



Propulsione Aerospaziale

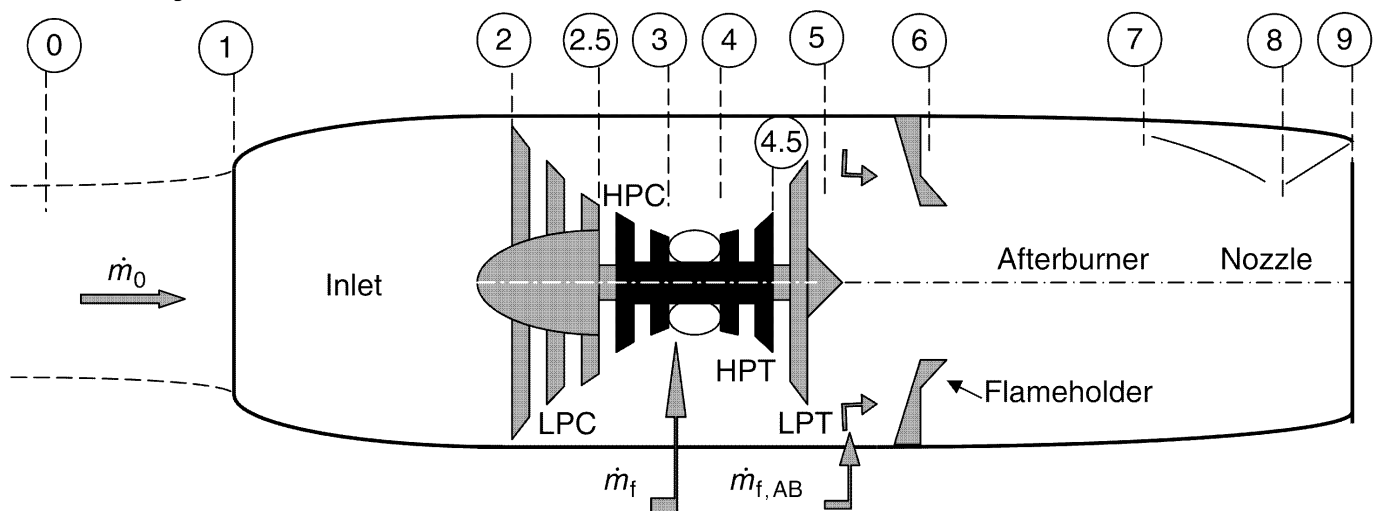
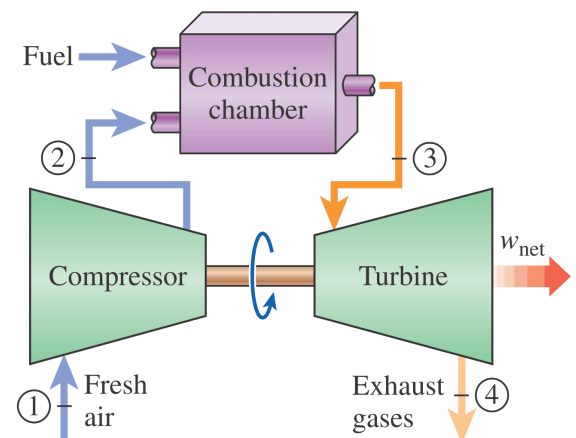
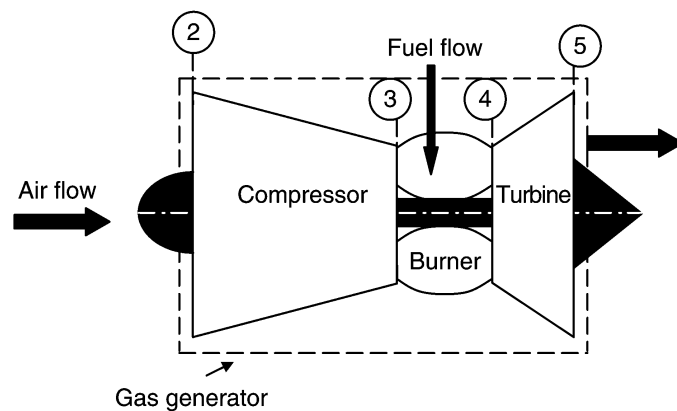
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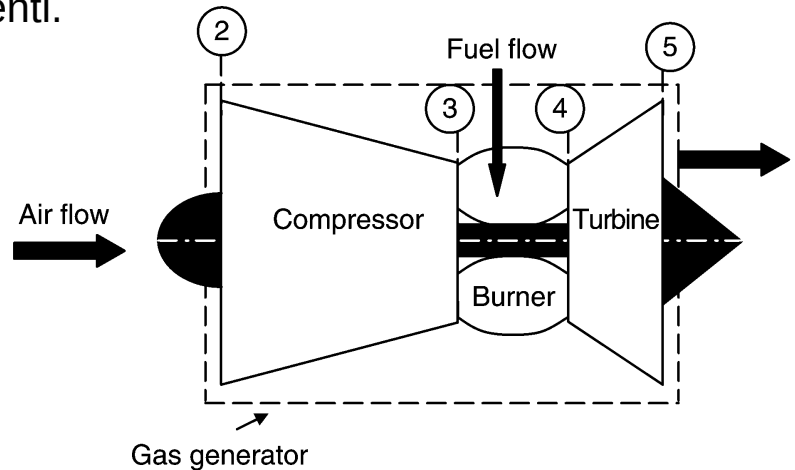
Generatore di gas



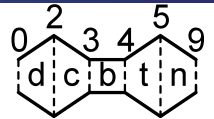
Generatore di gas

I parametri fondamentali di un **generatore di gas** sono:

- Il **rapporto di compressione** del compressore $\pi_c = \frac{p_{t3}}{p_{t2}}$;
- La **portata di aria** nel compressore $\dot{m}_0 \left[\frac{kg}{s} \right]$;
- La **portata di combustibile** $\dot{m}_f$ o la **temperatura all'ingresso della turbina** $T_{t4} [K]$;
- Il **potere calorifico** del combustibile $Q_R \left[\frac{kJ}{kg} \right]$;
- I **rendimenti** dei singoli componenti.



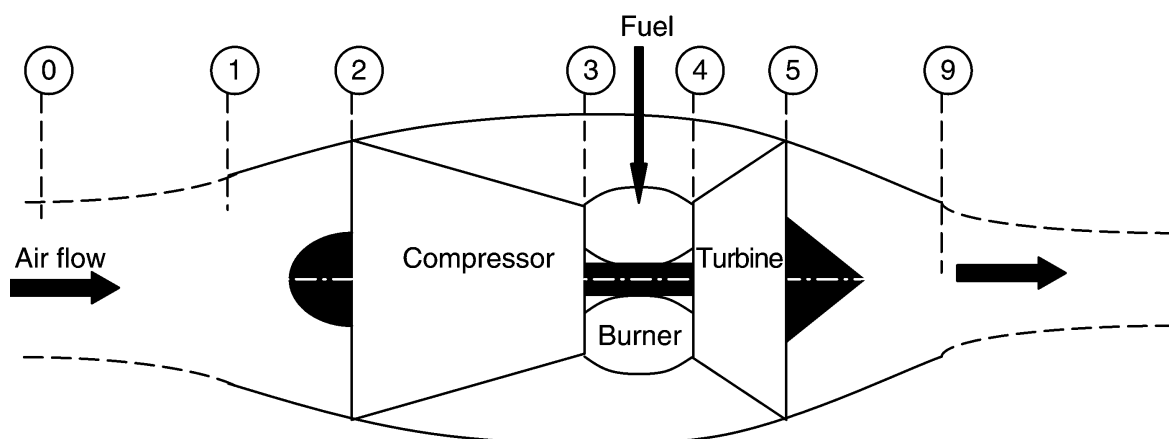
Turbogetto



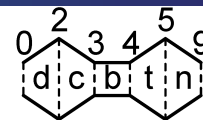
I **componenti** di un **turbogetto** sono:

- Presa d'aria (PA);
- Compressore;
- Camera di combustione (CC);
- Turbina;
- Ugello.

In questa prima fase questi **componenti** verranno esaminati solo da un punto di vista **termodinamico**.



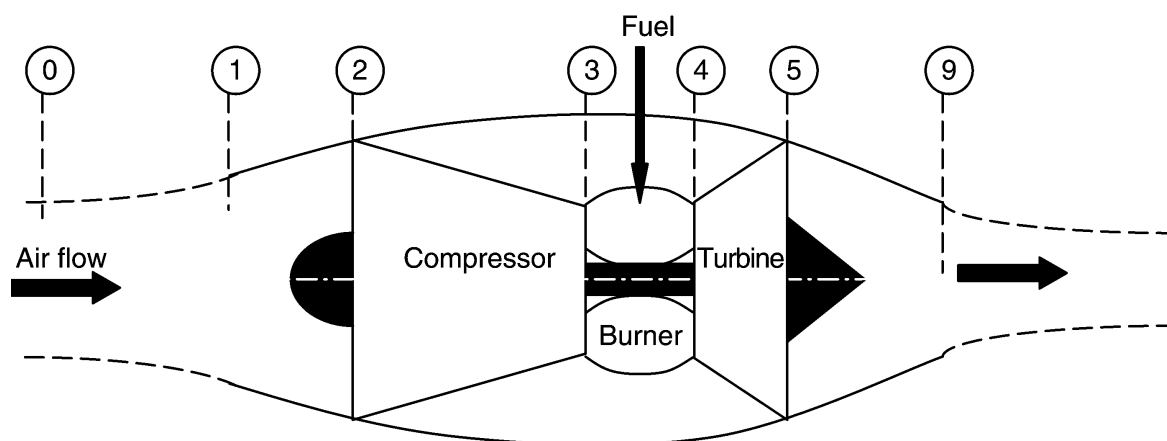
Generatore di gas



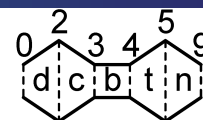
Per semplificare la trattazione si utilizzeranno le seguenti **notazioni**:

$$k = \frac{\gamma - 1}{\gamma} \quad \psi = 1 + \frac{\gamma - 1}{2} M^2 = \frac{T_t}{T} = \frac{h_t}{h} \quad \theta = \frac{h_t}{h_0}$$

mentre con π e τ si indicano i **rapporti** di **pressione** e **entalpia** (tendenzialmente totali) valle-monte dei singoli componenti.



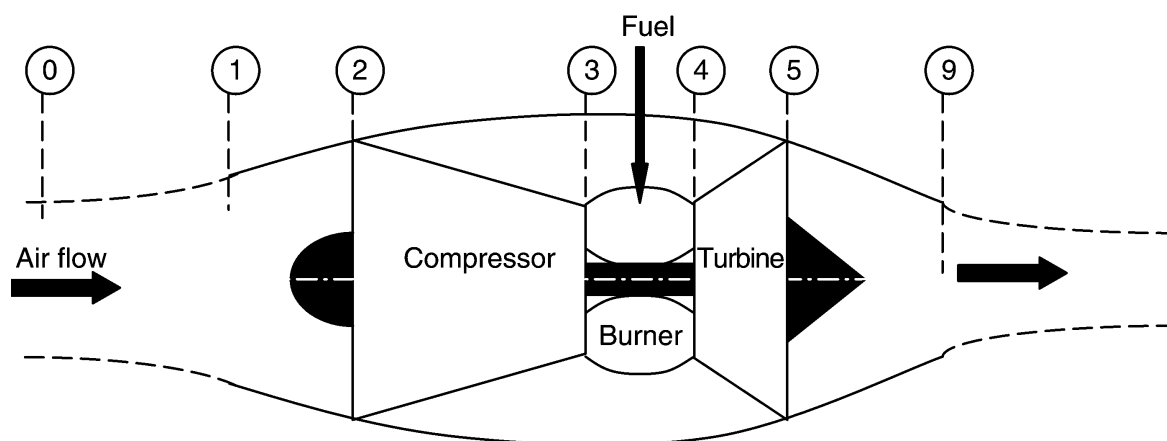
Generatore di gas



In dettaglio:

$$\begin{aligned} \tau_r &= \frac{h_{t0}}{h_0} & \tau_d &= \frac{h_{t2}}{h_{t0}} & \tau_c &= \frac{h_{t3}}{h_{t2}} & \tau_b &= \frac{h_{t4}}{h_{t3}} & \tau_t &= \frac{h_{t5}}{h_{t4}} & \tau_n &= \frac{h_{t9}}{h_{t5}} \\ \pi_r &= \frac{p_{t0}}{p_0} & \pi_d &= \frac{p_{t2}}{p_{t0}} & \pi_c &= \frac{p_{t3}}{p_{t2}} & \pi_b &= \frac{p_{t4}}{p_{t3}} & \pi_t &= \frac{p_{t5}}{p_{t4}} & \pi_n &= \frac{p_{t9}}{p_{t5}} \end{aligned}$$

$$\text{Inoltre: } \tau_\lambda = \theta_t = \theta_4 = \frac{h_{t4}}{h_0} = c_{pt} \frac{T_{t4}}{c_p T_0}$$



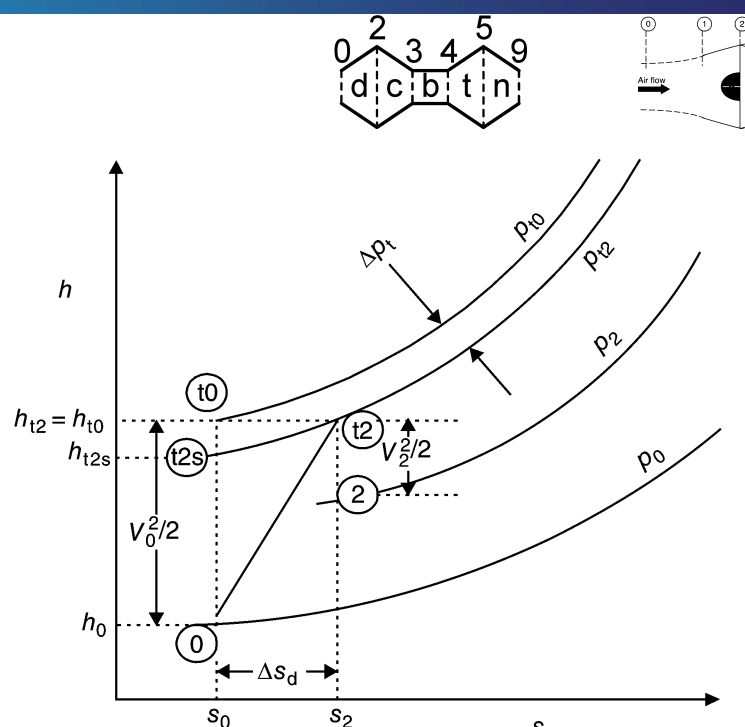
Il funzionamento della PA in condizioni ideali è **adiabatico** e **reversibile**. In **condizioni reali** la reversibilità viene a mancare ma l'ipotesi di **adiabaticità** è ragionevolmente **soddisfatta**.

