

## Esercizio Mattingly 9.35

|              |           |       |                |        |
|--------------|-----------|-------|----------------|--------|
| m            | 40        | kg/s  | Cz2=Cz3        |        |
| $\omega$     | 1000      | rad/s | M1             | 0.3    |
| Ro           | 0.4       | m     | M2             | 1.15   |
| Tt1          | 1780      | K     | $\alpha_{1,3}$ | 0      |
| pt1          | 1,400,000 | Pa    | $\omega_r$     | 0.08   |
| $\Delta T_t$ | 230       | K     | $\omega_s$     | 0.04   |
| R            | 287       | J/kgK |                |        |
| cp           | 1243.6667 |       | $\gamma$       | 1.3    |
| $\Psi^*$     | 0.7607961 |       | k              | 0.2308 |

$$k = \frac{\gamma - 1}{\gamma} = \frac{0.3}{1.3} = 0.2308 \quad K = \frac{\gamma + 1}{2(\gamma - 1)} = \frac{2.3}{0.6} = 3.833$$

### Sezione 1

$$\psi_1 = 1 + \frac{\gamma - 1}{2} M_1^2 = 1 + 0.15 \cdot 0.3^2 = 1.0135$$

$$T_1 = \frac{T_{t1}}{\psi_1} = \frac{1780}{1.0135} = 1756.3 \cdot K$$

$$p_1 = \frac{p_{t1}}{\psi_1^k} = \frac{1400}{1.0135^{0.2308}} = 1321.0 \cdot kPa$$

$$a_{t1} = \sqrt{\gamma R T_{t1}} = 1.3 \cdot 287 \cdot 1780 = 814.9 \cdot m/s$$

$$a_1 = \sqrt{\gamma R T_1} = 1.3 \cdot 287 \cdot 1756 = 809.5 \cdot m/s$$

$$C_1 = M_1 \cdot a_1 = 0.3 \cdot 809.5 = 242.8 \cdot m/s$$

$$C_{\theta 1} = C_1 \cdot \sin \alpha_1 = 0 \cdot m/s \quad C_{z1} = C_1 \cdot \cos \alpha_1 = 242.8 \cdot m/s$$

$$U = \omega r_0 = 1000 \cdot 0.4 = 400 \cdot \frac{m}{s}$$

$$\dot{m} = \frac{p_{t1} \frac{A^*}{A_{1R}} A_{1R} \psi^*}{a_{t1}}$$

$$\Psi(M) = \gamma M \psi^{-K}$$

$$\Psi_1 = \gamma M_1 \psi_1^{-K} = 1.3 \cdot 0.3 \cdot 1.014^{-3.83} = 0.371$$

$$A_1 = \frac{A_{1R}}{\cos \alpha} = \frac{\dot{m} a_{t1}}{\cos \alpha p_{t1} \Psi_1} = \frac{40 \cdot 815}{1400 \cdot 10^3} \frac{1}{0.371} = 0.06285 m^2$$

### Sezione 2

$$M_2 = 1.15 \quad T_{t2} = T_{t1} = 1780 \cdot K \quad \omega_s = 0.04$$

$$\psi_2 = 1 + \frac{\gamma - 1}{2} M_2^2 = 1 + 0.15 \cdot 1.15^2 = 1.1984$$

$$T_2 = \frac{T_{t2}}{\psi_2} = \frac{1780}{1.1984} = 1485K$$

$$a_{t2} = \sqrt{\gamma R T_{t2}} = 1.3 \cdot 287 \cdot 1780 = 814.9m/s$$

$$a_2 = \sqrt{\gamma R T_2} = 1.3 \cdot 287 \cdot 1485 = 744.4m/s$$

$$C_2 = M_2 \cdot a_2 = 1.15 \cdot 744.4 = 856.1m/s$$

$$\omega_s = \frac{p_{t1} - p_{t2}}{p_{t2} - p_2} = \omega_s = \frac{1 - \frac{p_{t2}}{p_{t1}}}{\frac{p_{t2}}{p_{t1}} \left(1 - \frac{p_2}{p_{t2}}\right)} \rightarrow$$

