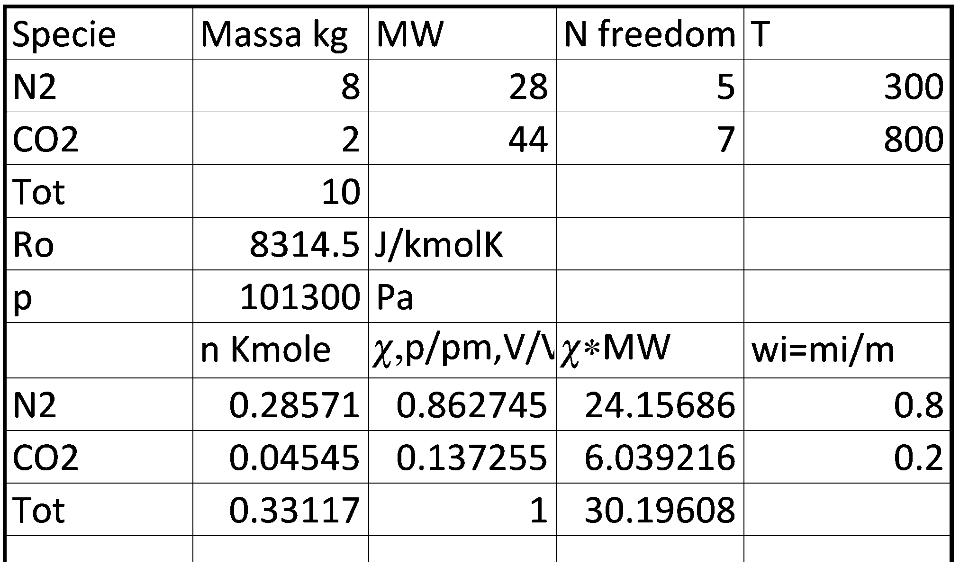
Miscele by Tom

Due gas (vedi tabella) sono miscelati a volume costante. Determinare: $n, \chi, MW_m, w_i, \bar{c}_{vm}, \bar{c}_{pm}, c_v, c_p, R, T_m, p_i, V, V_i\ e\ \gamma_m$.



Specie	Massa kg	MW	N freedom	T
N2	8	28	5	300
CO2	2	44	7	800
Tot	10			
Ro	8314.5	J/kmolK		
p	101300	Pa		
	n Kmole	χ,p/pm,V/\	χ*MW	wi=mi/m
N2	0.28571	0.862745	24.15686	0.8
CO2	0.04545	0.137255	6.039216	0.2
Tot	0.33117	1	30.19608	

$$n_{N_2} = \frac{m_{N_2}}{MW_{N_2}} = \frac{8}{28} = 0.286 \cdot kmol \qquad n_{CO_2} = \frac{m_{CO_2}}{MW_{CO_2}} = \frac{2}{44} = 0.0455 \cdot kmol$$

$$n_m = n_{N_2} + n_{CO_2} = 0.286 + 0.0455 = 0.332 \cdot kmol$$

$$\chi_{N_2} = \frac{n_{N_2}}{n_m} = \frac{0.286}{0.332} = 0.861 \qquad \chi_{\mathrm{CO}_2} = 1 - \chi_{N_2} = 1 - 0.861 = 0.139$$

$$MW_m = \chi_{N_2} MW_{N_2} + \chi_{CO_2} MW_{CO_2} = \frac{m_m}{n_m} = \frac{10}{0.332} = 30.1 \cdot \frac{kg}{kmol}$$

$$w_{N_2} = \frac{m_{N_2}}{m_m} = \frac{8}{10} = 0.8 \qquad \ldots$$

$$\bar{c}_{vN_2} = \bar{R}\frac{5}{2} = 8314.5 \cdot \frac{5}{2} = 20.8 \cdot \frac{kJ}{kmole \cdot K} \qquad \bar{c}_{vCO_2} = \bar{R}\frac{7}{2} = 8314.5 \cdot \frac{7}{2} = 29.1 \cdot \frac{kJ}{kmole \cdot K}$$