Come si vede dalla figura una conversione **reversibile** ($0 \rightarrow t_0$) dell'energia cinetica porterebbe ad una **pressione** di ristagno **maggiore** di quella raggiunta in un processo reale. Poiché il processo è adiabatico l'entalpia totale (di ristagno) rimane invariata ($h_{t_0} = h_{t_2}$).

Presatura d'aria

Quindi se la trasformazione fosse **reversibile** si raggiungerebbe la pressione di ristagno t_2 con una minore conversione di energia cinetica.

La **differenza** fra l'entalpia dei punti t_0 e t_{2s} rappresenta proprio l'aliquota di energia cinetica che, a causa di **irreversibilità** è stata **convertita inutilmente** in **energia termica**.



Si può quindi definire un **rendimento adiabatico** del diffusore come:

$$\eta_d = \frac{h_{t2s} - h_0}{h_{t2} - h_0}$$



Presatura d'aria

ricordando che: $p_{t2s} = p_{t2}$, $h = c_p T$, $T_t = T \left(1 + \frac{\gamma-1}{2} M^2\right) = T\psi$ e per flussi isentropici $\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_1}{p_2}\right)^k$ si ha:

$$\eta_d = \frac{h_{t2s} - h_0}{h_{t2} - h_0} = \frac{\frac{h_{t2s}}{h_0} - 1}{\frac{h_{t2}}{h_0} - 1} = \frac{\frac{T_{t2s}}{T_0} - 1}{\frac{T_{t0}}{T_0} - 1} = \frac{\left(\frac{p_{t2s}}{p_0}\right)^k - 1}{\tau_r - 1} = \frac{\left(\frac{p_{t2}}{p_0}\right)^k - 1}{\frac{\gamma-1}{2} M_0^2}$$

dove si τ_r è il **rapporto di temperatura** associato all'effetto dinamico (**ram**). L'equazione precedente risolta in termini di pressione da:

$$\frac{p_{t2}}{p_0} = \left(1 + \eta_d \frac{\gamma-1}{2} M_0^2\right)^{1/k}$$

che chiaramente restituisce la relazione isentropica quando il rendimento è unitario.



Presa d'aria



Un ulteriore parametro che descrive le prestazioni della presa d'aria è il **rapporto fra le pressioni di ristagno** a monte del compressore e quella asintotica.

$$\pi_d = \frac{p_{t2}}{p_{t0}}$$

$$\frac{p_{t2}}{p_0} = \left(1 + \eta_d \frac{\gamma - 1}{2} M_0^2 \right)^{1/k}$$

chiaramente i due indici sono correlati:

$$\pi_d = \frac{p_{t2}}{p_0} \frac{p_0}{p_{t0}} = \left(\frac{1 + \eta_d \frac{\gamma - 1}{2} M_0^2}{1 + \frac{\gamma - 1}{2} M_0^2} \right)^{1/k} = \frac{\left(1 + \eta_d \frac{\gamma - 1}{2} M_0^2 \right)^{1/k}}{\pi_r}$$

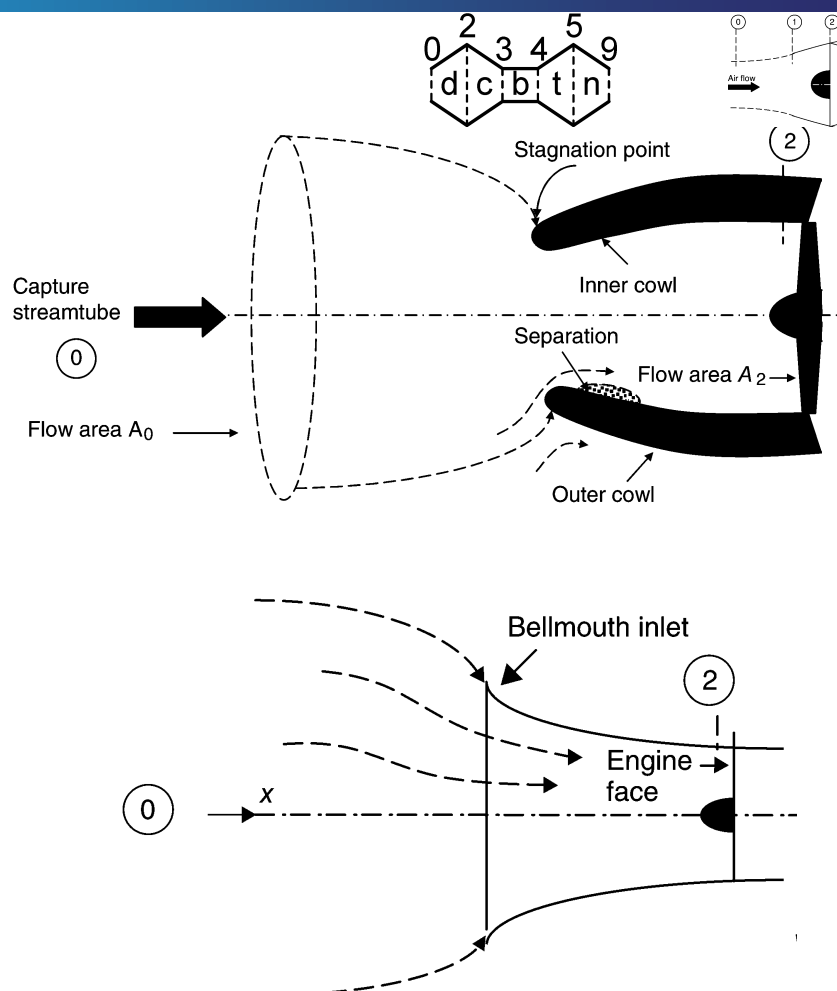
dove si π_r è il rapporto di pressione associato all'effetto di **pressione dinamica** (ram effect). Chiaramente al tendere del rendimento a uno anche $\pi_d \rightarrow 1$.

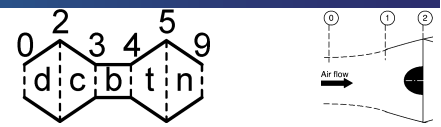


Presa d'aria

Evidentemente **al punto fisso** la situazione è completamente diversa.

Quando si effettuano prove del motore a terra, per evitare separazioni, conviene aggiungere un ugello convergente a monte della presa d'aria del motore.





Example 4.1

An aircraft is flying at an altitude where the ambient static pressure is $p_0 = 10$ kPa and the flight Mach number is $M_0 = 0.85$. The total pressure at the engine face is measured to be $p_{t2} = 15.88$ kPa. Assuming the inlet is adiabatic and $\gamma = 1.4$, calculate

- the inlet total pressure recovery π_d
- the inlet adiabatic efficiency η_d
- the nondimensional entropy rise caused by the inlet $\Delta s_d/R$

SOLUTION

We first calculate the flight total pressure p_{t0} , and from definition of π_d (i.e., Equation 4.6), the inlet total pressure recovery.

$$p_{t0} = p_0 [1 + (\gamma - 1)M_0^2/2]^{\frac{\gamma}{\gamma-1}} = 10 \text{ kPa} [1 + 0.2(0.85)^2]^{3.5} = 16.04 \text{ kPa}$$

$$\pi_d \equiv p_{t2}/p_{t0} = 15.88/16.04 = 0.990$$

Inlet adiabatic efficiency η_d is calculated from Equation 4.4

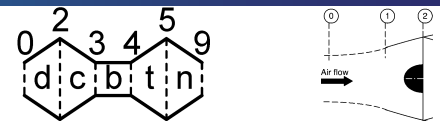
$$\eta_d = \left[\left(\frac{p_{t2}}{p_0} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] / \left(\frac{\gamma-1}{2} M_0^2 \right) = [(15.88/10)^{0.2857} - 1] / [0.2(0.85)^2] \cong 0.9775$$

The entropy rise is linked to the inlet total pressure loss parameter π_d via Equation 4.2,

$$\Delta s_d/R = -\ln(\pi_d) = -\ln(0.99) \cong 0.010$$



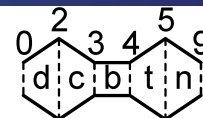
Presa d'aria



- the inlet flow may be considered to be adiabatic, that is, $h_{t2} = h_{t0}$
- the inlet flow is always irreversible, that is, $p_{t2} < p_{t0}$, with viscous dissipation in the boundary layer and in a shock as the sources of irreversibility ($s_2 > s_0$)
- there are two figures of merit that describe the extent of *losses* in the inlet and these are η_d and π_d
- the two figures of merit are related (via Equation 4.6b)
- in cruise, $A_0 < A_2$ and hence a diffusing passage and at low speed or takeoff, $A_0 > A_2$, that is, a nozzle
- the outer nacelle geometry of an inlet dictates the drag divergence and high angle of attack characteristics of the inlet and is crucial for the installed performance.

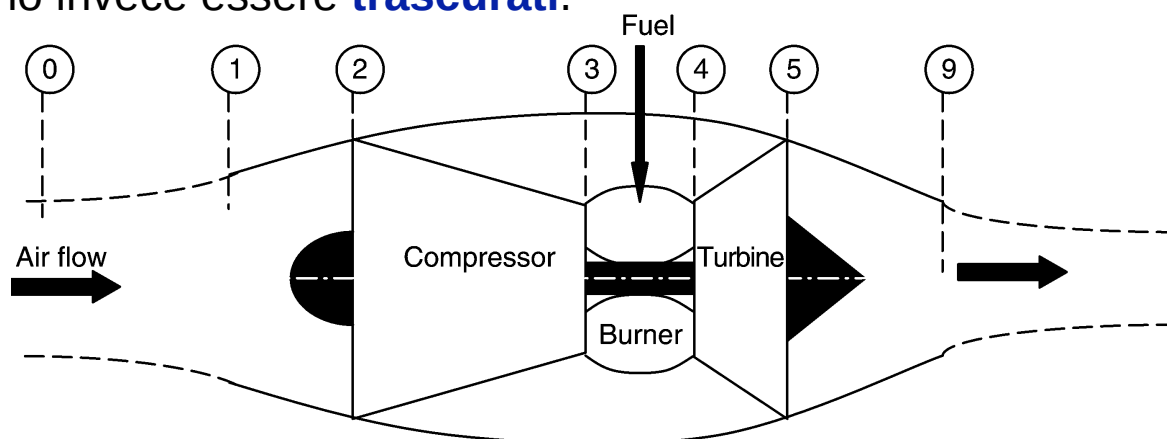


Compressore

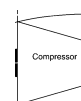
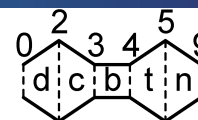


Il **compressore** è una **macchina operatrice** quindi fornisce energia nel **modo lavoro** al fluido e, chiaramente, necessita di una potenza esterna che normalmente viene fornita dalla turbina attraverso l'albero. All'avviamento il compressore deve essere alimentato da una **fonte esterna**.

Anche il funzionamento di un compressore reale è essenzialmente **adiabatico**. A causa delle elevate velocità gli **effetti dissipativi** associati agli sforzi viscosi e, eventualmente, alle onde d'urto **non** possono invece essere **trascurati**.



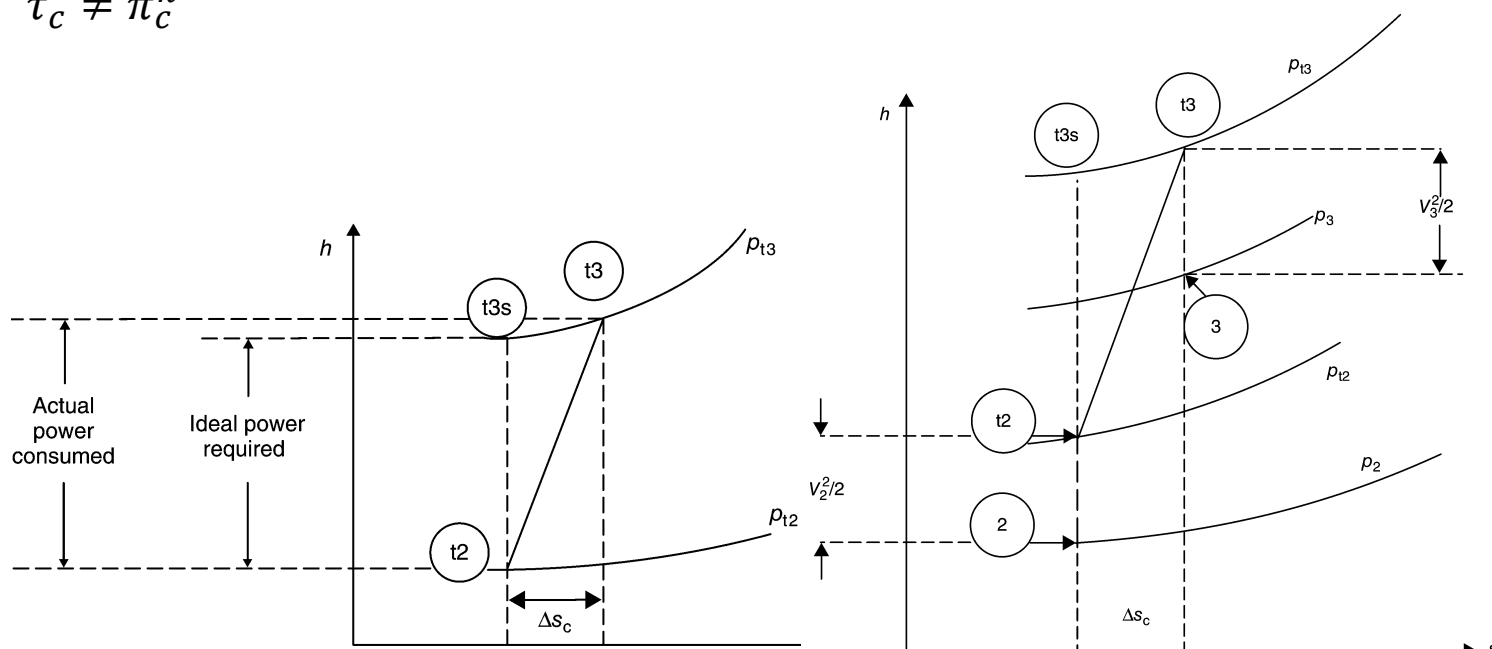
Compressore



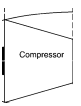
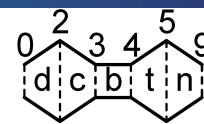
Dal punto di vista puramente termodinamico il comportamento di un **compressore** è molto **simile** al quello delle **prese d'aria**. Una differenza è che normalmente $V_2 \cong V_3$.

