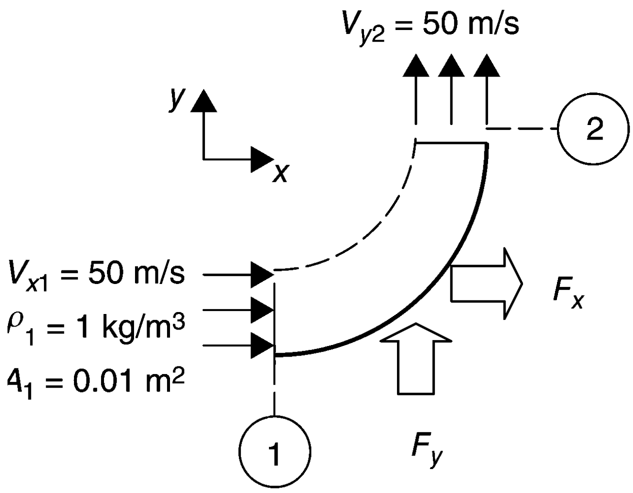
Farokhi problem 2.32

Consider the flow of perfect gas (air) on a vane, as shown. Apply momentum principles to the fluid to calculate the components of the force that act on the vane. Assume the flow is uniform, and due to low speeds, $p_1 = p_2 = p_0$, where p_0 is the ambient pressure.



Nell'ipotesi che *p* sia costante avremo che anche *V* è costante.
Indicando con I l'impulso totale si ha:

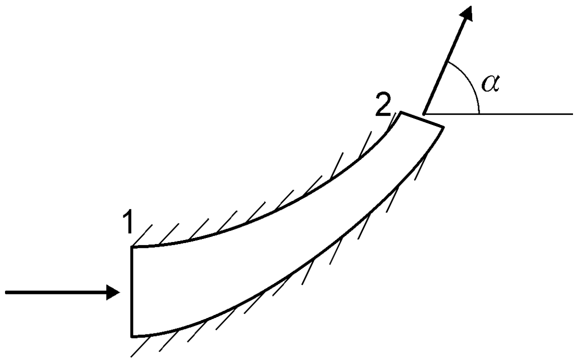
$F_x = (\underline{I_1} - \underline{I_2})_x \qquad F_y = (\underline{I_1} - \underline{I_2})_y$

Dove F è la forza esercitata dal fluido sulla parete.

$F_x = A_1[(p_1 - p_0) + \rho_1 \cdot V_{x1}^2] = 0.01(0 + 1 \cdot 50^2) = 25.00N$

$F_y = -A_2[(p_2 - p_0) + \rho_2 \cdot V_{y2}^2] = -0.01(0 + 1 \cdot 50^2) = -25.00N$

Rimuovendo l'ipotesi di $\boldsymbol{p = p_a}$



$A_2 = 0.0050m^2 \qquad \alpha = 45° \qquad p_1 = 1bar$

$$V_2 = \frac{V_1 A_1}{A_2} = \frac{50 \cdot 0.01}{0.0050} = 100 \cdot \frac{m}{s}$$

$$p_2 = p_1 + \frac{\rho}{2}(V_1^2 - V_2^2) = 1 \cdot 10^5 + 0.5(50^2 - 100^2) = 96.25 \cdot kPa$$

$(p_2 - p_0) = 96.25 - 100 = -3.75 \cdot kPa$

$I_1 = A_1[(p_1 - p_0) + \rho_1 \cdot V_1^2] = 0.01(0 + 1 \cdot 50^2) = 25.00N$

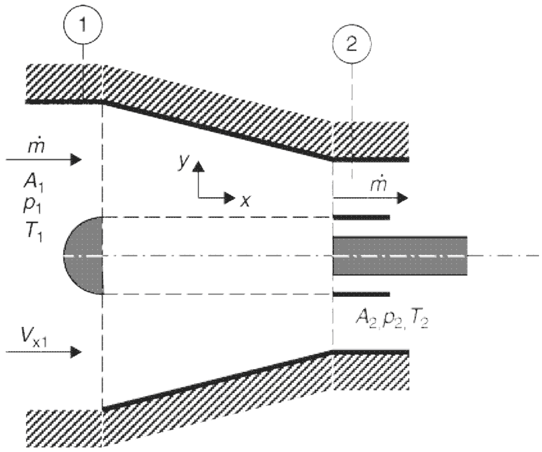
$I_2 = A_2[(p_2 - p_0) + \rho_2 \cdot V_2^2] = 0.005(-3750 + 1 \cdot 100^2) = 31.25N$

$$I_{x2} = I_2 \cdot \cos(\alpha) = 31.25 \cdot \frac{\sqrt{2}}{2} = 22.10 \cdot N$$

