Mattingly example 3.3

Consider both a two-stage vehicle and a three-stage vehicle for the launch of the 900-kg (2000-lbm) payload. Each stage uses a liquid H_2 -O_2 chemical rocket (C ¼ 4115 m/s, 13,500 ft/s), and the DV total of 14,300 m/s (46,900 ft/s) is split evenly between the stages. Suppose $\delta = m_{str}/m_0 = 0.03$.

MPayLoad	900
c	4115
DV	14300
$\delta = m_{str}/m_0$	0.03

#Stadi	1	2	3	
Dv	14,300	7,150	4,767	m/s
mpay	900	900	900	kg
MR	0.03096	0.1760	0.3140	
lambda	9.590E-04	0.1460	0.2840	
m01	938,465	6,166	3,169	kg
mstr1	28,154	185	95	kg
mp1	909,411	5,081	2,174	kg
m02		42,250	11,159	kg
mstr2		1,267	335	kg
mp2		34,816	7,655	kg
m03			39,291	kg
mstr3			1,179	kg
mp3			26,953	kg

Mono stadio

$$\Delta V = -\bar{c}\ln\frac{m_f}{m_0} = \bar{c}\ln\frac{1}{MR} \rightarrow MR = e^{-\frac{\Delta V}{c}} = e^{-\frac{14300}{4115}} = 0.0309590$$

$$\lambda = \frac{m_{pay}}{m_0} = MR - \frac{m_{str}}{m_0} = MR - \delta = 0.03096 - .03000 = 9.590 \cdot 10^{-4}$$

$$m_0 = \frac{m_{pay}}{\lambda} = \frac{900}{9.590 \cdot 10^{-4}} = 938.5 \cdot 10^3 kg$$

$$m_{str} = \delta m_0 = 0.0300 \cdot 938.5 \cdot 10^3 = 28{,}150 kg$$

$$m_p = m_0(1 - MR) = 938.5 \cdot 10^3(1 - 0.03096) = 909.4 \cdot 10^3 kg$$

Due stadi

$$\frac{\Delta V}{2} = -\bar{c}\ln\frac{m_f}{m_0} = \bar{c}\ln\frac{1}{MR} \rightarrow MR = e^{-\frac{\Delta V}{2c}} = e^{-\frac{14300}{2\cdot 4115}} = 0.1760$$

$$\lambda = \frac{m_{pay}}{m_0} = MR - \frac{m_{str}}{m_0} = MR - \delta = 0.1760 - .03000 = 0.1460$$

$$m_{01} = \frac{m_{pay1}}{\lambda} = \frac{900}{0.1460} = 6{,}166 kg$$

$$m_{str1} = \delta m_{01} = 0.0300 \cdot 6{,}166 = 185 kg$$

$$m_{p1} = m_{01}(1 - MR) = \ 6{,}166(1 - 0.1760) = 5{,}081kg$$
$$m_{pay2} = m_{01} \rightarrow m_{02} = \frac{m_{pay2}}{\lambda} = \frac{6{,}166}{0.1460} = 42{,}250kg$$
$$m_{str1} = \delta m_{01} = 0.0300 \cdot 42{,}250 = 1267kg$$
$$m_{p1} = m_{01}(1 - MR) = \ 42{,}250(1 - 0.1760) = 34{,}816kg$$

Tre stadi

Procedendo in modo analogo si costruisce la tabella.

Mattingly example 3.7

The space shuttle main engine (SSME) operates for up to 520 s in one mission at altitudes over 100 miles. The nozzle expansion ratio 1 is 77:1, and the inside exit diameter is 2.30m. Assume a calorically perfect gas with the following properties:

Ae/Ag	77	
De	2.298	m
γ	1.25	
pc	2.068E+07	Pa
Tt	4083	K
R	602.6	J/kgK
M2	5.092	

We want to calculate the following:
1) Characteristic velocity c^*
2) Mass flow rate of gases through the nozzle.
3) Pressure at which the nozzle is "on-design."
4) Pressure at which the nozzle is just separated (assume separation occurs when p_a>3.5 p_{r3}).
5) Thrust coefficient C_F, specific impulse and thrust for p_0=0, p_s.

