

Mattingly example 3.3

Consider both a two-stage vehicle and a three-stage vehicle for the launch of the 900-kg (2000-lbm) payload. Each stage uses a liquid H_2 - O_2 chemical rocket ($C = 4115$ m/s, 13,500 ft/s), and the DV total of 14,300 m/s (46,900 ft/s) is split evenly between the stages. Suppose $\delta = m_{str}/m_0 = 0.03$.

MPayload	900
c	4115
DV	14300
$\delta = m_{str}/m_0$	0.03

#Stadi	1	2	3	
Dv	14,300	7,150	4,767	m/s
mpay	900	900	900	kg
MR	0.03096	0.1760	0.3140	
lambda	9.590E-04	0.1460	0.2840	
m01	938,465	6,166	3,169	kg
mstr1	28,154	185	95	kg
mp1	909,411	5,081	2,174	kg
m02		42,250	11,159	kg
mstr2		1,267	335	kg
mp2		34,816	7,655	kg
m03			39,291	kg
mstr3			1,179	kg
mp3			26,953	kg

Mono stadio

$$\Delta V = -\bar{c} \ln \frac{m_f}{m_0} = \bar{c} \ln \frac{1}{MR} \rightarrow MR = e^{-\frac{\Delta V}{\bar{c}}} = e^{-\frac{14300}{4115}} = 0.0309590$$

$$\lambda = \frac{m_{pay}}{m_0} = MR - \frac{m_{str}}{m_0} = MR - \delta = 0.03096 - .03000 = 9.590 \cdot 10^{-4}$$

$$m_0 = \frac{m_{pay}}{\lambda} = \frac{900}{9.590 \cdot 10^{-4}} = 938.5 \cdot 10^3 kg$$

$$m_{str} = \delta m_0 = 0.0300 \cdot 938.5 \cdot 10^3 = 28,150 kg$$

$$m_p = m_0(1 - MR) = 938.5 \cdot 10^3(1 - 0.03096) = 909.4 \cdot 10^3 kg$$

Due stadi

Stadio finale (1)

$$\frac{\Delta V}{2} = -\bar{c} \ln \frac{m_f}{m_0} = \bar{c} \ln \frac{1}{MR} \rightarrow MR = e^{-\frac{\Delta V}{2\bar{c}}} = e^{-\frac{14300}{2 \cdot 4115}} = 0.1760$$

$$\lambda = \frac{m_{pay}}{m_0} = MR - \frac{m_{str}}{m_0} = MR - \delta = 0.1760 - .03000 = 0.1460$$

$$m_{01} = \frac{m_{pay1}}{\lambda} = \frac{900}{0.1460} = 6,166 kg$$

$$m_{str1} = \delta m_{01} = 0.0300 \cdot 6,166 = 185 kg$$

$$m_{p1} = m_{01}(1 - MR) = 6,166(1 - 0.1760) = 5,081 kg$$

$$m_{pay2} = m_{01} \rightarrow m_{02} = \frac{m_{pay2}}{\lambda} = \frac{6,166}{0.1460} = 42,250kg$$

$$m_{str1} = \delta m_{01} = 0.0300 \cdot 42,250 = 1267kg$$

$$m_{p1} = m_{01}(1 - MR) = 42,250(1 - 0.1760) = 34,816kg$$

Tre stadi

Procedendo in modo analogo si costruisce la tabella.

Mattingly example 3.7

The space shuttle main engine (SSME) operates for up to 520 s in one mission at altitudes over 100 miles. The nozzle expansion ratio 1 is 77:1, and the inside exit diameter is 2.30m. Assume a calorically perfect gas with the following properties:

Ae/Ag	77	
De	2.298	m
γ	1.25	
pc	2.068E+07	Pa
Tt	4083	K
R	602.6	J/kgK
M2	5.092	

We want to calculate the following:

1) Characteristic velocity c^*

$$k = \frac{\gamma - 1}{\gamma} = \frac{0.25}{1.25} = 0.20 \quad K = \frac{\gamma + 1}{2(\gamma - 1)} = \frac{2.25}{0.5} = 4.5$$

$$\psi = 1 + \frac{\gamma - 1}{2} M^2 \quad \psi^* = 1 + \frac{0.25}{2} = 1.125 \quad \Psi = \gamma M \psi^{-K} \quad \Psi^* = 1.25 \cdot 1.125^{-4.5} = 0.736$$

$$A_e = \frac{\pi}{4} D_e^2 = \frac{3.141}{4} 2.298^2 = 4.15 \cdot m^2 \quad A_{th} = \frac{A_{th}}{A_e} A_e = \frac{4.15}{77} = 0.0539 \cdot m^2$$

$$a_t = \sqrt{\gamma R T_t} = \sqrt{1.4 \cdot 602.6 \cdot 4083} = 1754 \cdot \frac{m}{s}$$

$$c^* = \frac{p_c A_{th}}{\dot{m}_p} = \frac{a_t}{\Psi^*} = \frac{1754}{0.736} = 2384 \cdot \frac{m}{s}$$

