# Mattingly problem 4.3 (4.1)

The inlet for a high-bypass-ratio turbofan engine has an area  $A_1 = 6.0m^2$  and is designed to have an inlet Mach number  $M_1 = 0.6$ . Determine the additive drag at the flight conditions of sea-level static test and Mach number of 0.8 at 12-km altitude.

$$\begin{split} M_1 &= 0.6 \qquad M_0 = 0 \qquad p_0 = 1.013 \cdot 10^5 \cdot Pa \qquad A_1 = 6 \cdot m^2 \\ D_{add} &= \int_0^1 (p - p_0) dA_x = \dot{m}_1 V_1 + (p_1 - p_0) A_1 - \dot{m}_0 V_0 \\ p_1 &= \frac{p_1}{p_{t1}} \frac{p_{t1}}{p_0} p_0 = (\psi_1)^{-\frac{1}{k}} \cdot 1 \cdot 1.013 \cdot 10^5 = (1.072)^{-\frac{1}{0.2856}} \cdot 1 \cdot 1.013 \cdot 10^5 = 79.4 \cdot kPa \\ \psi_1 &= 1 + \frac{\gamma - 1}{2} M_1^2 = 1 + 0.2 \cdot 0.6^2 = 1.072 \\ \dot{m}_1 V_1 &= \rho_1 V_1^2 A_1 = \rho_1 V_1^2 A_1 \frac{p_1}{p_1} = V_1^2 A_1 \frac{\gamma p_1}{\gamma R T_1} = \gamma p_1 M_1^2 A_1 = 1.4 \cdot 79.4 \cdot 10^3 \cdot 0.6^2 \cdot 6 \\ &= 24.0 \cdot 10^4 \cdot N \\ (p_1 - p_0) A_1 &= (79.4 - 101.3) 6 \cdot 10^3 = -131 \cdot 10^3 \cdot N \\ D_{add} &= 24.0 \cdot 10^4 - 13.1 \cdot 10^4 = 109 \cdot kN \end{split}$$

#### Mach number of 0.8 at 12-km altitude

$$\begin{split} M_{1} &= 0.6 \qquad M_{0} = 0.8 \qquad p_{0} = 19.4 \cdot kPa \qquad A_{1} = 6 \cdot m^{2} \\ M_{0} &= 0.8 \qquad \xrightarrow{ISO} \qquad \frac{A_{0}}{A^{*}} = 1.038 \qquad \frac{p_{0}}{p_{t0}} = 0.656 \\ M_{1} &= 0.6 \qquad \xrightarrow{ISO} \qquad \frac{A_{1}}{A^{*}} = 1.188 \qquad \frac{p_{1}}{p_{t1}} = 0.784 \\ p_{1} &= \frac{p_{1}}{p_{t1}} \frac{p_{t1}}{p_{t0}} \frac{p_{0}}{p_{0}} = \frac{0.784}{0.656} \cdot 19.4 = 23.2 \cdot kPa \\ A_{0} &= \frac{A_{0}}{A^{*}} \frac{A^{*}}{A_{1}} A_{1} = \frac{1.038}{1.188} 6 = 5.24 \cdot m^{2} \\ \dot{m}_{1}V_{1} &= \rho_{1}V_{1}^{2}A_{1} = \rho_{1}V_{1}^{2}A_{1} \frac{p_{1}}{p_{1}} = V_{1}^{2}A_{1} \frac{\gamma p_{1}}{\gamma RT_{1}} = \gamma p_{1}M_{1}^{2}A_{1} = 1.4 \cdot 23.2 \cdot 0.6^{2} \cdot 6 = 70.1 \cdot kN \\ m_{0}V_{0} &= \gamma p_{0}M_{0}^{2}A_{0} = 1.4 \cdot 19.4 \cdot 0.8^{2} \cdot 5.24 = 91.1 \cdot kN \\ (p_{1} - p_{0})A_{1} &= (23.2 - 19.4)6 \cdot 10^{3} = 22.7 \cdot kN \\ D_{add} &= 70.1 + 22.7 - 91.1 = 1.688 \cdot kN \end{split}$$

Consider a subsonic inlet at a flight cruise Mach number of 0.8. The captured streamtube undergoes a prediffusion external to the inlet lip, with an area ratio  $A_0/A_1 = 0.92$  as shown. Calculate



