Mattingly example 3.3

Consider both a two-stage vehicle and a three-stage vehicle for the launch of the 900-kg (2000-lbm) payload. Each stage uses a liquid H_2 -O_2 chemical rocket (C = 4115 m/s, 13,500 ft/s), and the DV total of 14,300 m/s (46,900 ft/s) is split evenly between the stages. Suppose $\delta = m_{str}/m_0 = 0.03$.

MPayLoad	900
c	4115
DV	14300
$\delta = m_{str}/m_0$	0.03

#Stadi	1	2	3	
Dv	14,300	7,150	4,767	m/s
mpay	900	900	900	kg
MR	0.03096	0.1760	0.3140	
lambda	9.590E-04	0.1460	0.2840	
m01	938,465	6,166	3,169	kg
mstr1	28,154	185	95	kg
mp1	909,411	5,081	2,174	kg
m02		42,250	11,159	kg
mstr2		1,267	335	kg
mp2		34,816	7,655	kg
m03			39,291	kg
mstr3			1,179	kg
mp3			26,953	kg

Mono stadio

$$m_0 = m_p + m_f = m_p + m_{pay} + m_{str}$$

$$MR = \frac{m_f}{m_o} \qquad \lambda = \frac{m_{pay}}{m_0} \qquad \delta = \frac{m_{str}}{m_0} \qquad MR = \lambda + \delta$$

$$\Delta V = -\overline{c}\ln\frac{m_f}{m_0} = \overline{c}\ln\frac{1}{MR} \rightarrow MR = e^{-\frac{\Delta V}{c}} = e^{-\frac{14300}{4115}} = 0.0310$$

$$\lambda = \frac{m_{pay}}{m_0} = MR - \frac{m_{str}}{m_0} = MR - \delta = 0.0310 - .03000 = 1.\cdot 10^{-3}$$

$$m_0 = \frac{m_{pay}}{\lambda} = \frac{m_{pay}}{MR - \delta} = \frac{m_{pay}}{e^{-\frac{\Delta V}{\overline{c}}} - \delta} = \frac{900}{1.\cdot 10^{-3}} = 900 \cdot 10^3 \cdot kg$$

$$m_{str} = \delta m_0 = 0.0300 \cdot 900 \cdot 10^3 = 27.0 \cdot 10^3 \cdot kg$$

$$m_p = m_0(1 - MR) = 900 \cdot 10^3(1 - 0.0310) = 872 \cdot 10^3 \cdot kg$$

Due stadi

Stadio finale (1)

$$\frac{\Delta V}{2} = -\overline{c}\ln\frac{m_f}{m_0} = \overline{c}\ln\frac{1}{MR} \rightarrow MR = e^{-\frac{\Delta V}{2c}} = e^{-\frac{14300}{2\cdot 4115}} = 0.1760$$

$$\lambda = \frac{m_{pay}}{m_0} = MR - \frac{m_{str}}{m_0} = MR - \delta = 0.1760 - .03000 = 0.1460$$

$$m_{01} = \frac{m_{pay}}{\lambda} = \frac{m_{pay}}{MR - \delta} = \frac{m_{pay}}{e^{-\frac{\Delta V}{2\bar{c}}} - \delta} = \frac{900}{0.1460} = 6{,}160 \cdot kg$$

$$m_{str1} = \delta m_{01} = 0.0300 \cdot 6{,}160 = 185 \cdot kg$$

$$m_{p1} = m_{01}(1 - MR) = 6{,}160(1 - 0.1760) = 5{,}080 \cdot kg$$

$$m_{pay2} = m_{01} \rightarrow m_{02} = \frac{m_{pay2}}{\lambda} = \frac{m_{pay}}{\lambda^2} = \frac{m_{pay}}{\left(e^{-\frac{\Delta V}{2\bar{c}}} - \delta\right)^2} = \frac{6{,}160}{0.1460} = 42.200 \cdot kg$$

$$m_{str1} = \delta m_{01} = 0.0300 \cdot 42.20.0 = 1266 kg$$

$$m_{p1} = m_{01}(1 - MR) = 42{,}200(1 - 0.1760) = 34.800 \cdot kg$$

Tre stadi

Procedendo in modo analogo si costruisce la tabella.

