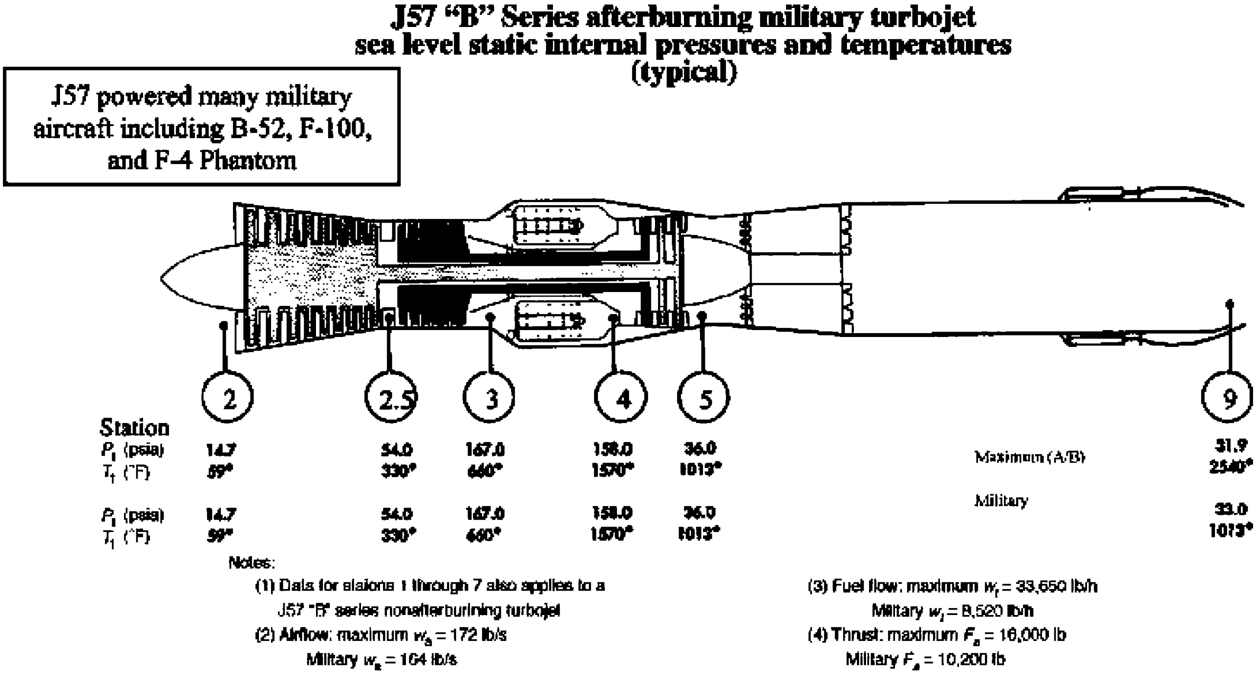
Farokhi problem 3.1

The total pressures and temperatures of the gas in an afterburning turbojet engine are shown (J57 “B” from Pratt & Whitney, 1988). The mass flow rates for the air and fuel are also indicated at two engine settings, the Maximum Power and the Military Power. Use the numbers specified in this engine to calculate:



Dati				Si		cp(J/kgK)	1,005	
QR(BTU/lbm	18,600	QR(ft2/s2)	4.65E+08	QR(kJ/kg)	43300	cp(BTU/lbm	0.24	
	AB	MIL		AB	MIL	SI	AB	MIL
m2(lbm/s)	172	164	m2(lbm/s)	172	164	m2(kg/s)	78.00	74.40
mf(lbm/hr)	33,650	8,520	mf(lbm/s)	9.35	2.37	mf(kg/s)	4.240	1.075
Fn(lbf)	16,000	10,200	Fn(lb ft/s2)	515,000	328,000	Fn(N)	71,200	45,400
	AB		MIL		SI	AB	MIL	
Stazioni	pt(psi)	T(F)	pt(psi)	T(F)	pt(kPa)	T(K)	pt(kPa)	T(K)
2	14.7	59	14.7	59	101.3	288.0	101.3	288.0
2.5	54.0	330	54.0	330	372.0	439.0	372.0	439.0
3	167	660	167	660	1,151.0	622.0	1,151.0	622.0
4	158	1,570	158	1,570	1,089.0	1,128.0	1,089.0	1,128.0
5	36.0	1,013	36.0	1,013	248.0	818.0	248.0	818.0
9	32	2,540	33	1,013	220	1,666.0	227	818.0

