Mattingly 9.12

Air enters a compressor stage that has the following properties:

$$\dot{m} = 50 \text{ kg/s}, \quad \omega = 800 \text{ rad/s}, \quad r = 0.5 \text{ m}$$

$$M_1 = M_3 = 0.5, \quad \alpha_1 = \alpha_3 = 40 \text{ deg}, \quad T_{t1} = 290 \text{ K}$$

$$P_{t1} = 101.3 \text{ kPa}, \quad u_2/u_1 = 1.0, \quad T_{t3} - T_{t1} = 45 \text{ K}$$

$$\phi_{cr} = 0.10, \quad \phi_{cs} = 0.03, \quad \sigma = 1$$

Note: For air, use $\gamma = 1.4$ and $R = 0.286 \text{ kJ/(kg} \cdot \text{K)}$. Make and fill out a table of flow properties like Table 9.3, and determine the diffusion factors, degree of reaction, stage efficiency, polytropic efficiency, and flow areas and associated hub and tip radii at stations 1, 2, and 3.

m	50	kg/s	Cz1=Cz2=Cz3	
ω	800	rad/s	M1=M3	0.5
Ro	0.5	m	α1,3	40
Tt1	290	K	ω_r	0.1
pt1	101,300	Pa	ω_s	0.03
ΔTt	45	K	σ_r	1
R	287	J/kgK	γ	1.4

$$k = \frac{\gamma - 1}{\gamma} = \frac{1.4 - 1}{1.4} = 0.286 \qquad R = 287 \cdot \frac{J}{kg \cdot K} \qquad c_p = \frac{R}{k} = 1004 \cdot \frac{J}{kg \cdot K}$$

Sezione 1

$$\psi_1 = 1 + \frac{\gamma - 1}{2} M_1^2 = 1 + .2 \cdot 0.5^2 = 1.050$$

$$T_1 = \frac{T_{t1}}{\psi_1} = \frac{290}{1.050} = 276 \cdot K \qquad a_1 = \sqrt{\gamma R T_1} = \sqrt{1.4 \cdot 287 \cdot 276} = 333 \cdot \frac{m}{s}$$

$$a_{t1} = \sqrt{\gamma R T_{t1}} = \sqrt{1.4 \cdot 287 \cdot 290} = 341 \cdot \frac{m}{s} \qquad p_1 = \frac{p_{t1}}{\psi_1^{\frac{1}{k}}} = \frac{101.3}{1.050^{\frac{1}{0.286}}} = 85.4 \cdot kPa$$

$$C_1 = M_1 a_1 = 0.5 \cdot 333 = 166.5 \cdot \frac{m}{s} \qquad C_{z1} = C_1 \cos \alpha_1 = 166.5 \cdot \cos 40 = 127.5 \cdot \frac{m}{s}$$

$$C_{\theta 1} = C_1 \sin \alpha_1 = 166.5 \cdot \sin 40 = 107.0 \cdot \frac{m}{s} \qquad U = \omega R_o = 800 \cdot 0.5 = 400 \cdot \frac{m}{s}$$

$$M_1 \xrightarrow{ISO} \frac{A_R}{A_1^*} = 1.340$$

$$\dot{m} = \frac{p_{t1} \frac{A_1^*}{A_R} A_R \Psi^*}{a_{t1}} \qquad A_R = \frac{\dot{m} a_{t1}}{p_{t1} \frac{A_1^*}{A_R} \Psi^*} = \frac{50 \cdot 341}{101{,}300 \cdot \frac{0.81}{1.340}} = 0.278 \cdot m^2$$

$$A_1 = \frac{A_R}{\cos \alpha_1} = \frac{0.278}{\cos 40} = 0.363 \cdot m^2$$

Sezione 1r

$$W_{\theta 1} = C_{\theta 1} - U = 107.0 - 400 = -293 \cdot \frac{m}{s}$$

$$W_1 = \sqrt{W_{\theta 1}^2 + W_{z1}^2} = \sqrt{293^2 + 127.5^2} = 320 \cdot \frac{m}{s}$$