Mattingly example 3.7

The space shuttle main engine (SSME) operates for up to 520 s in one mission at altitudes over 100 miles. The nozzle expansion ratio 1 is 77:1, and the inside exit diameter is 2.30m. Assume a calorically perfect gas with the following properties:

Ae/Ag	77	
De	2.298	m
γ	1.25	
pc	2.068E+07	Pa
Tt	4083	K
R	602.6	J/kgK
M2	5.092	

We want to calculate the following:

1) Characteristic velocity c^*

$$k = \frac{\gamma - 1}{\gamma} = \ = \frac{0.25}{1.25} = 0.20 \qquad K = \frac{\gamma + 1}{2(\gamma - 1)} = \frac{2.25}{0.5} = 4.5$$

$$\psi = 1 + \frac{\gamma - 1}{2} M^2 \qquad \psi^* = 1 + \frac{.25}{2} = 1.125 \qquad \Psi = \gamma M \psi^{-K}$$

$$\Psi^* = 1.25 \cdot 1.125^{-4.5} = 0.736$$

$$A_e = \frac{\pi}{4} D_e^2 = \frac{3.14}{4} 2.30^2 = 4.15 \cdot m^2 \qquad A_{th} = \frac{A_{th}}{A_e} A_e = \frac{4.15}{77} = 0.0539 \cdot m^2$$

$$a_t = \sqrt{\gamma R T_t} = \sqrt{1.4 \cdot 602.6 \cdot 4080} = 1752 \cdot \frac{m}{s}$$

$$c^* = \frac{p_c A_{th}}{\dot{m}_p} = \frac{a_t}{\Psi^*} = \frac{1752}{0.736} = 2380 \cdot \frac{m}{s}$$

2) Mass flow rate of gases through the nozzle.

$$\dot{m} = \frac{p_c A_{th} \Psi^*}{a_t} = \frac{p_c A_{th}}{c^*} = \frac{2.070 \cdot 10^7 \cdot 0.0539}{2380} = 466 \cdot \frac{kg}{s}$$

3) Pressure at which the nozzle is "on-design."

$$\psi_2 = 1 + \frac{\gamma - 1}{2} M_2^2 = 1 + 0.125 \cdot 5.09^2 = 4.24$$

$$p_2 = p_c \psi^{-\frac{1}{k}} = 2.068 \cdot 10^7 \cdot 4.24^{-\frac{1}{0.2}} = 15.11 \cdot kPa$$

4) Pressure at which the nozzle is just separated (assume separation occurs when p_a>3.5 p_{r3}).

$$p_s = p_2 \cdot 3.5 = 15.11 \cdot 3.5 = 52.9 \cdot kPa$$

5) Thrust coefficient C_F, specific impulse and thrust for p_0=0, p_s.

$$C_F = \frac{\dot{m}_p V_2 + (p_2 - p_0)A_2}{p_c A_{\text{th}}} = \sqrt{\frac{2\gamma^2}{\gamma - 1}\left(\frac{2}{\gamma + 1}\right)^{\frac{(\gamma+1)}{(\gamma-1)}}\left[1 - \left(\frac{p_2}{p_c}\right)^k\right]} + \frac{(p_2 - p_0)}{p_c}\frac{A_2}{A_{\text{th}}}$$

$$C_F = \sqrt{\frac{2 \cdot 1.25^2}{.25}\left(\frac{2}{2.25}\right)^{\frac{2.25}{.25}}[1 - 4.24^{-1}]} + \frac{(p_2 - p_0)}{p_c}77 = 1.819 + \frac{(p_2 - p_0)}{p_c}77$$

