Fa 11.10

A turbojet engine has the following design parameters (which is at takeoff):

$M_0 = 0, p_0 = 101.3\ kPa, T_0 = 15C, \gamma_c = 1.4, c_{pc} = 1004 J/kg\ K, \pi_d = 0.95,$

$\pi_c = 30, e_c = 0.90, Q_R = 42{,}600\ kJ/kg, \pi_b = 0.95, \eta_b = 0.98,$

$T_{t4} = 1700\ C, \eta_m = 0.98, e_t = 0.85, \gamma_t = 1.33, c_{pt} = 1156 J/kgK,$

$\pi_n = 0.90, p_9 = p_0$

This engine powers an aircraft that cruises at $M_0 = 0.80$ at an altitude where $T_0 = -35C$, $p_0 = 20\ kPa$. The turbine entry temperature at cruise is $T_{t4} = 1500C$. Assume that the engine has the same component efficiencies at cruise and takeoff, and the nozzle is perfectly expanded at cruise, as well. Calculate

(a) the exhaust velocity V_9 (in m/s) at the design point, i.e., at takeoff

	2	3	4	5	9		
	diff	comp	CC	Tur	No		
c_p	1004			1156	J/kgs	Mz	0.5
γ	1.4			1.33		N	7460
π	0.95	30	0.95		0.9	m2	77
η, $e_{c,t}$		0.9	0.98	0.85		Tref	288.15
Tt				**1973**		Pref	101,300
M0	**0**		**po/p9**	1	QR	42600	kJ/kgK
T0	**288**	**K**	**p0**	**101,300**	Pa	η_m	0.98
k	0.286	K	3	0.248			
R	287			287			
a0	340	m/s	V0	0	m/s		
$\tau_r=T_{t0}=0/T0(\psi)$	1						
Section	c	t	λ				t
τ	2.94	0.757	7.89			π	0.267661
Section	0	2	3	4	5	7	9
Tt	288	288	848	1973	1494		1494
pt	101,300	96,235	2,887,050	2,742,698	734,114		660,702
Section	f	Pt9/p9	M9	Tt9/T9	T9/T0	a9	
3	0.03622	6.52	1.89	1.592	3.26	598	m/s
		V9		V9/a0	V9/a0 eff	F/ma0	Num
		1134	m/s	3.33	3.33	3.45	1331784
Section	t	th	p	0	TSFC*1e3		
h	0.870	0.432	0.000	0.0000	0.0308		
δ_2	θ_2	Θ	mc2	Nc2	θ4	Nc4	
0.950	0.999	6.85	81.0	7462	6.85	2851	

$$k = \frac{\gamma - 1}{\gamma} = \frac{1.4 - 1}{1.4} = 0.286 \qquad k_t = \frac{\gamma_t - 1}{\gamma_t} = \frac{1.33 - 1}{1.33} = 0.248$$

$$R = kc_p = 1004 \cdot 0.286 = 287 \frac{J}{kg \cdot K} \qquad R_t = k_t c_{pt} = 1156 \cdot 0.248 = 287 \frac{J}{kg \cdot K}$$

Effetto Ram

$$a_0 = \sqrt{\gamma R T_0} = \sqrt{1.4 \cdot 287 \cdot 288} = 340 \frac{m}{s} \qquad V_0 = M_0 a_0 = 0 \frac{m}{s}$$

$$\tau_r = \psi_0 = 1 + \frac{\gamma - 1}{2} M_0^2 = 1$$

$$T_{t0} = T_0 \tau_r = 288 \cdot 1 = 288 \cdot K \qquad p_{t0} = p_0 \tau_r^{\frac{1}{k}} = 1.013 \cdot 10^5 \cdot 1^{\frac{1}{0.286}} = 101.3 \cdot kPa$$

Diffusore

$$p_{t2} = p_{t0} \pi_d = 101.3 \cdot 0.95 = 96.2 \cdot kPa$$

Compressore

$$\tau_c = \pi_c^{\frac{k}{e_c}} = 30^{\frac{0.286}{0.9}} = 2.95 \qquad p_{t3} = p_{t2} \pi_c = 96.2 \cdot 30 = 2890 \cdot kPa$$

$$T_{t3} = T_{t2} \tau_c = 288 \cdot 2.95 = 850 \cdot K$$

Camera di Combustione

$$\tau_\lambda = \frac{c_{pt} T_{t4}}{c_p T_0} = \frac{1154 \cdot 1973}{1004 \cdot 288} = 7.89$$

$$f = \frac{\tau_\lambda - \tau_c \tau_r}{Q_R \eta_b / (c_p T_0) - \tau_\lambda} = \frac{7.89 - 2.95 \cdot 1.0}{\frac{42.6 \cdot 10^6 \cdot 0.98}{1004 \cdot 288} - 7.89} = 0.0362$$

$$\tau_b = \frac{\tau_\lambda}{\tau_c \tau_r} = \frac{7.89}{2.94 \cdot 1.0} = 2.67$$

$$p_{t4} = p_{t3} \pi_b = 2890 \cdot 0.95 = 2750 \cdot kPa$$

Turbina

$$\tau_t = 1 - \frac{(\tau_c - 1)\tau_r}{\eta_m (1 + f) \tau_\lambda} = 1 - \frac{1.95 \cdot 1.0}{0.98 \cdot 1.036 \cdot 7.89} = 0.757$$

$$\pi_t = \tau_t^{\frac{1}{k_t e_t}} = 0.757^{\frac{1}{0.248 \cdot 0.85}} = 0.267$$

$$p_{t5} = p_{t4} \pi_t = 2750 \cdot 0.267 = 734 \cdot kPa \qquad T_{t5} = T_{t4} \tau_t = 1973 \cdot 0.757 = 1494 \cdot K$$