$F_x = I_1 - I_{x2} = 25.00 - 22.10 = 2.90 \cdot N$

$F_y = -I_{y2} = -22.10 \cdot N$

Farokhi Example 2.14



$\pi_c = 20 \quad \gamma = 1.4 \quad V_1 = V_2 \quad \frac{F_x}{p_1 A_1} = ?$

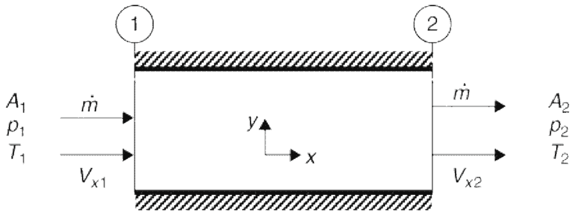
$$\frac{A_2}{A_1} = \frac{\rho_1}{\rho_2} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} = \pi_c^{-\frac{1}{\gamma}} = 20^{-\frac{1}{1.4}} = 0.118$$

$$\dot{m} = \rho V A = (\rho V A)_1 = (\rho V A)_2$$

$$\frac{F_x}{p_1 A_1} = \frac{A_1 p_1 + A_1 \rho_1 V_1^2 - A_2 p_2 - A_2 \rho_2 V_2^2}{p_1 A_1}$$

$$= 1 - \pi_c \frac{A_2}{A_1} = 1 - 20 \cdot 0.118 = -1.36$$

Farokhi Example 2.15



$\tau_b = 1.8 \quad \gamma = 1.4 \quad p_1 = p_2 \quad \frac{F_x}{\dot{m} V_1} = ?$

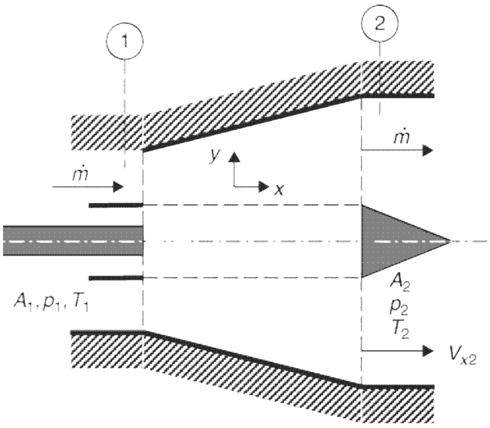
$$\frac{\rho_1}{\rho_2} = \frac{V_2}{V_1} = \frac{T_2}{T_1} = \tau_b = 1.8$$

$$\frac{F_x}{\dot{m} V_1} = \frac{A_1 p_1 + A_1 \rho_1 V_1^2 - A_2 p_2 - A_2 \rho_2 V_2^2}{\dot{m} V_1}$$

$$= \frac{\dot{m} V_1 - \dot{m} V_2}{\dot{m} V_1} = 1 - \frac{V_2}{V_1} = 1 - 1.8$$

$$= -0.8$$

Farokhi Example 2.16



$\tau_t = 0.79 \quad \gamma = 1.4 \quad V_1 = V_2 \quad \frac{F_x}{p_1 A_1} = ?$

$$\frac{A_2}{A_1} = \frac{\rho_1}{\rho_2} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma - 1}} = \tau_t^{\frac{-1}{\gamma - 1}} = 0.79^{-\frac{1}{0.4}} = 1.803$$

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma - 1}} = 0.79^{\frac{1.4}{0.4}} = 0.438 = \pi_t$$

$$\dot{m} = \rho V A = (\rho V A)_1 = (\rho V A)_2$$

$$\frac{F_x}{p_1 A_1} = \frac{A_1 p_1 + A_1 \rho_1 V_1^2 - A_2 p_2 - A_2 \rho_2 V_2^2}{p_1 A_1} = 1 - \pi_t \frac{A_2}{A_1}$$

$$= 1 - 0.438 \cdot 1.803 = 0.210$$

Farokhi problem 2.34

A scramjet combustor has a supersonic inlet condition and a choked exit. The combustor flow area increases linearly in the flow direction, as shown.