L'aumento di pressione non seguirà una isentropica, quindi:

$$\tau_c \neq \pi_c^k$$



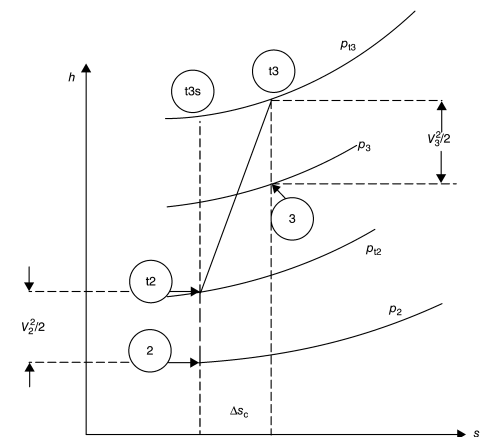
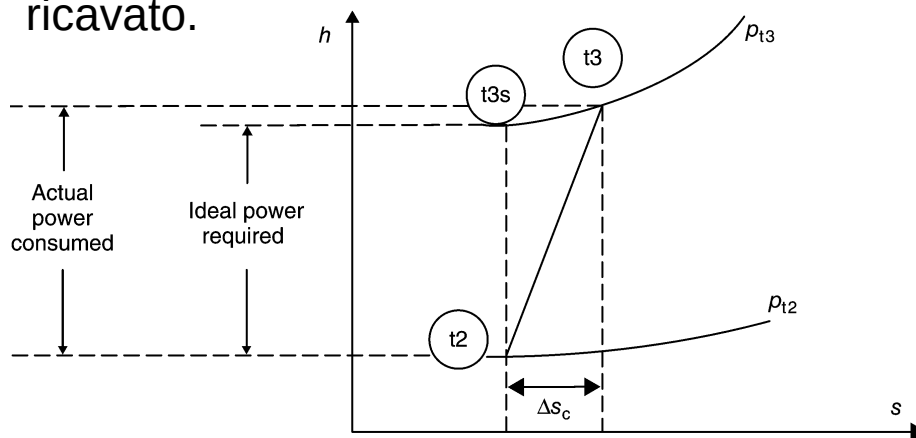
Compressore



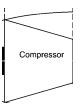
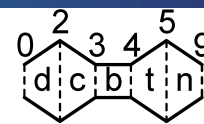
Si può quindi definire un **rendimento adiabatico** come il rapporto fra le differenze di entalpia di un processo isentropico e di quello reale, ovvero, il rapporto tra la **potenza assorbita** da un compressore **ideale** e quella necessaria per il compressore **reale**:

$$\eta_c = \frac{h_{t3s} - h_{t2}}{h_{t3} - h_{t2}} = \frac{T_{t3s}/T_{t2} - 1}{T_{t3}/T_{t2} - 1} = \frac{\left(\frac{p_{t3}}{p_{t2}}\right)^k - 1}{\tau_c - 1} = \frac{\pi_c^k - 1}{\tau_c - 1}$$

Normalmente si suppone che η_c e π_c siano noti mentre τ_c deve essere ricavato.



Compressore



Un secondo parametro che misura l'efficienza di un compressore è il **rendimento politropico**:

$$e_c = \frac{dh_{ts}}{dh_t}$$

Per piccoli rapporti di compressione le due definizioni di rendimento tendono a coincidere. Per una trasformazione isentropica:

$$dh_{ts} = dp_t / \rho_t$$

quindi:

$$e_c = \frac{1}{\rho_t c_p} \frac{dp_t}{dT_t} = \frac{RT_t}{p_t c_p} \frac{dp_t}{dT_t} = k \frac{dp_t/p_t}{dT_t/T_t} \rightarrow \frac{dp_t}{p_t} = \frac{e_c}{k} \frac{dT_t}{T_t}$$

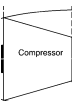
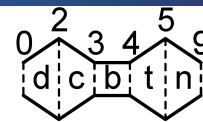
che integrata da:

$$\frac{p_{t3}}{p_{t2}} = \pi_c = \left(\frac{T_{t3}}{T_{t2}}\right)^{\frac{e_c}{k}} = \tau_c^{\frac{e_c}{k}} \rightarrow \tau_c = \pi_c^{\frac{k}{e_c}}$$



Compressore

$$\tau_c = \pi_c^{\frac{\gamma-1}{\gamma e_c}}$$



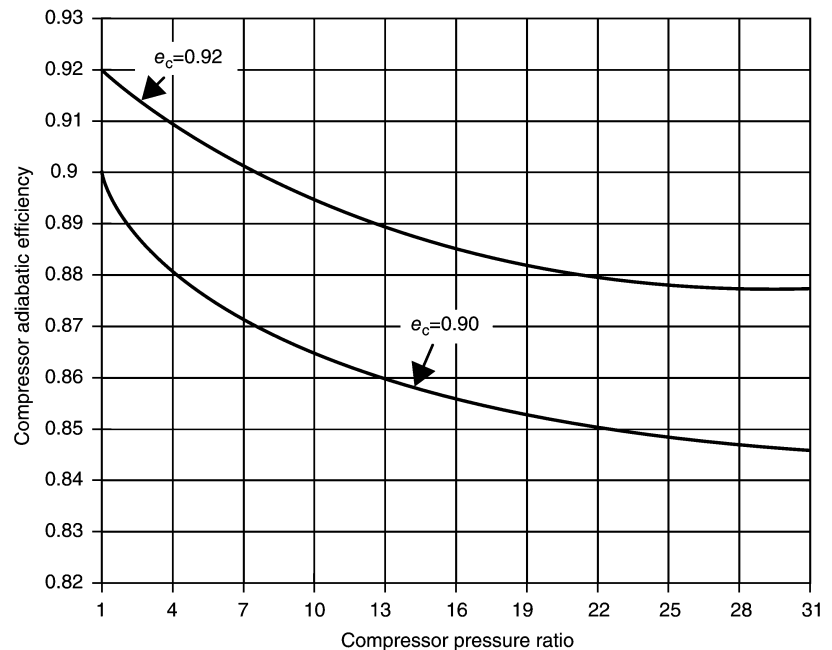
Essendo e_c minore di uno il **rapporto di temperature** è maggiore di quello isentropico a parità di rapporto di pressione imposto.

Valori tipici di e_c sono del ordine di 88-92%.

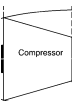
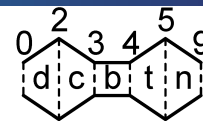
Sostituendo questa espressione nella relazione del rendimento adiabatico si ha:

$$\eta_c = \frac{\pi_c^k - 1}{\tau_c - 1} = \frac{\pi_c^k - 1}{\frac{k}{\pi_c^{e_c}} - 1}$$

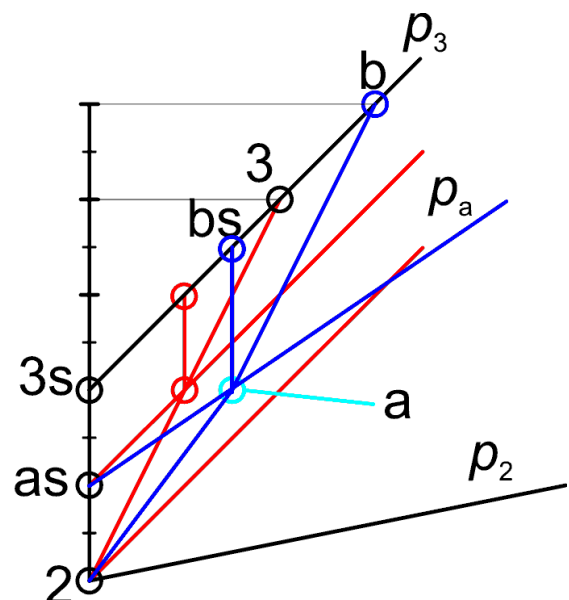
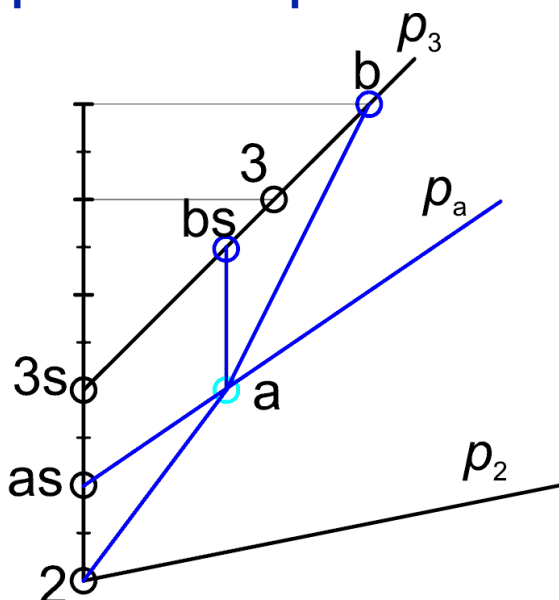
Il **rendimento politropico** è **indipendente** dal **rapporto di compressione** al contrario di quello adiabatico. Per questo motivo è il parametro che ci permette una progettazione preliminare del motore.



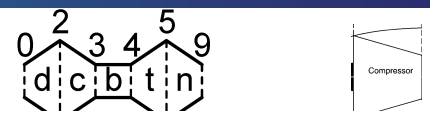
Compressore



Il **rendimento politropico** può essere visto come il rendimento adiabatico costante associato ad una **trasformazione per piccoli stadi** successivi. Ad ognuno di questi stadi sarà associato un aumento della temperatura il che, considerando la divergenza delle isobare, porta ad un **rendimento adiabatico inferiore** al **crescere** del **rapporto di compressione**.



Compressore



A multistage axial-flow compressor has a mass flow rate of 50 kg/s and a total pressure ratio of 35. The compressor polytropic efficiency is $e_c = 0.90$. The inlet flow condition to the compressor is described by $T_{12} = 288$ K and $p_{12} = 100$ kPa. Assuming the flow in the compressor is adiabatic, and constant gas properties throughout the com-

pressor are assumed, i.e., $\gamma = 1.4$ and $c_p = 1004$ J/kg · K, calculate

- compressor exit total temperature T_{13} in K
- compressor adiabatic efficiency η_c
- compressor shaft power $\mathcal{G}_c$ in MW

SOLUTION

Following Equation 4.20, we relate compressor total temperature and pressure ratio via polytropic efficiency,

$$\tau_c = \pi_c^{\frac{\gamma-1}{\gamma e_c}} = (35)^{0.31746} \cong 3.0916$$

Therefore, the exit total temperature is $T_{13} = 3.0916 T_{12} = 3.0916(288 \text{ K}) \cong 890.4$ K.

Compressor adiabatic efficiency is related to the polytropic efficiency and compressor pressure ratio, via Equation 4.13

$$\eta_c = \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\tau_c - 1} = \frac{35^{0.2857} - 1}{3.0916 - 1} \approx 0.8422$$

Therefore, compressor adiabatic efficiency is $\eta_c \cong 84.22\%$.

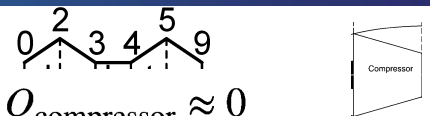
Compressor shaft power is proportional to the mass flow rate (i.e., the size of the compressor) as well as the total enthalpy rise across the compressor, according to

$$\begin{aligned} \mathcal{G}_c &= \dot{m}(h_{13} - h_{12}) = \dot{m}c_p(T_{13} - T_{12}) \\ &= (50 \text{ kg/s})(1004 \text{ J/kg} \cdot \text{K})(890.4 - 288) \text{ K} \\ &\approx 30.24 \text{ MW} \end{aligned}$$

Therefore the shaft power delivered to the compressor is $\mathcal{G}_c \approx 30.24$ MW



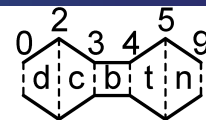
Compressore



- a real compressor flow may be considered adiabatic, that is, $Q_{\text{compressor}} \approx 0$
- the energy transfer to the fluid due to the shaft in a compressor is several orders of magnitude higher than any heat transfer that takes place through the casing, thus heat transfer is neglected
- viscous dissipation in the wall boundary layer and shocks account for the sources of irreversibility in a compressor
- there are two figures of merit that describe the compressor efficiency, one is the adiabatic compressor efficiency η_c (sometimes referred to as “isentropic” efficiency) and the second is the polytropic or small-stage efficiency e_c
- the two compressor efficiencies are interrelated, that is, $\eta_c = \eta_c(\pi_c, e_c)$
- the compressor polytropic efficiency is independent of compressor pressure ratio π_c
- the compressor adiabatic efficiency is a function of π_c and decreases with increasing pressure ratio
- to achieve a high-pressure ratio in a compressor, multistage and multistage configurations are needed
- in a gas turbine engine, the compressor power is derived from a shaft that is connected to a turbine.



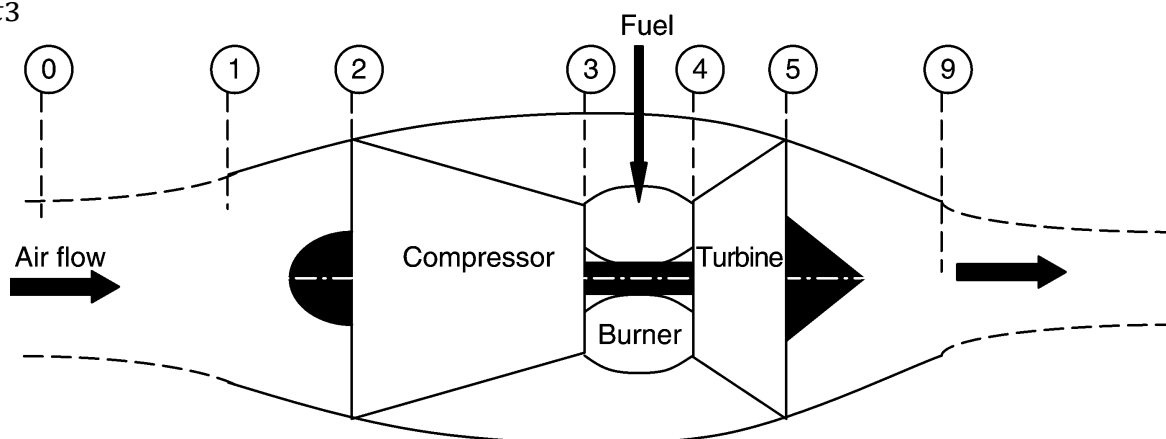
Camera di combustione



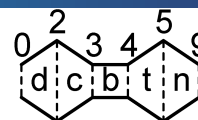
Nella **camera di combustione** (CC) l'aria proveniente dal compressore viene miscelata con il combustibile e attraverso una **reazione esotermica** si ha un aumento di temperatura ed una **variazione delle proprietà del gas** (γ e c_p).