$$A_e = \frac{\pi}{4}\, D_e^2 = \frac{3.141 \cdot 2.298^2}{4} = 4.148m^2 \rightarrow A_{th} = \frac{A_e}{77} = \frac{4.148}{77} = 0.05387m^2$$
$$a_t = \sqrt{\gamma R T_t} = \sqrt{1.25 \cdot 602.6 \cdot 4083} = 1754\frac{m}{s}$$
$$k = \frac{\gamma - 1}{\gamma} = \frac{0.25}{1.25} = 0.20 \qquad K = \frac{(\gamma + 1)}{2(\gamma - 1)} = \frac{2.25}{0.5} = 4.5$$
$$\psi = 1 + \frac{\gamma - 1}{2}M^2 \qquad \psi^* = \frac{\gamma + 1}{2} = \frac{2.25}{2} = 1.125$$
$$\Psi(\gamma, M) = \gamma M \psi^{-K} \qquad \Psi^* = 1.25 \cdot 1.125^{-4.5} = 0.7357$$
$$c^* = \frac{p_c A_{\text{th}}}{\dot{m}_p} = \frac{a_t}{\Psi^*} = \frac{\sqrt{\gamma R T_c}}{\gamma \left(\frac{2}{\gamma + 1}\right)^{\frac{(\gamma+1)}{2(\gamma-1)}}} = \frac{1754}{0.7357} = 2384\frac{m}{s}$$

$$\dot{m} = \frac{p_c A_{th} \Psi^*}{a_t} = \frac{2.068 \cdot 10^7 \cdot 0.05387 \cdot 0.7357}{1754} = 467.5 \frac{kg}{s}$$

$$\psi_2 = 1 + \frac{\gamma - 1}{2} M_2^2 = 1 + \frac{0.25}{2} 5.092^2 = 4.240$$

$$p_2 = p_c \psi^{-\frac{1}{k}} = 2.068 \cdot 10^7 \cdot 4.240^{-5} = 15.09 kPa$$

$$p_s = p_2 \cdot 3.5 = 15.09 \cdot 3.5 = 52.80 kPa$$

$$C_F = \frac{\dot{m}_p V_2 + (p_2 - p_0) A_2}{p_c A_{th}} = \sqrt{\frac{2\gamma^2}{\gamma - 1} \left(\frac{2}{\gamma + 1}\right)^{\frac{(\gamma+1)}{(\gamma-1)}} \left[1 - \left(\frac{p_2}{p_c}\right)^k\right]} + \frac{(p_2 - p_0)}{p_c} \frac{A_2}{A_{th}}$$

$$C_F = \sqrt{\frac{2 \cdot 1.25^2}{0.25} \left(\frac{2}{2.25}\right)^{\frac{2.25}{0.25}} [1 - 4.240^{-1}]} + \frac{(p_2 - p_0)}{p_c} 77 = 1.819 + \frac{(p_2 - p_0)}{p_c} 77$$

$$p_0 = 0 \rightarrow C_F = 1.819 + \frac{15.09}{2.068 \cdot 10^4} 77 = 1.875$$

$$F = C_F p_c A_{\text{th}} = 1.875 \cdot 2.068 \cdot 10^7 \cdot 0.05387 = 2090 kN$$

$$I_s = \frac{F}{g_0 \dot{m}_p} = \frac{2090 \cdot 1000}{9.807 \cdot 467.5} = 455.8 s$$

$$p_0 = 52.80 \rightarrow C_F = 1.819 + \frac{15.09 - 52.80}{2.068 \cdot 10^4} 77 = 1.679$$

$$F = C_F p_c A_{\text{th}} = 1.679 \cdot 2.068 \cdot 10^7 \cdot 0.05387 = 1871 kN$$

$$I_s = \frac{F}{g_0 \dot{m}_p} = \frac{1871 \cdot 1000}{9.807 \cdot 467.5} = 408.0 s$$

Farokhi problem 12.2

A solid rocket motor has a design chamber pressure of 10 MPa, an end-burning grain with n = 0.4 and $\dot{r}$ = 3cm/s at the design chamber pressure and design grain temperature of 15C. The temperature sensitivity of the burning rate is σ_p= 0.002/C, and chamber pressure sensitivity to initial grain temperature is π_K=0.005/C. The nominal effective burn time for the rocket is 120 s, i.e., at design conditions.