2) Mass flow rate of gases through the nozzle.

$$\dot{m} = \frac{p_c A_{th} \Psi^*}{a_t} = \frac{2.068 \cdot 10^7 \cdot 0.0539 \cdot 0.736}{1754} = 468 \cdot \frac{kg}{s}$$

3) Pressure at which the nozzle is “on-design.”

$$\psi_2 = 1 + \frac{\gamma - 1}{2} M_2^2 = 1 + 0.125 \cdot 5.092^2 = 4.24$$

$$p_2 = p_c \psi^{-\frac{1}{k}} = 2.068 \cdot 10^7 \cdot 4.24^{-\frac{1}{0.2}} = 15.09 \cdot kPa$$

4) Pressure at which the nozzle is just separated (assume separation occurs when $p_a > 3.5 p_{r3}$).

$$p_s = p_2 \cdot 3.5 = 15.09 \cdot 3.5 = 52.8 \cdot kPa$$

5) Thrust coefficient C_F , specific impulse and thrust for $p_0=0$, p_s .

$$C_F = \frac{\dot{m}_p V_2 + (p_2 - p_0) A_2}{p_c A_{th}} = \sqrt{\frac{2\gamma^2}{\gamma - 1} \left(\frac{2}{\gamma + 1}\right)^{\frac{(\gamma+1)}{(\gamma-1)}} \left[1 - \left(\frac{p_2}{p_c}\right)^k\right]} + \frac{(p_2 - p_0)}{p_c} \frac{A_2}{A_{th}}$$

$$C_F = \sqrt{\frac{2 \cdot 1.25^2}{.25} \left(\frac{2}{2.25}\right)^{\frac{2.25}{.25}} [1 - 4.24^{-1}]} + \frac{(p_2 - p_0)}{p_c} 77 = 1.819 + \frac{(p_2 - p_0)}{p_c} 77$$

$$p_0 = 0 \quad C_F = 1.819 + \frac{15.09}{2.068 \cdot 10^4} 77 = 1.875$$

$$F = C_F p_c A_{th} = 1.875 \cdot 2.068 \cdot 10^7 \cdot 0.0539 = 2090 \cdot kN$$

$$I_s = \frac{F}{g_0 \dot{m}_p} = \frac{2090}{9.81 \cdot 468} 10^3 = 455 \cdot s$$

$$p_0 = 52.8 \quad C_F = 1.819 + \frac{15.09 - 52.8}{2.068 \cdot 10^4} 77 = 1.679$$

$$F = C_F p_c A_{th} = 1.679 \cdot 2.068 \cdot 10^7 \cdot 0.0539 = 1871 \cdot kN$$

$$I_s = \frac{F}{g_0 \dot{m}_p} = \frac{1871}{9.81 \cdot 468} 10^3 = 408 \cdot s$$

Farokhi problem 12.2

A solid rocket motor has a design chamber pressure of 10 MPa, an end-burning grain with $n = 0.4$ and $\dot{r} = 3\text{cm/s}$ at the design chamber pressure and design grain temperature of 15C. The temperature sensitivity of the burning rate is $\sigma_p = 0.002/C$, and chamber pressure sensitivity to initial grain temperature is $\pi_K = 0.005/C$. The nominal effective burn time for the rocket is 120 s, i.e., at design conditions. Calculate

(a) the new chamber pressure and burning rate when the initial grain temperature is 75C;

$$\sigma_p = \frac{1}{\dot{r}} \left(\frac{\delta \dot{r}}{\delta T} \right)_p \frac{123465 \pi_K}{p} = \frac{1}{p} \left(\frac{\delta p}{\delta T} \right)_K$$

$$\Delta T = T_{c2} - T_{c1} = 75 - 15 = 60 \cdot C$$

$$\Delta p = p_{c1} \pi_K \Delta T = 10 \cdot 0.005 \cdot 60 = 3 \cdot MPa$$

$$p_{c2} = p_{c1} + \Delta p = 10 + 3 = 13 \cdot MPa$$

$$\dot{r} = a p_c^n \quad \dot{r}_{2p} = \dot{r}_1 \left(\frac{p_{c2}}{p_{c1}} \right)^n = 3 \cdot 1.3^{0.4} = 3.33 \cdot \frac{cm}{s}$$

$$\dot{r}_2 = \dot{r}_{2p} (1 + \sigma_p \Delta T) = 3.33 (1 + 0.002 \cdot 60) = 3.73 \cdot \frac{cm}{s}$$