In un combustore **ideale** si suppone che gli **effetti dissipativi** siano **trascurabili** e che il numero di **Mach** sia **nullo**, in queste ipotesi il processo è isobaro:

$$\pi_b = \frac{p_{t4}}{p_{t3}} = 1$$



Camera di combustione

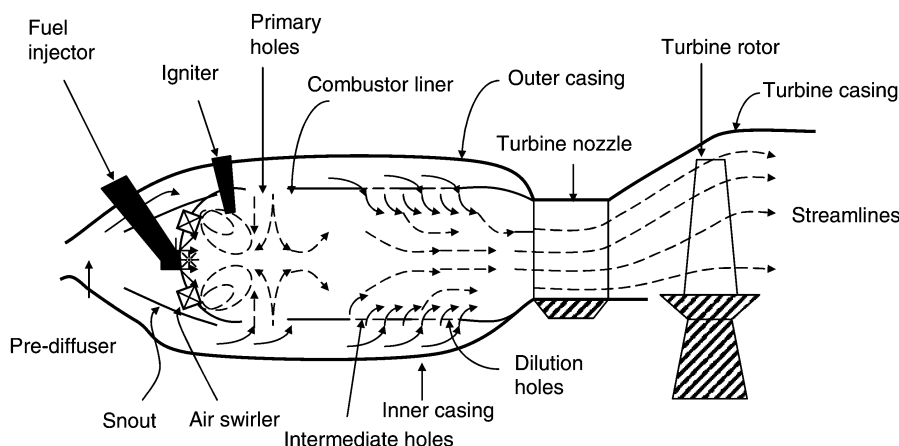


In realtà a causa degli attriti, del miscelamento turbolento e delle reazioni chimiche ad un numero di Mach basso ma diverso da zero il processo porta ad una **caduta della pressione di ristagno**.

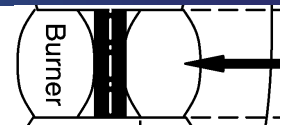
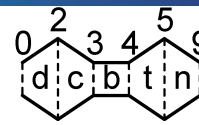
Supponendo che la perdita di pressione sia proporzionale alla pressione dinamica si trova la seguente relazione:

$$\pi_b = \frac{p_{t4}}{p_{t3}} = \frac{p_{t3} - \epsilon \frac{1}{2} \rho V_b^2}{p_{t3}} \approx 1 - \epsilon \frac{\gamma \rho V_b^2}{2 \gamma p_{t3}} \approx 1 - \epsilon \frac{\gamma}{2} M_b^2$$

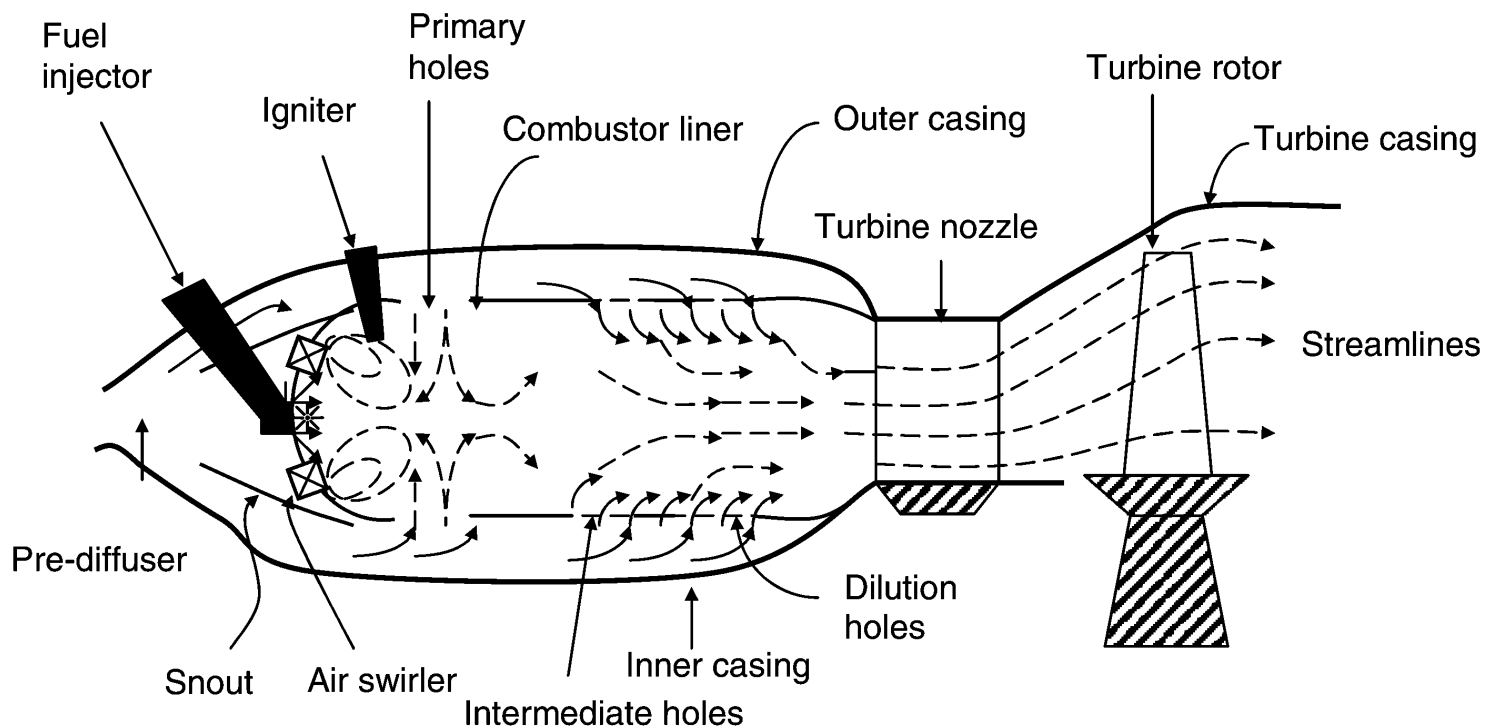
dove tipici valori della costante ϵ sono compresi fra 1 e 2 (e.g. per $M_b = 0.2$, $\epsilon = 2$ si ha $\pi_b \approx .95$).



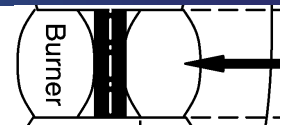
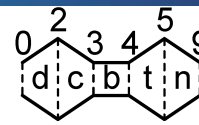
Camera di combustione



In figura è mostrato lo schema di un combustore.



Camera di combustione



In questa fase si analizzerà la camera di combustione solo dal punto di vista globale come variazioni delle condizioni fra le sezioni 3 e 4.

Da un bilancio di massa:

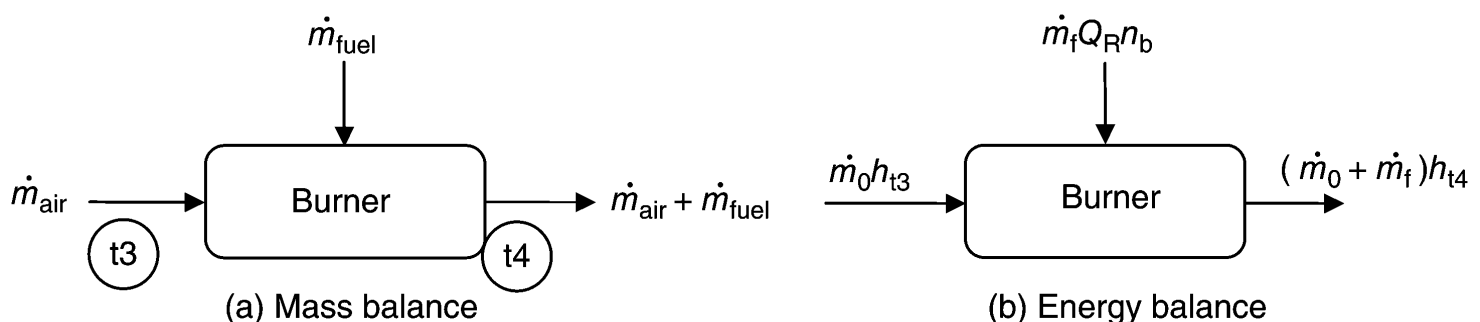
$$\dot{m}_4 = \dot{m}_0 + \dot{m}_f = (1 + f)\dot{m}_0$$

dove f è il **rapporto** fra la **portata** di **combustibile** e quella **d'aria**. Il bilancio di energia invece:

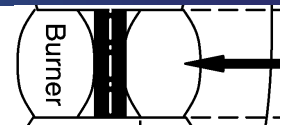
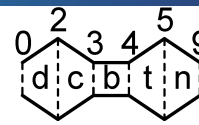
$$\dot{m}_0 h_{t3} + \dot{m}_f Q_R \eta_b = (\dot{m}_0 + \dot{m}_f) h_{t4} = (1 + f) \dot{m}_0 h_{t4}$$

dove $Q_R \left[\frac{kJ}{kg} \right]$ è il **potere calorifico del combustibile** (**fuel heating value**) e si

è introdotto il **rendimento del combustore**: $\eta_b = \frac{Q_{R.Actual}}{Q_{R.Ideal}}$ che normalmente è circa 98-99%.



Camera di combustione



La temperatura limite T_{t4} è funzione della tecnologia di **raffreddamento** utilizzata e tipici valori sono dell'ordine di **1,750-2,000K**.

Dal **bilancio di energia** si può ricavare f :

$$h_{t3} + \dot{m}_f / \dot{m}_0 Q_R \eta_b = h_{t3} + f Q_R \eta_b = (1 + f) h_{t4} \quad \rightarrow \quad f = \frac{h_{t4} - h_{t3}}{Q_R \eta_b - h_{t4}}$$

Ricordando che

$$\tau_\lambda = \frac{h_{t4}}{h_0} \text{ si ha:}$$

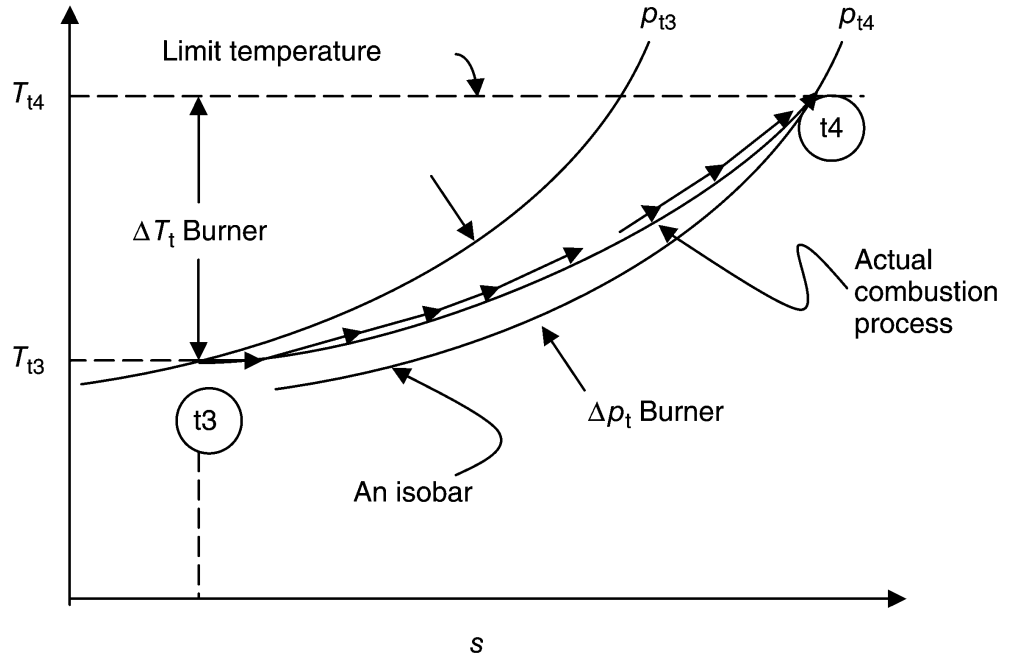
$$f = \frac{\tau_\lambda - \tau_c \tau_r}{Q_R \eta_b / h_0 - \tau_\lambda}$$

inoltre:

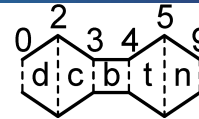
$$\tau_b = \frac{h_{t4}}{h_{t3}} = \frac{\tau_\lambda}{\tau_r \tau_c}$$

Dove:

$$\tau_c = \frac{h_{t3}}{h_{t2}} \quad \tau_r = \frac{h_{t2}}{h_0}$$



Camera di combustione



A gas turbine combustor has inlet condition $T_{t3} = 800$ K, $p_{t3} = 2$ Mpa, air mass flow rate of 50 kg/s, $\gamma_3 = 1.4$, $c_{p3} = 1004$ J/kg · K.

A hydrocarbon fuel with ideal heating value $Q_R = 42,000$ kJ/kg is injected in the combustor at a rate of 1 kg/s. The burner efficiency is $\eta_b = 0.995$ and the total pressure at the combustor exit is 96% of the inlet total pressure,

i.e., combustion causes a 4% loss in total pressure. The gas properties at the combustor exit are $\gamma_4 = 1.33$ and $c_{p4} = 1156$ J/kg · K. Calculate

- fuel-to-air ratio f
- combustor exit temperature T_{t4} in K and p_{t4} in MPa

SOLUTION

The air and fuel flow rates are specified at 50 and 1 kg/s, respectively, in the problem, therefore, $f = 1/50 = 0.02$ or 2%.

We calculate combustor exit temperature by energy balance,

$$\dot{m}_0 h_{t3} + \dot{m}_f Q_R \eta_b = (\dot{m}_0 + \dot{m}_f) h_{t4} = \dot{m}_0 (1 + f) h_{t4}$$

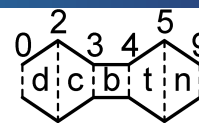
Therefore

$$h_{t3} + f Q_R \eta_b = (1 + f) h_{t4}$$

$$T_{t4} = \frac{(c_{p3}/c_{p4})T_{t3} + f Q_R \eta_b / c_{p4}}{1 + f} = \frac{(1004/1156) 800 \text{ K} + 0.02 (42000 \text{ kJ/kg}) (0.995) / 1.156 \text{ kJ/kg} \cdot \text{K}}{1.02}$$

Therefore, combustor exit temperature is $T_{t4} \cong 1390$ K and $p_{t4} = 0.96(2 \text{ MPa}) = 1.92 \text{ MPa}$

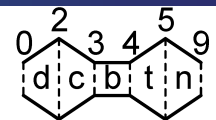




- the fuel is characterized by its heating value Q_R (maximum releasable thermal energy per unit mass)
- the burner is characterized by its efficiency η_b , and its total pressure ratio π_b
- burning at finite Mach number, frictional losses on the walls and turbulent mixing are identified as the sources of irreversibility, that is, losses, in a burner
- the fuel-to-air ratio f and the burner exit temperature T_{t4} are the thrust control/engine design parameters
- the application of the energy balance across the burner yields either f or T_{t4} .