Ugello

$$p_{t9} = p_{t5} \pi_n = 734 \cdot 0.90 = 661 \cdot kPa \qquad T_{t9} = T_{t5} = 1494 \cdot K$$

$$\frac{p_{t9}}{p_9} = \pi_n \pi_t \pi_b \pi_c \pi_d \pi_r \frac{p_0}{p_9} = \frac{p_{t9}}{p_0} \frac{p_0}{p_9} = \frac{661}{101.3} \cdot 1 = 6.53$$

$$M_9 = \sqrt{\frac{2}{\gamma_t - 1}\left[\left(\frac{p_{t9}}{p_9}\right)^{k_t} - 1\right]} = \sqrt{\frac{2}{0.33}(6.53^{0.248} - 1)} = 1.895$$

$$\frac{T_{t9}}{T_9} = \psi_9 = 1 + \frac{\gamma_t - 1}{2} M_9^2 = \left(\frac{p_{t9}}{p_9}\right)^{k_t} = 6.53^{0.248} = 1.593 \qquad T_9 = \frac{T_9}{T_{t9}} T_{t9} = \frac{1494}{1.593} = 938 \cdot K$$

$$a_9 = \sqrt{\gamma_t R_9 T_9} = \sqrt{1.33 \cdot 287 \cdot 938} = 598 \cdot \frac{m}{s}$$

$$V_9 = M_9 a_9 = 1.895 \cdot 598 = 1133 \cdot \frac{m}{s} \qquad V_{9.e} = V_9\left(1 + \frac{1 - \frac{p_0}{p_9}}{\gamma_9 M_9^2}\right) = V_9 \qquad \frac{V_9}{a_0} = \frac{1133}{340} = 3.33$$

Spinta e rendimenti

(b) the thermal efficiency η_{th} at the design point

(c) thrust-specific fuel consumption at the design point

(d) the compressor pressure ratio at cruise

(e) the exhaust velocity V_9 (in m/s) at cruise

$$\frac{F_u}{\dot{m}_0 a_0} = (1 + f)\frac{V_{9.\mathrm{e}}}{a_0} - M_0 = 1.036 \cdot 3.33 - 0 = 3.45$$

$$\eta_{\mathrm{th}} = \frac{\Delta K\dot{E}}{\mathcal{P}_t} = \frac{a_0^2[(1 + f)(V_9/a_0)^2 - M_0^2]}{2 f Q_R} = \frac{B}{2 f Q_R}$$

$$\eta_p = \frac{F_i V_0}{\Delta K\dot{E}} \approx \frac{2 F_u V_0/\dot{m}_0}{a_0^2[(1 + f)(V_9/a_0)^2 - M_0^2]} = \frac{2 F_u V_0/\dot{m}_0}{B}$$

$$B = a_0^2[(1 + f)(V_9/a_0)^2 - M_0^2] = 1.036 \cdot 1134^2 - 0^2 = 1.330 \cdot 10^6 \cdot \frac{m^2}{s^2}$$

$$\eta_{\mathrm{th}} = \frac{B}{2 f Q_R} = \frac{1.330 \cdot 10^6}{2 \cdot 0.0362 \cdot 42.600 \cdot 10^6} = 0.431$$

$$\eta_p = \frac{2 a_0 V_0 \frac{F_u}{\dot{m}_0 a_0}}{B} = \frac{2 \cdot 340 \cdot 0 \cdot 3.45}{1.330 \cdot 10^6} = 0 \qquad \eta_o = \eta_p \eta_{\mathrm{th}} = 0.43 \cdot 0 = 0.$$

$$TSFC = \frac{\dot{m}_f}{F_u} = \frac{f}{F_u/\dot{m}_0} = \frac{f}{a_0 \frac{F_u}{\dot{m}_0 a_0}} = \frac{0.0362}{340 \cdot 3.45} = 3.09 \cdot 10^{-5} \cdot \frac{kg}{Ns} = 0.0309 \cdot \frac{kg}{kNs}$$

Parametri corretti

$$\delta_i = \frac{p_{ti}}{p_{ref}} \qquad \theta_i = \frac{T_{ti}}{T_{ref}} \qquad \Theta = \frac{T_{t4}}{T_{t2}}$$

$$\dot{m}_{ci} = \frac{\dot{m}_i\sqrt{\theta_i}}{\delta_i} \qquad N_{c2} = N_2/\sqrt{\theta_2} \qquad N_{c4} = N_4/\sqrt{\theta_4}$$

$$\delta_2 = \frac{p_{t2}}{p_{ref}} = \frac{96.2}{101.3} = 0.950 \qquad \theta_2 = \frac{T_{t2}}{T_{ref}} = \frac{288}{288.15} = 0.999 \qquad \Theta = \frac{T_{t4}}{T_{t2}} = \frac{1973}{288} = 6.85$$

$$\theta_4 = \frac{T_{t4}}{T_{ref}} = \frac{1973}{288.15} = 6.85$$

Si supponga che in condizioni di progetto si abbia:

$$M_z = 0.5, N = 7460 \cdot rpm, \dot{m}_2 = 77 \cdot \frac{kg}{s}$$

$$\dot{m}_{c2} = \frac{\dot{m}_2\sqrt{\theta_2}}{\delta_2} = \frac{77\sqrt{0.999}}{0.95} = 81.0 \cdot \frac{kg}{s} \qquad N_{c2} = N_2/\sqrt{\theta_2} = \frac{7460}{\sqrt{0.999}} = 7462 \cdot \mathrm{rpm}$$

$$N_{c4} = N_4/\sqrt{\theta_4} = \frac{7460}{\sqrt{6.85}} = 2850 \cdot \mathrm{rpm}$$

Off design

Le condizioni di fuori progetto sono:

$M_0 = 0.80,\ T_0 = -35C = -35 + 273 = 238 \cdot K,\ p_0 = 20 \cdot kPa, T_{t4} = 1500C = 1773 \cdot K.$