$$\bar{c}_{vm} = \bar{c}_{vN_2}\chi_{N_2} + \bar{c}_{vCO_2}\chi_{CO_2} = 20.8 \cdot 0.861 + 29.1 \cdot (1 - 0.861) = 21.9 \cdot \frac{kJ}{kmole \cdot K}$$

$$\bar{c}_{pm} = \bar{c}_{vm} + \bar{R} = 21.9 + 8.3145 = 30.2 \cdot \frac{kJ}{kmole \cdot K}$$

$$c_{v_{N_2}} = \frac{\bar{c}_{vN_2}}{MW_{N_2}} = \frac{20.8}{28} = 743 \cdot \frac{J}{kg \cdot K} \qquad \ldots.$$

$$c_{v_m} = \frac{\bar{c}_{vm}}{MW_m} = \frac{21.9}{30.1} = 728 \cdot \frac{J}{kg \cdot K}$$

$$R = \frac{\bar{R}}{MW_m} = \frac{8314.5}{30.1} = 276 \cdot \frac{J}{kg \cdot K}$$

$$T_M = \frac{c_{v_{N_2}} w_{N_2} T_{N2} + c_{v_{CO_2}} w_{CO_2} T_{CO2}}{c_{vm}} = \frac{743 \cdot 0.8 \cdot 300 + 661 \cdot 0.2 \cdot 800}{728} = 390 \cdot K$$

$$p_{N2} = p_m \chi_{N_2} = 101.3 \cdot 0.861 = 87.2 \cdot kPa \qquad \ldots$$

$$V_m = \frac{n_{N_2}\bar{R}T_m}{p_{N_2}} = \frac{n_m\bar{R}T_m}{p_m} = \frac{0.332 \cdot 8314.5 \cdot 393}{101{,}300} = 10.63 \cdot m^3$$

$$V_{N_2} = V_m \chi_{N_2} = \frac{n_{N_2}\bar{R}T_m}{p_m} = 10.63 \cdot 0.865 = 9.152 \cdot m^3 \qquad \ldots$$

$$\gamma_m = \frac{\bar{c}_{pm}}{\bar{c}_{vm}} = \frac{30.2}{21.9} = 1.379$$

j/kmolK	cv	cv*χ	Cp	cp*χ
N2	20786.3	17933.24	29100.75	25106.529
CO2	29100.8	3994.221	37415.25	5135.4265
	medio	21927.46		30241.956
	J/kgK			
	cv	cv wi	cp	cp wi
N2	742.366	593.8929	1039.313	831.45
CO2	661.381	132.2761	850.3466	170.06932
	Medio	726.169		1001.5193
γ	1.37918			
	wi cv T	wi cp T	R	R wi
N2	178168	249435	296.9464	237.55714
CO2	105821	136055.5	188.9659	37.793182
Somma	283989	385490.5		275.35032
Tm	391.078	384.9057		
		V costante	p costante	
	Vm	10.63016	10.46238	
	pi	Vi	Vi	
N2	87396.1	9.171114	9.026366	
CO2	13903.9	1.459041	1.436013	
		10.63016	10.46238	

Farokhi problem 7.4

Write the chemical reaction for the complete combustion of JP-4 and air. JP-4 has the formula $CH_{1.93}$. Also, calculate the stoichiometric fuel-to-air ratio for this blended jet fuel.