a) the fuel-to-air ratio f in the primary burner and the afterburner, at both power settings

$$f = \dot{m}_f/\dot{m}_2 = 4.24/78.0 = 0.0544 \qquad f_{Mil} = \dot{m}_f/\dot{m}_2 = 1.075/74.4 = 0.0145$$

b) the low- and high-pressure spool compressor pressure ratios and the turbine pressure ratio (note that these remain constant with the two power settings)

$$\pi_{cLp} = p_{t2.5}/p_{t2} = 372/101.3 = 3.67$$

$$\pi_{cHp} = p_{t3}/p_{t2.5} = 1.151/372 = 3.09 \quad \pi_t = p_{t5}/p_{t4} = 248/1089 = 0.228$$

c) the exhaust velocity for both power settings by assuming the specified thrust is based on the nozzle gross thrust (because of sea level static) and neglecting any pressure thrust at the nozzle exit

$$F = \dot{m}_9 V_9 \rightarrow V_9 = \frac{F}{\dot{m}_9} = \frac{71.200}{4.24 + 78.0} = 866 \cdot \frac{m}{s}$$

$$F_{Mil} = \dot{m}_9 V_9 \rightarrow V_9 = \frac{F}{\dot{m}_9} = \frac{45.400}{1.075 + 74.40} = 602 \cdot \frac{m}{s}$$

d) the thermal efficiency of this engine for both power settings (at the sea level static operation), assuming the fuel heating value is 18,600 BTU/lbm and c_p=0.24BTU/lbmR. Explain the lower thermal efficiency of the Maximum power setting

$$\eta_{\text{th}} = \frac{(1+f)V_9^2 - V_0^2}{2fQ_R} = \frac{1.054 \cdot 866^2}{2 \cdot 0.0544 \cdot 43.300 \cdot 10^6} = 16.8\%$$

$$\eta_{\text{th}_{Mil}} = \frac{(1+f)V_9^2 - V_0^2}{2fQ_R} = \frac{1.014 \cdot 602^2}{2 \cdot 0.0145 \cdot 43.300 \cdot 10^6} = 29.4\%$$

e) the thrust specific fuel consumption in lbm/h/lbf in both power settings

$$TSFC = \frac{\dot{m}_f}{F_u} = \frac{4.24}{71.200} = 59.6 \cdot \frac{mg}{N} \qquad TSFC_{Mil} = \frac{\dot{m}_f}{F_u} = \frac{1.075}{45.400} = 23.7 \cdot \frac{mg}{N}$$

f) the Carnot efficiency of a corresponding engine, i.e., operating at the same temperature limits, in both settings

$$\eta_{Carnot} = 1 - \frac{T_{min}}{T_{max}} = 1 - \frac{288}{1666} = 82.7\% \qquad \eta_{Carnot_{Mil}} = 1 - \frac{T_{min}}{T_{max}} = 1 - \frac{288}{1128} = 74.5\%$$

g) the comparison of percent thrust increase to percent fuel flow rate increase when we turn the afterburner on

$$\%F = 100\frac{F_{AB} - F_{Mil}}{F_{Mil}} = 100\ \frac{71.200 - 45.400}{45.400} = 56.8\%$$

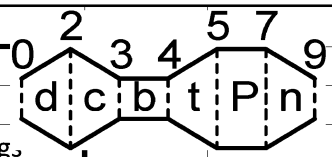
$$\%\dot{m}_f = 100\frac{\dot{m}_{fAB} - \dot{m}_{fMil}}{\dot{m}_{fMil}} = 100\ \frac{4.24 - 1.075}{1.075} = 295\%$$

h) why don't we get proportional thrust increase with fuel flow increase (when it is introduced in the afterburner), i.e., doubling the fuel flow in the engine (through afterburner use) does not double the thrust.

