1}}{p_0}\, p_0 = \frac{1}{2.12}\cdot 0.937\cdot p_0 = 44.2\cdot kPa$$

Punto r_2

$$M_{r_2} = 0.547 \xrightarrow{\text{RF}} \quad \frac{p_{r_2}}{p^*_{r_2}} = 1.691 \qquad \frac{T_{0r_2}}{T^*_0} = 0.756$$

$$p^*_{r_2} = p_O = \frac{p^*_{r_2}}{p_{r_2}}\frac{p_{r_2}}{p_0}\, p_0 = \frac{1}{1.691}\cdot 0.512\cdot p_0 = 30.3\cdot kPa$$

Punto r_3

$$M_{r_3} = 2.20 \xrightarrow{\text{RF}} \quad \frac{p_{r_3}}{p^*_{r_3}} = 0.309 \qquad \frac{T_{0r_3}}{T^*_0} = \frac{T_{0r_2}}{T^*_0} = 0.756$$

$p^*_{r_3} = p_O = p^*_{r_2}$ (siccome i punti r_3 ed r_2 hanno stesso G ed I, ovvero sono sulla stessa curva di Rayleigh)

Punto r_1

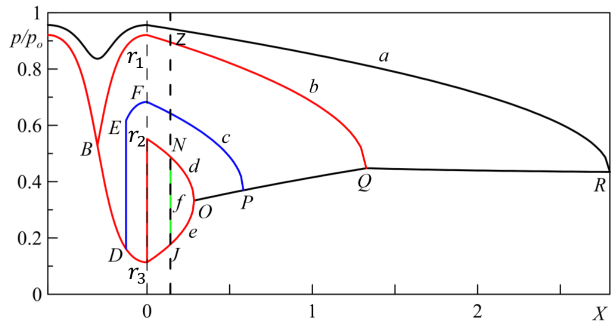
$$p^*_{r_1} = 44.2\cdot kPa \qquad \frac{T_{0r_1}}{T^*_0} = 0.358$$

$$\frac{q^*}{c_p T_{01}} = \frac{c_p(T^*_0 - T_{01})}{c_p T_{01}} = \frac{T^*_0}{T_{01}} - 1 = \frac{1}{0.358} - 1 = 1.793$$

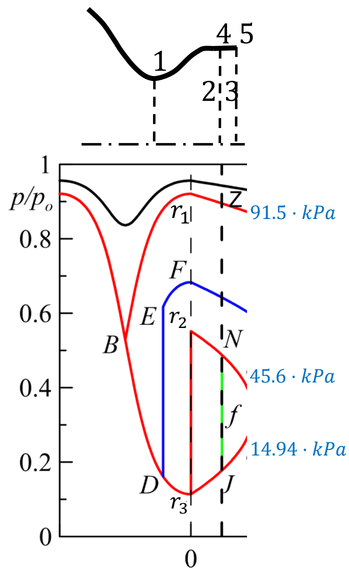
Punto r_2, r_3

$$p^*_{r_2} = p^*_{r_3} = 30.3\cdot kPa \qquad \frac{T_{0r_2}}{T^*_0} = \frac{T_{0r_3}}{T^*_0} = 0.756$$

$$\frac{q^*}{c_p T_{01}} = \frac{c_p(T^*_0 - T_{01})}{c_p T_{01}} = \frac{T^*_0}{T_{01}} - 1 = \frac{1}{0.756} - 1 = 0.323$$



Caso a): $\frac{q}{c_p T_{01}} = 0.150 \quad \rightarrow \quad \frac{T_{05}}{T_{02}} = 1 + 0.150 = 1.150$



Punto Z

$$\frac{T_{05}}{T_0^*} = \frac{T_{05}}{T_{02}}\frac{T_{02}}{T_0^*} = 1.150 \cdot 0.358 = 0.412$$

$$\frac{T_{05}}{T_0^*} = 0.412 \quad \xrightarrow[(M<1)]{\text{RF}} \quad M_5 = 0.335 \qquad \frac{p_5}{p^*} = 2.07$$