Calculate

(a) the new chamber pressure and burning rate when the initial grain temperature is 75C;

(b) the corresponding reduction in burn time Δt_b in seconds.

$$\Delta T = T_{c2} - T_{c1} = 75 - 15 = 60C$$

$$\Delta p = p_{c1} \pi_K \Delta T = 10 \cdot 0.005 \cdot 60 = 3MPa$$

$$p_{c2} = p_{c1} + \Delta p = 10 + 3 = 13MPa$$

Lo spessore del grano è:

$$h = \dot{r}_1 t_{b1} = 3 \cdot 120 = 360cm$$

$$\dot{r}_{2p} = \dot{r}_1 \left(\frac{p_{c2}}{p_{c1}}\right)^n = 3 \cdot \left(\frac{13}{10}\right)^{0.4} = 3.332\ cm/s$$

$$\dot{r}_2 = \dot{r}_{2p}(1 + \pi_K \Delta T) = 3.332\frac{cm}{s} \cdot (1 + 0.002 \cdot 60) = 3.732\frac{cm}{s}$$

$$t_{b2} = \frac{h}{\dot{r}_2} = \frac{360}{3.732} = 96.47s$$

$$\Delta t_b = t_{b2} - t_{b1} = 120 - 96.47 = 23.53s$$

Farokhi problem 12.20

Consider a scramjet in a Mach 6 flight. The fuel for this engine is hydrogen with Q_R=120,000kJ/kg. The inlet uses multiple oblique shocks with a total pressure recovery following MIL-E-5008B standards for M 0 > 5, i.e.,

$$\pi_d = \frac{800}{M_0^4 + 935}$$

The combustor entrance Mach number is M2=2.6. Use frictionless, constant-pressure heating, i.e., p4= p2, to simulate the combustor with combustor exit Mach number M4= 1.0. All component parameters and gas constants are shown in the schematic drawing below.

Calculate

(a) Inlet static temperature ratio T2/T0

(b) combustor exit temperature T4 in K

(c) fuel-to-air ratio f

(d) nozzle exit Mach number M10

(e) nondimensional ram drag Dram/p0A1 (note that A0= A1)

(f) nondimensional gross thrust Fg/p0A1

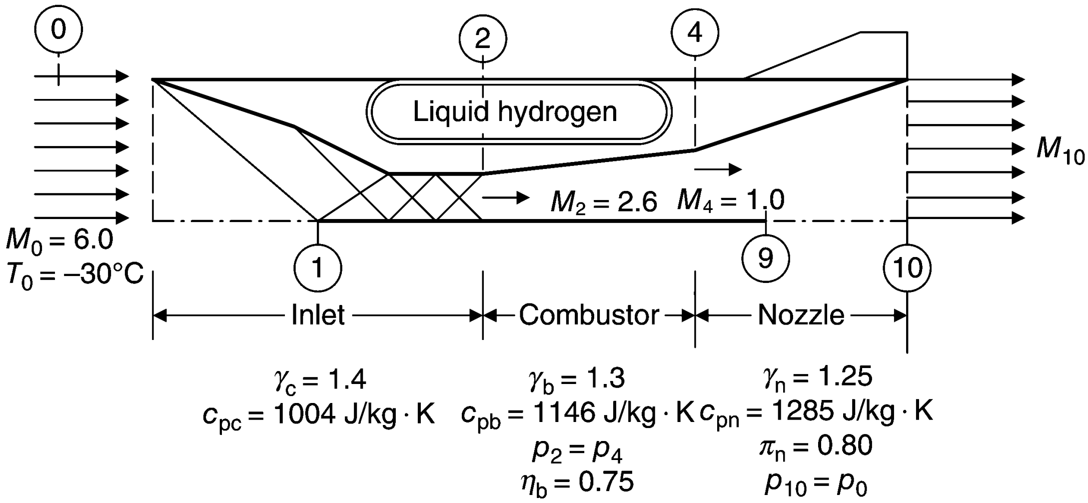
(g) fuel-specific impulse Is in seconds

(h) combustor area ratio A4/A2

(i) nozzle area ratio A10/A4

(j) thermal efficiency

(k) propulsive efficiency



$$k = \frac{\gamma - 1}{\gamma} \rightarrow k_0 = 0.2857,\ \ k_{\rm b} = 0.2308,\ \ k_{\rm n} = 0.20$$

$$K = \frac{\gamma + 1}{2(\gamma - 1)} \rightarrow K_0 = 3.00,\ \ K_2 = 3.833,\ \ K_{10} = 4.50$$