$$CH_{1.93} + x(O_2 + 3.76N_2) \rightarrow CO_2 + yH_2O + z3.76N_2$$

$$x = z \qquad 2y = 1.93 \qquad 2x = 2 + y$$

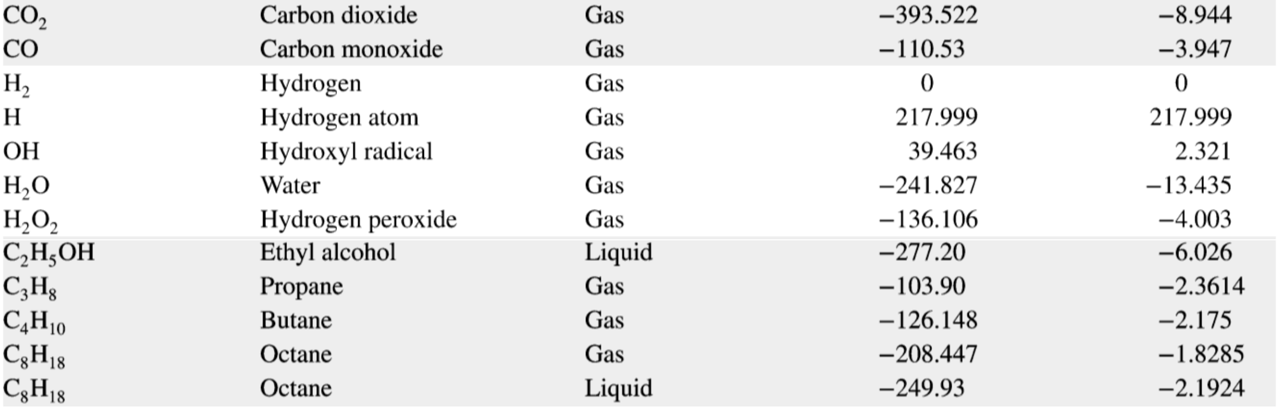
$$2x = 2 + \frac{1.93}{2} \quad \rightarrow \quad 2x = \frac{4 + 1.93}{2} \quad \rightarrow \quad x = \frac{5.93}{4}$$

$$CH_{1.93} + \frac{5.93}{4}(O_2 + 3.76N_2) \rightarrow CO_2 + \frac{1.93}{2}H_2O + \frac{5.93}{4}3.76N_2$$

$$f = \frac{\dot{m}_f}{\dot{m}_0} = \frac{1 \cdot 12 + 1 \cdot 1.93}{\frac{5.93}{4}(2 \cdot 16 + 3.76 \cdot 2 \cdot 14)} = 0.0684$$

Farokhi problem 7.5

Calculate the lower and higher heating values of octane, C_8H_{18},in the stoichiometric chemical reaction with oxygen at a reference temperature of 298.16 K and the pressure of 1 bar ($h_{VapH_2O} = 2243 kJ/kg$).



CO_2	Carbon dioxide	Gas	−393.522	−8.944
CO	Carbon monoxide	Gas	−110.53	−3.947
H_2	Hydrogen	Gas	0	0
H	Hydrogen atom	Gas	217.999	217.999
OH	Hydroxyl radical	Gas	39.463	2.321
H_2O	Water	Gas	−241.827	−13.435
H_2O_2	Hydrogen peroxide	Gas	−136.106	−4.003
C_2H_5OH	Ethyl alcohol	Liquid	−277.20	−6.026
C_3H_8	Propane	Gas	−103.90	−2.3614
C_4H_{10}	Butane	Gas	−126.148	−2.175
C_8H_{18}	Octane	Gas	−208.447	−1.8285
C_8H_{18}	Octane	Liquid	−249.93	−2.1924

$$C_8H_{18} + \frac{25}{2}O_2 \rightarrow 8CO_2 + 9H_2O$$

$$Q_{ext} = \sum_j \left[n_j \bar{c}_{pj}\left(T_2 - T_f\right)\right]_{prod} - \sum_i \left[n_i \bar{c}_{pi}\left(T_1 - T_f\right)\right]_{rea} + \Delta H_{RPf}$$

$$Q_{ext} = \Delta H_{RPf} = \sum_j \left[n_j \Delta \bar{\mathrm{h}}_{fj}^0\right]_{prod} - \sum_i \left[n_i \Delta \bar{\mathrm{h}}_{fi}^0\right]_{rea} = -8 \cdot 394 - 9 \cdot 242 - (-208)$$

$$= -5120 \cdot \frac{kJ \cdot kmol}{gmol} = -5.120 \cdot MJ$$

$$LHV = \frac{-Q_{ext}}{m_f} = \frac{5114 \cdot 10^3}{12 \cdot 8 + 1 \cdot 18} = 44.5 \cdot 10^3 \cdot \frac{kJ}{kg}$$

$$HHV = LHV + \frac{m_{H_2O}}{m_{C_8H_{18}}} \mathrm{h}_{VapH_2O} = 44.5 \cdot 10^3 + \frac{18 \cdot 9}{12 \cdot 8 + 1 \cdot 18} 2243 = 47.7 \cdot 10^3 \cdot \frac{kJ}{kg}$$