(a) Cp (i.e., the pressure coefficient) at the stagnation point

$$c_{p} = \frac{p_{t0} - p_{0}}{\frac{1}{2}\rho_{0}V_{0}^{2}} = \frac{\gamma p_{0}}{\frac{1}{2}\gamma\rho_{0}V_{0}^{2}} \left(\frac{p_{t0}}{p_{0}} - 1\right) = \frac{a_{0}^{2}}{\frac{1}{2}\gamma V_{0}^{2}} \left(\frac{p_{t0}}{p_{0}} - 1\right) = \frac{2}{\gamma M_{0}^{2}} \left(\frac{p_{t0}}{p_{0}} - 1\right)$$

$$M_{0} \xrightarrow{ISO} \frac{p_{0}}{p_{t0}} = 0.656 \qquad \frac{T_{0}}{T_{t0}} = 0.887 \qquad \frac{A_{0}}{A^{*}} = 1.038$$

$$c_{p} = \frac{2}{\gamma M_{0}^{2}} \left(\frac{p_{t0}}{p_{0}} - 1\right) = \frac{2}{1.4 \cdot 0.8^{2}} \left(\frac{1}{0.656} - 1\right) = 1.171$$
(b) inlet lip Mach number M<sub>1</sub>

$$\frac{A_{1}}{P_{0}} = \frac{A_{1}A_{0}}{P_{0}} = \frac{1.038}{P_{0}} = 1.128 \qquad \frac{ISO}{P_{0}} \qquad M_{0} = 0.658 \qquad \frac{T_{1}}{P_{0}} = 0.920 \qquad \frac{p_{1}}{P_{0}}$$

 $\frac{1}{A^*} = \frac{1}{A_0} \frac{1}{A^*} = \frac{1}{0.92} = 1.128 \qquad \longrightarrow \qquad M_1 = 0.658 \qquad \frac{1}{T_{t1}} = 0.920 \qquad \frac{p_1}{p_{t1}} = 0.748$ (c) lip contraction ratio  $A_1/A_{th}$  for a throat Mach number  $M_{th} = 0.75$  (assume  $p_{t,th}/p_{t1} = 1$ )

$$M_{th} = 0.75 \xrightarrow{ISO} \frac{A_{th}}{A^*} = 1.062 \quad \frac{A_1}{A_{th}} = \frac{A_1}{A^*} \frac{A^*}{A_{th}} = \frac{1.128}{1.062} = 1.062$$

(d) the diffuser area ratio  $A_2/A_{th}$  if  $M_2 = 0.5$  and  $p_{t2}/p_{t,th} = 0.98$ 

$$M_{2} = 0.5 \xrightarrow{ISO} \frac{A_{2}}{A^{*}} = 1.340 \qquad \frac{A_{2}}{A_{th}}\Big|_{ideal} = \frac{A_{2}}{A^{*}}\frac{A^{*}}{A_{th}} = \frac{1.340}{1.062} = 1.261$$

$$\frac{A_{2}}{A_{th}} = \frac{A_{2}}{A_{th}}\Big|_{ideal} \cdot \frac{p_{t,th}}{p_{t2}} = \frac{1.261}{0.98} = 1.287$$
(e) the nondimensional inlet additive drag  $D_{add}/p_{0}A_{1}$ 

$$D_{add} = \dot{m}_{1}V_{1} + (p_{1} - p_{0})A_{1} - \dot{m}_{0}V_{0}$$

$$\frac{a_1}{a_0} = \sqrt{\frac{T_1}{T_{t1}} \frac{T_{t0}}{T_0}} = \sqrt{\frac{0.920}{0.887}} = 1.018 \qquad \frac{p_1}{p_0} = \frac{p_1}{p_{t1}} \frac{p_{t1}}{p_{t0}} \frac{p_{t0}}{p_0} = 0.748 \cdot 1\frac{1}{0.656} = 1.140$$

$$\begin{aligned} \frac{D_{add}}{p_0 A_1} &= \frac{(p_1 - p_0)A_1}{p_0 A_1} + \frac{\dot{m}_0}{p_0 A_1} (V_1 - V_0) = \left(\frac{p_1}{p_0} - 1\right) + \frac{\gamma \rho_0 V_0^2 A_0}{\gamma p_0 A_1} \frac{(V_1 - V_0)}{V_0} \\ &= \left(\frac{p_1}{p_0} - 1\right) + \frac{\gamma M_0^2 A_0}{A_1} \left(\frac{V_1}{V_0} - 1\right) = \left(\frac{p_1}{p_0} - 1\right) + \gamma \frac{A_0}{A_1} M_0 \left(M_1 \frac{a_1}{a_0} - M_0\right) \\ &= 0.140 + 1.4 \cdot 0.92 \cdot 0.8 (0.658 \cdot 1.018 - 0.8) = 6.424 \cdot 10^{-3} \end{aligned}$$