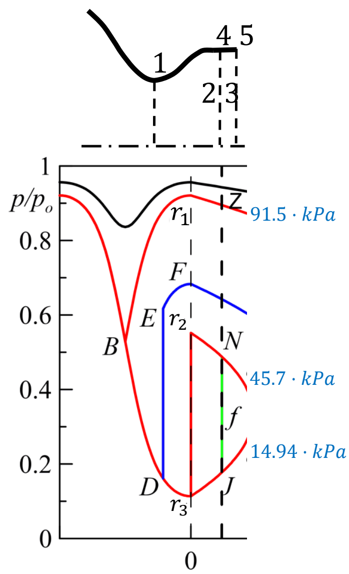
$$p_Z = p_5 = \frac{p_5}{p^*}p^* = 2.07 \cdot 44.2 = \quad 91.5 \cdot kPa$$

Analogamente:

Punto N $\quad \frac{T_{05}}{T_0^*} = \frac{T_{05}}{T_{02}}\frac{T_{02}}{T_0^*} = 1.150 \cdot 0.756 = 0.869 \quad \xrightarrow[(M<1)]{\text{RF}} \quad M_5 = 0.651 \quad \frac{p_5}{p^*} = 1.506 \Rightarrow p_5 = 45.6 \cdot kPa$

Punto J $\quad \frac{T_{05}}{T_0^*} = \frac{T_{05}}{T_{02}}\frac{T_{02}}{T_0^*} = 1.150 \cdot 0.756 = 0.698 \quad \xrightarrow[(M>1)]{\text{RF}} \quad M_5 = 1.662 \quad \frac{p_5}{p^*} = 0.493 \Rightarrow p_5 = 14.94 \cdot kPa$

Caso a): $\frac{q}{c_p T_{01}} = 0.150 \quad \rightarrow \quad \frac{T_{05}}{T_{02}} = 1 + 0.150 = 1.150$



Analizziamo i 2 sottocasi proposti nella traccia:

i. $p_a = 95.0 \cdot kPa$

ii. $p_a = 60.0 \cdot kPa$

i. $p_a > pz$: il moto è completamente subsonico (funzionamento alla Venturi);

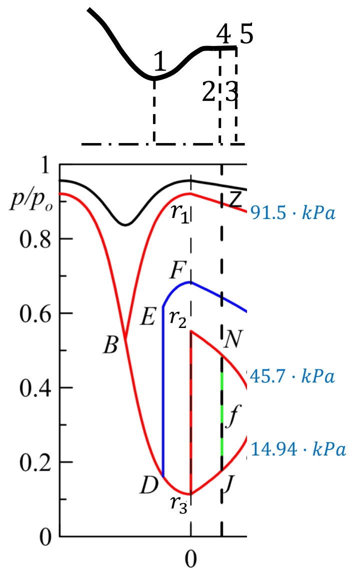
ii. $p_V < p_a < p_Z$: è presente un'onda d'urto nel divergente 1-2;

Caso a) i.: $\frac{q}{c_p T_{01}} = 0.15 \quad \rightarrow \quad \frac{T_{05}}{T_{02}} = 1 + 0.15 = 1.15, \; p_a = 95 \cdot kPa$

Il moto è completamente subsonico (funzionamento alla Venturi),

Si procede iterativamente assegnando il numero di Mach all'uscita del condotto.