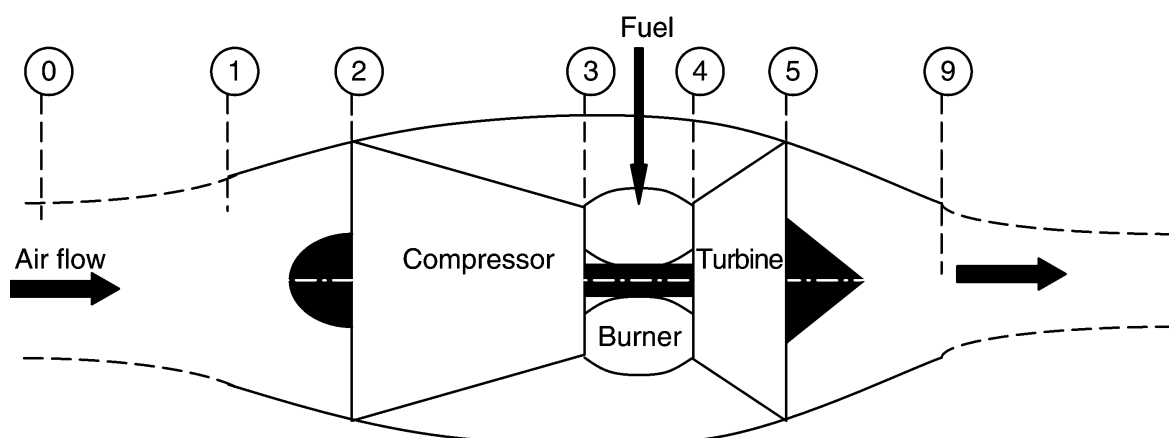


Turbina

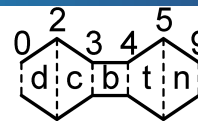


La turbina è una **macchina motrice** quindi prende energia nel modo lavoro dal fluido e la trasferisce attraverso un albero al compressore e alle altre macchine operatrici. Evidentemente all'interno di una turbina **il fluido espande** riducendo sia la **pressione** che la **temperatura**.

A causa delle **alte temperature** a valle della CC i primi stadi della turbina (HPT) devono essere **raffreddati** con aria presa dal compressore (e.g. fra il compressore di alta (HPC) e di bassa LPT); questo effetto normalmente si trascura nell'analisi preliminare.

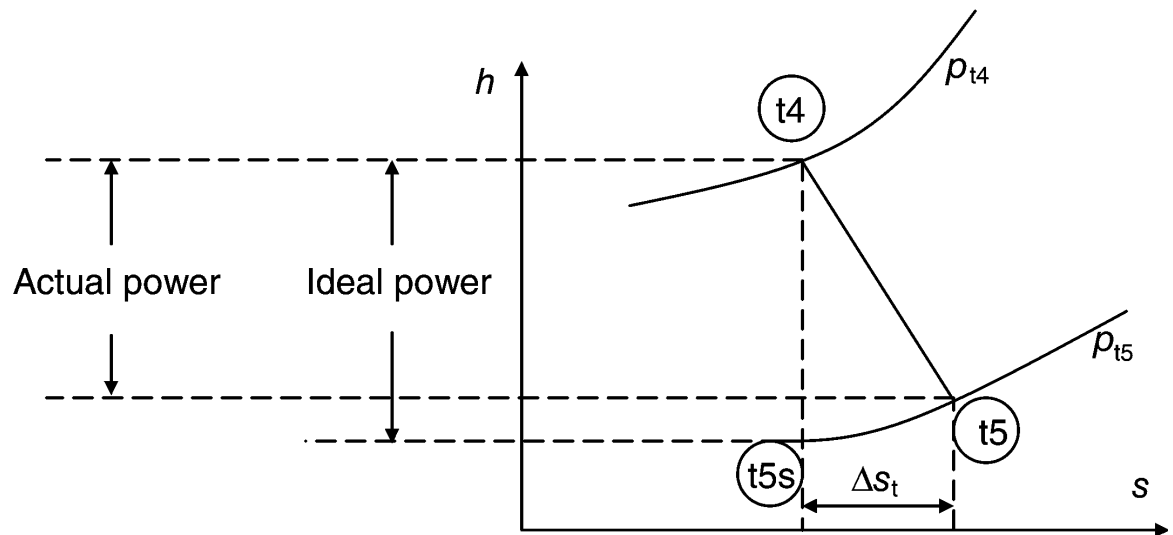


Turbina

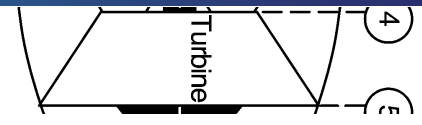
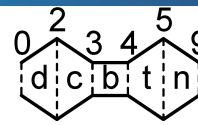


Anche nel flusso attraverso una turbina esistono degli effetti dissipativi. Quindi il processo **non è isentropico** ed in analogia con i compressori si possono definire sia un rendimento adiabatico che politropico:

$$\eta_t = \frac{h_{t4} - h_{t5}}{h_{t4} - h_{t5s}} = \frac{1 - T_{t5}/T_{t4}}{1 - T_{t5s}/T_{t4}} = \frac{1 - \tau_t}{1 - \left(\frac{p_{t5}}{p_{t4}}\right)^{k_t}} = \frac{1 - \tau_t}{1 - \pi_t^{k_t}} \quad k_t = \frac{\gamma_t - 1}{\gamma_t}$$



Turbina

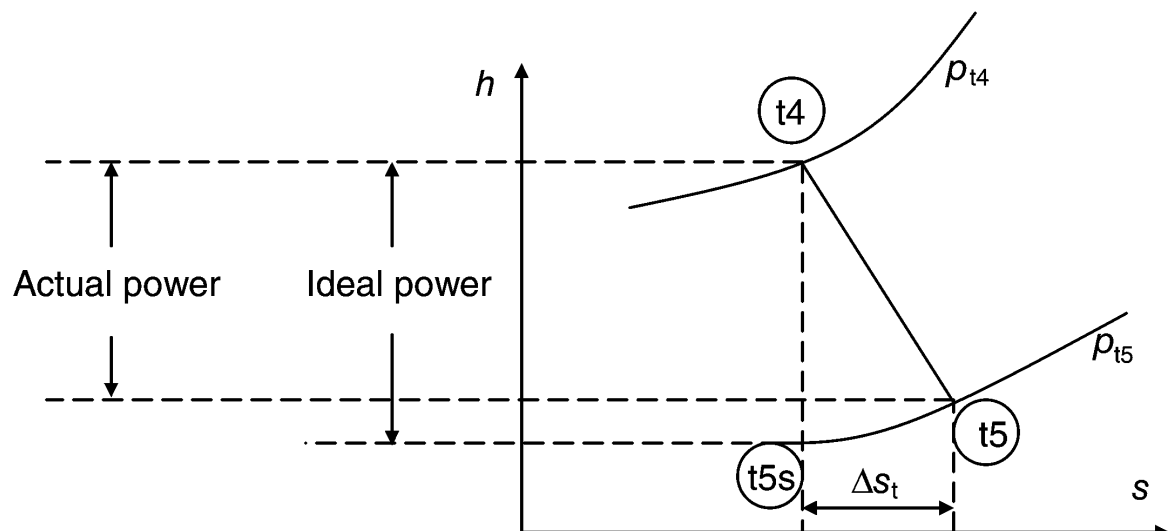


Come già fatto per i compressori dal **rendimento politropico**:

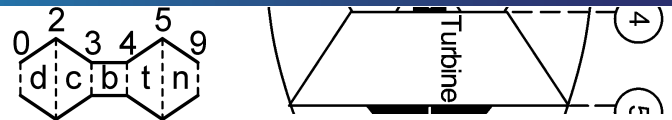
$$e_t = \frac{dh_t}{dh_{ts}} \rightarrow \tau_t = \pi_t^{k_t e_t} \rightarrow \pi_t = \tau_t^{\frac{1}{k_t e_t}}$$

Sostituendo quest'ultima equazione nell'espressione del **rendimento adiabatico**:

$$\eta_t = \frac{1 - \tau_t}{1 - \tau_t^{\frac{1}{e_t}}}$$



Turbina



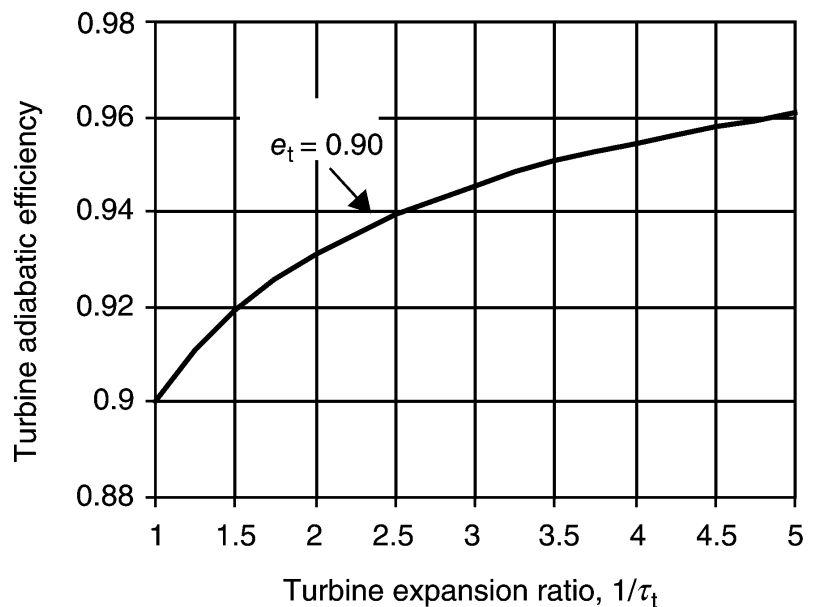
Come già fatto per i compressori dal **rendimento politropico**:

$$e_t = \frac{dh_t}{dh_{ts}} \rightarrow \tau_t = \pi_t^{k_t e_t} \rightarrow \pi_t = \tau_t^{\frac{1}{k_t e_t}}$$

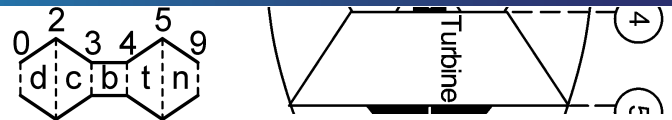
Sostituendo quest'ultima equazione nell'espressione del **rendimento adiabatico**:

$$\eta_t = \frac{1 - \tau_t}{1 - \tau_t^{\frac{1}{e_t}}}$$

Il comportamento per una **turbina** è **opposto** a quello del **compressore**.



Turbina



Il rapporto delle temperature si ricava da un **bilancio delle potenze** necessarie per il **compressore** e per le **altre utenze** (potenza elettrica e potenze dissipate nei cuscinetti). Nell'ipotesi **ideale** che la potenza generata dalla turbina sia assorbita solo dal compressore:

$$\mathcal{P}_t = \mathcal{P}_c \rightarrow \dot{m}_0(1+f)(h_{t4} - h_{t5}) = \dot{m}_0(h_{t3} - h_{t2})$$

Ovvero in forma adimensionale:

$$(1+f)(h_{t4} - h_{t5})/h_0 = (h_{t3} - h_{t2})/h_0$$

$$(1+f)(1 - \tau_t)\tau_\lambda = (\tau_c - 1)\tau_r$$

Quindi:

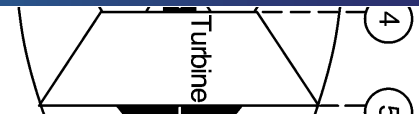
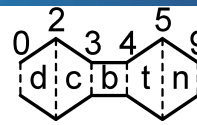
$$\tau_t = 1 - \frac{(\tau_c - 1)\tau_r}{(1+f)\tau_\lambda}$$

Dove:

$$\tau_t = \frac{T_{t5}}{T_{t4}} = \frac{h_{t5}}{h_{t4}} \quad \tau_c = \frac{h_{t3}}{h_{t2}} = \frac{T_{t3}}{T_{t2}} \quad \tau_r = \frac{h_{t2}}{h_0} = \frac{T_{t2}}{T_0} \quad \tau_\lambda = \frac{h_{t4}}{h_0}$$



Turbina



Normalmente le potenze aggiuntive:

$$\mathcal{P}_c = \mathcal{P}_t - \Delta\mathcal{P}_b - \Delta\mathcal{P}_e$$

vengono trattate come **aliquota** della potenza del **compressore**:

$$\mathcal{P}_c = \eta_m \mathcal{P}_t$$

Dove è stato introdotto il **rendimento meccanico** che dovrebbe essere assegnato a priori. La formula precedente diventa:

$$\tau_t = 1 - \frac{(\tau_c - 1)\tau_r}{\eta_m(1 + f)\tau_\lambda}$$



Turbina

Consider an uncooled gas turbine with its inlet condition the same as the exit condition of the combustor described in Example 4.3. The turbine adiabatic efficiency is 88%. The turbine produces a shaft power to drive the compressor and other accessories at $\mathcal{P}_t = 45$ MW. Assuming that the gas properties in the turbine are the same as the burner exit in Example 4.3, calculate

- turbine exit total temperature T_{t5} in K
- turbine polytropic efficiency, e_t
- turbine exit total pressure p_{t5} in kPa
- turbine shaft power $\mathcal{P}_t$ based on turbine expansion ΔT_t

SOLUTION

The turbine shaft power is proportional to the mass flow rate through the turbine, which from Example 4.3 is 51 kg/s (50 for air and 1 for fuel flow rate), as well the total enthalpy drop, i.e.,

$$\mathcal{P}_t = \dot{m}_t(h_{t4} - h_{t5})$$

Therefore, we isolate h_{t5} from above equation to get

$$\begin{aligned} h_{t5} &= h_{t4} - \mathcal{P}_t / \dot{m}_t = c_{p4}T_{t4} - 45 \times 10^6 / 51 \\ &= 1.156 \text{ kJ/kg} \cdot \text{K} (1390 \text{ K}) \\ &\quad - 45,000 \text{ kW} / 51 \text{ kg/s} \approx 724.5 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} T_{t5} &= h_{t5} / c_{p5} \\ &= (724.5 \text{ kJ/kg}) / (1.156 \text{ kJ/kg} \cdot \text{K}) \approx 626.7 \text{ K} \end{aligned}$$