Si parte con

Effetto Ram

$$a_0 = \sqrt{\gamma R T_0} = \sqrt{1.4 \cdot 287 \cdot 238} = 309\frac{m}{s} \qquad V_0 = M_0 a_0 = 0.8 \cdot 309 = 247\frac{m}{s}$$

$$\tau_r = \psi_0 = 1 + \frac{\gamma - 1}{2} M_0^2 = 1 + .2 \cdot 0.8^2 = 1.128$$

$$T_{t0} = T_{t2} = T_0 \tau_r = 238 \cdot 1.128 = 268 \cdot K \qquad p_{t0} = p_0 \tau_r^{\frac{1}{k}} = 20 \cdot 1.128^{\frac{1}{0.286}} = 30.5 \cdot kPa$$

Diffusore

$$p_{t2} = p_{t0}\pi_d = 30.5 \cdot 0.95 = 29.0 \cdot kPa$$

Parametri corretti Off design

$$\delta_2 = \frac{p_{t2}}{p_{ref}} = \frac{29.0}{101.3} = 0.286 \qquad \theta_2 = \frac{T_{t2}}{T_{ref}} = \frac{268}{288.15} = 0.932 \qquad \Theta = \frac{T_{t4}}{T_{t2}} = \frac{1773}{268} = 6.62$$

$$\theta_4 = \frac{T_{t4}}{T_{ref}} = \frac{1773}{288.15} = 6.15$$

$$\tau_\lambda = \frac{c_{pt} T_{t4}}{c_p T_0} = \frac{1154 \cdot 1773}{1004 \cdot 238} = 8.58$$

Off design
(d) the compressor pressure ratio at cruise

$$\tau_c - 1 = \eta_m (1 + f) \frac{c_{p4}}{c_{p2}} \frac{T_{t4}}{T_{t2}} (1 - \tau_t)$$

$$\tau_c = 1 + 0.98(1. + f)\frac{1156}{1004} 6.62(1 - 0.757) = 1 + (1 + f)1.815 = 1 + 1.036 \cdot 1.815 = 2.88$$

$$f=\frac{\tau_\lambda-\tau_c\tau_r}{\frac{Q_R\eta_b}{(c_pT_0)}-\tau_\lambda}=\frac{8.58-\tau_c\cdot 1.128}{\frac{42.6\cdot 10^6\cdot 0.98}{1004\cdot 238}-8.58}=\frac{8.58-\tau_c\cdot 1.128}{166.1}=5.16\cdot 10^{-2}-6.79\cdot 10^{-3}\tau_c$$
$$=0.0320$$

$$\tau_c=1+(1+f)1.80=1+1.032\cdot 1.815=2.87$$

$$f=\frac{\tau_\lambda-\tau_c\tau_r}{Q_R\eta_b/(c_pT_0)-\tau_\lambda}=5.166\cdot 10^{-2}-6.791\cdot 10^{-3}\tau_c=0.0321$$

$$\tau_c=2.87 \qquad f=0.0321$$

$$\frac{\dot m_{c2,OD}}{\dot m_{c2,D}}\approx\frac{\pi_{c,OD}}{\pi_{c,D}}\sqrt{\frac{\Theta_D}{\Theta_{OD}}} \qquad \dot m_{c2,OD}=\dot m_{c2,D}\frac{\pi_{c,OD}}{\pi_{c,D}}\sqrt{\frac{\Theta_D}{\Theta_{OD}}}=81.0\frac{27.6}{30}\sqrt{\frac{6.85}{6.62}}=75.8\cdot\frac{kg}{s}$$

$$\dot m_2=\frac{\dot m_{c2}\delta_2}{\sqrt{\theta_2}}=75.8\cdot\frac{0.286}{\sqrt{0.930}}=22.5\cdot\frac{kg}{s}$$

$$N_{c4,OD}\approx N_{c4,D}\approx 2850\cdot \text{rpm} \quad\rightarrow\quad N_4=\sqrt{\theta_4}N_{c4}=\sqrt{6.15}\cdot 2850=7070\cdot rpm$$

$$\frac{\dot m_{c2,OD}}{\dot m_{c2,D}}=\frac{Mz_{2,OD}\psi_{z2,OD}^{-K_2}}{M_{z2,D}\psi_{z2,D}^{-K_2}} \quad\rightarrow\quad Mz_{2,OD}\psi_{z2,OD}^{-K_2}=\frac{\dot m_{c2,OD}}{\dot m_{c2,D}}\,M_{z2,D}\psi_{z2,D}^{-K_2}$$

$$K_2=\frac{\gamma+1}{2(\gamma-1)}=\frac{2.4}{0.8}=3$$

$$Mz_{2,OD}\psi_{z2,OD}^{-K_2}=\frac{75.8}{81}0.5(1+0.2\cdot 0.5^2)^{-3}=0.404$$

$$Mz_{2,OD}=0.5 \quad Mz_{2,OD}\psi_{z2,OD}^{-K_2}=0.5(1+0.2\cdot 0.5^2)^{-3}=0.432 \quad e=0.432-0.404=0.0279$$

$$Mz_{2,OD}=0.4 \quad Mz_{2,OD}\psi_{z2,OD}^{-K_2}=0.4(1+0.2\cdot 0.4^2)^{-3}=0.364 \quad e=0.364-0.404=-0.0401$$

$$Mz_{2,OD}=\frac{0.5\cdot(-0.0401)-0.4\cdot 0.0279}{(-0.0401)-0.0279}=0.459$$

$$Mz_{2,OD}=0.457$$

Compressore

$$\pi_c=\tau_c^{\frac{e_c}{k}}=2.87^{\frac{0.9}{0.286}}=27.6 \qquad p_{t3}=p_{t2}\pi_c=29\cdot 27.6=800\cdot kPa$$

$$T_{t3}=T_{t2}\tau_c=268\cdot 2.87=769\cdot K$$

Camera di Combustione

$$f=0.0321 \qquad \tau_b=\frac{\tau_\lambda}{\tau_c\tau_r}=\frac{8.58}{2.87\cdot 1.128}=2.65$$

$$p_{t4}=p_{t3}\pi_b=799\cdot 0.95=760\cdot kPa$$

Turbina

$$\tau_t=0.757 \qquad \pi_t=\tau_t^{\frac{1}{k_te_t}}=0.757^{\frac{1}{0.248\cdot 0.85}}=0.267$$

$$p_{t5}=p_{t4}\pi_t=759\cdot 0.267=203\cdot kPa \qquad T_{t5}=T_{t4}\tau_t=1773\cdot 0.757=1342\cdot K$$