Supponendo che $M_5 = 0.300$ ($< M_Z = 0.335$).



$$M_5 = 0.300 \xrightarrow{\text{RF}} \quad \frac{T_{05}}{T_0^*} = 0.347 \qquad \frac{p_5}{p^*} = 2.13$$

$$\frac{T_{02}}{T_0^*} = \frac{T_{02}}{T_{05}}\frac{T_{05}}{T_0^*} = \frac{0.347}{1.150} = 0.302 \; \xrightarrow[(M<1)]{\text{RF}} \; \frac{p_2}{p^*} = 2.17 \qquad M_2 = 0.275 \xrightarrow{\text{ISO}} \frac{p_2}{p_{0_2}} = 0.949$$

Infine:

$$p_5 = \frac{p_5}{p^*}\frac{p^*}{p_2}\frac{p_2}{p_{0_2}} p_{0_2} = \frac{2.13}{2.17} \cdot 0.949 \cdot 100 = 93.2 \cdot kPa \quad \Rightarrow \quad e = \frac{p_5 - p_a}{p_a} = -1.895\%$$

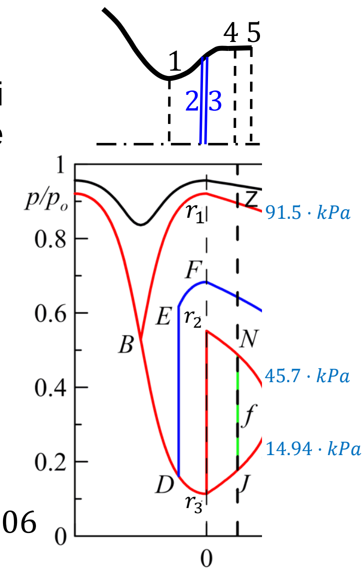
Si procede con il metodo di falsa posizione.

$$\frac{q}{c_p T_{01}} = 0.150 \qquad p_0 = 100 \cdot kPa \quad p_a = 95.0 \cdot kPa$$

		dalle tabelle (RF) entrando con M_5		$= \frac{T_{02}}{T_{05}}\frac{T_{05}}{T_0^*}$	dalle tabelle (RF) entrando con $\frac{T_{02}}{T_0^*}$		da (ISO) con M_2	$p_5 = p_5/p^* \cdot p^*/p_2 \cdot p_2/p_{0_2} \cdot p_{0_2}$	$= \frac{p_5 - p_a}{p_a}$
tenta-tivo #	M_5	$\frac{p_5}{p^*}$	$\frac{T_{05}}{T_0^*}$	$\frac{T_{02}}{T_0^*}$	M_2	$\frac{p_2}{p^*}$	$\frac{p_2}{p_{0_2}}$	p_5 (kPa)	**errore (%)**
(1)	0.335	0.411	2.07	0.357	0.306	2.12	0.937	91.5	-3.68
(2)	0.3	0.347	2.13	0.302	0.275	2.17	0.949	93.2	-1.895
(3)	0.263	0.28	2.19	0.243	0.242	2.22	0.96	94.7	-0.316
(4)	0.256	0.267	2.2	0.232	0.236	2.23	0.962	94.9	-0.105
(5)	0.253	0.262	2.2	0.228	0.233	2.23	0.963	95.000	0

$$M_5^{(n)} = \frac{M_5^{(n-1)} e^{(n-2)} - M_5^{(n-2)} e^{(n-1)}}{e^{(n-2)} - e^{(n-1)}}$$

Caso a) ii.: $\frac{q}{c_p T_{01}} = 0.15 \quad \rightarrow \quad \frac{T_{05}}{T_{02}} = 1 + 0.150 = 1.150,\ p_a = 60.0 \cdot kPa$



È presente un'onda d'urto nel divergente 1-2. In questo caso al contrario di quello che succedeva nel moto alla Fanno e per ugello convergente divergente si deve procedere per tentativi.

Avendo già trovato i punti N e Z si può immediatamente applicare il metodo di falsa posizione.