Farokhi problem 7.9

One mole of octane is burned with 120% theoretical air. Assuming that the octane and air enter the combustion chamber at 25C and the excess oxygen and nitrogen in the reaction will not dissociate, calculate
(a) the fuel–air ratio
(b) the equivalence ratio ϕ
(c) the adiabatic flame temperature T_{af}
Assume: $\bar{c}_{p\mathrm{CO_2}} = 61.9kJ/kmol\cdot K, \bar{c}_{p\mathrm{O_2}} = 37.8kJ/kmol\cdot K,\ \bar{c}_{p\mathrm{N_2}} = 33.6kJ/kmol\cdot K\ \bar{c}_{p\mathrm{H_2O}} = 52.3kJ/kmol\cdot K$
(d) Evaluate also the flame temperature by considering non constant specific heat.

%Oss	1.2		FA7.9					
Reagenti	n	MW	h0f	Prodotti	n	MW	h0f	cp
C8H18	1	114	-208.447	CO2	8	44	-393.522	61.9
O2	15	32		H2O	9	18	-241.827	52.3
N2	56.4	28		N2	56.4	28		33.6
Tot	72.4			O2	2.5	32		37.8
Tot	298.15			Tot	75.9			
mf	114		ΔH	-5.116E+06				
mair	2059.2			Stima	Valore medio esatto			
f	0.05536		Cpm	38.938603	36.4777			
fst	0.06643		T	2029.2633	2146.04			
ϕ	0.83333		Dif	-324,038				
Tf guess	2029		Err%	-6.334E+00				

$$C_8H_{18} + x(O_2 + 3.76N_2) \rightarrow 8CO_2 + 9H_2O + x3.76N_2$$

$$2x = 16 + 9 \quad \rightarrow \quad x = 25/2 = 12.5$$

$$C_8H_{18} + 12.5(O_2 + 3.76N_2) \rightarrow 8CO_2 + 9H_2O + 12.5\cdot 3.76N_2$$

$$\alpha = 1.2$$

$$C_8H_{18} + \alpha x(O_2 + 3.76N_2) \rightarrow 8CO_2 + 9H_2O + yO_2 + \alpha x3.76N_2$$

$$y = x(\alpha - 1) = 12.5\cdot 0.2 = 2.5$$

$$C_8H_{18} + 15(O_2 + 3.76N_2) \rightarrow 8CO_2 + 9H_2O + 2.5O_2 + 15\cdot 3.76N_2$$

(a) the fuel–air ratio

$$f = \frac{\dot{m}_f}{\dot{m}_0} = \frac{8\cdot 12 + 18\cdot 1}{15(2\cdot 16 + 3.76\cdot 2\cdot 14)} = \frac{114}{2060} = 0.0553$$

(b) the equivalence ratio ϕ

$$\phi = \frac{f}{f_{st}} = \frac{0.0554}{\dfrac{8\cdot 12 + 18\cdot 1}{12.5(2\cdot 16 + 3.76\cdot 2\cdot 14)}} = \frac{1}{\alpha} = 0.833$$

(c) the adiabatic flame temperature T_{af}

$$Q_{ext} = \sum_j \left[n_j\bar{c}_{pj}(T_2 - T_f)\right]_{prod} - \sum_i \left[n_i\bar{c}_{pi}(T_1 - T_f)\right]_{rea} + \Delta H_{RPf}$$

$$Q_{ext} = 0 = \sum_j \left[n_j \bar{c}_{pj}\left(T_2 - T_f\right)\right]_{prod} + \Delta H_{RPf}$$