$$p_0 = 0 \qquad C_F = 1.819 + \frac{15.11}{2.070 \cdot 10^4}77 = 1.875$$

$$\frac{F}{\dot{m}} = c^* C_F = 2380 \cdot 1.875 = 4468 \cdot \frac{N \cdot s}{kg}$$

$$I_s = \frac{F}{\mathrm{g}_0 \dot{m}_p} = \frac{4468}{9.81} = 455 \cdot s$$

$$p_0 = 52.8 \qquad C_F = 1.819 + \frac{15.09 - 52.8}{2.068 \cdot 10^4}77 = 1.679$$

$$\frac{\dot{F}}{\dot{m}} = c^* C_F = 2380 \cdot 1.875 = 3990 \cdot \frac{N \cdot s}{kg}$$

$$I_s = \frac{F}{\mathrm{g}_0 \dot{m}_p} = \frac{3990}{9.81} = 407 \cdot s$$

Farokhi problem 12.2

A solid rocket motor has a design chamber pressure of 10 MPa, an end-burning grain with n = 0.4 and $\dot{r}$ = 3cm/s at the design chamber pressure and design grain temperature of 15C. The temperature sensitivity of the burning rate is $\sigma_p = 0.002/C$, and chamber pressure sensitivity to initial grain temperature is $\pi_K = 0.005/C$. The nominal effective burn time for the rocket is 120 s, i.e., at design conditions.
Calculate

(a) the new chamber pressure and burning rate when the initial grain temperature is 75C;

$$\sigma_p = \frac{1}{\dot{r}}\left(\frac{\delta \dot{r}}{\delta T}\right)_p \qquad \pi_K = \frac{1}{p}\left(\frac{\delta p}{\delta T}\right)_K$$

$$\Delta T = T_{c2} - T_{c1} = 75 - 15 = 60 \cdot C$$

$$\Delta \mathrm{p} = p_{c1}\pi_K \Delta T = 10 \cdot 0.005 \cdot 60 = 3 \cdot MPa$$

$$p_{c2} = p_{c1} + \Delta p = 10 + 3 = 13 \cdot MPa$$

$$\dot{r} = a p_c^n \qquad \dot{r_{2p}} = \dot{r}_1 \left(\frac{p_{c2}}{p_{c1}}\right)^n = 3 \cdot 1.3^{0.4} = 3.33 \cdot \frac{cm}{s}$$

$$\dot{r}_2 = \dot{r}_{2p}\left(1 + \sigma_p \Delta T\right) = 3.33(1 + 0.002 \cdot 60) = 3.73 \cdot \frac{cm}{s}$$

(b) the corresponding reduction in burn time *Δt_b* in seconds.

$$h = \dot{r}_1 t_{b1} = 3 \cdot 120 = 360 \cdot cm$$

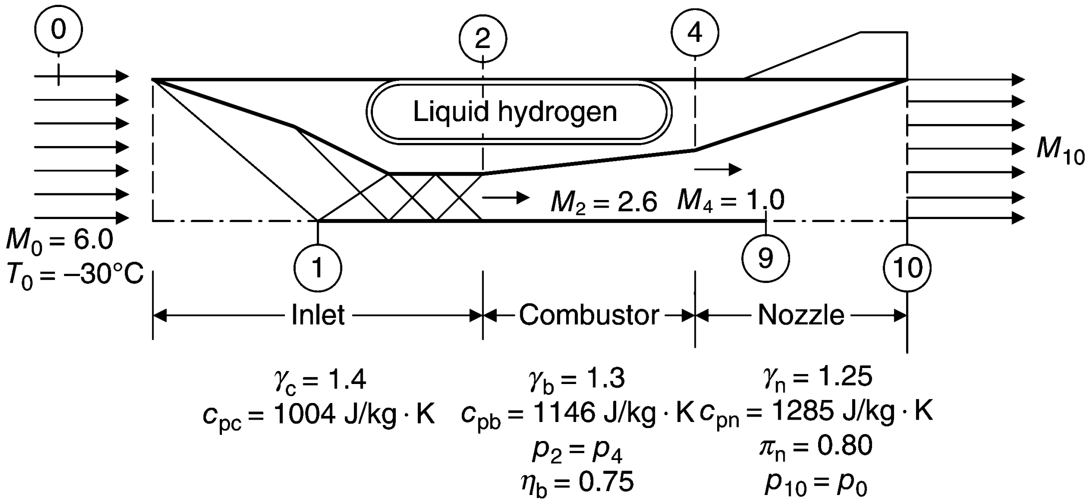
$$t_{b2} = \frac{h}{\dot{r}_2} = \frac{360}{3.73} = 96.5 \cdot s \qquad \Delta t_b = t_{b1} - t_{b2} = 120 - 96.5 = 23.5 \cdot s$$