$$M_Z = 0.335, p_Z = 91.5 \cdot kPa \qquad M_2^{(n-2)} = M_B = 1, e^{(n-2)} = 52.5\%$$
$$M_N = 0.655, p_Z = 45.5 \cdot kPa \quad \rightarrow \quad M_2^{(n-1)} = M_{r_3} = 2.20, e^{(n-1)} = -24.2\%$$

da cui:

$$M_2^{(n)} = \frac{M_2^{(n-1)} e^{(n-2)} - M_2^{(n-2)} e^{(n-1)}}{e^{(n-2)} - e^{(n-1)}} = \quad 1.821$$

$$M_2 = 1.821 \xrightarrow{\text{NSW}} \frac{p_{03}}{p_{02}} = 0.803 \qquad \frac{A_4}{A_4^*} = \frac{A_4}{A_1}\frac{A_1}{A_4^*} = \frac{A_4}{A_1}\frac{p_{03}}{p_{02}} = 2 \cdot 0.803 = 1.606$$

$$\frac{A_4}{A_4^*} = 1.606 \xrightarrow[(M<1)]{\text{ISO}} M_4 = 0.395 \qquad \frac{p_4}{p_{04}} = 0.898 \xrightarrow{\text{RF}} \frac{p_4}{p^*} = 1.970 \qquad \frac{T_{04}}{T_0^*} = 0.520$$

$$\frac{T_{05}}{T_0^*} = \frac{T_{05}}{T_{04}}\frac{T_{04}}{T_0^*} = 0.598 \xrightarrow{\text{RF}} \begin{matrix} M_5 = 0.441 \\ \frac{p_5}{p^*} = 1.888 \end{matrix} \quad p_5 = \frac{p_5}{p^*}\frac{p^*}{p_4}\frac{p_4}{p_{04}}\frac{p_{04}}{p_{02}} p_0 = \frac{1.888}{1.970} 0.898 \cdot 0.803 \cdot 100 = 69.1 \cdot kPa$$

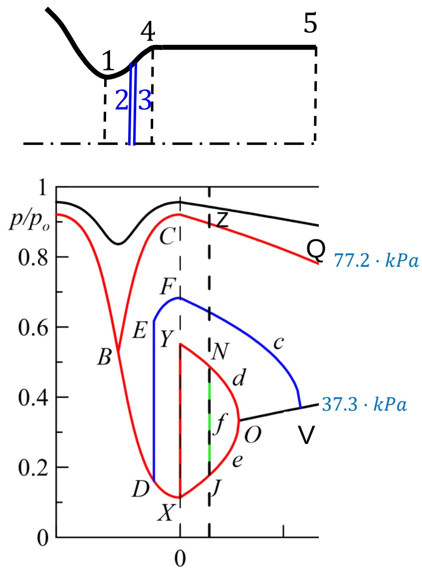
Si procede con il metodo di falsa posizione.

$$\frac{q}{c_p T_{01}} = 0.150 \qquad \frac{T_{05}}{T_{04}} = 1.150 \qquad p_a = 60.0 \cdot kPa$$

		(NSW)	$= \frac{A_4}{A_1}\frac{p_{03}}{p_{02}}$	(ISO)	(ISO)	(RF) M_4	(RF) M_4	$= \frac{T_{05}}{T_{04}}\frac{T_{04}}{T_0^*}$	(RF)	(RF)	$\frac{p_5}{p^*}\frac{p^*}{p_4}\frac{p_4}{p_{04}}\frac{p_{04}}{p_{02}} p_0$	$= \frac{p_5 - p_a}{p_a}$
#	M_2	$\frac{p_{03}}{p_{02}}$	$\frac{A_4}{A_4^*}$	M_4	$\frac{p_4}{p_{04}}$	$\frac{p_4}{p^*}$	$\frac{T_{04}}{T_0^*}$	$\frac{T_{05}}{T_0^*}$	M_5	$\frac{p_5}{p^*}$	p_5 (kPa)	**errore (%)**
(1)	1	1	2	0.306	0.937	2.12	0.358	0.412	0.335	2.07	91.5	52.5
(2)	2.20	0.628	1.256	0.549	0.815	1.688	0.759	0.873	0.655	1.499	45.5	-24.2
(3)	1.821	0.803	1.606	0.395	0.898	1.970	0.520	0.598	0.44	1.888	69.1	15.17
(4)	1.967	0.736	1.472	0.441	0.875	1.886	0.599	0.689	0.498	1.781	60.8	1.333
(5)	1.981	0.73	1.46	0.445	0.873	1.879	0.606	0.697	0.504	1.77	60.0	0
(6)	1.981	0.73	1.46	0.445	0.873	1.879	0.606	0.697	0.504	1.77	60.0	0

$$M_2^{(n)} = \frac{M_2^{(n-1)} e^{(n-2)} - M_2^{(n-2)} e^{(n-1)}}{e^{(n-2)} - e^{(n-1)}}$$