	AB	MIL	SI	AB	MIL	
f	0.0544	0.0145	f	0.0544	0.0145	
π_{cLP}	3.670	3.670	π_{cLP}	3.670	3.670	
π_{cHP}	3.090	3.090	π_{cHP}	3.090	3.090	
π_{c}	11.340	11.340		11.340	11.340	
π_{t}	0.228	0.228	π_{t}	0.228	0.228	No change in π_{t} for AB On /off
V9(ft/s)	2,840	1,972	V9(m/s)	866	602	
η_{th}	0.168	0.294	η_{th}	0.168	0.294	
TSFC(lbm/h	2.100	0.8350	TSFC(mg/s N)	59.600	23.700	
η_{carnot}	0.827	0.744	η_{carnot}	0.827	0.745	
%F	56.9		%F	56.8		
%mf	295.0		%mf	294.0		

TurboJet by Tom

Turbo Jet By TomLevel 4							
	2	3	4	5	7	9	
	diff	comp	CC	Tur	AB	No	
c_p	1004			1152		1243	J/kg
γ	1.4			1.33		1.3	
π	0.96	10	0.95		0.98	0.97	
$\eta, e_{c,t}$		0.9	0.99	0.9	0.99		
Tt				1750		2250	
M0	2		po/p9	1	QR	42800	kJ/kgK
T0	250	K	p0	101,300	Pa	η_m	0.99



$$k = \frac{\gamma - 1}{\gamma} = \frac{1.4 - 1}{1.4} = 0.286 \qquad k_t = \frac{1.33 - 1}{1.33} = 0.248 \qquad k_9 = \frac{1.3 - 1}{1.3} = 0.231$$

$$R = kc_p = 1004 \cdot 0.286 = 287 \frac{J}{kg \cdot K} \qquad R_t = k_t c_{pt} = 1152 \cdot 0.248 \frac{J}{kg \cdot K} = 286$$

$$R_9 = k_9 c_{p9} = 1243 \cdot 0.231 = 287 \frac{J}{kg \cdot K}$$

$$\pi_{AB.off} = 1$$

Effetto Ram

$$a_0 = \sqrt{\gamma R T_0} = \sqrt{1.4 \cdot 287 \cdot 250} = 317 \frac{m}{s} \qquad V_0 = M_0 a_0 = 2 \cdot 317 = 634 \frac{m}{s}$$

$$\tau_r = \psi_0 = 1 + \frac{\gamma - 1}{2} M_0^2 = 1.8$$

$$T_{t0} = T_0 \tau_r = 250 \cdot 1.8 = 450 \cdot K \qquad p_{t0} = p_0 \tau_r^{\frac{1}{k}} = 1.013 \cdot 10^5 \cdot 1.8^{\frac{1}{0.286}} = 7.91 \cdot 10^5 \cdot Pa$$

Diffusore

$$p_{t2} = p_{t0} \pi_d = 7.91 \cdot 10^5 \cdot 0.96 = 7.59 \cdot 10^5 Pa$$

Compressore

$$\tau_c = \pi_c^{\frac{k}{e_c}} = 10^{\frac{0.286}{0.9}} = 2.08 \qquad p_{t3} = p_{t2} \pi_c = 7.59 \cdot 10^5 \cdot 10 = 7.59 \cdot 10^6 \cdot Pa$$

$$T_{t3} = T_{t2} \tau_c = 450 \cdot 2.08 = 936 \cdot K$$

Camera di Combustione

$$\tau_\lambda = \frac{c_{pt} T_{t4}}{c_p T_0} = \frac{1152 \cdot 1750}{1004 \cdot 250} = 8.03$$