Ao/A1	0.92			
γ	1.4			
k	3.5			
К	3			
pt2/pt0	0.98			
	0	*	th	2
М	0.8	1	0.75	0.5
$\psi$	1.128	1.2	1.1125	1.05
Ψ	0.78035	0.810185	0.762587	0.6046863
			Con le tabelle	
pt0/p0	1.52434	M0	$\pi 0/\pi \tau 0$	0.6560216
			T0/Tt0	0.8865248
Cpre	1.1704		A0/A*	1.03823
A0/A*	1.03823		Cpre	1.1704018
A1/A*	1.12851	Mth	Ath/A*	1.0624171
M1	0.65778		A1/A*	1.1285109
$\psi 1$	1.08654		M1	0.6577826
Ath/A*	1.06242		A1/A $\tau\eta$	1.0622107
A1/Ath	1.06221	M2	A2/A*	1.3398438
A2/A*	1.33984		A2/Ath Ide	1.2611278
A2/Ath Ide	1.26113		$A2/A\tau\eta$	1.2868651
A2/Ath	1.28687	M1	p1/pt0	0.7479045
			T1/Tt0	0.9203564
$\psi 0/\psi 1$	1.03816		p1/p0	1.1400608
Dadd/poA1	0.00633		T1/T0	1.038162
			Dadd/poA1	0.0063316

A normal-shock inlet is operating in a supercritical mode, as shown. Flight Mach number is  $M_0 = 1.6$ . The inlet capture area ratio  $A_0/A_1 = 0.90$  and the diffuser area ratio  $A_2/A_1 = 1.2$  and  $p_{t2}/p_{t1} = 0.95$  Calculate

(a)  $M_1, M_2$ 

(b) inlet total pressure recovery  $\pi_d$ , i.e.  $p_{t2}/p_{t0}$ 



6.1	7	
Input	M0	1.6
Input	A0/A1	0.9
Input	A2/A1	2
Input	pt2/pt1	0.95
NSW	M0v	0.668437
NSW	ptv/ptm	0.8952
lso	A/A*	1.119211
	A1/A*	1.243567
lso	M1	0.557724
	A2/A*	2.362778
lso	M2	0.254573
	pt2/pt0	0.85044

An isentropic convergent–divergent supersonic inlet is designed for  $M_0 = 1.6$ . Calculate the inlet's

(a) area contraction ratio  $A_1/A_{th}$ 

(b) subsonic Mach number where the throat first chokes

$$M_0 = 1.6$$
  $\xrightarrow{Iso}$   $\frac{A_0}{A_0^*} = \frac{A_1}{A_1^*} = \frac{A_1}{A_{th}} = 1.250$   $\xrightarrow{Iso}$   $M_{sub} = 0.557$ 

(c) percent spillage at  $M_0 = 0.7$ 

(d) percent spillage at  $M_0 = 1.6$  (in the unstarted mode)



$$M_{sub} = 0.557 \xrightarrow{NSW} M_{over} = 2.13$$

(f) throat Mach number after the inlet was started, with still M overspeed as the flight Mach number

$$M_{over} = 2.13$$
  $\xrightarrow{ISO}$   $\frac{A_1}{A^*} = 1.935$   $\frac{A_{th}}{A^*} = \frac{A_{th}}{A_1} \frac{A_1}{A^*} = \frac{1.935}{1.250} = 1.549$ 

$$\frac{A_{th}}{A^*} = 1.549$$

 $\xrightarrow{ISO} M_{th} = 1.895$ 

6.18		
Input	M0	1.6
NSW	pt2/pt0	0.8952
iso	A/A*	1.250235
iso	Msub crit	0.553164
Input	M0	0.7
iso	A/A*	1.094373
M=07	%spillage	12.46664
M=1.6	%spillage	10.47997
Nsw	M over	2.155573
iso	A/A*	1.927934
	Ath/A*	1.542057
iso	Mth	1.889287

**6.23** A Kantrowitz–Donaldson inlet is designed for  $M_{\rm D} = 1.7$ . Calculate

- (a) the inlet contraction ratio  $A_1/A_{\rm th}$
- (b) the throat Mach number after the inlet self started
- (c) the total pressure recovery with the best backpressure.