Ugello

$$p_{t9} = p_{t5}\pi_n = 203 \cdot 0.90 = 182.7 \cdot kPa \qquad T_{t9} = T_{t5} = 1342 \cdot K$$

$$\frac{p_{t9}}{p_9} = \pi_n\pi_t\pi_b\pi_c\pi_d\pi_r\frac{p_0}{p_9} = \frac{p_{t9}}{p_0}\frac{p_0}{p_9} = \frac{182.7}{20.0} \cdot 1 = 9.14$$

$$M_9 = \sqrt{\frac{2}{\gamma_t - 1}\left[\left(\frac{p_{t9}}{p_9}\right)^{k_t} - 1\right]} = \sqrt{\frac{2}{0.33}(9.14^{0.248} - 1)} = 2.10$$

$$\frac{T_{t9}}{T_9} = \psi_9 = 1 + \frac{\gamma_t - 1}{2}M_9^2 = \left(\frac{p_{t9}}{p_9}\right)^{k_t} = 9.14^{0.248} = 1.731$$

$$T_9 = \frac{T_9}{T_{t9}}T_{t9} = \frac{1342}{1.731} = 775 \cdot K$$

$$a_9 = \sqrt{\gamma_t R_9 T_9} = \sqrt{1.33 \cdot 287 \cdot 775} = 544 \cdot \frac{m}{s}$$

$$V_9 = M_9 a_9 = 2.10 \cdot 544 = 1142 \cdot \frac{m}{s} \qquad V_{9.e} = V_9\left(1 + \frac{1 - \frac{p_0}{p_9}}{\gamma_9 M_9^2}\right) = V_9 \qquad \frac{V_9}{a_0} = \frac{1142}{309} = 3.70$$

Spinta e rendimenti
(e) the exhaust velocity V_9 (in m/s) at cruise
(f) the thermal efficiency η_{th} at cruise
(g) the propulsive efficiency η_p at cruise
(h) the thrust-specific fuel consumption at cruise

$$\frac{F_u}{\dot{m}_0 a_0} = (1 + f)\frac{V_{9.\mathrm{e}}}{a_0} - M_0 = 1.032 \cdot 3.70 - 0.8 = 3.02$$

$$\eta_{\mathrm{th}} = \frac{\Delta K\dot{E}}{\mathcal{P}_t} = \frac{a_0^2[(1 + f)(V_9/a_0)^2 - M_0^2]}{2fQ_R} = \frac{B}{2fQ_R}$$

$$\eta_p = \frac{F_i V_0}{\Delta K\dot{E}} \approx \frac{2F_u V_0/\dot{m}_0}{a_0^2[(1 + f)(V_9/a_0)^2 - M_0^2]} = \frac{2F_u V_0/\dot{m}_0}{B}$$

$$B = a_0^2[(1 + f)(V_9/a_0)^2 - M_0^2] = 1.032 \cdot 1142^2 - 247^2 = 1.285 \cdot 10^6 \cdot \frac{m^2}{s^2}$$

$$\eta_{\mathrm{th}} = \frac{B}{2fQ_R} = \frac{1.285 \cdot 10^6}{2 \cdot 0.0322 \cdot 42.600 \cdot 10^6} = 0.468$$

$$\eta_p = \frac{2a_0 V_0 \frac{F_u}{\dot{m}_0 a_0}}{B} = \frac{2 \cdot 309 \cdot 247 \cdot 3.02}{1.285 \cdot 10^6} = 0.59 \qquad \eta_o = \eta_p\eta_{\mathrm{th}} = 0.468 \cdot 0.359 = 0.17$$

$$TSFC = \frac{\dot{m}_f}{F_u} = \frac{f}{F_u/\dot{m}_0} = \frac{f}{a_0\frac{F_u}{\dot{m}_0 a_0}} = \frac{0.0322}{309 \cdot 3.02} = 3.45 \cdot 10^{-5} \cdot \frac{kg}{Ns} = 0.0345 \cdot \frac{kg}{kNs}$$

	2	3	4	5	9		
	diff	comp	CC	Tur	No		
c_p	1004			1156	J/kgs		
γ	1.4			1.33			
π	0.95		0.95		0.9		
η, $e_{c,t}$		0.9	0.98	0.85			
Tt				**1773**			
M0	**0.8**		**po/p9**	1	QR	42600	kJ/kgK
T0	**238**	**K**	**p0**	**20,000**	Pa	η_m	0.98
k	0.286	K	3	0.248			
R	287			287			
a0	309	m/s	V0	247	m/s		
$\tau_r = T_{t0} = 0$ /	1.128						
Section	c	t	λ			c	t
τ	2.87	0.757	8.58		π	27.6	0.268
Section	0	2	3	4	5	7	9
Tt	268	268	770	1773	1343		1343
pt	30,487	28,962	798,965	759,017	203,159		182,843
Section	f	Pt9/p9	M9	Tt9/T9	T9/T0	a9	
3	0.032166	9.14	2.11	1.732	3.26	544	m/s
		V9		V9/a0	V9/a0 eff	F/ma0	Num
		1145	m/s	3.70	3.70	3.02	1292645
Section	t	th	p	0	TSFC*1e3		
h	0.870	0.472	0.358	0.1687	0.0344		
δ_2	θ_2	Θ	mc2	Nc2	θ4	Nc4	N
0.286	0.932	6.60	75.9	7326	6.15	2851	7072
Calcolo τ_c iterativo con f variabile							
τ_c	2.87	2.87	2.87	2.87			
f	0.0321	0.0322	0.0322	0.0322			
Rhs	Mz2	0.5	0.4	0.459685	0.457566	0.457455	
0.40451	e	0.027409	-0.04058	0.001493	7.46E-05	-1.6E-07	