Farokhi problem 12.20

Consider a scramjet in a Mach 6 flight. The fuel for this engine is hydrogen with Q_R=120,000kJ/kg. The inlet uses multiple oblique shocks with a total pressure recovery following MIL-E-5008B standards for $M_0 > 5$, i.e.,

$$\pi_d = \frac{800}{M_0^4 + 935}$$

The combustor entrance Mach number is $M_2 = 2.6$. Use frictionless, constant-pressure heating, i.e., $p_4 = p_2$, to simulate the combustor with combustor exit Mach number $M_4 = 1.0$. All component parameters and gas constants are shown in the schematic drawing below.



Calculate

(a) Inlet static temperature ratio T_2/T_0

$$k_0 = \frac{\gamma - 1}{\gamma} = \frac{0.4}{1.4} = 0.286 \qquad K_0 = \frac{\gamma + 1}{2(\gamma - 1)} = \frac{2.4}{0.8} = 3.00$$

$$k_b = \frac{\gamma - 1}{\gamma} = \frac{0.3}{1.3} = 0.231 \qquad K_b = \frac{\gamma + 1}{2(\gamma - 1)} = \frac{2.3}{0.6} = 3.83$$

$$k_n = \frac{\gamma - 1}{\gamma} = \frac{0.25}{1.25} = 0.200 \qquad K_n = \frac{\gamma + 1}{2(\gamma - 1)} = \frac{2.25}{0.5} = 4.50$$

$$T_0 = 273.15 - 30 = 243 \cdot K \qquad a_0 = \sqrt{1.4 \cdot 0.286 \cdot 1004 \cdot 243} = 313 \cdot \frac{m}{s}$$

$$V_0 = M_0 a_0 = 6 \cdot 313 = 1878 \cdot \frac{m}{s}$$

$$\psi = 1 + \frac{\gamma - 1}{2} M^2 \quad \rightarrow \quad \psi_0 = 1 + 0.2 \cdot 6^2 = 8.20 \qquad \psi_2 = 1 + 0.2 \cdot 2.6^2 = 2.35$$

$$\frac{T_2}{T_0} = \frac{T_2}{T_{t2}} \frac{T_{t0}}{T_0} = \frac{\psi_0}{\psi_2} = \frac{8.20}{2.35} = 3.49 \qquad T_2 = \frac{T_2}{T_0} T_0 = 3.49 \cdot 243 = 848 \cdot K$$

$$T_{t2} = \psi_2 T_2 = 2.35 \cdot 848 = 1993 \cdot K$$

$$\frac{p_{t0}}{p_0} = \psi_0^{\frac{1}{k_0}} = 8.20^{\frac{1}{0.286}} = 1567 \qquad \frac{p_{t2}}{p_2} = \psi_0^{\frac{1}{k_0}} = 2.35^{\frac{1}{0.286}} = 19.84$$

$$\pi_d = \frac{800}{M_0^4 + 935} = \frac{800}{6^4 + 935} = 0.359$$

$$\frac{p_2}{p_0} = \frac{p_2}{p_{t2}}\frac{p_{t2}}{p_{t0}}\frac{p_{t0}}{p_0} = \frac{1567 \cdot 0.359}{19.84} = 28.4$$

$$\frac{p_{t2}}{p_0} = \frac{p_{t2}}{p_{t0}}\frac{p_{t0}}{p_0} = \pi_d \frac{p_{t0}}{p_0} = 0.359 \cdot 1567 = 563$$

