Esercizio Mattingly 9.35

m	40	kg/s	Cz2=Cz3	
ω	1000	rad/s	M1	0.3
Ro	0.4	m	M2	1.15
Tt1	1780	K	$\alpha 1,3$	0
pt1	1,400,000	Pa	ω_r	0.08
ΔTt	230	K	ω_s	0.04
R	287	J/kgK		
cp	1243.6667		γ	1.3
Ψ^*	0.7607961		k	0.2308

$$k = \frac{\gamma - 1}{\gamma} = \frac{0.3}{1.3} = 0.231 \qquad K = \frac{\gamma + 1}{2(\gamma - 1)} = \frac{2.3}{0.6} = 3.83$$

Sezione 1

$$\psi_1 = 1 + \frac{\gamma - 1}{2} M_1^2 = 1 + 0.15 \cdot 0.3^2 = 1.014$$

$$T_1 = \frac{T_{t1}}{\psi_1} = \frac{1780}{1.014} = 1755 \cdot K$$

$$p_1 = \frac{p_{t1}}{\psi_1^{\frac{1}{k}}} = \frac{1400}{1.014^{\frac{1}{0.231}}} = 1318 \cdot kPa$$

$$a_{t1} = \sqrt{\gamma R T_{t1}} = 1.3 \cdot 287 \cdot 1780 = 815 \cdot m/s$$

$$a_1 = \sqrt{\gamma R T_1} = 1.3 \cdot 287 \cdot 1755 = 809 \cdot m/s$$

$$C_1 = M_1 \cdot a_1 = 0.3 \cdot 809 = 243 \cdot m/s$$

$$C_{\theta 1} = C_1 \cdot \sin \alpha_1 = 0 \cdot m/s \qquad C_{z1} = C_1 \cdot \cos \alpha_1 = 243 \cdot m/s$$

$$U = \omega r_0 = 1000 \cdot 0.4 = 400 \cdot \frac{m}{s}$$

$$\dot{m} = \frac{p_{t1} \frac{A^*}{A_{1R}} A_{1R} \Psi^*}{a_{t1}} = \frac{p_{t1} A_{1R} \Psi_1}{a_{t1}}$$

$$\Psi(M) = \gamma M \psi^{-K}$$

$$\Psi_1 = \gamma M_1 \psi_1^{-K} = 1.3 \cdot 0.3 \cdot 1.014^{-3.83} = 0.370$$

$$A_1 = \frac{A_{1R}}{\cos \alpha} = \frac{\dot{m} a_{t1}}{\cos \alpha \, p_{t1}} \frac{1}{\Psi_1} = \frac{40 \cdot 815}{1400 \cdot 10^3} \frac{1}{0.370} = 0.0629 \cdot m^2$$

Sezione 2

$$M_2 = 1.15 \qquad T_{t2} = T_{t1} = 1780 \cdot K \qquad \omega_s = 0.04$$

$$\psi_2 = 1 + \frac{\gamma - 1}{2} M_2^2 = 1 + 0.15 \cdot 1.15^2 = 1.198$$

$$T_2 = \frac{T_{t2}}{\psi_2} = \frac{1780}{1.198} = 1486K$$

$$a_{t2} = \sqrt{\gamma R T_{t2}} = \sqrt{1.3 \cdot 287 \cdot 1780} = 815 \cdot m/s$$

$$a_2 = \sqrt{\gamma R T_2} = \sqrt{1.3 \cdot 287 \cdot 1486} = 745 \cdot m/s$$

$$C_2 = M_2 \cdot a_2 = 1.15 \cdot 745 = 857 m/s$$

$$\omega_s = \frac{p_{t1} - p_{t2}}{p_{t2} - p_2} = \omega_s = \frac{1 - \frac{p_{t2}}{p_{t1}}}{\frac{p_{t2}}{p_{t1}} \left(1 - \frac{p_2}{p_{t2}}\right)} \rightarrow$$

$$p_{t2} = \frac{p_{t1}}{1 + \omega_s \left(1 - \psi_2^{-\frac{1}{k}}\right)} = \frac{1400 \cdot 10^3}{1 + 0.04 \left(1 - 1.198^{\frac{-1}{0.231}}\right)} = 1{,}370 KPa$$

$$p_2 = p_{t2} \psi_2^{\frac{-1}{k}} = 1370 \cdot 1.198^{\frac{-1}{0.231}} = 627 \cdot kPa$$

Dall'equazione di Eulero (con $C_{\theta 3} = 0$):

$$\Delta h_t = -U \Delta C_\theta = -U(C_{\theta 3} - C_{\theta 2}) = U C_{\theta 2}$$

$$C_{\theta 2} = -\frac{\Delta h_t}{U} = \frac{c_p \Delta T_t}{U} = 1242 \cdot \frac{230}{400} = 714 \cdot m/s$$

$$\alpha_2 = \operatorname{asin} \frac{C_{\theta 2}}{C_2} = \operatorname{asin} \frac{714}{857} = 56.4°$$

$$C_{z2} = C_2 \cdot \cos \alpha_2 = 857 \cdot \cos 56.4 = 474 \cdot m/s$$

$$\Psi_2 = \gamma M_2 \psi_2^{-K} = 1.3 \cdot 1.15 \cdot 1.198^{-3.83} = 0.748$$

$$A_2 = \frac{A_{2R}}{\cos \alpha_2} = \frac{\dot{m} a_{t2}}{\cos \alpha_2 \, p_{t2}} \frac{1}{\Psi_2} = \frac{40 \cdot 815}{0.5498 \cdot 1{,}370 \cdot 10^3} \frac{1}{0.748} = 0.0575 \cdot m^2$$