$$f = \frac{\tau_\lambda - \tau_c \tau_r}{Q_R \eta_b / (c_p T_0) - \tau_\lambda} = \frac{8.03 - 2.08 \cdot 1.8}{\frac{42.8 \cdot 10^6 \cdot 0.99}{1004 \cdot 250} - 8.03} = 0.0267$$

$$\tau_b = \frac{\tau_\lambda}{\tau_c \tau_r} = \frac{8.03}{2.08 \cdot 1.8} = 2.14$$

$$p_{t4} = p_{t3}\pi_b = 7.59 \cdot 10^6 \cdot 0.95 = 7.21 \cdot 10^6 \cdot Pa$$

Turbina

$$\tau_t = 1 - \frac{(\tau_c - 1)\tau_r}{\eta_m (1+f)\tau_\lambda} = 1 - \frac{1.08 \cdot 1.8}{0.99 \cdot 1.027 \cdot 8.03} = 0.762 \qquad \pi_t = \tau_t^{\frac{1}{k_t e_t}} = 0.763^{\frac{1}{0.248 \cdot 0.90}} = 0.296$$

$$p_{t5} = p_{t4}\pi_t = 7.21 \cdot 10^6 \cdot 0.296 = 2.13 \cdot 10^6 \cdot Pa$$

$$T_{t5} = T_{t4}\tau_t = 1750 \cdot 0.762 = 1334 \cdot K$$

Ugello

$$p_{t9} = p_{t5}\pi_n = 2.13 \cdot 10^6 \cdot 0.97 = 2.07 \cdot 10^6 \cdot Pa \qquad T_{t9} = T_{t5} = 1334 \cdot K$$

$$\frac{p_{t9}}{p_9} = \pi_n \pi_t \pi_b \pi_c \pi_d \pi_r \frac{p_0}{p_9} = \frac{p_{t9}}{p_0}\frac{p_0}{p_9} = \frac{20.7}{1.013} \cdot 1 = 20.4$$

$$M_9 = \sqrt{\frac{2}{\gamma_t - 1}\left[\left(\frac{p_{t9}}{p_9}\right)^{k_t} - 1\right]} = \sqrt{\frac{2}{0.33}(20.63^{0.248} - 1)} = 2.60$$

$$\frac{T_{t9}}{T_9} = \psi_9 = 1 + \frac{\gamma_t - 1}{2}M_9^2 = \left(\frac{p_{t9}}{p_9}\right)^{k_t} = 20.^{0.248} = 2.12$$

$$T_9 = \frac{T_9}{T_{t9}}T_{t9} = \frac{1335}{2.12} = 630 \cdot K \qquad a_9 = \sqrt{\gamma_t R_9 T_9} = \sqrt{1.33 \cdot 286 \cdot 630} = 490 \cdot \frac{m}{s}$$

$$V_9 = M_9 a_9 = 2.60 \cdot 490 = 1274 \cdot \frac{m}{s} \qquad V_{9.e} = V_9\left(1 + \frac{1 - \frac{p_0}{p_9}}{\gamma_9 M_9^2}\right) = V_9 \qquad \frac{V_9}{a_0} = \frac{1274}{317} = 4.02$$

Spinta e rendimenti

$$\frac{F_u}{\dot{m}_0 a_0} = (1+f)\frac{V_{9.\mathrm{e}}}{a_0} - M_0 = 1.027 \cdot 4.02 - 2 = 2.13$$

$$\eta_{\mathrm{th}} = \frac{\Delta K\dot{E}}{\mathcal{P}_t} = \frac{a_0^2[(1+f)(V_9/a_0)^2 - M_0^2]}{2fQ_R} = \frac{B}{2fQ_R}$$

$$\eta_p = \frac{F_i V_0}{\Delta K\dot{E}} \approx \frac{2F_u V_0/\dot{m}_0}{a_0^2[(1+f)(V_9/a_0)^2 - M_0^2]} = \frac{2F_u V_0/\dot{m}_0}{B}$$