$$p_{t2} = \frac{p_{t1}}{1 + \omega_s \left(1 - \psi_2^{-\frac{1}{k}}\right)} = \frac{1400 \cdot 10^3}{1 + 0.04 \left(1 - 1.1984^{\frac{-1}{0.2308}}\right)} = 1,370KPa$$

$$p_2 = p_{t2} \psi_2^{\frac{-1}{k}} = 1370 \cdot 1.1984^{\frac{-1}{0.2308}} = 625.5kPa$$

Dall'equazione di Eulero:

$$C_{\theta 2} = -\frac{\Delta h_t}{U} = \frac{c_p \Delta T_t}{U} = 1243.7 \cdot \frac{230}{400} = 715.1m/s$$

$$\alpha_2 = \arcsin \frac{C_{\theta 2}}{C_2} = \arcsin \frac{715.1}{856.1} = 56.65^\circ$$

$$C_{z2} = C_2 \cdot \cos \alpha_2 = 470.7m/s$$

$$\Psi_2 = \gamma M_2 \psi_2^{-K} = 1.3 \cdot 1.15 \cdot 1.198^{-3.83} = 0.748$$

$$A_2 = \frac{A_{2R}}{\cos \alpha_2} = \frac{\dot{m} a_{t2}}{\cos \alpha_2 p_{t2} \Psi_2} = \frac{40 \cdot 815}{0.5498 \cdot 1,370 \cdot 10^3} \frac{1}{0.748} = 0.05792m^2$$

## Sezione 2r

$$W_{\theta 2} = C_{\theta 2} - U = 715.1 - 400 = 315.1 \frac{m}{s}$$

$$W_2 = \sqrt{W_{\theta 2}^2 + W_{z2}^2} = \sqrt{470.7^2 + 315.1^2} = 566.4 \frac{m}{s}$$

$$\beta_2 = \arctan \frac{W_{\theta 2}}{W_{z2}} = \arctan \frac{315.1}{470.7} = 33.80^\circ$$

$$M_{2R} = \frac{W_2}{a_2} = \frac{566.4}{744.4} = 0.7609$$

$$\psi_{2R} = 1 + \frac{\gamma - 1}{2} M_{2R}^2 = 1 + 0.15 \cdot 0.7609^2 = 1.0868$$

$$T_{t2R} = T_2 \psi_{2R} = 1485 \cdot 1.0868 = 1614K$$

$$p_{t2R} = p_2 \psi_{2R}^{\frac{1}{k}} = 625.5 \cdot 1.0868^{\frac{1}{0.2308}} = 897,3 kPa$$

### Sezione 3

$$C_{z3} = C_{z2} = 470.7 m/s \quad \alpha = 0 \rightarrow C_3 = C_{z3} = 470.7 m/s \quad C_{\theta_3} = 0 m/s$$

$$T_{t3} = T_{t2} - \Delta T_t = 1780 - 230 = 1550 K$$

$$T_3 = T_{t3} - \frac{C_3^2}{2c_p} = 1550 - \frac{470.7^2}{2 \cdot 1244} = 1460.9 K$$

$$\psi_3 = \frac{T_{t3}}{T_3} = \frac{1550}{1460.9} = 1.0610$$

$$a_3 = \sqrt{\gamma R T_3} = 1.3 \cdot 287 \cdot 1460.9 = 738.3 m/s$$

$$a_{t3} = \sqrt{\gamma R T_{t3}} = 1.3 \cdot 287 \cdot 1550 = 760.5 m/s$$

$$M_3 = \frac{C_3}{a_3} = \frac{470.7}{738.3} = 0.6375$$

### Sezione 3r

$$W_{\theta 3} = C_{\theta 3} - U = -400 = -400 \frac{m}{s}$$

$$W_3 = \sqrt{W_{\theta 3}^2 + W_{z3}^2} = \sqrt{400^2 + 470.7^2} = 617.7 \frac{m}{s}$$