Fa 11.19

A high bypass ratio separate-flow turbofan engine with convergent nozzles has the following design point parameters (at takeoff, standard sea-level conditions):

$M_0 = 0, p_0 = 101.33\ kPa, T_0 = 288\ K, \gamma_c = 1.4, c_{pc} = 1004 J/kgK,$

$\pi_d = 0.98, \dot{m}_{c2} = 600 kg/s\ , M_{z2} = 0.5, \pi_f = 1.8, e_f = 0.90, \alpha = 5.0, \pi_{cH} = 14, e_{cH} = 0.90,$

$T_{t4} = 1873K, Q_R = 42.8 MJ/kgK\ , \pi_b = 0.95, \eta_b = 0.99,$

$e_{tH} = 0.85, \eta_{mH} = 0.995, e_{tL} = 0.89, \eta_{mL} = \ 0.995, \gamma_t = \ 1.33, c_{pt} = \ 1464 kJ/kgK$

$\pi_n = 0.98, \pi_{nf} = 0.98, p_8 = \ p_{18} = \ p_0, M_8 = \ M_{18} = \ 1$

The off-design point is the cruise condition described by:

$M_0 = 0.84, p_0 = \ 20\ kPa, T_0 = \ 253\ K, T_{t4} = 1573K$

Calculate C_1, C_2 and C_3 parameters at the design point. Then use Equation 11.67 to solve, i.e., to iterate for, $\tau_{cH_{OD}}$. Then, calculate $\tau_{f_{OD}}$ and α_{OD}. Use polytropic efficiencies to get π_f and π_{cH} at off-design.

	2	3	4	4.5	5	9	13	19
	diff	compH	CC	TH	TL	No	Fan	No FAn
c_p	1004			1146.4				J/kgK
γ	1.4			1.33				
π	0.98	14	0.95			0.98	1.8	0.98
$\eta, e_{c,t}$		0.9	0.99	0.85	0.89		0.9	
Tt			1873					
k	0.286			0.248			Mz	0.5
R	287			284	kJ/kgK		N	7460
M0	0	alpha	5	QR	42800	kJ/kgK	m2	462
p0	101,330	Pa		η_{mH}	0.995		Tref	288
T0	288	K		η_{mL}	0.995		Pref	101,300
a0	340	m/s	V0	0	m/s			
$\tau_r = T_{t0} = 0 / T$	1							
Section	f	c_H	c	t_H	t_L	t	λ	b
τ	1.205	2.31	2.79	0.793	0.797	0.632	7.43	2.6659911
			25.2	0.333	0.357	0.1190		
Section	0	2	13/2.5	3	4	5	9	19
Tt	288	288	347	802	1873	1183	1183	347
pt	101,330	99,303	178,746	2,502,446	2,377,323	282,898	277,240	175,171
Core		f	0.0334	M9 Eff	1			
Pt9/p9	M9	M9 Eff	Tt9/T9	P9	po/p9		1.284	
2.74	1.311	1.311	1.284	101,330	1.000			
T9	a9	V9	V9/a0	V9/a0 eff	F/ma0	F/maira0		Fc/Ft
922	591	774	2.28	2.28	2.35	0.392		0.335
Fan								
Pt19/p19	M19	M19 Eff	Tt19/T19	P19	po/p19			
1.729	0.920	0.920	1.169	101,330	1			
T19	a19	V19	V19/a0	V19/a0 eff	F/ma0	F/maira0		FF/Ft
297	345	318	0.934	0.934	4.67	0.778		0.665
Num	Section	t	th	p	0	TSFC*1e3	F/ma0	F/maira0
1,124,127	η	0.88217	0.393684	0	0	0.0140	7.02295	1.1704919
δ_2	θ_2	Θ	mc2	Nc2	θ4	Nc4		
0.980	1.000	6.50	471.3	7460	6.50	2925		
C1	C2	C3						
0.213	0.1658	28.20						

$$k = \frac{\gamma - 1}{\gamma} = \frac{1.4 - 1}{1.4} = 0.286 \qquad k_t = \frac{\gamma_t - 1}{\gamma_t} = \frac{1.33 - 1}{1.33} = 0.248$$

$$R = kc_p = 1004 \cdot 0.286 = 287 \cdot \frac{J}{kg \cdot K} \qquad R_t = k_t c_{pt} = 1146 \cdot 0.248 \frac{J}{kg \cdot K} = 284 \cdot \frac{J}{kg \cdot K}$$

Effetto Ram

$$a_0 = \sqrt{\gamma R T_0} = \sqrt{1.4 \cdot 287 \cdot 288} = 340 \cdot \frac{m}{s}$$

$$V_0 = M_0 a_0 = 0 \cdot 340 = 0 \ \frac{m}{s} \qquad \tau_r = \psi_0 = 1 + \frac{\gamma - 1}{2} M_0^2 = 1$$

$$T_{t0} = T_0\tau_r = 288 \cdot 1 = 288 \cdot K \qquad p_{t0} = p_0\tau_r^{\frac{1}{k}} = 1.013 \cdot 10^5 \cdot 1^{\frac{1}{0.286}} = 101.3 \cdot kPa$$