Turbine polytropic efficiency e_t may be related to its adiabatic efficiency and τ_t via equation

$$\eta_t = \frac{1 - \tau_t}{1 - \tau_t^{1/e_t}}$$

The turbine expansion parameter $\tau_t = 626.7/1390 = 0.4509$; and if we isolate e_t from above equation we get

$$\begin{aligned} e_t &= \ln(\tau_t) / \ln[1 - (1 - \tau_t)/\eta_t] \\ &= \ln(0.4509) / \ln[1 - 0.5491/0.88] \cong 0.8144 \end{aligned}$$

We know that turbine pressure and temperature ratios are related by the polytropic efficiency via

$$\pi_t = \tau_t^{\frac{\gamma}{(\gamma-1)e_t}} = (0.4509)^{\frac{1.33}{0.33(0.8144)}} \cong 0.01941$$

The turbine exit total pressure is therefore $p_{t4} \cdot \pi_t$. We had found $p_{t4} = 1.92$ MPa in Example 4.3, therefore,

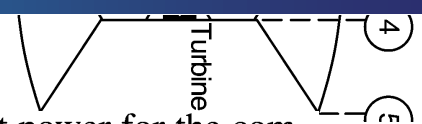
$$p_{t5} = 0.01941(1.92 \text{ MPa}) = 37.26 \text{ kPa}$$

The turbine shaft power is the product of the turbine mass flow rate and the total enthalpy drop across the turbine, i.e.,

$$\begin{aligned} \mathcal{P}_t &= \dot{m}_t c_{pt}(T_{t4} - T_{t5}) \\ &= 51 \text{ kg/s} (1156 \text{ J/kg} \cdot \text{K})(1390 - 626.7) \text{ K} \cong 45 \text{ MW} \end{aligned}$$



Turbina

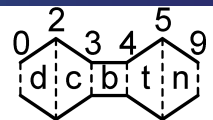


- the flow expansion in the turbine produces the needed shaft power for the compressor and other propulsion system or aircraft needs, for example, an electric generator
- there are two figures of merit in a turbine, which measure the extent of irreversibility in a turbine, η_t and e_t and they are related
- the gas expansion in an uncooled turbine is treated as adiabatic
- the frictional losses on the blades and the casing as well as any shock losses, in relative supersonic passages, are the sources of irreversibility in an uncooled turbine
- turbulent mixing losses between the coolant and the hot gas is an added source of irreversibility in a cooled turbine
- the turbine entry temperature T_{t4} is a design parameter that sets the stage for the turbine material and cooling requirements
- the power balance between the turbine, the compressor, and other known power drainage, establishes the turbine expansion ratio τ_t
- cooling of the high-pressure turbine is achieved through compressor air bleed that is injected through the blade root in the rotor and the casing for the turbine nozzle
- the turbine nozzle is choked (i.e., the throat Mach number is 1), over a wide operating range of the engine, and as such is the first *throttle station* of the engine.

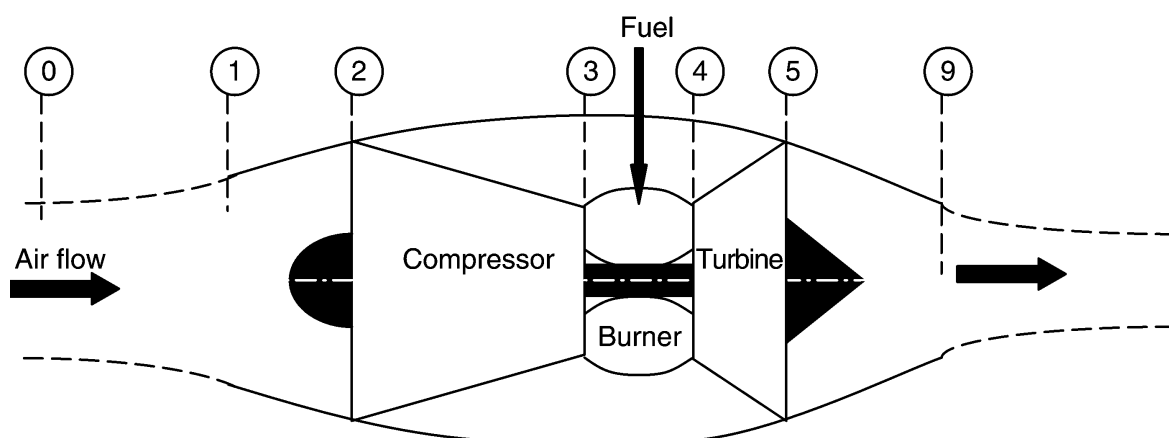


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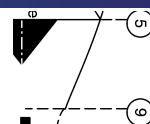
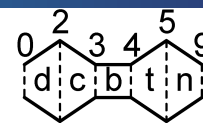
Ugello



La funzione dell'ugello è di **accelerare** i **gas combusti** efficientemente. L'espansione dei gas nel ugello provoca un **gradiente di pressione favorevole**, quindi lo strato limite è sottile e stabile.



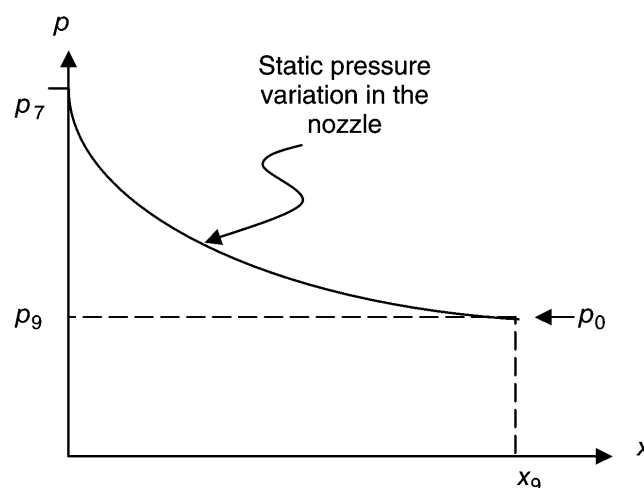
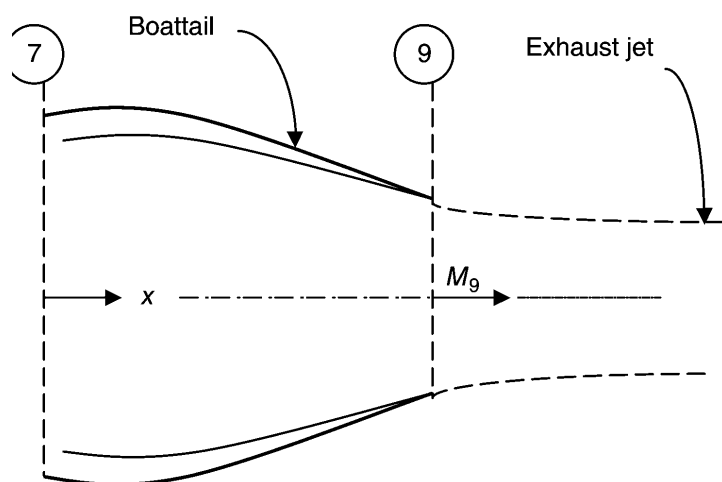
Ugello



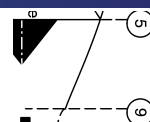
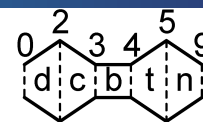
La funzione dell'ugello è di **accelerare** i **gas combusti** efficientemente. L'espansione dei gas nell'ugello provoca un **gradiente di pressione favorevole**, quindi lo strato limite è sottile e stabile.

Evidentemente anche la **forma esterna** (Boattail) è importante per ridurre le perdite all'esterno.

Le curve di funzionamento di un ugello convergente **dovrebbero essere note**, di seguito si farà un brevissima ripetizione.



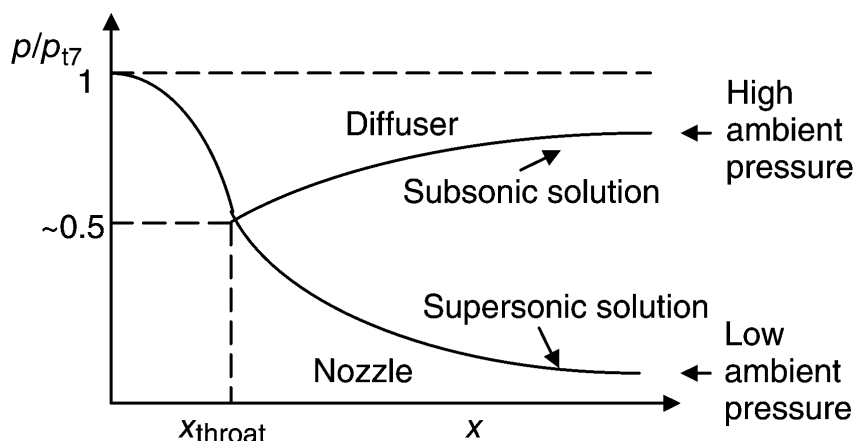
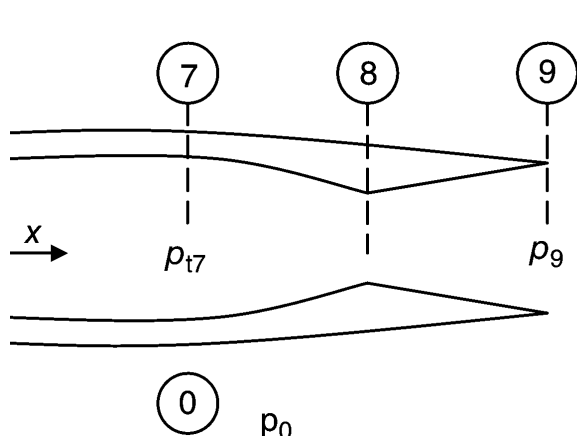
Ugello



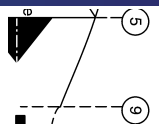
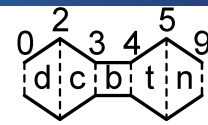
Nel caso di ugelli convergenti divergenti potrebbe non essere soddisfatta la condizione di **Kutta**.

La simbologia è leggermente diversa. Si definisce il rapporto di pressione nell'ugello (**Nozzle Pressure Ratio**):

$$NPR = \frac{p_{t7}}{p_0}$$



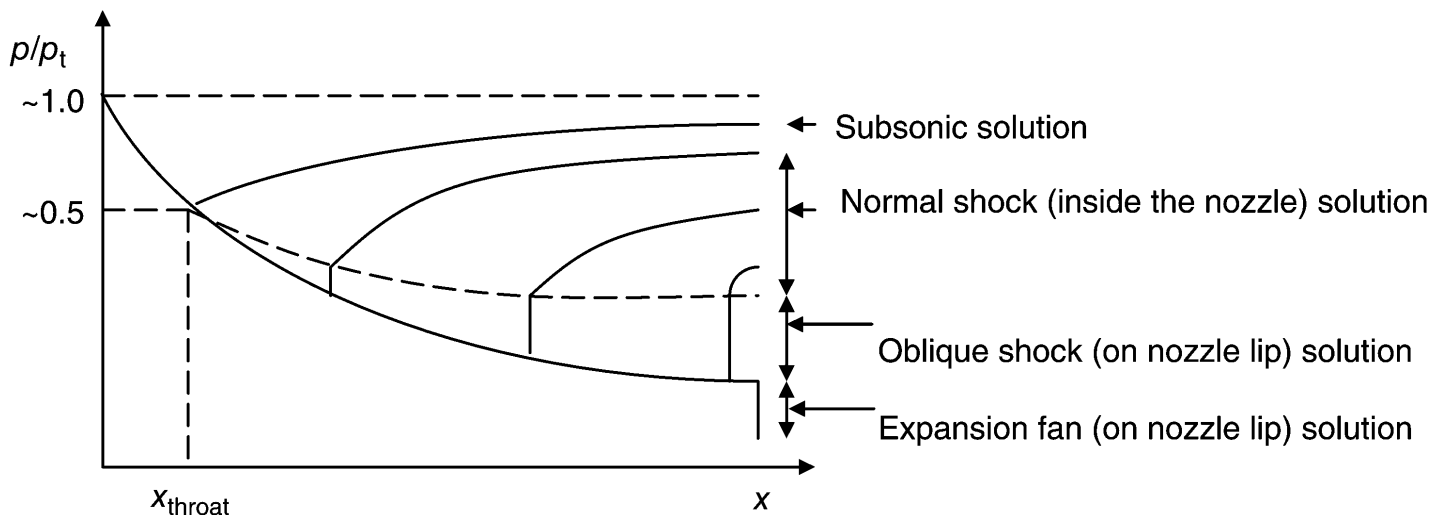
Ugello



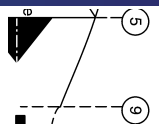
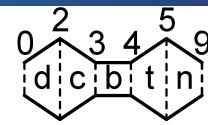
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Ugello



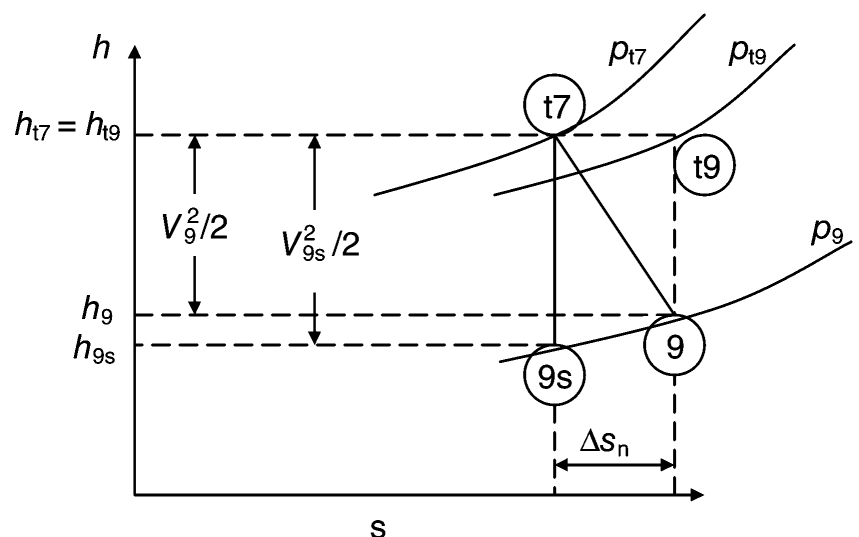
Anche per l'ugello si può definire un **rendimento adiabatico**:

$$\eta_n = \frac{h_{t7} - h_9}{h_{t7} - h_{9s}} = \frac{V_9^2/2}{V_{9s}^2/2} = \frac{1 - \frac{h_9}{h_{t7}}}{1 - \frac{h_{9s}}{h_{t7}}} = \frac{1 - \left(\frac{p_9}{p_{t9}}\right)^{k_9}}{1 - \left(\frac{p_9}{p_{t7}}\right)^{k_9}}$$

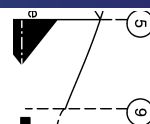
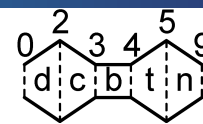
Dove si è utilizzata la costanza delle entalpie $h_{t7} = h_{t9}$.