$$B = a_0^2[(1+f)(V_9/a_0)^2 - M_0^2] = 317^2[1.027 \cdot 4.02^2 - 4] = 1.265 \cdot 10^6 \cdot \frac{m^2}{s^2}$$

$$\eta_{\mathrm{th}} = \frac{B}{2fQ_R} = \frac{1.26 \cdot 10^6}{2 \cdot 0.0266 \cdot 42.800 \cdot 10^6} = 0.553$$

$$\eta_p = \frac{2a_0 V_0 \frac{F_u}{\dot{m}_0 a_0}}{B} = \frac{2 \cdot 317 \cdot 634 \cdot 2.13}{1.26 \cdot 10^6} = 0.677 \qquad \eta_o = \eta_p \eta_{\mathrm{th}} = 0.553 \cdot 0.677 = 0.374$$

$$TSFC = \frac{\dot{m}_f}{F_u} = \frac{f}{F_u/\dot{m}_0} = \frac{f}{a_0 \frac{F_u}{\dot{m}_0 a_0}} = \frac{0.0266}{317 \cdot 2.13} = 3.94 \cdot 10^{-5} \cdot \frac{kg}{Ns} = 0.0395 \cdot \frac{kg}{kNs}$$

Post bruciatore acceso

$$p_{t7} = p_{t5}\pi_{AB} = 2.13 \cdot 10^6 \cdot 0.98 = 2.09 \cdot 10^6 \cdot Pa$$

$$\tau_{\lambda.AB} = \frac{c_{p9}T_{t7}}{c_pT_0} = \frac{1243 \cdot 2250}{1004 \cdot 250} = 11.14$$

$$f_{AB} = \frac{(1+f)(\tau_{\lambda AB} - \tau_\lambda \tau_t)}{\frac{Q_{R.AB}\eta_{AB}}{c_pT_0} - \tau_{\lambda.AB}} = \frac{1.027(11.14 - 8.03 \cdot 0.762)}{\frac{42.800 \cdot 10^6 \cdot 0.99}{1004 \cdot 250} - 11.14} = 0.0327$$

Ugello

$$p_{t9} = p_{t7}\pi_n = 2.09 \cdot 10^6 \cdot 0.97 = 2.03 \cdot 10^6 \cdot Pa \qquad T_{t9} = T_{t7} = 2250 \cdot K$$

$$\frac{p_{t9}}{p_9} = \pi_n\pi_{AB}\pi_t\pi_b\pi_c\pi_d\pi_r\frac{p_0}{p_9} = \frac{p_{t9}}{p_0}\frac{p_0}{p_9} = \frac{20.3}{1.013} \cdot 1 = 20.0$$

$$M_9 = \sqrt{\frac{2}{\gamma_9 - 1}\left[\left(\frac{p_{t9}}{p_9}\right)^{k_9} - 1\right]} = \sqrt{\frac{2}{0.30}(20.0^{0.231} - 1)} = 2.58$$

$$\frac{T_{t9}}{T_9} = \psi_9 = 1 + \frac{\gamma_9 - 1}{2}M_9^2 = \left(\frac{p_{t9}}{p_9}\right)^{k_9} = 20.0^{0.231} = 1.998$$

$$T_9 = \frac{T_9}{T_{t9}}T_{t9} = \frac{2250}{1.998} = 1125 \cdot K \qquad a_9 = \sqrt{\gamma_9 R_9 T_9} = \sqrt{1.30 \cdot 287 \cdot 1125} = 648 \cdot \frac{m}{s}$$

$$V_9 = M_9 a_9 = 2.58 \cdot 648 = 1672 \cdot \frac{m}{s} \qquad V_{9.e} = V_9\left(1 + \frac{1 - \frac{p_0}{p_9}}{\gamma_9 M_9^2}\right) = V_9 \qquad \frac{V_9}{a_0} = \frac{1672}{317} = 5.27$$