(b) combustor exit temperature T_4 in K

Dalla conservazione della massa e dal bilancio della QM supponendo $p_2 = p_4$, $\gamma_2 = \gamma_4 = \gamma_b, R_2 = R_4 = R_b$ e trascurando gli effetti viscosi si ha:

$$\dot{m}_4 = (1+f)\dot{m}_2 \qquad \dot{m} = \rho V A = \frac{\gamma p \rho V A}{\gamma p} = \frac{\gamma p V A}{a^2} = \frac{\gamma p M A}{a}$$

$$\frac{p_2 M_2 A_2}{a_2} = \frac{p_4 M_4 A_4}{a_4} \quad \xrightarrow{p_2 = p_4} \quad \sqrt{\frac{T_4}{T_2}} = \frac{M_4 A_4}{M_2 A_2}$$

Se la pressione è costante sulla superfice del volume di controllo e gli effetti viscosi sono trascurabili (spinta trascurabile), l'equazione di bilancio della QM si riduce a:

$$\rho_2 V_2^2 A_2 = \dot{m} V_2 = \dot{m} V_4 = \rho_4 V_4^2 A_4 \quad \rightarrow \quad V_2 = V_4$$

$$\sqrt{\frac{T_4}{T_2}} = \frac{a_4}{a_2} = \frac{a_4}{a_2}\frac{V_2}{V_4} = \frac{M_2}{M_4} = 2.6 \quad \rightarrow \quad \frac{T_4}{T_2} = 2.6^2 = 6.76$$

Sostituendo nella relazione precedente si trova anche il rapporto delle aree:

$$\sqrt{\frac{T_4}{T_2}} = \frac{M_2}{M_4} = \frac{M_4 A_4}{M_2 A_2} \quad \rightarrow \quad \frac{A_4}{A_2} = \frac{M_2^2}{M_4^2}$$

$$T_4 = \frac{T_4}{T_2} T_2 = 2.6^2 \cdot 848 = 5730K$$

$$\psi = 1 + \frac{\gamma - 1}{2} M^2 \quad \rightarrow \quad \psi_4 = 1 + 0.15 \cdot 1 = 1.15$$

$$T_{t4} = \psi_4 T_4 = 1.15 \cdot 5731 = 6590 \cdot K$$

(c) fuel-to-air ratio f

$$\dot{m}_f Q_R \eta_b = \dot{m}_0 c_{pb}[(1+f)T_{t4} - T_{t2}] \quad \rightarrow$$

$$f = \frac{T_{t4} - T_{t2}}{\frac{Q_R \eta_b}{c_{pb}} - T_{t4}} = \frac{6590 - 1992}{120 \cdot 10^6 \cdot \frac{0.75}{1146} - 6590} = 0.0639$$

Mentre il libro suppone che:

$$f = \frac{T_4 - T_2}{\frac{Q_R \eta_b}{c_{pb}}} = \frac{5730 - 847}{120 \cdot 10^6 \cdot \frac{0.75}{1146}} = 0.0621$$