$$\beta_1 = \text{atan}\frac{W_{\theta 1}}{W_{z1}} = \text{atan} - \frac{293}{127.5} = -66.5°$$

$$M_{1R} = \frac{W_1}{a_1} = \frac{320}{333} = 0.961 \qquad \psi_{1R} = 1 + \frac{\gamma - 1}{2} M_{1R}^2 = 1 + .2 \cdot 0.961^2 = 1.185$$

$$T_{t1R} = T_1 \psi_{1R} = 276 \cdot 1.185 = 327 \cdot K \qquad p_{t1R} = p_1 \psi_{1R}^{\frac{1}{k}} = 85.4 \cdot 1.185^{\frac{1}{0.286}} = 154.6 \cdot kPa$$

Sezione 2r

$$\omega_r = \frac{p_{t1R} - p_{t2R}}{\frac{1}{2}\rho_1 W_1^2} = \frac{p_{t1R} - p_{t2R}}{\frac{1}{2}\frac{\rho_1}{p_1} p_1 W_1^2} = \frac{p_{t1R} - p_{t2R}}{\frac{1}{2}\gamma p_1 M_{1r}^2}$$

$$p_{t2R} = p_{t1R} - \omega_r \frac{1}{2}\gamma p_1 M_{1r}^2 = 154.6 - 0.1\frac{1.4 \cdot 85{,}400 \cdot 0.961^2}{2} = 149.1 \cdot kPa$$

$$T_{t2} = T_{t1} + \Delta T_t = 290 + 45 = 335 \cdot K \qquad \frac{T_{t2}}{T_{t1}} = 1 + \frac{U^2}{c_p T_{t1}}\left[1 + \frac{C_{z1}}{U}(\tan \beta_2 - \tan \alpha_1)\right]$$

$$c_p(T_{t2} - T_{t1}) - U^2 = U C_{z1}(\tan \beta_2 - \tan \alpha_1)$$

$$\tan \beta_2 = \tan\alpha_1 + \frac{c_p(T_{t2} - T_{t1}) - U^2}{U C_{z1}} = \tan 40 + \frac{1004(335 - 290) - 400\text{^}2}{400 \cdot 127.5} = -1.412$$

$$\beta_2 = \text{atan} -1.411 = -54.7°$$

$$W_{\theta 2} = C_{z2} \tan \beta_2 = 127.5 \cdot (-1.412) = \quad -179.9 \cdot \frac{m}{s}$$

$$W_2 = \sqrt{W_{\theta 2}^2 + W_{z2}^2} = \sqrt{179.9^2 + 127.5^2} = 221 \cdot \frac{m}{s}$$

$$T_2 = T_{t2R} - \frac{W^2}{2c_p} = 327 - \frac{221^2}{2 \cdot 1004} = 303 \cdot K$$

$$a_2 = \sqrt{\gamma R T_2} = \sqrt{1.4 \cdot 287 \cdot 303} = 349 \cdot \frac{m}{s} \qquad M_{2R} = \frac{W_2}{a_2} = \frac{221}{349} = 0.633$$

$$p_2 = p_{t2R}\left(\frac{T_2}{T_{t2R}}\right)^{\frac{1}{k}} = 149.2 \cdot \left(\frac{303}{327}\right)^{\frac{1}{0.286}} = 114.2 \cdot kPa$$