$$\Delta H_{RPf} = \sum_j \left[n_j \Delta \bar{\mathrm{h}}^0_{fj}\right]_{prod} - \sum_i \left[n_i \Delta \bar{\mathrm{h}}^0_{fi}\right]_{rea} = 10^3\left(-8 \cdot 394 - 9 \cdot 242 - (-1 \cdot 208)\right)$$

$$= -5120 \cdot \frac{MJ}{kmol_{C_8H_{18}}}$$

$$n_m = 8 + 9 + 2.5 + 15 \cdot 3.76 = 75.9$$

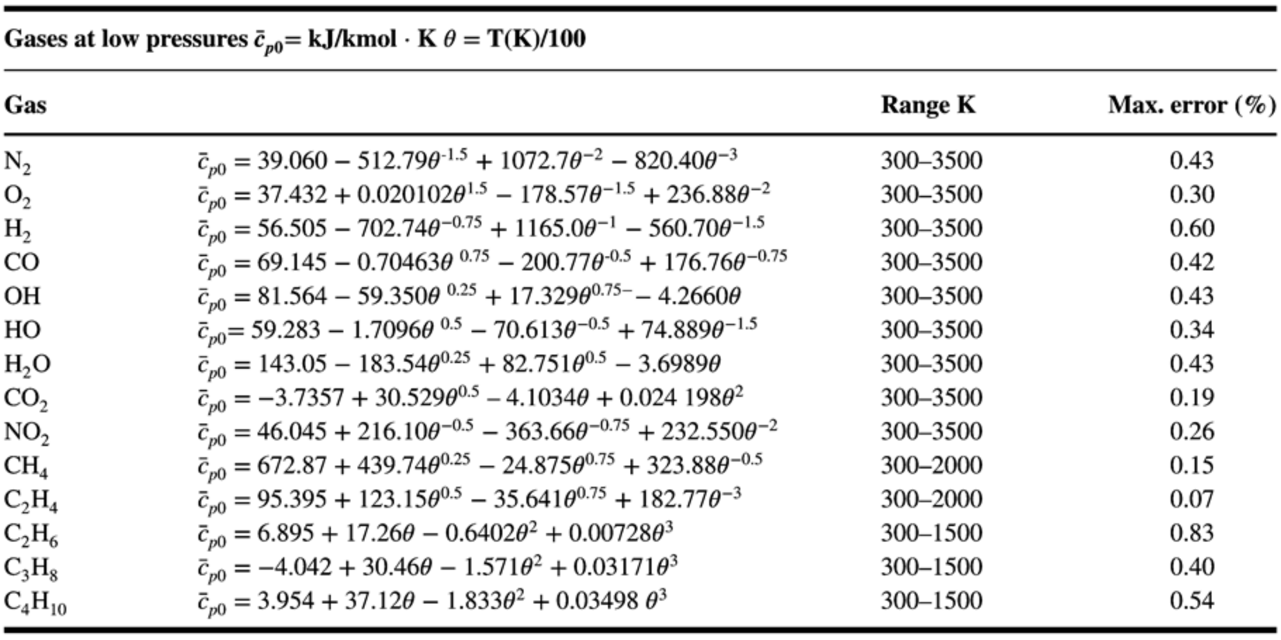
$$c_{pm_{prod}} = \sum \chi_j \bar{c}_{pj} = \frac{8 \cdot 61.9 + 9 \cdot 52.3 + 2.5 \cdot 37.8 + 15 \cdot 3.76 \cdot 33.6}{75.9} = 38.9 \cdot \frac{kJ}{kmol \cdot K}$$

$$n_m c_{pm_{prod}}\left(T_2 - T_f\right) = -\Delta H_{RPf}$$

$$T_2 = T_f - \frac{\Delta H_{RPf}}{n_m c_{pm_{prod}}} = 298.15 + \frac{5120 \cdot 10^3}{75.9 \cdot 38.9} = 2030 \cdot K$$