6.23		
Input	M0	1.7
iso	A/A*	1.337606
Nsw	Μv	0.640544
iso	A1/Ath=A,	1.144618
	Ath/A*	1.168605
iso	Mth	1.488692
NSW	ptv/pt0	0.933331

A convergent nozzle experiences  $\pi_n$  of 0.98, the gas ratio of specific heats  $\gamma = 1.30$ , and the gas constant is R = 291 J/kgK. First, calculate the minimum nozzle pressure ratio that will choke the expanding nozzle, i.e.,  $NPR_{crit}$ . This nozzle operates, however, at a higher NPR than the critical, namely, NPR=4. 2 and with an inlet stagnation temperature of  $T_{t7} = 939K$ . Assuming this nozzle operates in  $p_0 = 100kPa$  ambient static pressure, calculate:

$$\gamma = 1.3 \qquad k = \frac{\gamma - 1}{\gamma} = \frac{0.3}{1.3} = 0.2308 \qquad \psi(M = 1) = 1 + \frac{\gamma - 1}{2} = 1 + 0.15 = 1.15$$
$$\frac{p_{t8}}{p_8} = \psi^{\frac{1}{k}} = 1.15 \frac{1}{0.2308} = 1.832 \qquad NPR_{crit} = \frac{p_{t7}}{p_0} = \frac{p_{t7}}{p_8} = \frac{p_{t7}}{p_{t8}} \frac{p_{t8}}{p_8} = \frac{1}{\pi_n} \frac{p_{t8}}{p_8} = \frac{1.832}{0.98} = 1.870$$

(a) the exit static pressure and temperature  $p_9$  and  $T_9$ , respectively

$$NPR = \frac{p_{t7}}{p_0} = 4.2 \qquad p_{t7} = NPR \cdot p_0 = 4.2 \cdot 100 = 420 \cdot kPa$$
$$p_{t8} = p_{t7}\pi_n = 420 \cdot 0.98 = 412 \cdot kPa \qquad p_9 = p_8 = \frac{p_8}{p_{t8}}p_{t8} = \frac{420}{1.832} = 225 \cdot kPa$$
$$T_9 = T_8 = \frac{T_{t7}}{\psi} = \frac{939}{1.15} = 817 \cdot K$$

(b) the actual exit velocity  $V_9$  in m/s

$$V_9 = a_9 = \sqrt{\gamma R T_9} = \sqrt{1.3 \cdot 291 \cdot 817} = 556 \cdot \frac{m}{s}$$

(c) nozzle adiabatic efficiency  $\eta_n$ 

$$\eta_n = \frac{\left(NPR\frac{p_0}{p_9}\right)^{k_9} - \pi_n^{-k_9}}{\left(NPR\frac{p_0}{p_9}\right)^{k_9} - 1} = \frac{1.870^{0.2308} - 0.98^{-0.2308}}{1.870^{0.2308} - 1} = \frac{1.155 - 0.98^{-0.2308}}{1.155 - 1} = 0.972$$

(d) the ideal exit velocity  $V_{9s}$  in m/s

 $V_{9s} = V_9$ 

(e) percentage gross thrust gain, had we used a convergent-divergent nozzle with perfect expansion

$$\begin{aligned} \frac{F_{u_{conv}}}{\dot{m}_8 a_8} &= \frac{V_8}{a_8} + A_8 \frac{p_8 - p_0}{\dot{m}_8 a_8} = \frac{V_8}{a_8} + \frac{p_8 - p_0}{p_8} \frac{p_8 A_8}{\dot{m}_8 a_8} = \frac{V_8}{a_8} + \frac{p_8 - p_0}{p_8} \gamma p_8 \frac{A_8}{\gamma \rho_8 V_8 A_8 a_8} \\ &= \frac{V_8}{a_8} + \frac{p_8 - p_0}{p_8} \frac{a_8^2}{\gamma V_8 a_8} = 1 + \left(1 - \frac{100}{225}\right) \frac{1}{1.3} = 1.427 \end{aligned}$$