Sezione 2

$$C_{\theta 2} = W_{\theta 2} + U = -179.9 + 400 = 220 \cdot \frac{m}{s}$$

$$C_2 = \sqrt{C_{\theta 2}^2 + C_{z2}^2} = \sqrt{220^2 + 127.5^2} = 254 \cdot \frac{m}{s}$$

$$\alpha_2 = \operatorname{atan}\frac{C_{\theta 2}}{C_{z2}} = \operatorname{atan}\frac{220}{127.5} = 59.9°$$

$$M_2 = \frac{C_2}{a_2} = \frac{254}{349} = 0.728 \cdot \frac{m}{s} \qquad \psi_2 = 1 + \frac{\gamma - 1}{2} M_2^2 = 1 + .2 \cdot 0.728^2 = 1.106$$

$$T_{t2} = 335 \cdot K \qquad p_{t2} = p_2 \psi_2^{\frac{1}{k}} = 114.2(1.106)^{\frac{1}{0.286}} = 162.4 \cdot kPa$$

$$a_{t2} = \sqrt{\gamma R T_{t2}} = \sqrt{1.4 \cdot 287 \cdot 335} = 367 \cdot \frac{m}{s}$$

$$M_2 \quad \xrightarrow{ISO} \quad \frac{A_{R2}}{A_2^*} = 1.075 \qquad A_{R_2} = \frac{\dot{m} a_{t2}}{p_{t2} \frac{A_2^*}{A_{R2}} \Psi^*} = \frac{50 \cdot 367}{162{,}400 \cdot \frac{0.81}{1.075}} = 0.1500 \cdot m^2$$

$$A_2 = \frac{A_{R2}}{\cos \alpha_2} = \frac{0.1500}{\cos 59.9} = 0.299 \cdot m^2$$

Sezione 3

$$T_{t3} = T_{t2} = 335 \cdot K$$

$$p_{t3} = p_{t2} - \omega_s \frac{1}{2} \gamma p_2 M_2^2 = 162.4 - 0.03 \, \frac{1.4 \cdot 114.200 \cdot 0.728^2}{2} = 161.1 \cdot kPa$$

$$M_3 = M_2 = 0.5 \qquad \psi_3 = \psi_1 = 1.050 \qquad T_3 = \frac{T_{t3}}{\psi_3} = \frac{335}{1.050} = 319 \cdot K$$

$$a_3 = \sqrt{\gamma R T_3} = \sqrt{1.4 \cdot 287 \cdot 319} = 358 \cdot \frac{m}{s}$$

$$a_{t3} = a_{t2} = 367 \cdot \frac{m}{s} \qquad p_3 = p_{t3} \psi_3^{-\frac{1}{k}} = 161.1 \cdot (1.050)^{-\frac{1}{0.286}} = 135.8 \cdot kPa$$

$$C_3 = M_3 \cdot a_3 = 0.5 \cdot 358 = 179 \cdot \frac{m}{s}$$

$$C_{z3} = C_3 \cos \alpha_3 = 179 \cdot \cos 40 = 137 \cdot \frac{m}{s}$$

$$C_{\theta 3} = C_3 \sin \alpha_3 = 179 \cdot \sin 40 = 115.1 \cdot \frac{m}{s}$$

$$M_3 \quad \xrightarrow{ISO} \quad \frac{A_{R3}}{A_3^*} = 1.340 \qquad A_{R2} = \frac{\dot{m} a_{t3}}{p_{t3} \frac{A_3^*}{A_{R3}} \Psi^*} = \frac{50 \cdot 367}{161{,}100 \cdot \frac{0.81}{1.340}} = 0.1884 \cdot m^2$$

$$A_3 = \frac{A_{R3}}{\cos \alpha_1} = \frac{0.1880}{\cos 40} = 0.246 \cdot m^2$$