Diffusore

$$p_{t2} = p_{t0}\pi_d = 101.3 \cdot 0.98 = 99.3 \cdot kPa$$

Fan

$$\tau_f = \pi_f^{\frac{k}{e_f}} = 1.8^{\frac{0.286}{0.90}} = 1.205 \qquad p_{t13} = p_{t2}\pi_f = 99.3 \cdot 1.80 = 178.7 \cdot kPa$$

$$T_{t13} = T_{t2}\tau_f = 288 \cdot 1.205 = 347 \cdot K$$

Compressore (HP)

$$\tau_{cH} = \pi_{cH}^{\frac{k}{e_{cH}}} = 14.0^{\frac{0.286}{0.90}} = 2.31$$

$$\pi_c = \pi_f\pi_{cH} = 1.8 \cdot 14.0 = 25.2 \qquad \tau_c = \tau_f\tau_{cH} = 1.205 \cdot 2.31 = 2.78$$

$$p_{t3} = p_{t2}\pi_c = 99.3 \cdot 25.2 = 2500 \cdot kPa$$
$$T_{t3} = T_{t2}\tau_c = 288 \cdot 2.78 = 801 \cdot K$$

Camera di Combustione

$$\tau_\lambda = \frac{c_{pt}T_{t4}}{c_pT_0} = \frac{1146 \cdot 1873}{1004 \cdot 288} = 7.42$$

$$f = \frac{\tau_\lambda - \tau_c\tau_r}{Q_R\eta_b/(c_pT_0) - \tau_\lambda} = \frac{7.42 - 2.78 \cdot 1}{\frac{42.8 \cdot 10^6 \cdot 0.99}{1004 \cdot 288} - 7.42} = 0.0334$$

$$\tau_b = \frac{\tau_\lambda}{\tau_c\tau_r} = \frac{7.42}{2.78 \cdot 1} = 2.67$$

$$p_{t4} = p_{t3}\pi_b = 2500 \cdot 0.95 = 2380 \cdot kPa$$

Turbina

$$\tau_{tH} = 1 - \frac{\tau_f(\tau_{cH} - 1)}{\eta_{mH}(1+f)\frac{\tau_\lambda}{\tau_r}} = 1 - \frac{1.205 \cdot 1.31}{0.995 \cdot 1.033\frac{7.42}{1}} = 0.793$$

$$\tau_{tL} = 1 - \frac{(1+\alpha)(\tau_f - 1)}{\eta_{mL}(1+f)\frac{\tau_{tH}\tau_\lambda}{\tau_r}} = 1 - \frac{6 \cdot 0.205}{0.995 \cdot 1.033\frac{0.793 \cdot 7.42}{1}} = 0.797$$

$$\pi_{tH} = \tau_{tH}^{\frac{1}{k_te_{tH}}} = 0.793^{\frac{1}{0.248 \cdot 0.85}} = 0.333 \qquad \pi_{tL} = \tau_{tL}^{\frac{1}{k_te_{tL}}} = 0.797^{\frac{1}{0.248 \cdot 0.89}} = 0.358$$

$$\tau_t = \tau_{tH}\tau_{tL} = 0.797 \cdot 0.793 = 0.632 \qquad \pi_t = \pi_{tH}\pi_{tL} = 0.333 \cdot 0.358 = 0.1192$$

$$p_{t5} = p_{t4}\pi_t = 2380 \cdot 0.1192 = 284 \cdot kPa \qquad T_{t5} = T_{t4}\tau_t = 1873 \cdot 0.632 = 1183 \cdot K$$

Ugello

$$p_{t9} = p_{t5}\pi_n = 284 \cdot 0.98 = 278 \cdot kPa \qquad T_{t9} = T_{t5} = 1183 \cdot K$$

$$\frac{p_{t9}}{p_9} = \pi_n\pi_t\pi_b\pi_c\pi_d\pi_r\frac{p_0}{p_9} = \frac{p_{t9}}{p_0}\frac{p_0}{p_9} = \frac{278}{101.3} \cdot 1 = 2.74$$

$$M_9 = \sqrt{\frac{2}{\gamma_t - 1}\left[\left(\frac{p_{t9}}{p_9}\right)^{k_t} - 1\right]} = \sqrt{\frac{2}{0.33}(2.74^{0.248} - 1)} = 1.312$$

Supersonico, si dovrebbe correggere la spinta

$$\frac{T_{t9}}{T_9} = \psi_9 = 1 + \frac{\gamma_t - 1}{2}M_9^2 = \left(\frac{p_{t9}}{p_9}\right)^{k_t} = 2.74^{0.248} = 1.284 \qquad T_9 = \frac{T_9}{T_{t9}}T_{t9} = \frac{1183}{1.284} = 921 \cdot K$$

$$a_9 = \sqrt{\gamma_t R_9 T_9} = \sqrt{1.33 \cdot 284 \cdot 921} = 590 \cdot \frac{m}{s} \qquad V_9 = M_9 a_9 = 1.312 \cdot 590 = 774 \cdot \frac{m}{s}$$

$$V_{9.e} = V_9\left(1 + \frac{1 - \frac{p_0}{p_9}}{\gamma_9 M_9^2}\right) = V_9 \qquad \frac{V_9}{a_0} = \frac{774}{340} = 2.28$$