The inlet and exit flow conditions are:

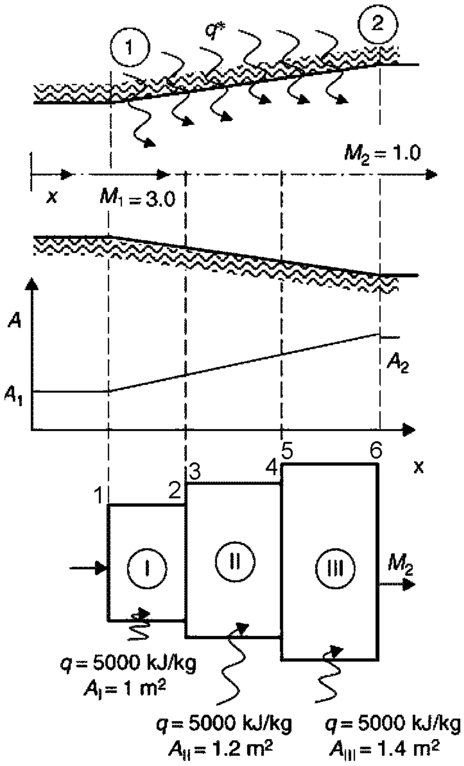
$M_1 = 3.0,\ \ p_1 = 1bar,\ \ T_1 = 1000K\ ,\ \ A_1 = 1\ m^2$

$M_2 = 1.0,\ \ A_2 = 1.4\ m^2,\ \ \gamma = 1.4,\ \ \ R = 287 \dfrac{J}{kg \cdot K}$

The total heat release due to combustion, per unit flow rate in the duct, is initially assumed to be 15 MJ/kg. If we divide the combustor into three constant-area sections, with stepwise jumps in the duct area, we may apply Rayleigh flow principles to each segment, as shown. The heat release per segment is then 1⁄3 of the total heat release in the duct, i.e., 5,000 kJ/kg.

As the exit condition of a segment needs to be matched to the inlet condition of the following segment, we propose to satisfy continuity equation at the boundary through an isentropic step area expansion, i.e., p_t, T_t remain the same and only the Mach number jumps isentropically through area expansion.

If we march from the inlet condition toward the exit with the assumed heat release rates, we calculate the exit Mach number M_2. Since the exit flow is specified to be choked, then we need to adjust the total heat release in order to get a choked exit. Calculate the critical heat release in the above duct that leads to thermal choking of the flow.



In realtà si utilizza una simbologia diversa, nominando le varie sezioni partendo da 1 fino a quella finale che sarà la numero 6 inoltre si suppone che $M_6 = 1.2$.

$$Q_{12} = 500 \cdot \frac{kJ}{K} \qquad A_3 = A_4 = 1.2 \cdot m^2 \quad A_5 = A_6 = 1.4 \cdot m^2$$

$$M_1 = 3.0 \ \overset{ISO}{\longrightarrow} \ \frac{T_1}{T_{01}} = 0.357 \quad T_{01} = \frac{T_{01}}{T_1} T_1 = \frac{1000}{0.357} = 2800K$$

$$M_1 = 3.0 \ \overset{\text{RA}}{\rightarrow} \ \frac{T_{01}}{T_0^*} = 0.654 \quad T_{02} = T_{01} + \frac{Q_{12}}{c_p} = 2800 + \frac{500}{1.0045} = 3298 \cdot K$$

$$\frac{T_{02}}{T_0^*} = \frac{T_{02}}{T_{01}} \frac{T_{01}}{T_0^*} = \frac{3298}{2800} 0.654 = 0.770 \ \overset{\text{RA}}{\rightarrow} \ M_2 = 2.121$$

$$M_2 = 2.121 \ \overset{ISO}{\longrightarrow} \ \frac{A_2}{A^*} = 1.870 \quad \frac{A_3}{A^*} = \frac{A_3}{A_2} \frac{A_2}{A^*} = 1.2 \cdot 1.870 = 2.24 \ \overset{ISO}{\longrightarrow} \ M_3 = 2.325$$

$$M_3 = 2.325 \ \overset{\text{RA}}{\rightarrow} \ \frac{T_{03}}{T_0^*} = 0.736 \quad T_{04} = T_{03} + \frac{Q_{34}}{c_p} = 3298 + \frac{500}{1.0045} = 3796 \cdot K$$