Sezione 2r

$$W_{\theta 2} = C_{\theta 2} - U = 714 - 400 = 314 \cdot \frac{m}{s}$$

$$W_2 = \sqrt{w_{\theta 2}^2 + W_{z2}^2} = \sqrt{474^2 + 314^2} = 569 \frac{m}{s}$$

$$\beta_2 = \operatorname{atan} \frac{W_{\theta 2}}{W_{z2}} = \operatorname{atan} \frac{314}{474} = 33.5°$$

$$M_{2R} = \frac{W_2}{a_2} = \frac{569}{745} = 0.764$$

$$\psi_{2R} = 1 + \frac{\gamma - 1}{2} M_{2R}^2 = 1 + 0.15 \cdot 0.764^2 = 1.088$$

$$T_{t2R} = T_2 \psi_{2R} = 1486 \cdot 1.088 = 1617 \cdot K$$

$$p_{t2R} = p_2 \psi_{2R}^{\frac{1}{k}} = 627 \cdot 1.088^{\frac{1}{0.231}} = 903 \cdot kPa$$

Sezione 3

$$C_{z3} = C_{z2} = 474 \cdot m/s \qquad \alpha = 0 \rightarrow C_3 = C_{z3} = 474 \cdot m/s \qquad C_{\theta_3} = 0 m/s$$

$$T_{t3} = T_{t2} - \Delta T_t = 1780 - 230 = 1550K$$

$$T_3 = T_{t3} - \frac{C_3^3}{2c_p} = 1550 - \frac{474^2}{2 \cdot 1244} = 1460 \cdot K$$

$$\psi_3 = \frac{T_{t3}}{T_3} = \frac{1550}{1460} = 1.062$$

$$a_3 = \sqrt{\gamma R T_3} = \sqrt{1.3 \cdot 287 \cdot 1460} = 738 \cdot m/s$$

$$a_{t3} = \sqrt{\gamma R T_{t3}} = \sqrt{1.3 \cdot 287 \cdot 1550} = 760 \cdot m/s$$

$$M_3 = \frac{C_3}{a_3} = \frac{474}{738} = 0.641$$

Sezione 3r

$$W_{\theta 3} = C_{\theta 3} - U = -400 = -400 \frac{m}{s}$$

$$W_3 = \sqrt{w_{\theta 3}^2 + W_{z3}^2} = \sqrt{400^2 + 473^2} = 619 \cdot \frac{m}{s}$$

$$\beta_3 = \text{atan} \frac{W_{\theta 3}}{W_{z3}} = \text{atan} \frac{-400}{473} = -40.2°$$

$$M_{3R} = \frac{W_3}{a_3} = \frac{619}{738} = 0.840$$

$$\psi_{3R} = 1 + \frac{\gamma - 1}{2} M_{3R}^2 = 1 + 0.15 \cdot 0.840^2 = 1.106$$

$$T_{t3R} = T_{t2R} = 1615K$$

$$p_{t3r} = \frac{p_{t2r}}{1 + \omega_r \left(1 - \psi_{3R}^{-\frac{1}{k}}\right)} = \frac{900 \cdot 10^3}{1 + 0.08 \left(1 - 1.106^{\frac{-1}{0.231}}\right)} = 878 \cdot kPa$$

$$p_3 = p_{t3R} \psi_{3R}^{\frac{-1}{k}} = 878 \cdot 1.106^{\frac{-1}{0.231}} = 568 \cdot kPa$$

Sezione 3

$$p_{t3} = p_3 \psi_3^{\frac{1}{k}} = 568 \cdot 1.062^{\frac{1}{0.231}} = 737 \cdot kPa$$

$$\Psi_3 = \gamma M_3 \psi_3^{-K} = 1.3 \cdot 0.642 \cdot 1.062^{-3.83} = 0.663$$

$$A_3 = \frac{A_{3R}}{\cos\alpha_3} = \frac{\dot{m}a_{t3}}{\cos\alpha_3\, p_{t3}}\frac{1}{\Psi_3} = \frac{40\cdot 760}{737\cdot 10^3}\frac{1}{0.663} = 0.0622\cdot m^2$$

Parametri dello stadio

$$^{\circ}R = \frac{h_2 - h_3}{h_1 - h_3} = \frac{1486 - 1460}{1755 - 1460} = 0.0881$$

$$\eta = \frac{1-\tau_t}{1-\pi_t^k} = \frac{1-\frac{1550}{1780}}{1-\left(\frac{737}{1400}\right)^{0.231}} = 0.938$$

$$e_c = \frac{\ln\tau_t}{k\ln\pi_t} = \frac{\ln\frac{1550}{1780}}{0.231\ln\frac{737}{1400}} = 0.933$$

$$\Psi_c = \frac{c_p\Delta T_c}{U^2} = \frac{1242\cdot 230}{400^2} = 1.785$$

	1	2	2R	3R	3
Tt	**1780**	1780.0	1614.3	1614.3	**1550.0**
T	1756.3	1485.3	1485.3	1460.9	1460.9
pt	**1,400,000**	1,370,211	897,283	872,762	731,764
p	1,320,966	625,488	625487.9	566,246	470.7
M	**0.3**	**1.1500**	0.7609	0.8366	0.6375
C/W	242.8	856.1	566.4	617.7	470.7
W/C_z	242.8	470.7	470.7	470.7	470.7
W/C_θ	0.0	715.1	315.1	-400.0	0.0
α	**0**	56.65			**0**
β			33.80	-40.36	