Ugello Fan

$$p_{t19} = p_{t13}\pi_{nf} = 178.7 \cdot 0.98 = 175.1 \cdot kPa \qquad T_{t19} = T_{t13} = 347 \cdot K$$

$$\frac{p_{t19}}{p_{19}} = \frac{p_{t19}}{p_0}\frac{p_0}{p_{19}} = \frac{175.1}{101.3} \cdot 1 = 1.728$$

$$M_{19} = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{p_{t19}}{p_{19}}\right)^{k} - 1\right]} = \sqrt{\frac{2}{0.40}(1.729^{0.286} - 1)} = 0.920$$

$$\frac{T_{t19}}{T_{19}} = \psi_{19} = 1 + \frac{\gamma - 1}{2}M_{19}^2 = \left(\frac{p_{t19}}{p_{19}}\right)^{k} = 1.729^{0.286} = 1.169$$

$$T_{19} = \frac{T_{19}}{T_{t19}}T_{t19} = \frac{347}{1.169} = 297 \cdot K \qquad a_{19} = \sqrt{\gamma R T_{19}} = \sqrt{1.40 \cdot 287 \cdot 297} = 345\frac{m}{s}$$

$$V_{19} = M_{19}a_{19} = 0.920 \cdot 345 = 317 \cdot \frac{m}{s} \quad V_{19.e} = V_{19}\left(1 + \frac{1 - \frac{p_0}{p_{19}}}{\gamma M_{19}^2}\right) = V_{19} \quad \frac{V_{19}}{a_0} = \frac{317}{340} = 0.932$$

Spinta e rendimenti

$$\frac{F_{u.core}}{\dot{m}_{air}a_0} = \frac{(1+f)}{1+\alpha}\frac{V_{9.e}}{a_0} - \frac{M_0}{1+\alpha} = \frac{1.033}{6.0}2.28 - 0 = 0.393$$

$$\frac{F_{u.Fan}}{\dot{m}_{air}a_0} = \frac{\alpha}{1+\alpha}\frac{V_{19.e}}{a_0} - \frac{\alpha M_0}{1+\alpha} = \frac{5.0}{6.0}0.932 - 0 = 0.777$$

$$\frac{F_u}{\dot{m}_{air}a_0} = \frac{F_{u.core}}{\dot{m}_{air}a_0} + \frac{F_{u.Fan}}{\dot{m}_{air}a_0} = 0.393 + 0.777 = 1.170$$

$$\frac{F_{u.core}}{F_u} = \frac{0.393}{1.170} = 33.6\% \qquad \frac{F_{u.Fan}}{F_u} = \frac{0.777}{1.170} = 66.4\%$$

$$TSFC = \frac{f}{F_u/\dot{m}_0} = \frac{f}{(1+f)V_{9.e} + \alpha V_{19.e} - V_0} = \frac{0.0334}{1.033 \cdot 774 + 5.0 \cdot 317 - 0} = 14.01 \cdot 10^{-6}$$

$$N = (1+f)V_{9.e}^2 + \alpha V_{19.e}^2 - (1+\alpha)V_0^2 = 1.033 \cdot 774^2 + 5.0 \cdot 318^2 = 1.122 \cdot 10^6$$

$$\eta_{th} = \frac{\Delta K\dot{E}}{\mathcal{P}_t} = \frac{(1+f)V_{9.e}^2 + \alpha V_{19.e}^2 - (1+\alpha)V_0^2}{2fQ_R} = \frac{N}{2fQ_R} = \frac{1.122 \cdot 10^6}{2 \cdot 0.0334 \cdot 42.8 \cdot 10^6} = 39.2\%$$

$$\eta_p = \frac{2V_0\{[(1+f)V_{9.e}] + \alpha V_{19.e} - (1+\alpha)V_0\}}{(1+f)V_{9.e}^2 + \alpha V_{19.e}^2 - (1+\alpha)V_0^2} = \frac{2V_0[(1+f)V_9 + \alpha V_{19} - (1+\alpha)V_0]}{N} = 0$$

Parametri corretti

$$\delta_i = \frac{p_{ti}}{p_{ref}} \quad \theta_i = \frac{T_{ti}}{T_{ref}} \quad \Theta = \frac{T_{t4}}{T_{t2}} \quad \dot{m}_{ci} = \frac{\dot{m}_i\sqrt{\theta_i}}{\delta_i} \quad N_{c2} = N_2/\sqrt{\theta_2} \quad N_{c4} = N_4/\sqrt{\theta_4}$$

$$\delta_2 = \frac{p_{t2}}{p_{ref}} = \frac{99.3}{101.3} = 0.980 \quad \theta_2 = \frac{T_{t2}}{T_{ref}} = \frac{288}{288} = 1 \quad \Theta = \frac{T_{t4}}{T_{t2}} = \frac{1873}{288} = 6.50$$

Off design

Le condizione di fuori progetto sono:

$M_0 = 0.84$, $p_0 = 20\ kPa$, $T_0 = 253\ K$, $T_{t4} = 1573K$

Effetto Ram

$$a_0 = \sqrt{\gamma R T_0} = \sqrt{1.4 \cdot 287 \cdot 253} = 319 \cdot \frac{m}{s} \qquad V_0 = M_0 a_0 = 0.84 \cdot 319 = 268\ \frac{m}{s}$$

$$\tau_r = \psi_0 = 1 + \frac{\gamma - 1}{2} M_0^2 = 1 + 0.2 \cdot 0.84^2 = 1.141$$

$$T_{t0} = T_0 \tau_r = 253 \cdot 1.141 = 289 \cdot K \qquad p_{t0} = p_0 \tau_r^{\frac{1}{k}} = 20 \cdot 1.141^{\frac{1}{0.286}} = 31.7 \cdot kPa$$

$$\tau_\lambda = \frac{c_{pt} T_{t4}}{c_p T_0} = \frac{1146 \cdot 1573}{1004 \cdot 253} = 7.10$$