Spinta e rendimenti

$$\frac{F_u}{\dot{m}_0 a_0} = (1 + f + f_{AB})\frac{V_{9.e}}{a_0} - M_0 = (1 + 0.0266 + 0.0327) \cdot 5.27 - 2 = 3.58$$

$$\eta_{th} = \frac{\Delta K\dot{E}}{\mathcal{P}_t} = \frac{a_0^2[(1+f)(V_9/a_0)^2 - M_0^2]}{2fQ_R} = \frac{B}{2fQ_R}$$

$$\eta_p = \frac{F_i V_0}{\Delta K\dot{E}} \approx \frac{2F_u V_0/\dot{m}_0}{a_0^2[(1+f)(V_9/a_0)^2 - M_0^2]} = \frac{2F_u V_0/\dot{m}_0}{B}$$

$$B = a_0^2[(1 + f + f_{AB})(V_9/a_0)^2 - M_0^2] = 317^2[(1 + 0.0266 + 0.0327) \cdot 5.27^2 - 4]$$
$$= 2.55 \cdot 10^6 \cdot \frac{m^2}{s^2}$$

$$\eta_{th} = \frac{B}{2fQ_R} = \frac{2.55 \cdot 10^6}{2 \cdot (0.0266 + 0.0327) \cdot 42.800 \cdot 10^6} = 0.502$$

$$\eta_p = \frac{2a_0 V_0 \frac{F_u}{\dot{m}_0 a_0}}{B} = \frac{2 \cdot 317 \cdot 634 \cdot 3.59}{2.55 \cdot 10^6} = 0.564 \qquad \eta_o = \eta_p\eta_{th} = 0.502 \cdot 0.564 = .283$$

$$TSFC = \frac{\dot{m}_f}{F_u} = \frac{f}{F_u/\dot{m}_0} = \frac{f}{a_0\frac{F_u}{\dot{m}_0 a_0}} = \frac{(0.0266 + 0.0327)}{317 \cdot 3.58} = 5.23 \cdot 10^{-5} \cdot \frac{kg}{Ns} = 0.0523 \cdot \frac{kg}{kNs}$$

k	0.28571			0.24812		0.23077	
R	286.857			285.835		286.846	kJ/kgK
a0	316.86	m/s	V0	633.719	m/s		
$\tau_r = T_{t0} = 0$/	1.8						
Section	c	t	λ				t
τ	2.07711	0.76251	8.03187			π	0.29696
Section	0	2	3	4	5	7	9
Tt	450	450	934.701	1750	1334.4		1334.4
pt	792,617	760,912	7.61E+06	7.23E+06	2.15E+06		2.08E+06
Section	f	Pt9/p9	M9	Tt9/T9	T9/T0	a9	
3	0.0267	2.06E+01	2.60209	2.11719	2.52107	489.492	m/s
5	0	V9		V9/a0	V9/a0 eff	F/ma0	Num
		1273.7	m/s	4.01977	4.01977	2.1271	1.26E+06
Section	t	th	p	0	TSFC*1e3		
η	0.913	0.55303	0.67581	0.37374	0.039617		

AB ON

Section	c	t	λ	λ_{AB}			t
τ	2.07711	0.76251	8.03187	11.1424		π	0.29696
Section	0	2	3	4	5	7	9
Tt	450	450	934.701	1750	1334.4	2250	2250
pt	792,617	760,912	7.61E+06	7.23E+06	2.15E+06	2.10E+06	2.04E+06
Section	f	Pt9/p9	M9	Tt9/T9	T9/T0	a9	
3	0.0267	2.01E+01	2.58154	1.99966	4.50078	647.754	m/s
5	0.03268	V9		V9/a0	V9/a0 eff	F/ma0	Num
		1672.2	m/s	5.27743	5.27743	3.59079	2.56E+06
Section	t	th	p	0	TSFC*1e3		
η	0.913	0.50381	0.56315	0.28372	0.052187		