Moltiplicando numeratore e denominatore per:

$$\left(\frac{p_{t7}}{p_9}\right)^{k_9}$$



Ugello



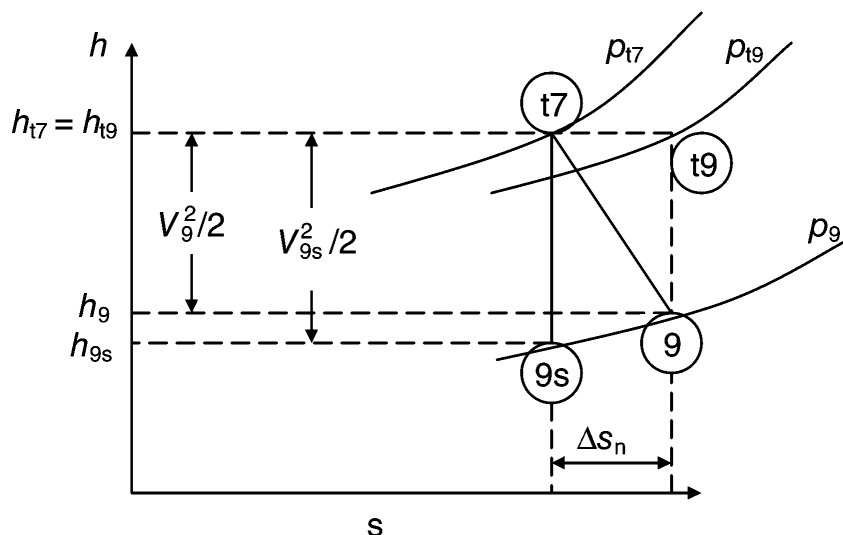
$$\eta_n = \frac{1 - \left(\frac{p_9}{p_{t9}}\right)^{k_9}}{1 - \left(\frac{p_9}{p_{t7}}\right)^{k_9}} = \frac{\left(\frac{p_{t7}}{p_9}\right)^{k_9} - \pi_n^{-k_9}}{\left(\frac{p_{t7}}{p_9}\right)^{k_9} - 1}$$

Con il rapporto fra le pressioni totali nell'ugello dato da:

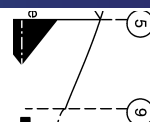
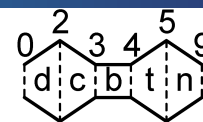
$$\pi_n = \frac{p_{t9}}{p_{t7}}$$

Il rendimento si può anche esprimere attraverso NPR:

$$\frac{p_{t7}}{p_9} = \frac{p_{t7}}{p_0} \frac{p_0}{p_9} = NPR \frac{p_0}{p_9}$$



Ugello



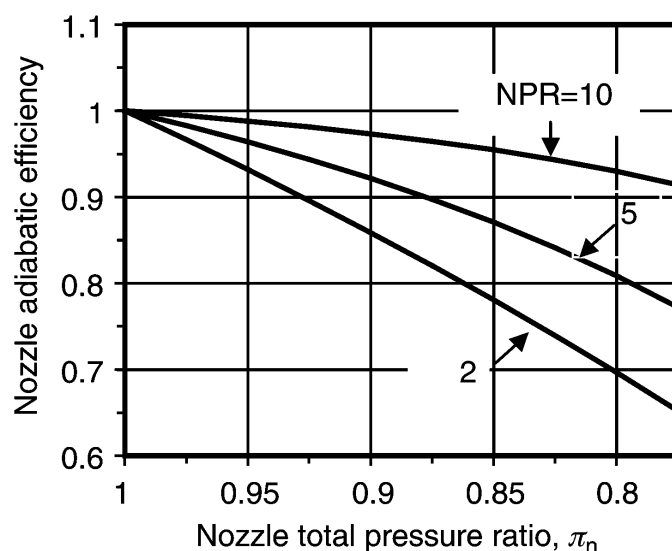
$$\eta_n = \frac{\left(\frac{p_{t7}}{p_9}\right)^{k_9} - \pi_n^{-k_9}}{\left(\frac{p_{t7}}{p_9}\right)^{k_9} - 1} = \frac{\left(NPR \frac{p_0}{p_9}\right)^{k_9} - \pi_n^{-k_9}}{\left(NPR \frac{p_0}{p_9}\right)^{k_9} - 1}$$

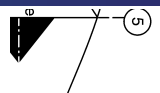
$$\frac{p_{t7}}{p_9} = \frac{p_{t7}}{p_0} \frac{p_0}{p_9} = NPR \frac{p_0}{p_9}$$

$$\pi_n = \frac{p_{t9}}{p_{t7}}$$

Dove il rapporto p_0/p_9 è unitario per **espansione corretta**. In queste condizioni il rendimento adiabatico è diagrammato nella figura.

Nel caso di ugello isentropico chiaramente π_n è unitario.





Consider a convergent–divergent nozzle with a pressure ratio $\text{NPR} = 10$. The gas properties are $\gamma = 1.33$ and $c_p = 1,156 \text{ J/kg} \cdot \text{K}$ and remain constant in the nozzle. The nozzle adiabatic efficiency is $\eta_n = 0.94$. Calculate

- nozzle total pressure ratio π_n
- nozzle area ratio A_9/A_8 for a perfectly expanded nozzle
- nozzle exit Mach number M_9 (perfectly expanded)

SOLUTION

We may use Equation 4.64 that relates the figures of merit of a nozzle (η_n and π_n) with NPR ,

$$\eta_n = \frac{\left\{ \text{NPR} \left(\frac{p_0}{p_9} \right) \right\}^{\frac{\gamma-1}{\gamma}} - \pi_n^{-\frac{\gamma-1}{\gamma}}}{\left\{ \text{NPR} \left(\frac{p_0}{p_9} \right) \right\}^{\frac{\gamma-1}{\gamma}} - 1}$$

For a perfectly expanded nozzle, $p_0 = p_9$, therefore, we get the following expression for π_n

$$\pi_n = \left[\text{NPR}^{\frac{\gamma-1}{\gamma}} - \eta_n \left(\text{NPR}^{\frac{\gamma-1}{\gamma}} - 1 \right) \right]^{\frac{\gamma}{\gamma-1}} \cong 0.8335$$

The entropy rise in an adiabatic nozzle is a function of the total pressure ratio π_n , according to (Equation 4.59)

$$\Delta s_n/R = -\ell n \pi_n \cong -\ell n(0.8335) \approx 0.1822$$

We can calculate the local Mach number M_9 if we know the total and static pressures p_{t9} and p_9 simultaneously. From the NPR and the total pressure ratio, we may write

$$p_{t9}/p_9 = (p_{t9}/p_{t7})(p_{t7}/p_0)(p_0/p_9) = 0.8335(10)(1) = 8.335$$

From the general expression for total pressure and Mach number

$$p_t/p = (1 + (\gamma - 1)M^2/2)^{\frac{\gamma}{\gamma-1}}$$

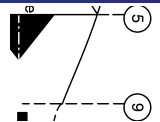
We can isolate Mach number as follows

$$M_9 = \sqrt{\frac{2}{\gamma-1} \left[(p_{t9}/p_9)^{\frac{\gamma-1}{\gamma}} - 1 \right]} = \sqrt{\frac{2}{0.33} \left[(8.335)^{0.3/1.33} - 1 \right]} \cong 2.05$$

Using continuity, we calculate $A_9/A_8 \sim 1.8242$



Ugello



- the primary function of a nozzle is to accelerate the gas efficiently
- the gross thrust parameter F_g signifies nozzle's contribution to the thrust production
- the gross thrust reaches a maximum when the nozzle is perfectly expanded; that is, $p_9 = p_0$
- real nozzle flows may still be considered as adiabatic
- a nozzle pressure ratio (NPR) that causes a Mach-1 flow at the throat (i.e., choking condition) is called the *critical nozzle pressure ratio*, and as a rule of thumb, we may remember an $(\text{NPR})_{\text{crit}}$ of ~ 2
- there are two efficiency parameters that quantify losses or the degree of irreversibility in a nozzle and they are related
- nozzle losses manifest themselves as the total pressure loss
- all subsonic exhaust streams have $p_{\text{jet}} = p_{\text{ambient}}$
- a perfect nozzle expansion means that the nozzle exit (static) pressure and the ambient pressure are equal
- an imperfect nozzle expansion is caused by a mismatch between the nozzle area ratio and the altitude of operation
- underexpansion is caused by smaller-than-necessary nozzle area ratio, leading to $p_9 > p_0$
- overexpansion is caused by larger-than-necessary nozzle area ratio, leading to $p_9 < p_0$.



Ricapitolando

In figura sono mostrati i valori tipici delle grandezze caratteristiche in ventenni successivi (1945-65, 1965-85, 1985-05, 2005-2025).

Component	Figure of merit	Type ^a	Level of technology ^b			
			1	2	3	4
Diffuser	$\pi_{d\max}$	A	0.90	0.95	0.98	0.995
		B	0.88	0.93	0.96	0.98
		C	0.85	0.90	0.94	0.96
Compressor	e_c		0.80	0.84	0.88	0.90
Fan	e_f		0.78	0.82	0.86	0.89
Burner	π_b		0.90	0.92	0.94	0.95
	η_b		0.88	0.94	0.99	0.999
Turbine	e_t	Uncooled	0.80	0.85	0.89	0.90
		Cooled		0.83	0.87	0.89
Afterburner	π_{AB}		0.90	0.92	0.94	0.95
	η_{AB}		0.85	0.91	0.96	0.99
Nozzle	π_n	D	0.95	0.97	0.98	0.995
		E	0.93	0.96	0.97	0.98
		F	0.90	0.93	0.95	0.97
Mechanical shaft	η_m	Shaft only	0.95	0.97	0.99	0.995
		With power takeoff	0.90	0.92	0.95	0.97
Maximum T_{t4}		(K)	1110	1390	1780	2000
		(R)	2000	2500	3200	3600
Maximum T_{t7}		(K)	1390	1670	2000	2220
		(R)	2500	3000	3600	4000

^aA = subsonic aircraft with engines in nacelles D = fixed-area convergent nozzle
 B = subsonic aircraft with engine(s) in airframe E = variable-area convergent nozzle
 C = supersonic aircraft with engine(s) in airframe F = variable-area convergent-divergent nozzle
^bNotes: Stealth may reduce $\pi_{d\max}$, π_{AB} , and π_n . The levels of technology can be thought of as representing the technical capability for 20-yr increments in time beginning in 1945. Thus level 3 of technology presents typical component design values for the time period 1985–2005.



Ricapitolando

Component	Figure of merit	Type ^a	Level of technology ^b			
			45-65 1	65-85 2	85-05 3	05-25 4
Diffuser	$\pi_{d\max}$	A	0.90	0.95	0.98	0.995
		B	0.88	0.93	0.96	0.98
		C	0.85	0.90	0.94	0.96
Compressor	e_c		0.80	0.84	0.88	0.90
Fan	e_f		0.78	0.82	0.86	0.89
Burner	π_b		0.90	0.92	0.94	0.95
	η_b		0.88	0.94	0.99	0.999
Turbine	e_t	Uncooled	0.80	0.85	0.89	0.90
		Cooled		0.83	0.87	0.89
Afterburner	π_{AB}		0.90	0.92	0.94	0.95
	η_{AB}		0.85	0.91	0.96	0.99
Nozzle	π_n	D	0.95	0.97	0.98	0.995
		E	0.93	0.96	0.97	0.98
		F	0.90	0.93	0.95	0.97



Ricapitolando

45-65 65-85 85-05 05-25

Afterburner	π_{AB}		0.90	0.92	0.94	0.95
	η_{AB}		0.85	0.91	0.96	0.99
Nozzle		D	0.95	0.97	0.98	0.995
		E	0.93	0.96	0.97	0.98
		F	0.90	0.93	0.95	0.97
Mechanical shaft	η_m	Shaft only	0.95	0.97	0.99	0.995
		With power takeoff	0.90	0.92	0.95	0.97
Maximum T_{t4}		(K)	1110	1390	1780	2000
		(R)	2000	2500	3200	3600
Maximum T_{t7}		(K)	1390	1670	2000	2220
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^aA = subsonic aircraft with engines in nacelles

D = fixed-area convergent nozzle

B = subsonic aircraft with engine(s) in airframe

E = variable-area convergent nozzle

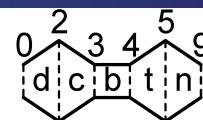
C = supersonic aircraft with engine(s) in airframe

F = variable-area convergent-divergent nozzle

^bNotes: Stealth may reduce π_{dmax} , π_{AB} , and π_n . The levels of technology can be thought of as representing the technical capability for 20-yr increments in time beginning in 1945. Thus level 3 of technology presents typical component design values for the time period 1985–2005.