Off design

Si parte con

$$C_1 = \frac{\tau_r \tau_f}{\tau_\lambda}(\tau_{cH} - 1) = \frac{1 \cdot 1.205}{7.43} 1.31 = 0.213$$

$$C_2 = \frac{\tau_r}{\tau_\lambda}(1+\alpha)(\tau_f - 1) = \frac{1}{7.42} 6 \cdot 0.205 = 0.1658$$

$$C_3 = \alpha \pi_{cH} \sqrt{\frac{\tau_r \tau_f}{\tau_\lambda}} = 5 \cdot 14 \sqrt{\frac{1 \cdot 1.205}{7.42}} = 28.2$$

$$\left(1 + \frac{C_3}{\tau_{cH}^{\frac{e_{cH}}{k}}} \sqrt{\frac{\tau_{cH} - 1}{C_1}}\right)\left(\frac{C_1}{(\tau_{cH} - 1)} - \frac{\tau_r}{\tau_\lambda}\right) = C_2$$

$$\left(1 + \frac{28.2}{\tau_{cH}^{\frac{0.9}{0.286}}} \sqrt{\frac{\tau_{cH} - 1}{0.213}}\right)\left(\frac{0.213}{(\tau_{cH} - 1)} - \frac{1.141}{7.10}\right) = 0.1658$$

$$\left(1 + \frac{61.1}{\tau_{cH}^{3.15}} \sqrt{\tau_{cH} - 1}\right)\left(\frac{0.213}{(\tau_{cH} - 1)} - 0.1607\right) = 0.1658$$

Si itera

$$\tau_{cH} = 2.31 \ \rightarrow \quad e = \left(1 + \frac{61.1}{2.31^{3.15}}\sqrt{1.31}\right)\left(\frac{0.213}{1.31} - 0.1607\right) - 0.1658 = -0.1544$$

$$\tau_{cH} = 2.20 \ \rightarrow \quad e = \left(1 + \frac{61.1}{2.20^{3.15}}\sqrt{1.20}\right)\left(\frac{0.213}{1.20} - 0.1607\right) - 0.1658 = -0.0552$$

$$\tau_{cH} = \frac{2.20 \cdot (-.1544) - 2.31(-.0552)}{-.1544 + .0552} = 2.14$$

$$\tau_{cH} = 2.14 \ \rightarrow \quad e = \left(1 + \frac{61.1}{2.14^{3.15}}\sqrt{1.14}\right)\left(\frac{0.213}{1.14} - 0.1607\right) - 0.1658 = 0.01559$$

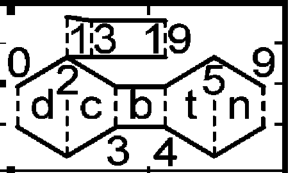
$$\tau_{cH} = \frac{2.20 \cdot .0156 - 2.14(-.0552)}{.0156 + .0552} = 2.15$$

$$\pi_{cH} = \tau_{cH}^{\frac{e_{cH}}{k}} = 2.15^{3.15} = 11.12$$

$$C_1 = \frac{\tau_r \tau_f}{\tau_\lambda}(\tau_{cH} - 1) \ \rightarrow \quad \tau_f = \frac{C_1}{\frac{\tau_r}{\tau_\lambda}(\tau_{cH} - 1)} = \frac{0.213}{.1607 \cdot 1.15} = 1.153$$

$$\pi_f = \tau_f^{\frac{e_f}{k}} = 1.153^{3.15} = 1.565$$

$$C_3 = \alpha \pi_{cH} \sqrt{\frac{\tau_r \tau_f}{\tau_\lambda}} \ \rightarrow \quad \alpha = \frac{C_3}{\pi_{cH}\sqrt{\frac{\tau_r \tau_f}{\tau_\lambda}}} = \frac{28.2}{11.12\sqrt{.1607 \cdot 1.153}} = 5.89$$

	2	3	4	4.5	5	9	13	19
	diff	comp	CC	TH	TL	No	Fan	No FAn
c_p	1004			1146.4				J/kgK
γ	1.4			1.33				
π	0.98	11.16729	0.95			0.98	1.553222	0.98
η, $e_{c,t}$		0.9	0.99	0.85	0.89		0.9	
Tt			**1573**					
k	0.285714			0.24812				
R	286.8571			284.4451	kJ/kgK			
M0	**0.84**	alpha	5.873285	QR	42800	kJ/kgK		
p0	**20,000**	Pa		η_m	0.995			
T0	**253**	K			9.95E-01			
a0	318.7551	m/s	V0	267.7543	m/s			
$\tau_r = T_{t0} = 0 / T0$	1.14112							
Section	f	c_H	c	t_H	t_L	t	λ	b
τ	1.150	2.15	2.47	0.792	0.795	0.629	7.10	2.514714
			17.3	0.330	0.354	0.1170		
Section	0	2	13/2.5	3	4	5	9	19
Tt	289	289	332	714	1573	990	990	332
pt	31,746	31,111	48,322	539,631	512,649	59,958	58,759	47,356
Core		f	0.0268	M9 Eff	1			
Pt9/p9	M9	M9 Eff	Tt9/T9	P9	po/p9		1.307	
2.94	1.363	1.363	1.307	20,000	1.000			
T9	a9	V9	V9/a0	V9/a0 eff	F/ma0	F/maira0		Fc/Ft
758	535	730	2.29	2.29	1.51	0.220		0.420
Fan								
Pt19/p19	M19	M19 Eff	Tt19/T19	P19	po/p19			
2.368	1.182	1.000	1.200	25,017	0.799446			
T19	a19	V19	V19/a0	V19/a0 eff	F/ma0	F/maira0		FF/Ft
277	333	333	1.046	1.196	2.09	0.304		0.580
Num	Section	t	th	p	0	TSFC*1e3	F/ma0	F/maira0
907,144	η	0.882417	0.39581	0.67727	0.26807	0.0233	3.599268	0.52366
δ_2	θ_2	Θ	mc2	Nc2	θ4	Nc4		
0.003	0.007	786.50		0	0.01	0		
τ_{CH}	2.31	2.2	2.137259	2.152326	2.15123	2.151205	2.151205	2.151205
e	-0.15645	-0.05641	0.017827	-0.0014	-3.1E-05	5.73E-08	-2.3E-12	
τ_{CH}	π_{CH}	τ_f	π_f	α				
2.151205	11.16729	1.15003	1.553222	5.873285				