Parametri dello stadio

$$°R = \frac{T_2 - T_1}{T_3 - T_1} = \frac{(303 - 276)}{319 - 276} = 0.628$$

$$D_r = 1 - \frac{W_2}{W_1} + \frac{|W_{\theta 2} - W_{\theta 1}|}{2 \sigma_r W_1} = 1 - \frac{221}{320} + \frac{293 - 179.9}{2 \cdot 1 \cdot 320} = 0.486 < 0.6$$

$$D_s = 1 - \frac{C_3}{C_2} + \frac{|C_{\theta 3} - C_{\theta 2}|}{2 \sigma_s C_2} = 1 - \frac{179}{254} + \frac{220 - 115.1}{2 \cdot 1 \cdot 254} = 0.503 < 0.6$$

$$\eta_c = \frac{\pi_c^k - 1}{\tau_c - 1} = \frac{\left(\frac{161.1}{101.3}\right)^{0.2856} - 1}{\frac{335}{290} - 1} = 0.914$$

$$\tau_c = \pi_c^{\frac{k}{e_c}} \qquad e_c = k\frac{\ln \pi_c}{\ln \tau_c} = 0.286\frac{\ln \frac{161.1}{101.3}}{\ln \frac{335}{290}} = 0.920$$

$$\Psi_c = \frac{c_p \Delta T_t}{U^2} = \frac{1004 \cdot 45}{400^2} = 0.282 \qquad \phi = \frac{C_z}{U} = \frac{127.5}{400} = 0.319$$

m	50	kg/s	Cz1=Cz2=Cz3		Ma9.12		
ω	800	rad/s	M1=M3	0.5			
Ro	0.5	m	$\alpha 1{,}3$	40			
Tt1	290	K	ω_r	0.1			
pt1	101,300	Pa	ω_s	0.03			
ΔTt	45	K	σ_r	1	Ψ^*	0.810185	
R	287	J/kgK	γ	1.4	k	0.2857	
ψ_1	1.050	T1	276.2	at1	341.4	p1	85,398
a1	333.1	C1	166.6	$C_{\theta 1}$	107.1	Cz1	127.6
A1	0.3637	U	400	$W_{\theta 1}$	-292.9	W1	319.5
$\beta 1$	-66.46	M1r	0.9591	ψ_{1r}	1.184	Tt1r	327.0
pt1r	154,230	pt2r	148,731	Tt2	335		
Tan(α_1)	0.8391	dum	-2.24926	Tan(β_2)	-1.410	$\beta 2$	-54.66
$W_{\theta 2}$	-179.9	W2	220.6	$C_{\theta 2}$	220.1	C2	254.4
α_2	59.90	T2	302.8	p2	113,616	a2	348.8
M2	0.7293	M2r	0.632395	pt2	161,847	at2	366.9
A2	0.2997	T3	319.0	pt3	160,578	a3	358.0
C3	179.0	$C_{\theta 3}$	115.1	Cz3	137.1	A3	0.246619
p3	135,370	°R	0.620634	Dr	0.48649	Ds	0.50264
η	0.9066192	ec	0.912492	Ψ_c	0.282516	ϕ	0.318987
	1	1R	2R	2	3		1.05
Tt	**290**	327.0	327.0	**335**	**335.0**		
T	276.2	276.2	302.8	302.8	319.0		
pt	**101,300**	154,230	148,731	161,847	135,370		
p	85,398	85,398	113,616	113,616	137.1		
M	**0.5**	0.9591	0.6324	0.7293	**0.5**		
C/W	166.6	319.5	220.6	254.4	179.0		
W/C_z	127.6	127.6	127.6	127.6	137.1		
W/C_θ	107.1	-292.9	-179.9	220.1	115.1		
α	**40**			59.90	**40**		
β		-66.46	-54.66				

	1	1R	2R	2	3		1.05
Tt	**290**	327.0	327.0	**335**	**335.0**		
T	276.2	276.2	302.8	302.8	319.0		
pt	**101,300**	154,230	148,731	161,847	135,370		
p	85,398	85,398	113,616	113,616	137.1		
M	**0.5**	0.9591	0.6324	0.7293	**0.5**		
C/W	166.6	319.5	220.6	254.4	179.0		
W/C_z	127.6	127.6	127.6	127.6	137.1		
W/C_θ	107.1	-292.9	-179.9	220.1	115.1		
α	**40**			59.90	**40**		
β		-66.46	-54.66				