Caso b): $\frac{q}{c_p T_{01}} = 1.00 \quad \rightarrow \quad \frac{T_{05}}{T_{02}} = 1 + 1.00 = 2.00$



Punto Q

$$\frac{T_{05}}{T_0^*} = \frac{T_{05}}{T_{02}}\frac{T_{02}}{T_0^*} = 2 \cdot 0.358 = \quad 0.716$$

$$\frac{T_{05}}{T_0^*} = 0.716 \quad \xrightarrow[(M<1)]{\text{RF}} \quad M_5 = \ 0.517 \qquad \frac{p_5}{p^*} = \ 1.746$$

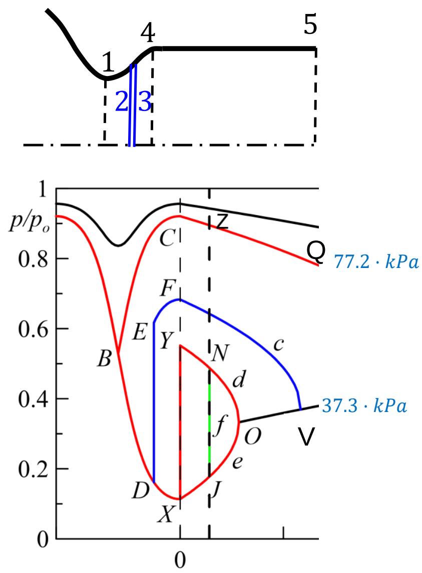
$$p_Q = p_5 = \frac{p_5}{p^*} p^* = 1.746 \cdot 44.2 = \quad 77.2 \cdot kPa$$

Punto V

$$\frac{T_{04}}{T_0^*} = \frac{T_{04}}{T_{05}} = 0.500 \quad \xrightarrow[(M<1)]{\text{RF}} \quad M_4 = \ 0.384 \qquad \frac{p_4}{p^*} = \ 1.989 \quad \xrightarrow{\text{ISO}} \quad \frac{p_4}{p_{04}} = \ 0.903 \qquad \frac{A_4}{A^*} = \ 1.644$$

$$\frac{p_{03}}{p_{02}} = \frac{A_2^*}{A_3^*} = \frac{A_1}{A_4}\frac{A_4}{A_4^*} = \frac{1.644}{2} = 0.822 \quad \xrightarrow{\text{NSW}} \quad M_2 = \ 1.779 \qquad p_5 = p^* = \frac{p^*}{p_4}\frac{p_4}{p_{04}}\frac{p_{04}}{p_{02}} p_0 = 37.3 \cdot kpa$$

Caso b): $\frac{q}{c_p T_{01}} = 1.00 \quad \rightarrow \quad \frac{T_{05}}{T_{02}} = 1 + 1.00 = 2.00$



Analizziamo i 2 sottocasi proposti nella traccia:

i. $p_a = 80.0 \cdot kPa$

ii. $p_a = 60.0 \cdot kPa$

i. $p_a > pz$: il moto è completamente subsonico (funzionamento alla Venturi);

ii. $p_V < p_a < p_Z$: è presente un'onda d'urto nel divergente 1-2;

Si procede con il metodo di falsa posizione.