$$\frac{T_{04}}{T_0^*} = \frac{T_{04}}{T_{03}} \frac{T_{03}}{T_0^*} = \frac{3796}{3298} 0.736 = 0.847 \ \overset{\text{RA}}{\rightarrow} \ M_4 = 1.755$$

$$M_4 = 1.755 \xrightarrow{ISO} \frac{A_4}{A^*} = 1.392 \quad \frac{A_5}{A^*} = \frac{A_5}{A_4}\frac{A_4}{A^*} = \frac{1.4}{1.2} \cdot 1.392 = 1.624 \xrightarrow{ISO} M_5 = 1.954$$

$$M_5 = 1.954 \xrightarrow{\text{RA}} \frac{T_{05}}{T_0^*} = 0.803 \quad T_{06} = T_{05} + \frac{Q_{56}}{c_p} = 3796 + \frac{500}{1.0045} = 4293 \cdot K$$

$$\frac{T_{06}}{T_0^*} = \frac{T_{06}}{T_{05}}\frac{T_{05}}{T_0^*} = \frac{4293}{3796} 0.803 = 0.908 \xrightarrow{\text{RA}} M_6 = 1.505$$

Si deve aumentare $Q_{12} = 600 \cdot \frac{kJ}{K}$

Q(kJ/kg)	500	Ra	Iso	Ra	Iso	Ra
Sezione	1	2	3	4	5	6
A	1	1	1.2	1.2	1.4	1.4
M	3	2.121	2.325	1.755	1.954	1.505
T0/T0*	0.654	0.770	0.736	0.847	0.803	0.908
Tt	2800	3298	3298	3796	3796	4293
A/A*	4.23	1.870	2.24	1.392	1.624	1.179
Err	0.305					

Q	600	Ra	Iso	Ra	Iso	Ra
Sezione	1	2	3	4	5	6
A	1	1	1.2	1.2	1.4	1.4
M	3	1.999	2.211	1.589	1.809	1.295
T0/T0*	0.654	0.793	0.754	0.887	0.834	0.959
Tt	2800	3397	3397	3995	3995	4592
A/A*	4.23	1.687	2.02	1.241	1.448	1.064
Err	0.095	Q	645.4			

Q	645.4	Ra	Iso	Ra	Iso	Ra
Sezione	1	2	3	4	5	6
A	1	1	1.2	1.2	1.4	1.4
M	3	1.948	2.162	1.517	1.749	1.185
T0/T0*	0.654	0.804	0.763	0.905	0.848	0.981
Tt	2800	3443	3443	4085	4085	4728
A/A*	4.23	1.616	1.94	1.188	1.386	1.026
Err	-0.015	Q	639.2			

Q	639.2	Ra	Iso	Ra	Iso	Ra
Sezione	1	2	3	4	5	6
A	1	1	1.2	1.2	1.4	1.4
M	3	1.955	2.169	1.527	1.757	1.202
T0/T0*	0.654	0.803	0.762	0.903	0.846	0.978
Tt	2800	3436	3436	4073	4073	4709
A/A*	4.23	1.626	1.95	1.195	1.394	1.031

Mattingly problem 3.11 (2.33)

At launch, the space shuttle main engine (SSME) has 1030 lbm/s of gas leaving the combustion chamber at $p_t = 3000psi, T_t = 7350R$. The exit area of the SSME nozzle is 77 times the throat area. If the flow through the nozzle is considered to be reversible and adiabatic (isentropic) with $R = 3800 ft^2/s^2 R$ and $\gamma = 1.25$ find the area of the nozzle throat and the exit Mach number.

$$\Psi(\gamma, M) = \frac{A^*}{A}\Psi^* = \gamma M \left(1 + \frac{\gamma - 1}{2}M^2\right)^{-\frac{(\gamma+1)}{2(\gamma-1)}} = \gamma M \psi^{-K}$$

$$\psi(\gamma, M) = 1 + \frac{\gamma - 1}{2}M^2 \qquad K(\gamma) = \frac{\gamma + 1}{2(\gamma - 1)} = 4.5 \quad \Psi^* = \gamma\left(\frac{\gamma + 1}{2}\right)^{-K} = 0.736$$

$$\dot{m}_p = 1030 \cdot \frac{lbm}{s} = 1030 \cdot 0.453 \cdot \frac{kg}{s} = 467 \cdot \frac{kg}{s}$$