(d) nozzle exit Mach number M_{10}

$$\frac{p_{t4}}{p_0} = \frac{p_{t4}}{p_4}\frac{p_2}{p_0} = \psi_4^{\frac{1}{k_b}}\frac{p_2}{p_0} = 1.15^{\frac{1}{0.231}} \cdot 28.4 = 52.0$$

$$\frac{p_{t10}}{p_0} = \frac{p_{t4}}{p_0}\pi_n = 52.0 \cdot 0.80 = 41.6 = \psi_{10}^{\frac{1}{k_n}} \quad \rightarrow \quad \psi_{10} = 41.6^{0.2} = 2.11$$

$$M_{10} = \sqrt{(\psi_{10} - 1)\frac{2}{\gamma_n - 1}} = \sqrt{1.11\frac{2}{0.25}} = 2.98$$

$$T_{10} = \frac{T_{t10}}{\psi_{10}} = \frac{T_{t4}}{\psi_{10}} = \frac{6590}{2.11} = 3120 \qquad a_{10} = \sqrt{1.25 \cdot 0.20 \cdot 1285 \cdot 3120} = 1001 \cdot \frac{m}{s}$$

$$V_{10} = M_{10}a_{10} = 2.98 \cdot 1001 = 2980 \cdot \frac{m}{s}$$

(e) nondimensional ram drag D_{ram}/p_0A_1 (note that $A_0 = A_1$)

$$\frac{D_{ram}}{p_0A_1} = \frac{\dot{m}_0V_0}{p_0A_1} = \frac{\rho_0V_0^2A_1}{p_0A_1} = \gamma_cM_0^2 = 1.4 \cdot 36 = 50.4$$

(f) nondimensional gross thrust F_g/p_0A_1

$$\frac{F_g}{p_0A_1} = \frac{\dot{m}_{10}V_{10}}{p_0A_1} = \frac{(1+f)\rho_0V_0A_1V_{10}}{p_0A_1} = \frac{\gamma_c(1+f)M_0V_{10}}{a_0} = \frac{1.4 \cdot 1.062 \cdot 6 \cdot 2980}{313} = 84.9$$

$$\frac{F_u}{p_0A_1} = \frac{F_g - D_{ram}}{p_0A_1} = 84.9 - 50.4 = 34.5$$

(g) fuel-specific impulse Is in seconds

$$I_s = \frac{F_u}{\dot{m}_fg_0} = \frac{\dot{m}_0[(1+f)V_{10} - V_0]}{f\dot{m}_0g_0} = \frac{1.062 \cdot 2980 - 1878}{0.0621 \cdot 9.81} = 2110 \cdot s$$

(h) combustor area ratio A_4/A_2

$$\frac{A_4}{A_2} = \frac{M_2^2}{M_4^2} = 2.6^2 = 6.76$$

(i) nozzle area ratio A_{10}/A_4

$$\frac{p_4M_4A_4}{\sqrt{T_{24}}} = \frac{p_{10}M_{10}A_{10}}{\sqrt{T_{10}}} \quad \rightarrow \quad \frac{A_{10}}{A_4} = \frac{p_4M_4\sqrt{T_{10}}}{p_{10}M_{10}\sqrt{T_4}} = 28.4\frac{1}{2.98}\sqrt{\frac{3120}{5730}} = 7.03$$

(j) thermal efficiency

$$\eta_{\text{th}} = \frac{\Delta K\dot{E}}{\mathcal{P}_t} = \frac{\left[(1+f)V_{10}{}^2 - V_0^2\right]}{2fQ_R} = \frac{(1.062 \cdot 2980^2 - 1878^2)/2}{0.0621 \cdot 120 \cdot 10^6} = \frac{2.950 \cdot 10^6}{7.450 \cdot 10^6} = 0.396$$

(k) propulsive efficiency

$$\eta_p = \frac{F_i V_0}{\Delta K\dot{E}} \approx \frac{\frac{2F_u V_0}{\dot{m}_0}}{\left[(1+f){V_{10}}^2 - V_0^2\right]} = \frac{[(1+f)V_{10} - V_0]V_0}{2.970 \cdot 10^6} = \frac{(1.062 \cdot 2980 - 1878)1878}{2.970 \cdot 10^6} = 0.820$$