■ **TABLE 7.2**

Molar Specific Heats of Various Gases



Gases at low pressures $\bar{c}_{p0}$= kJ/kmol · K θ = T(K)/100

Gas		Range K	Max. error (%)
N_2	$\bar{c}_{p0} = 39.060 - 512.79\theta^{-1.5} + 1072.7\theta^{-2} - 820.40\theta^{-3}$	300–3500	0.43
O_2	$\bar{c}_{p0} = 37.432 + 0.020102\theta^{1.5} - 178.57\theta^{-1.5} + 236.88\theta^{-2}$	300–3500	0.30
H_2	$\bar{c}_{p0} = 56.505 - 702.74\theta^{-0.75} + 1165.0\theta^{-1} - 560.70\theta^{-1.5}$	300–3500	0.60
CO	$\bar{c}_{p0} = 69.145 - 0.704463\theta^{0.75} - 200.77\theta^{-0.5} + 176.76\theta^{-0.75}$	300–3500	0.42
OH	$\bar{c}_{p0} = 81.564 - 59.350\theta^{0.25} + 17.329\theta^{0.75-} - 4.2660\theta$	300–3500	0.43
HO	$\bar{c}_{p0}= 59.283 - 1.7096\theta^{0.5} - 70.613\theta^{-0.5} + 74.889\theta^{-1.5}$	300–3500	0.34
H_2O	$\bar{c}_{p0} = 143.05 - 183.54\theta^{0.25} + 82.751\theta^{0.5} - 3.6989\theta$	300–3500	0.43
CO_2	$\bar{c}_{p0} = -3.7357 + 30.529\theta^{0.5} - 4.1034\theta + 0.024\,198\theta^2$	300–3500	0.19
NO_2	$\bar{c}_{p0} = 46.045 + 216.10\theta^{-0.5} - 363.66\theta^{-0.75} + 232.550\theta^{-2}$	300–3500	0.26
CH_4	$\bar{c}_{p0} = 672.87 + 439.74\theta^{0.25} - 24.875\theta^{0.75} + 323.88\theta^{-0.5}$	300–2000	0.15
C_2H_4	$\bar{c}_{p0} = 95.395 + 123.15\theta^{0.5} - 35.641\theta^{0.75} + 182.77\theta^{-3}$	300–2000	0.07
C_2H_6	$\bar{c}_{p0} = 6.895 + 17.26\theta - 0.6402\theta^2 + 0.00728\theta^3$	300–1500	0.83
C_3H_8	$\bar{c}_{p0} = -4.042 + 30.46\theta - 1.571\theta^2 + 0.03171\theta^3$	300–1500	0.40
C_4H_{10}	$\bar{c}_{p0} = 3.954 + 37.12\theta - 1.833\theta^2 + 0.03498\,\theta^3$	300–1500	0.54

Source: Adapted from Van Wylen and Sonntag 1985.

(d) Evaluate also the flame temperature by considering non constant specific heat.

$$\bar{h}_j = \left(\int_{T_f}^{T_2} \bar{c}_{pj}\,dT + \Delta \bar{\mathrm{h}}^0_{fj}\right) \qquad \overline{c_{pJ}} = \frac{1}{T_2 - T_f}\int_{T_f}^{T_2} \bar{c}_{pj}\,dT \qquad \theta = \frac{T}{100}$$

$$\bar{c}_{p_{CO_2}} = \frac{1}{T_2 - T_f}\int_{T_f}^{T_2} 100\sum\left(a_i \theta^{e_i}\right) d\left(\frac{T}{100}\right) = \frac{1}{T_2 - T_f}\left[100\frac{\sum a_i \theta^{e_i+1}}{e_i + 1}\right]_{T_f}^{T_2} = \frac{1}{T_2 - T_f}\left[T\frac{\sum a_i \theta^{e_i}}{e_i + 1}\right]_{T_f}^{T_2}$$

$$\bar{c}_{p_{CO_2}} = \frac{1}{T_2 - T_f}\left[-3.736 \cdot T + \frac{30.53}{1.5} \cdot T\left(\frac{T}{100}\right)^{0.5} - \frac{4.103}{2} \cdot T\left(\frac{T}{100}\right)^{1} + \frac{0.02420}{3} \cdot T\left(\frac{T}{100}\right)^{2}\right]_{T_f}^{T_2}$$