$$\begin{aligned} \frac{F_{u_{convdiv}}}{\dot{m}_8 a_8} &= \frac{V_9}{a_8} \frac{T_9}{T_{t9}} = \left(\frac{p_9}{p_{t9}}\right)^k = \left(\frac{p_9}{p_{t7}} \frac{p_{t7}}{p_{t9}}\right)^k = \left(\frac{1}{NPR} \frac{1}{\pi_n}\right)^k = \left(\frac{1}{4.2} \frac{1}{0.98}\right)^{0.2308} \\ &= 0.721 \end{aligned}$$

$$\begin{aligned} T_9 &= \frac{T_9}{T_{t9}} T_{t7} = 0.721 \cdot 939 = 677 \cdot K \qquad a_9 = \sqrt{\gamma RT_9} = \sqrt{1.3 \cdot 291 \cdot 677} = 506 \cdot \frac{m}{s} \\ \frac{T_{t9}}{T_9} &= \psi_9 = 1 + \frac{\gamma - 1}{2} M_9^2 \qquad M_9 = \sqrt{\frac{2}{\gamma - 1}} (\psi_9 - 1) = \sqrt{\frac{2}{0.3}} \left(\frac{1}{0.721} - 1\right) = 1.606 \end{aligned}$$

$$V_{9} = M_{9}a_{9} = 1.606 \cdot 506 = 813 \cdot \frac{m}{s}$$
  
$$\frac{F_{u_{conv_{div}}}}{\dot{m}_{8}a_{8}} = \frac{V_{9}}{a_{8}} = \frac{813}{556} = 1.462 \qquad \% F = \frac{F_{u_{conv_{div}}} - F_{u_{conv}}}{F_{u_{conv}}} = \frac{1.462 - 1.427}{1.427} = 2.4\%$$

(f) nozzle discharge coefficient  $C_{D8}$ 

$$C_{D8} = \frac{\dot{m}_8}{\dot{m}_{8ideal}} = \pi_n = 0.98$$

(g) draw a qualitative wave pattern in the exhaust plume

6.43			
Input	γ	1.3	
Input	R	291	J/K
	k	0.230769	
Input	М	1	
	$\psi$	1.15	
	pt/p	1.832416	
Input	$\pi_{ ext{cn}}$	0.98	
	NPRcr	1.869812	
Input	NPR	4.2	
Input	Tt7	939	К
Input	p0	100	kPa
	pt7	420	
Con perdite	pt9	411.6	
Output	p9	224.6215	
Output	Т9	816.5217	
Output	a9	555.7789	
Output	$\eta_{n}$	1.004673	
	F/m8a8	1.426774	
ideale	p9s	229.2056	
	Т9	816.5217	
	a9	555.7789	
ConDiv Ide	T9/Tt7	0.721436	
	Т9	677.4287	
	a9	506.2324	
	M9	1.604418	
	V9	812.2086	
	F/m8a8	1.461388	
	%	2.426007	

6.44 A convergent-divergent nozzle has a conical exhaust shape with the half-cone angle of  $\alpha = 25^{\circ}$ . Calculate the divergence loss  $C_A$  for this nozzle due to nonaxial exhaust flow. Assuming the same (half) divergence angle of 25°, but in a 2D rectangular nozzle, calculate the flow angularity loss and compare it to the conical case.

$$C_{A3D} = \frac{1 + \cos(\alpha)}{2} = \frac{1 + \cos(25)}{2} = 0.953 \qquad \qquad C_{A2D} = \frac{\sin(\alpha)}{\alpha} = \frac{\sin(25)}{25\frac{\pi}{180}} = 0.969$$

6.44		
Input	alpha	25
	CA	0.953154
	CA2D	0.96857
	%	1.617348

#### Mattingly 2.6

One method of reducing an aircarft's landing distance is through the use of thrust reversers. Consider the turbofan engine in Fig. P2.5 with thrust reverser of the bypass airstream. It is given that 1500 lbm/s of air at 60°F and 14.7 psia enters the engine at a velocity of 450 ft/s and that 1250 lbm/s of bypass air leaves the engine at 60 deg to the horizontal, velocity of 890 ft/s, and pressure of 14.7 psia. The remaining 250 lbm/s leaves the engine core at a velocity of 1200 ft/s and pressure of 14.7 psia. Determine the force on the strut  $F_x$ . Assume the outside of the engine sees a pressure of 14.7 psia.