$$T_0 = -30 + 273.15 = 243.15K$$

$$a_0 = \sqrt{\gamma_0 k_0 c_{p0} T_0} = \sqrt{1.40 \cdot 0.2867 \cdot 1004 \cdot 243.15} = 312.5\frac{m}{s}$$

$$V_0 = M_0 a_0 = 6 \cdot 312.5 = 1875 \frac{m}{s}$$

$$\psi = 1 + \frac{\gamma - 1}{2} M^2 \rightarrow \quad \psi_0 = 1 + 0.20 \cdot 36 = 8.2 \quad \psi_2 = 1 + 0.20 \cdot 2.6\text{^}2 = 2.352$$

$$\frac{T_2}{T_0} = \frac{T_2}{T_{t2}} \frac{T_{t0}}{T_0} = \frac{\psi_0}{\psi_2} = \frac{8.2}{2.352} = 3.486 \rightarrow T_2 = 3.486 \cdot 243.15 = 847.7K$$

$$T_{t2} = 847.7 \cdot 2.352 = 1994K$$

$$\pi_d = \frac{800}{M_0^4 + 935} = \frac{800}{36 + 935} = 0.3586$$

$$\frac{p_{t0}}{p_o} = \psi_0^{\frac{1}{k_0}} = 8.2^{\frac{1}{0.2857}} = 1579 \qquad \frac{p_{t2}}{p_2} = \psi_2^{\frac{1}{k_0}} = 2.352^{\frac{1}{0.2857}} = 19.95$$

$$\frac{p_2}{p_0} = \frac{p_2}{p_{t2}} \frac{p_{t0}}{p_o} = \frac{1579}{19.95} = 28.37$$

$$\frac{p_{t2}}{p_0} = \frac{p_{t2}}{p_2} \frac{p_2}{p_o} = 19.95 \cdot 28.37 = 566.2$$

Dalla conservazione della massa e della QM, per p2=p4, $\gamma_2 = \gamma_4 = \gamma_b$, $R_2 = R_4 = R_b$ e trascurabilità degli sforzi viscosi si avrebbe:

$$\rho V A = \frac{\gamma p \rho V A}{\gamma p} = \frac{\gamma p V A}{a^2} = \frac{\gamma p M A}{a} \rightarrow \frac{M_2 A_2}{\sqrt{T_2}} = (1 + f) \frac{M_4 A_4}{\sqrt{T_4}}$$

$$\sqrt{\frac{T_4}{T_2}} = (1 + f) \frac{M_4 A_4}{M_2 A_2}$$

$$A(p + \rho V^2) = pA(1 + \gamma M^2) = cost \rightarrow \frac{A_4}{A_2} = \frac{1 + \gamma_b M_2^2}{1 + \gamma_b M_4^2}$$

$$\sqrt{\frac{T_4}{T_2}} = (1 + f) \frac{M_4 A_4}{M_2 A_2} = (1 + f) \frac{M_4}{M_2} = (1 + f) \frac{1}{2.6} \frac{1 + 1.3 \cdot 2.6^2}{2.3} = (1 + f) 1.637$$

$$\frac{T_4}{T_2} = (1 + f)^2 \cdot 2.679$$

Seguendo il libro invece si suppone che:

$$\frac{A_4}{A_2} = \frac{M_2^2}{M_4^2} \rightarrow \sqrt{\frac{T_4}{T_2}} = \frac{M_2}{M_4} = 2.6 \rightarrow \frac{T_4}{T_2} = 6.76 \rightarrow T_4 = 6.76 \cdot 847.7 = 5731K$$

$$\psi = 1 + \frac{\gamma - 1}{2} M^2 \rightarrow \quad \psi_4 = 1 + 0.15 = 1.15$$

$$T_{t4} = T_4 \, \psi_4 = 5731 \cdot 1.15 = 6590K$$

Dalla conservazione dell'energia si avrebbe:

$$\dot{m}_f Q_R \eta_b = \dot{m}_0 c_{pb} [(1 + f) T_{t4} - T_{t2}] \rightarrow f = \frac{T_{t4} - T_{t2}}{Q_R \eta_b / c_{pb} - T_{t4}}$$

$$f = \frac{6590 - 1994}{120 \cdot 10^6 \cdot 0.75/1146 - 6590} = 0.06389$$

Seguendo il libro invece si suppone che:

$$f = \frac{c_{pb}(T_4 - T_2)}{Q_R \eta_b} = \frac{1146 \cdot (5731 - 847)}{120 \cdot 10^6 \cdot 0.75} = 0.06217$$

$$\frac{p_{t4}}{p_0} = \frac{p_{t4}}{p_4}\frac{p_2}{p_o} = \psi_4^{k_4}\frac{p_2}{p_o} = 1.15^{0.2308} 19.95 \cdot 28.37 = 51.99$$

$$\frac{p_{t10}}{p_0} = \frac{p_{t10}}{p_{10}} = \frac{p_{t4}}{p_0}\pi_n = 51.99 \cdot 0.8 = 41.59$$

$$\frac{p_{t10}}{p_{10}} = \psi_{10}^{\frac{1}{k_n}} \rightarrow \psi_{10} = 41.59^{0.2} = 2.108$$

$$T_{10} = \frac{T_{t10}}{\psi_{10}} = \frac{6590}{2.108} = 3127$$

$$M_{10} = \sqrt{(\psi_{10} - 1)\frac{2}{\gamma_n - 1}} = \sqrt{1.108\frac{2}{0.25}} = 2.977$$

$$a_{10} = \sqrt{\gamma_n k_n c_{pn} T_{10}} = \sqrt{1.250 \cdot 0.200 \cdot 11285 \cdot 3127} = 1002\frac{m}{s}$$

$$V_{10} = M_{10} a_{10} = 2.977 \cdot 1002 = 2983\frac{m}{s}$$

$$\frac{D_{ram}}{p_0 A_1} = \frac{\dot{m}_0 V_0}{p_0 A_1} = \frac{\rho_0 V_0^2 A_1}{p_0 A_1} = \gamma_n M_0^2 = 1.25 \cdot 36 = 50.40$$

$$\frac{F_g}{p_0 A_1} = \frac{\dot{m}_{10} V_{10}}{p_0 A_1} = (1+f)\frac{\rho_0 V_0 V_{10} A_1}{p_0 A_1} = (1+f)\frac{\gamma_0 M_0 V_{10}}{a_0} = 1.062\frac{1.4 \cdot 6 \cdot 2983}{312.5} = 85.18$$

$$\frac{F_u}{p_0 A_1} = \frac{F_g - D_{ram}}{p_0 A_1} = 85.18 - 50.40 = 34.78$$

$$I_s = \frac{F_u}{\dot{m}_f g} = \frac{\dot{m}_0[(1+f)V_{10} - V_0]}{\dot{m}_f g} = \frac{(1+f)V_{10} - V_0}{fg} = \frac{1.062 \cdot 2983 - 1875}{0.06217 \cdot 9.81} = 2122s$$

Dalla $\frac{M_2 A_2}{\sqrt{T_2}} = (1+f)\frac{M_4 A_4}{\sqrt{T_4}}$ trascurando f si ritrova

$$\frac{A_4}{A_2} = 6.76$$

$$\frac{p_{10} M_{10} A_{10}}{\sqrt{T_{10}}} = \frac{p_4 M_4 A_4}{\sqrt{T_4}}$$

$$\frac{A_{10}}{A_4} = \frac{p_4}{p_{10}}\frac{M_4}{M_{10}}\sqrt{\frac{T_{10}}{T_4}} = \frac{p_2}{p_0}\frac{M_4}{M_{10}}\sqrt{\frac{T_{10}}{T_4}} = 28.37\frac{1}{2.977}\sqrt{\frac{3127}{5731}} = 7.040$$

$$\eta_{th} = \frac{\Delta K\dot{E}}{\mathcal{P}_t} = \frac{(1+f)V_9^2 - V_0^2}{2fQ_R} = \frac{(1.062 \cdot 2983^2 - 1875^2)/2}{0.06217 \cdot 120 \cdot 10^6} = \frac{2.970 \cdot 10^6}{7.461 \cdot 10^6}$$
$$= 0.398$$
$$\eta_p = \frac{F_u V_0}{\Delta K\dot{E}} = \frac{2[(1+f)V_{10} - V_0]V_0}{(1+f)V_9^2 - V_0^2} = \frac{(1.062 \cdot 2983 - 1875)1875}{2.970 \cdot 10^6} = \frac{2.426 \cdot 10^6}{2.970 \cdot 10^6}$$
$$= 0.8170$$