$$\bar{c}_{p_{CO_2}} = \frac{[(-7{,}590 + 186{,}200 - 84{,}500 + 6{,}750) - (-1{,}115 + 10{,}470 - 1{,}822 + 21.4)]}{T_2 - T_f}$$

$$= \frac{[100{,}900 - 7{,}550]}{2030 - 298.15} = 53.9 \cdot \frac{kJ}{kmol \cdot K} \cdot K$$

$$T_2 = T_f - \frac{\Delta H_{RPf}}{n_m c_{pm_{prod}}} = 298.15 + \frac{5120 \cdot 10^3}{75.9 \cdot 36.5} = 2150 \cdot K$$

Si deve iterare e si ha:

$$T_2 = T_f - \frac{\Delta H_{RPf}}{n_m c_{pm_{prod}}} = 298.15 + \frac{5120 \cdot 10^3}{75.9 \cdot 36.9} = 2130 \cdot K$$

$$T_2 = T_f - \frac{\Delta H_{RPf}}{n_m c_{pm_{prod}}} = 298.15 + \frac{5120 \cdot 10^3}{75.9 \cdot 36.8} = 2130 \cdot K$$

Da https://cearun.grc.nasa.gov/ThermoBuild/:

$$h_{2029} - h_{298.15} = 93.19 \frac{kJ}{mole}$$

$$\bar{c}_{p_{CO_2}} = \frac{h_{2029} - h_{298.15}}{2030 - 298.15} = \frac{93.249 \cdot 1000}{2030 - 298.15} = 53.8 \cdot \frac{kJ}{kmol \cdot K} \cdot K$$

Prodotti	Δh	Δh n	cpm		
CO2	93,144	745,156	53.81429		
H2O	74,735	672,613	43.1781		
N2	57,157	3,223,638	33.02233		
O2	60,291	150,727	34.83301		
	Tot	4,792,134			
https://www.grc.nasa.gov/www/CEAWeb/ceaThermoBuild.htm					
	CO2	T1	298.15	T2	2029
Cost	Esp	Cp	Int	Cp	Int
-3.7357	0	-3.7357	-1113.8	-3.7357	-7,580
30.529	0.5	52.71449	10477.88	137.51612	186,013
-4.1034	1	-12.2343	-1823.83	-83.25799	-84,465
0.024198	2	0.215104	21.37778	9.9619319	6,738
		36.9596	7561.635	60.484363	100,706
				Diff	93,144
H2O		T1	298.15	T2	2029
Cost	Esp	Cp	Int	Cp	Int
143.05	0	143.05	42650.36	143.05	290,248
-183.54	0.25	-241.179	-57526	-389.5393	-632,300
82.751	0.5	142.8863	28401.04	372.7471	504,203
-3.6689	1	-10.9388	-1630.71	-74.44198	-75,521
		33.81853	11894.68	51.815856	86,630
					74,735
N2		T1	298.15	T2	2029
Cost	Esp	Cp	Int	Cp	Int
39.06	0	39.06	11645.74	39.06	79,253
-512.79	-1.5	-99.6064	59395.31	-5.610693	22,768
1072.7	-2	120.6726	-35978.5	2.6056386	-5,287
-820.4	-3	-30.9543	4614.515	-0.098215	100
		29.17186	39677.03	35.956731	96,834
					57,157
O2		T1	298.15	T2	2029
Cost	Esp	Cp	Int	Cp	Int
37.432	0	37.432	11160.35	37.432	75,950
0.020102	1.5	0.103488	12.34202	1.837225	1,491
-178.57	-1.5	-34.6862	20683.36	-1.953824	7,929
236.88	-2	26.64764	-7944.99	0.5753926	-1,167
		29.49696	23911.06	37.890794	84,202
					60,291

Farokhi problem 7.10

One mole of oxygen, O_2(g), is heated to 4000 K at the pressure of p_m. A fraction of the oxygen dissociates to oxygen atoms according to $xO_2 \rightarrow 2xO$. Assuming a state of equilibrium is reached in the mixture, calculate
(a) mole fraction of O_2 at equilibrium when p_m is 1 atm.
(b) mole fraction of O_2 at equilibrium when p_m is 10 atm.
Assume the equilibrium constant for the reaction $O_2 \leftrightarrow 2O$ is K_p= 2.19 atm at the temperature of 4000 K. Explain the effect of pressure on dissociation.