$$p_t = 3000 \cdot psi = 3000 \cdot 6895 \cdot Pa = 206.8 \cdot bar$$

$$T_t = 7350 \cdot R = \frac{7350}{1.8}K = 4083 \cdot K \qquad R = 635.4\frac{J}{kgK}$$

$$a_t = \sqrt{\gamma R T_t} = \sqrt{1.25 \cdot 635.4 \cdot 4083} = 1801 \cdot \frac{m}{s}$$

$$\dot{m} = \frac{p_t A_t \Psi^*}{a_t} \quad \rightarrow \quad A_t = \frac{\dot{m} a_t}{p_t \Psi^*} = \frac{467 \cdot 1801}{206.8 \cdot 10^5 \cdot 0.736} = 0.0553 \cdot m^2$$

$$M = 4 \quad \rightarrow \quad \psi = 1 + \frac{\gamma - 1}{2}M^2 = 1 + 0.125 * 4^2 = 3$$

$$\Psi = \gamma M \psi^{-K} = 1.25 * 4 * 3^{-4.5} = 0.0356$$

$$\frac{A}{A^*} = \frac{\Psi^*}{\Psi} = \frac{0.736}{0.0356} = 20.67 \quad e = 100 \cdot \frac{77 - 20.67}{77} = 73.15\%$$

$$M = 5 \quad \rightarrow \quad \psi = 1 + \frac{\gamma - 1}{2}M^2 = 1 + 0.125 * 4^2 = 4.125$$

$$\Psi = \gamma M \psi^{-K} = 1.25 * 4 * 3^{-4.5} = 0.0106$$

$$\frac{A}{A^*} = \frac{\Psi^*}{\Psi} = \frac{0.736}{0.0356} = 69.22 \quad e = 100 \cdot \frac{77 - 20.67}{77} = 10\%$$

$$M^{n+1} = \frac{M^n e^{n-1} - M^{n-1} e^n}{e^{n-1} - e^n} = \frac{5 \cdot 73.15 - 4 \cdot 10}{73.15 - 10} = 5.16$$

γ	1.25	$(\gamma-1)/2$	0.125	K	4.5
$\Psi*$	0.73574			A/A*	77
M	psi	Ψ	A/A*	Err	
4	3	0.035639	20.64	-73.1893	
5	4.125	0.010629	69.22	-10.0998	
5.16	4.32831	0.008834	83.29	8.16815	
5.089	4.23661	0.009592	76.7	-0.38674	
5.092	4.24073	0.009556	76.99	-0.0139	
5.092	4.24088	0.009555	77	2.5E-05	

Mattingly problem 4.1

Determinare la spinta per $p_a = 14.7psi$, alla quota di 44.600ft e nel vuoto.

$M = 5.09$

$p_0 = 14.7 \cdot psi = 14.7 \cdot 6895 = 1.013 \cdot 10^5 Pa = 1.013 \cdot bar$

$A_9 = A_t \cdot \frac{A_9}{A_t} = 0.0553 \cdot 77 = 4.26 \cdot m^2$

$T_9 = T_{9t} \cdot \psi^{-1} = 4083 \cdot 4.241^{-1} = \ 963 \cdot K$

$a_9 = \sqrt{\gamma R T_9} = \sqrt{1.25 \cdot \ 635.4 \cdot 963} = 874 \cdot \frac{m}{s}$

$V_9 = M_9 \cdot a_9 = 5.09 \cdot 874 = 4449 \cdot \frac{m}{s}$ $\quad p_9 = p_{9t} \cdot \psi^{-\frac{1}{k}} = 206.8 \cdot 4.241^{-5} = 0.151 \cdot bar$

$F = \dot{m} V_9 + A_9 (p_9 - p_0) = \ 467 \cdot 4449 + 4.26 \cdot (0.151 - 1.013) \cdot 10^5 = 1710 \cdot kN$

$p_a = 0 \quad F = 467 \cdot 4449 + 4.26 \cdot (0.151 - 0) \cdot 10^5 = 2141 \cdot kN$

$p_a = 15.1 \cdot kPa \quad F = 467 \cdot 4449 + 4.26 \cdot (0.151 - 0.151) \cdot 10^5 = 2080 \cdot kN$

Mattingly problem 4.3 (4.1)

The inlet for a high-bypass-ratio turbofan engine has an area $A_1 = 6.0 m^2$ and is designed to have an inlet Mach number $M_1 = 0.6$. Determine the additive drag at the flight conditions of sea-level static test and Mach number of 0.8 at 12-km altitude.