$$\beta_3 = \text{atan} \frac{W_{\theta 3}}{W_{z3}} = \text{atan} \frac{-400}{470.7} = -40.36^\circ$$

$$M_{3R} = \frac{W_3}{a_3} = \frac{617.7}{738.3} = 0.8366$$

$$\psi_{3R} = 1 + \frac{\gamma - 1}{2} M_{3R}^2 = 1 + 0.15 \cdot 0.8366^2 = 1.1050$$

$$T_{t3R} = T_{t2R} = 1614 K$$

$$p_{t3R} = \frac{p_{t2r}}{1 + \omega_r \left( 1 - \psi_{3R}^{\frac{-1}{k}} \right)} = \frac{897,3 \cdot 10^3}{1 + 0.08 \left( 1 - 1.1050^{\frac{-1}{0.2308}} \right)} = 872,8 kPa$$

$$p_3 = p_{t3R} \psi_{3R}^{\frac{-1}{k}} = 872,8 \cdot 1.1050^{\frac{-1}{0.2308}} = 566,2 kPa$$

### Sezione 3

$$p_{t3} = p_3 \psi_3^{\frac{1}{k}} = 566,2 \cdot 1.0610^{\frac{1}{0.2308}} = 731,8 kPa$$

$$\Psi_3 = \gamma M_3 \psi_3^{-K} = 1.3 \cdot 0.638 \cdot 1.061^{-3.83} = 0.661$$

$$A_3 = \frac{A_{3R}}{\cos \alpha_3} = \frac{\dot{m} a_{t3}}{\cos \alpha_3 p_{t3}} \frac{1}{\Psi_3} = \frac{40 \cdot 760.5}{731,8 \cdot 10^3} \frac{1}{0.661} = 0.06293 m^2$$

### Parametri dello stadio

$$\circ_R = \frac{h_2 - h_3}{h_1 - h_3} = \frac{1485.3 - 1460.9}{1756.3 - 1460.9} = 0.0826$$

$$\eta = \frac{1 - \tau_t}{1 - \pi_t^k} = \frac{1 - \frac{1550}{1780}}{1 - \left(\frac{732}{1400}\right)^{0.2308}} = 0.9293$$

$$e_c = \frac{\ln \tau_t}{k \ln \pi_t} = \frac{\ln \frac{1550}{1780}}{0.2308 \ln \frac{1400}{731,8}} = 0.9241$$

$$\Psi_c = \frac{c_p \Delta T_c}{U^2} = \frac{1244 \cdot 230}{400^2} = 1.788$$

|                  | 1                | 2             | 2R       | 3R      | 3             |
|------------------|------------------|---------------|----------|---------|---------------|
| Tt               | <b>1780</b>      | 1780.0        | 1614.3   | 1614.3  | <b>1550.0</b> |
| T                | 1756.3           | 1485.3        | 1485.3   | 1460.9  | 1460.9        |
| pt               | <b>1,400,000</b> | 1,370,211     | 897,283  | 872,762 | 731,764       |
| p                | 1,320,966        | 625,488       | 625487.9 | 566,246 | 470.7         |
| M                | <b>0.3</b>       | <b>1.1500</b> | 0.7609   | 0.8366  | 0.6375        |
| C/W              | 242.8            | 856.1         | 566.4    | 617.7   | 470.7         |
| W/C <sub>z</sub> | 242.8            | 470.7         | 470.7    | 470.7   | 470.7         |
| W/C <sub>θ</sub> | 0.0              | 715.1         | 315.1    | -400.0  | 0.0           |
| α                | <b>0</b>         | 56.65         |          |         | <b>0</b>      |
| β                |                  |               | 33.80    | -40.36  |               |