T(K)	$\frac{1}{2}\mathbf{O_2} \leftrightarrow \mathbf{O}$	$\frac{1}{2}\mathrm{H_2} \leftrightarrow \mathrm{H}$	$\frac{1}{2}\mathbf{H_2} + \frac{1}{2}\mathbf{O_2} \leftrightarrow \mathbf{OH}$	$\mathbf{H_2} + \frac{1}{2}\mathbf{O_2} \leftrightarrow \mathbf{H_2O}$
3500	-0.310	-0.231	0.160	0.712
4000	0.170	0.201	0.233	0.238

Logaritmo in base 10 delle cosanti d'equilibrio K

$$K = \frac{\chi_O}{\chi_{O_2}^{\frac{1}{2}}} p^{\frac{1}{2}} = 10^{0.170} = 1.479 \qquad K_p = \frac{\chi_O^2}{\chi_{O_2}} p = K^2 = 1.479^2 = 2.187$$

$$O_2 \rightarrow (1-x)O_2 + 2xO \qquad n = 1 - \mathrm{x} + 2\mathrm{x} = 1 + \mathrm{x} \qquad \chi_O = \frac{2\mathrm{x}}{1+\mathrm{x}} \qquad \chi_{O_2} = \frac{1-\mathrm{x}}{1+\mathrm{x}}$$

$$\frac{\left(\frac{2\mathrm{x}}{1+\mathrm{x}}\right)^2}{\frac{1-\mathrm{x}}{1+\mathrm{x}}} p = K^2 = 1.479^2 = 2.187 \qquad \frac{4x^2}{1+x} p = K_p(1-x) \qquad 4x^2 p = 2.187(1-x^2)$$

$$x^2(4p + 2.187) = 2.187 \qquad x = \sqrt{\frac{2.187}{4p + 2.187}}$$

$$p_m = 1 \qquad x = \sqrt{\frac{2.187}{4 + 2.187}} = 0.595$$

$$\chi_O = \frac{2 \cdot 0.595}{1 + 0.595} = 0.746 \qquad \chi_{O_2} = \frac{1 - 0.595}{1 + 0.595} = 0.254$$

$$p_m = 10 \qquad x = \sqrt{\frac{2.187}{40 + 2.187}} = 0.228$$

$$\chi_O = \frac{2 \cdot 0.228}{1 + 0.228} = 0.371 \qquad \chi_{O_2} = \frac{1 - 0.228}{1 + 0.228} = 0.629$$

In realtà se si considera la reazione $xO_2 \rightarrow 2xO$ la costante sarebbe K_x:

$$K_x = \frac{\chi_O^{2x}}{\chi_{O_2}^{x}} p^{2x-x} = \frac{\chi_O^{2x}}{\chi_{O_2}^{x}} p^{x} = K^{2x} = K_p^{x}$$

Si dovrebbe risolvere quindi:

$$\frac{\left(\frac{2\mathrm{x}}{1+\mathrm{x}}\right)^{2x}}{\left(\frac{1-\mathrm{x}}{1+\mathrm{x}}\right)^{x}} p^{x} = K_{x} = K^{2x} = K_{p}^{x}$$

Ma facendo la radice x esima di questa equazione si riottiene quella precedente:

$$\sqrt[x]{\frac{\left(\frac{2\mathrm{x}}{1+\mathrm{x}}\right)^{2x}}{\left(\frac{1-\mathrm{x}}{1+\mathrm{x}}\right)^{x}} p^{x}} = \frac{\left(\frac{2\mathrm{x}}{1+\mathrm{x}}\right)^{2}}{\frac{1-\mathrm{x}}{1+\mathrm{x}}} p = \sqrt[x]{K_{x}} = \sqrt[x]{K^{2x}} = \sqrt{K_{p}^{x}} = K_{p}$$