(b) the corresponding reduction in burn time Δt_b in seconds.

$$h = \dot{r}_1 t_{b1} = 3 \cdot 120 = 360 \cdot cm$$

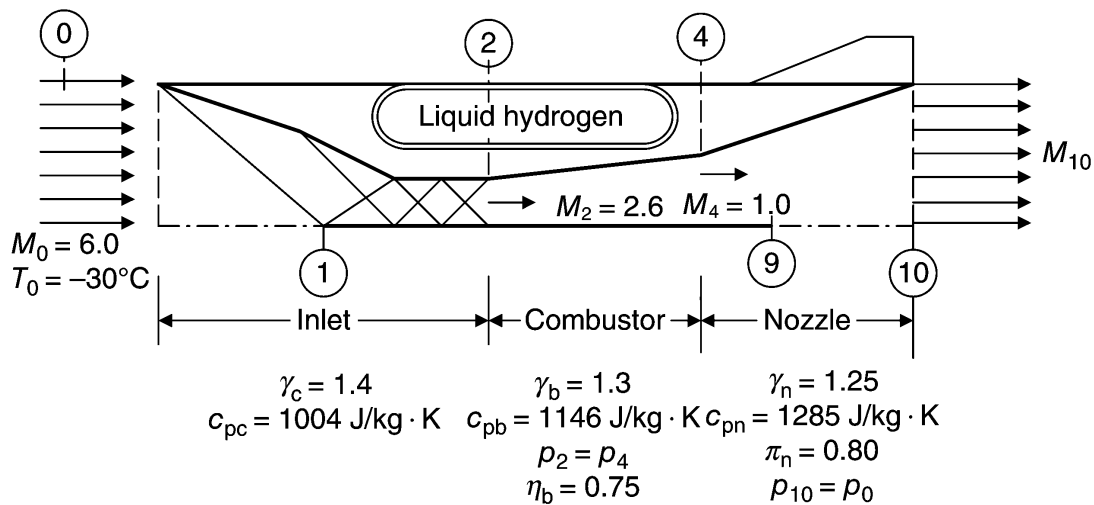
$$t_{b2} = \frac{h}{\dot{r}_2} = \frac{360}{3.73} = 96.5 \cdot s \quad \Delta t_b = t_{b1} - t_{b2} = 120 - 96.5 = 23.5 \cdot s$$

Farokhi problem 12.20

Consider a scramjet in a Mach 6 flight. The fuel for this engine is hydrogen with $Q_R=120,000\text{kJ/kg}$. The inlet uses multiple oblique shocks with a total pressure recovery following MIL-E-5008B standards for $M_0 > 5$, i.e.,

$$\pi_d = \frac{800}{M_0^4 + 935}$$

The combustor entrance Mach number is $M_2 = 2.6$. Use frictionless, constant-pressure heating, i.e., $p_4 = p_2$, to simulate the combustor with combustor exit Mach number $M_4 = 1.0$. All component parameters and gas constants are shown in the schematic drawing below.



Calculate

(a) Inlet static temperature ratio T_2/T_0

$$k_0 = \frac{\gamma - 1}{\gamma} = \frac{0.4}{1.4} = 0.2857 \quad K_0 = \frac{\gamma + 1}{2(\gamma - 1)} = \frac{2.4}{0.8} = 3$$

$$k_b = \frac{\gamma - 1}{\gamma} = \frac{0.3}{1.3} = 0.2308 \quad K_b = \frac{\gamma + 1}{2(\gamma - 1)} = \frac{2.3}{0.6} = 3.833$$

$$k_n = \frac{\gamma - 1}{\gamma} = \frac{0.25}{1.25} = 0.200 \quad K_n = \frac{\gamma + 1}{2(\gamma - 1)} = \frac{2.25}{0.5} = 4.50$$

$$T_0 = 273.15 - 30 = 243 \cdot \text{K} \quad a_0 = \sqrt{1.4 \cdot 0.2857 \cdot 1004 \cdot 243} = 312 \cdot \frac{\text{m}}{\text{s}}$$