$$\frac{q}{c_p T_{01}} = 1.00$$

$$p_0 = 100 \cdot kPa \quad p_a = 80.0 \cdot kPa$$

		dalle tabelle (RF) entrando con M_5		$= \frac{T_{02}}{T_{05}} \frac{T_{05}}{T_0^*}$	dalle tabelle (RF) entrando con $\frac{T_{02}}{T_0^*}$		da (ISO) con M_2	$p_5 = p_5/p^* \, p^*/p_2 \, p_2/p_{0_2} \, p_{0_2}$	$= \frac{p_5 - p_a}{p_a}$
tenta-tivo #	$\boldsymbol{M_5}$	$\boldsymbol{\frac{p_5}{p^*}}$	$\frac{T_{05}}{T_0^*}$	$\frac{T_{02}}{T_0^*}$	$\boldsymbol{M_2}$	$\boldsymbol{\frac{p_2}{p^*}}$	$\boldsymbol{\frac{p_2}{p_{0_2}}}$	$\boldsymbol{p_5}$ **(kPa)**	**errore (%)**
(1)	0.517	0.716	1.746	0.358	0.306	2.12	0.937	77.2	-3.5
(2)	0.3	0.347	2.13	0.1735	0.2	2.27	0.972	91.2	14
(3)	0.474	0.652	1.826	0.326	0.289	2.15	0.944	80.2	0.25
(4)	0.477	0.657	1.82	0.329	0.29	2.15	0.943	79.8	-0.25
(5)	0.476	0.655	1.822	0.328	0.29	2.15	0.943	79.900	-0.125
(6)	0.475	0.654	1.824	0.327	0.289	2.15	0.944	80.1	0.125

$$M_5^{(n)} = \frac{M_5^{(n-1)} e^{(n-2)} - M_5^{(n-2)} e^{(n-1)}}{e^{(n-2)} - e^{(n-1)}}$$

Si procede con il metodo di falsa posizione.

$$\frac{q}{c_p T_{01}} = 1.00 \qquad \frac{T_{05}}{T_{04}} = 2.00$$

$$p_a = 60.0 \cdot kPa$$

		(NSW)	$= \frac{A_4}{A_1} \frac{p_{03}}{p_{02}}$	(ISO)	(ISO)	(RF) M_4	(RF) M_4	$= \frac{T_{05}}{T_{04}} \frac{T_{04}}{T_0^*}$	(RF)	(RF)	$\frac{p_5}{p^*} \frac{p^*}{p_4} \frac{p_4}{p_{04}} \frac{p_{04}}{p_{02}} p_0$	$= \frac{p_5 - p_a}{p_a}$
#	$\boldsymbol{M_2}$	$\boldsymbol{\frac{p_{03}}{p_{02}}}$	$\boldsymbol{\frac{A_4}{A_4^*}}$	$\boldsymbol{M_4}$	$\boldsymbol{\frac{p_4}{p_{04}}}$	$\boldsymbol{\frac{p_4}{p^*}}$	$\boldsymbol{\frac{T_{04}}{T_0^*}}$	$\boldsymbol{\frac{T_{05}}{T_0^*}}$	$\boldsymbol{M_5}$	$\boldsymbol{\frac{p_5}{p^*}}$	$\boldsymbol{p_5}$ **(kPa)**	**errore (%)**
(1)	1	1	2	0.306	0.937	2.12	0.358	0.716	0.517	1.746	77.2	28.7
(2)	1.779	0.822	1.644	0.384	0.903	1.989	0.501	1.002	1.000	1.000	37.3	-37.8
(3)	1.336	0.973	1.946	0.315	0.934	2.11	0.374	0.748	0.541	1.702	73.3	22.2
(4)	1.500	0.93	1.86	0.332	0.927	2.08	0.406	0.812	0.594	1.606	66.6	11.00
(5)	1.661	0.872	1.744	0.358	0.915	2.03	0.454	0.908	0.699	1.425	56.0	-6.67
(6)	1.600	0.895	1.79	0.347	0.92	2.05	0.433	0.866	0.647	1.513	60.8	1.333

$$M_2^{(n)} = \frac{M_2^{(n-1)} e^{(n-2)} - M_2^{(n-2)} e^{(n-1)}}{e^{(n-2)} - e^{(n-1)}}$$