$$V_0 = M_0 a_0 = 6 \cdot 312 = 1875 \cdot \frac{\text{m}}{\text{s}}$$

$$\psi = 1 + \frac{\gamma - 1}{2} M^2 \quad \rightarrow \quad \psi_0 = 1 + 0.2 \cdot 6^2 = 8.20 \quad \psi_2 = 1 + 0.2 \cdot 2.6^2 = 2.35$$

$$\frac{T_2}{T_0} = \frac{T_2}{T_{t2}} \frac{T_{t0}}{T_0} = \frac{\psi_0}{\psi_2} = \frac{8.20}{2.35} = 3.49 \quad T_2 = \frac{T_2}{T_0} T_0 = 3.49 \cdot 243 = 848 \cdot \text{K}$$

$$T_{t2} = \psi_2 T_2 = 2.35 \cdot 848 = 1992 \cdot \text{K}$$

$$\frac{p_{t0}}{p_0} = \psi_0^{\frac{1}{k_0}} = 8.20^{\frac{1}{0.2857}} = 1579$$

$$\frac{p_{t2}}{p_2} = \psi_0^{\frac{1}{k_0}} = 2.35^{\frac{1}{0.2857}} = 19.95$$

$$\pi_d = \frac{800}{M_0^4 + 935} = \frac{800}{6^4 + 935} = 0.359$$

$$\frac{p_2}{p_0} = \frac{p_2}{p_{t2}} \frac{p_{t2}}{p_{t0}} \frac{p_{t0}}{p_0} = \frac{1579 \cdot 0.359}{19.95} = 28.37$$

$$\frac{p_{t2}}{p_0} = \frac{p_{t2}}{p_{t0}} \frac{p_{t0}}{p_0} = \pi_d \frac{p_{t0}}{p_0} = 0.359 \cdot 1579 = 567$$

(b) combustor exit temperature T_4 in K

Dalla conservazione della massa e dal bilancio della QM supponendo $p_2 = p_4$, $\gamma_2 = \gamma_4 = \gamma_b$, $R_2 = R_4 = R_b$ e trascurando gli effetti viscosi si ha:

$$\dot{m}_4 = (1 + f)\dot{m}_2 \quad \dot{m} = \rho VA = \frac{\gamma p \rho VA}{\gamma p} = \frac{\gamma p VA}{a^2} = \frac{\gamma p MA}{a}$$

$$\frac{p_2 M_2 A_2}{a_2} = \frac{p_4 M_4 A_4}{a_4} \quad \xrightarrow{p_2=p_4} \quad \sqrt{\frac{T_4}{T_2}} = \frac{M_4 A_4}{M_2 A_2}$$

Seguendo il libro e supponendo che la pressione sia costante sulla superficie del volume di controllo, si ha che anche la V è costante quindi:

$$\sqrt{\frac{T_4}{T_2}} = \frac{a_4}{a_2} = \frac{V_4}{M_4} \frac{M_2}{V_2} = \frac{M_2}{M_4} \rightarrow \sqrt{\frac{T_4}{T_2}} = \frac{M_2}{M_4} = \frac{M_4 A_4}{M_2 A_2}$$

$$\frac{A_4}{A_2} = \frac{M_2^2}{M_4^2} \quad \sqrt{\frac{T_4}{T_2}} = \frac{M_4 A_4}{M_2 A_2} \rightarrow \sqrt{\frac{T_4}{T_2}} = \frac{M_2}{M_4} = 2.6 \quad \frac{T_4}{T_2} = 2.6^2 = 6.76$$

$$T_4 = \frac{T_4}{T_2} T_2 = 2.6^2 \cdot 848 = 5731 K$$

$$\psi = 1 + \frac{\gamma - 1}{2} M^2 \rightarrow \psi_4 = 1 + 0.15 \cdot 1 = 1.15$$

$$T_{t4} = \psi_4 T_4 = 1.15 \cdot 5731 = 6590 \cdot K$$

(c) fuel-to-air ratio f

$$\dot{m}_f Q_R \eta_b = \dot{m}_0 c_{pb} [(1 + f) T_{t4} - T_{t2}] \rightarrow$$

$$f = \frac{T_{t4} - T_{t2}}{\frac{Q_R \eta_b}{c_{pb}} - T_{t4}} = \frac{6590 - 1992}{120 \cdot 10^6 \cdot \frac{0.75}{1146} - 6590} = 0.0639$$

Mentre il libro suppone che:

$$f = \frac{T_4 - T_2}{\frac{Q_R \eta_b}{c_{pb}} - T_4} = \frac{5731 - 847}{120 \cdot 10^6 \cdot \frac{0.75}{1146}} = 0.0622$$

(d) nozzle exit Mach number M_{10}

$$\frac{p_{t4}}{p_0} = \frac{p_{t4}}{p_4} \frac{p_2}{p_0} = \psi_4^{\frac{1}{\gamma_b}} \frac{p_2}{p_0} = 1.15^{0.231} \cdot 28.37 = 52.0$$

$$\frac{p_{t10}}{p_0} = \frac{p_{t4}}{p_0} \pi_n = 52.0 \cdot 0.80 = 41.6 = \psi_{10}^{\frac{1}{\gamma_n}} \rightarrow \psi_{10} = 41.6^{0.2} = 2.11$$

$$M_{10} = \sqrt{(\psi_{10} - 1) \frac{2}{\gamma_n - 1}} = \sqrt{1.11 \frac{2}{0.25}} = 2.98$$

$$T_{10} = \frac{T_{t10}}{\psi_{10}} = \frac{T_{t4}}{\psi_{10}} = \frac{6590}{2.11} = 3127 \quad a_{10} = \sqrt{1.25 \cdot 0.20 \cdot 1285 \cdot 3127} = 1002 \cdot \frac{m}{s}$$

$$V_{10} = M_{10} a_{10} = 2.98 \cdot 1002 = 2983 \cdot \frac{m}{s}$$

(e) nondimensional ram drag $D_{ram}/p_0 A_1$ (note that $A_0 = A_1$)

$$\frac{D_{ram}}{p_0 A_1} = \frac{\dot{m}_0 V_0}{p_0 A_1} = \frac{\rho_0 V_0^2 A_1}{p_0 A_1} = \gamma_c M_0^2 = 1.4 \cdot 36 = 50.4$$

(f) nondimensional gross thrust $F_g/p_0 A_1$

$$\frac{F_g}{p_0 A_1} = \frac{\dot{m}_{10} V_{10}}{p_0 A_1} = \frac{(1+f)\rho_0 V_0 A_1 V_{10}}{p_0 A_1} = \frac{\gamma_c (1+f) M_0 V_{10}}{a_0} = \frac{1.4 \cdot 1.0622 \cdot 6 \cdot 2983}{312} = 85.2$$

$$\frac{F_u}{p_0 A_1} = \frac{F_g - D_{ram}}{p_0 A_1} = 85.2 - 50.4 = 34.8$$

(g) fuel-specific impulse I_s in seconds

$$I_s = \frac{F_u}{\dot{m}_f g_0} = \frac{\dot{m}_0 [(1+f)V_{10} - V_0]}{f \dot{m}_0 g_0} = \frac{1.062 \cdot 2983 - 1875}{0.0622 \cdot 9.81} = 2122 \cdot s$$

(h) combustor area ratio A_4/A_2

$$\frac{A_4}{A_2} = \frac{M_2^2}{M_4^2} = 2.6^2 = 6.76$$

(i) nozzle area ratio A_{10}/A_4

$$\frac{p_4 M_4 A_4}{\sqrt{T_{24}}} = \frac{p_{10} M_{10} A_{10}}{\sqrt{T_{10}}} \rightarrow \frac{A_{10}}{A_4} = \frac{p_4 M_4 \sqrt{T_{10}}}{p_{10} M_{10} \sqrt{T_4}} = 28.4 \frac{1}{2.98} \sqrt{\frac{3127}{5731}} = 7.04$$

(j) thermal efficiency

$$\eta_{th} = \frac{\Delta K \dot{E}}{\mathcal{P}_t} = \frac{[(1+f)V_{10}^2 - V_0^2]}{2f Q_R} = \frac{(1.062 \cdot 2983^2 - 1875^2)/2}{0.0622 \cdot 120 \cdot 10^6} = \frac{2.970 \cdot 10^6}{7.461 \cdot 10^6} = 0.398$$

(k) propulsive efficiency

$$\eta_p = \frac{F_i V_0}{\Delta K \dot{E}} \approx \frac{\frac{2F_u V_0}{\dot{m}_0}}{[(1+f)V_{10}^2 - V_0^2]} = \frac{[(1+f)V_{10} - V_0]V_0}{2.970 \cdot 10^6} = \frac{(1.062 \cdot 2983 - 1875)1875}{2.970 \cdot 10^6} = 0.817$$