Rendimento termico

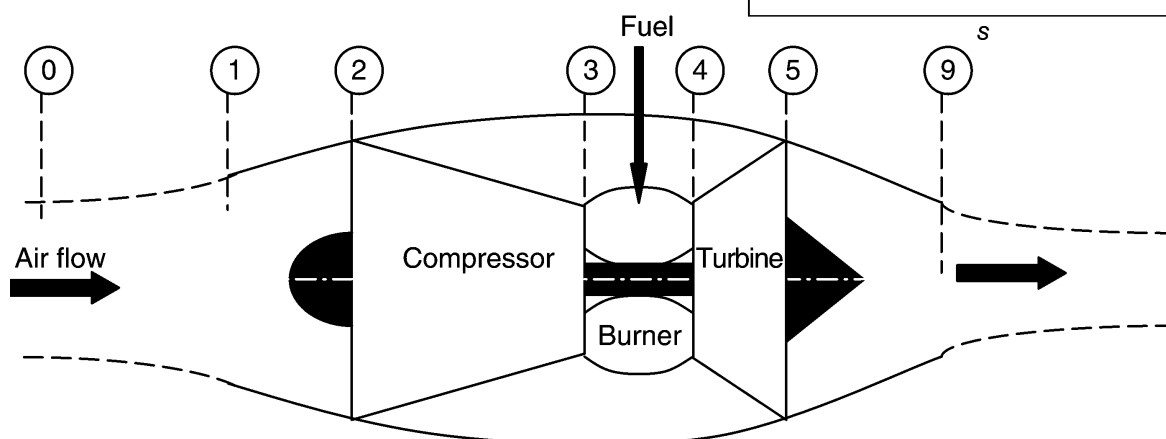
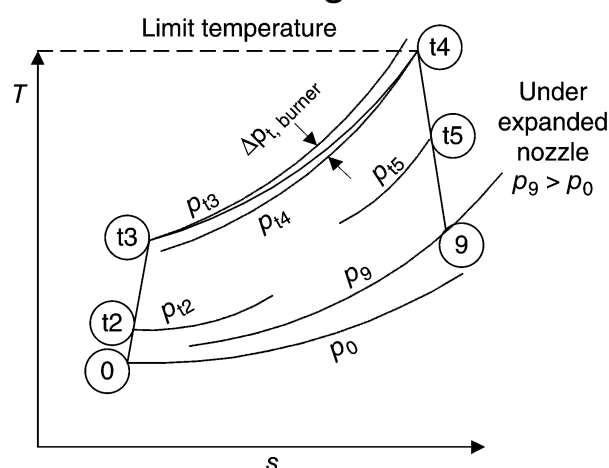


Come già detto il **rendimento termico** teorico di un turbogetto è:

$$\eta_{th} = 1 - \frac{T_0}{T_{t3}}$$

Che in termini di rapporti diventa:

$$\eta_{th} = 1 - \frac{1}{\frac{T_{t3}}{T_{t2}} \frac{T_{t2}}{T_0}} = 1 - \frac{1}{\tau_c \tau_r}$$



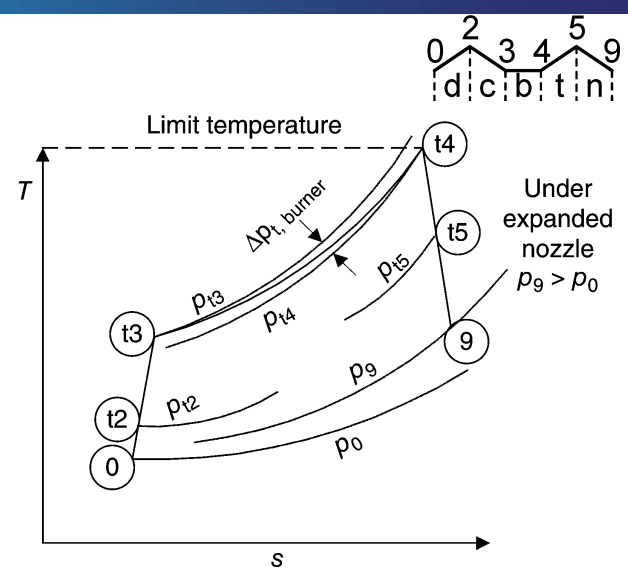
Rendimento del ciclo ideale

Ricordando che:

$$\tau_c = \pi_c^k \quad \tau_r = \left(1 + \frac{\gamma - 1}{2} M_0^2\right) = \psi_0$$

Si ha:

$$\eta_{th} = 1 - \frac{1}{\pi_c^k \left(1 + \frac{\gamma - 1}{2} M_0^2\right)} = 1 - \frac{1}{\pi_c^k \psi_0}$$

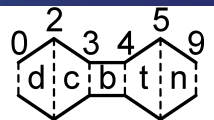


Per numero di Mach fissato il **rendimento aumenta** al crescere del **rapporto di pressioni** nel compressore. Però un aumento eccessivo della rapporto di pressione implica anche temperature più elevate e di conseguenza:

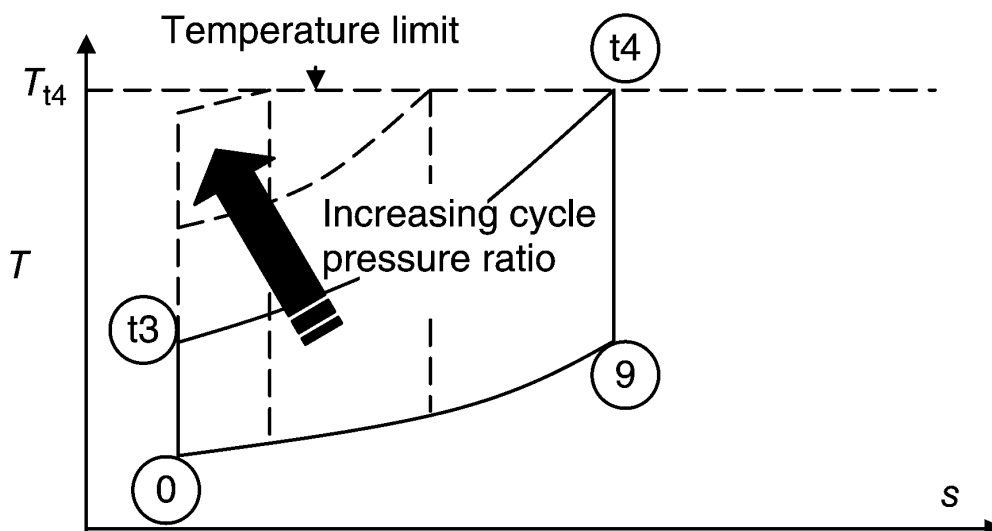
- Utilizzo di superleghe anche negli stadi finali del compressore (HPC);
- Raffreddamento del HPC;
- Passaggi anulari molto stretti con conseguenti grosse perdite secondarie nel HPC.



Rendimento del ciclo ideale



Un ulteriore problema è, che come già detto, il tendere della temperatura a valle del compressore alla temperatura limite porta una **diminuzione** del **lavoro** che può essere **estratto dal ciclo**.



$$\eta_{th} = 1 - \frac{1}{\pi_c^{\frac{\gamma-1}{\gamma}} \left(1 + \frac{\gamma-1}{2} M_0^2 \right)}$$

Mantenendo costante π_c l'aumento del numero di Mach provoca un aumento del rendimento.

Il **rendimento è minimo** al punto **fisso**.

Se M_0 aumenta molto può essere conveniente eliminare il compressore perché la compressione fornita dalla presa d'aria è sufficiente.



Rendimento del ciclo ideale

Ricordando la:

$$W_{net} = c_p T_1 \left[\left(\frac{T_3}{T_1} - \tau_c \right) (1 - \tau_c^{-1}) \right]$$

Che riscritta in un ambito più generale diventa:

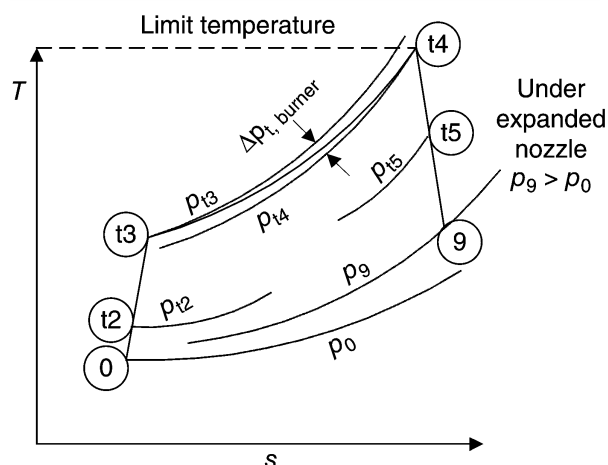
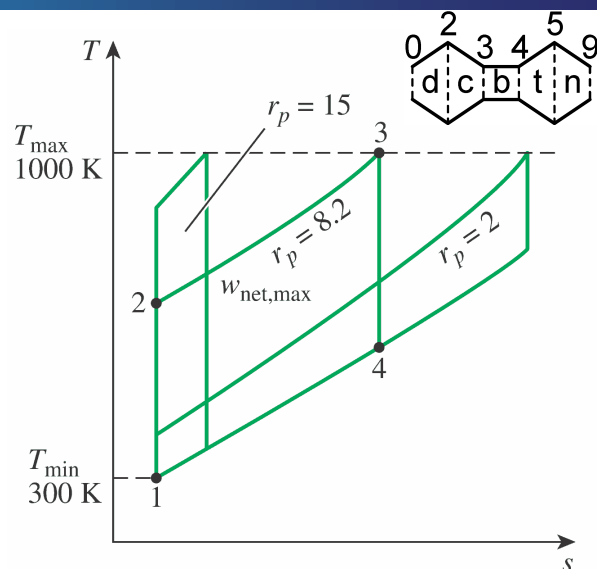
$$W_{net} = h_0 \left[(\tau_\lambda - \tau_c \tau_r) \left(1 - \frac{1}{\tau_c \tau_r} \right) \right]$$

Nel ciclo ideale si ha: $c_p = c_{pt} \rightarrow$

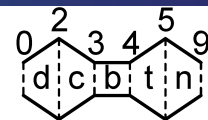
$$\tau_\lambda = \frac{T_{t4}}{T_0}$$

Derivando (rispetto a $\tau_c \tau_r$) si ottiene che il lavoro prodotto è massimo per:

$$\tau_c \tau_r = \sqrt{\tau_\lambda} \rightarrow \tau_c = \frac{\sqrt{\tau_\lambda}}{\tau_r} = \frac{\sqrt{\tau_\lambda}}{\psi_0}$$

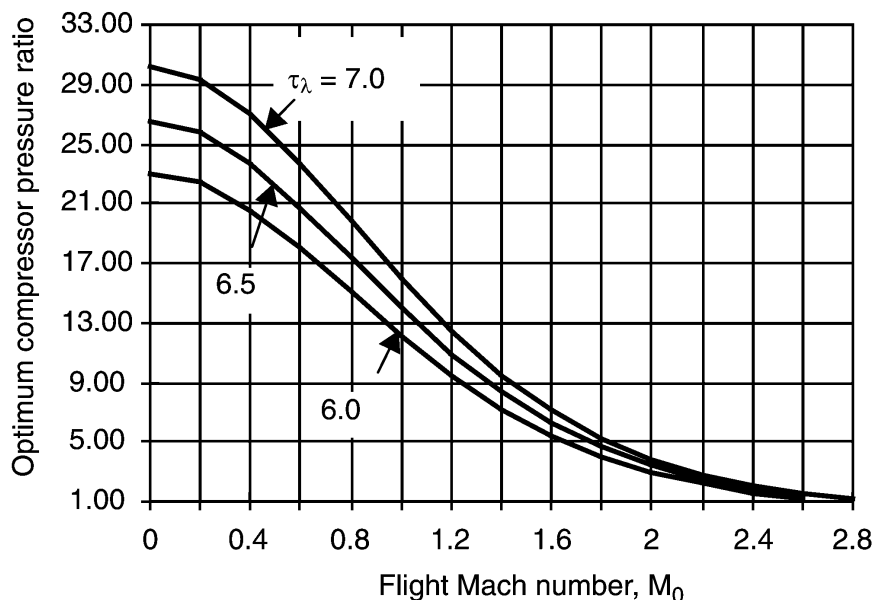


Rendimento del ciclo ideale

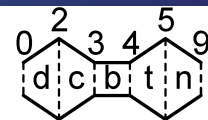


La relazione precedente può essere riscritta in termini di **rapporto di pressione ottimo** (per aumentare la **spinta**):

$$\pi_{c.max.F} = \left(\frac{\sqrt{\tau_\lambda}}{\tau_r} \right)^{\frac{1}{k}} = \left(\frac{\sqrt{\tau_\lambda}}{\psi_0} \right)^{\frac{1}{k}} = \left(\frac{\sqrt{\tau_\lambda}}{1 + \frac{\gamma-1}{2} M_0^2} \right)^{\frac{1}{k}}$$



Rendimento del ciclo ideale



La relazione precedente può essere riscritta in termini di **rapporto di pressione ottimo** (per aumentare la **spinta**):

$$\pi_{c.max.F} = \left(\frac{\sqrt{\tau_\lambda}}{\psi_0} \right)^{\frac{1}{k}} = \left(\frac{\sqrt{\tau_\lambda}}{1 + \frac{\gamma-1}{2} M_0^2} \right)^{\frac{1}{k}}$$

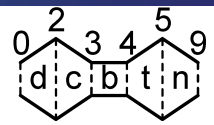
Se si impone che il rapporto di pressioni ottimo sia unitario (i.e. compressore assente) si può determinare il minimo numero di Mach di volo per cui un **RamJet** diventa più efficiente di un turbo getto:

$$M_{0.max} = \sqrt{\frac{2}{\gamma-1} (\sqrt{\tau_\lambda} - 1)}$$

Per aria si ha $M_{0.max} = 2.78$ e 2.87 per τ_λ rispettivamente uguale a 6.5 e 7.



Rendimento del ciclo ideale



Riprendendo l'equazione $\tau_c \tau_r = \sqrt{\tau_\lambda}$ si ha:

$$\sqrt{\tau_\lambda} = \sqrt{\frac{T_{t4}}{T_0}} = \sqrt{\frac{T_{t4}}{T_9} \frac{T_9}{T_0}}$$

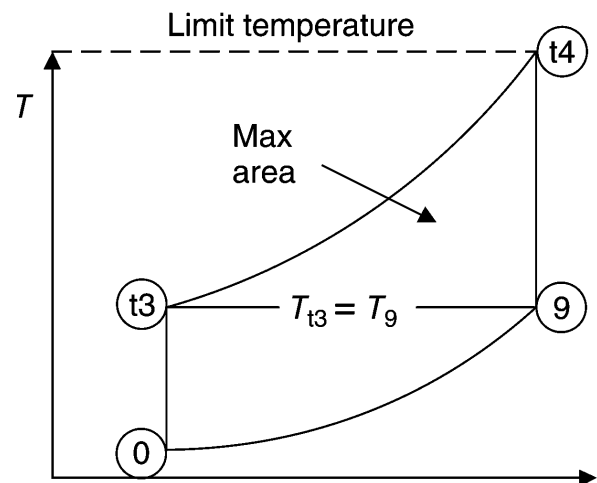
Per un ciclo ideale: $\frac{T_{t4}}{T_9} = \left(\frac{p_{t4}}{p_9}\right)^k = \left(\frac{p_{t3}}{p_0}\right)^k = \frac{T_{t3}}{T_0}$ da cui:

$$\sqrt{\tau_\lambda} = \sqrt{\frac{T_{t4}}{T_9} \frac{T_9}{T_0}} = \sqrt{\frac{T_{t3}}{T_0} \frac{T_9}{T_0}}$$

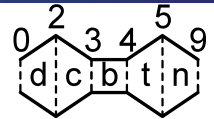
Quindi:

$$\tau_c \tau_r = \frac{T_{t3}}{T_{t2}} \frac{T_{t2}}{T_0} = \frac{T_{t3}}{T_0} = \sqrt{\frac{T_{t3}}{T_0} \frac{T_9}{T_0}} = \sqrt{\tau_\lambda}$$

$$T_{t3} = \sqrt{T_{t3} T_9} \rightarrow T_{t3} = T_9$$



Rendimento del ciclo ideale



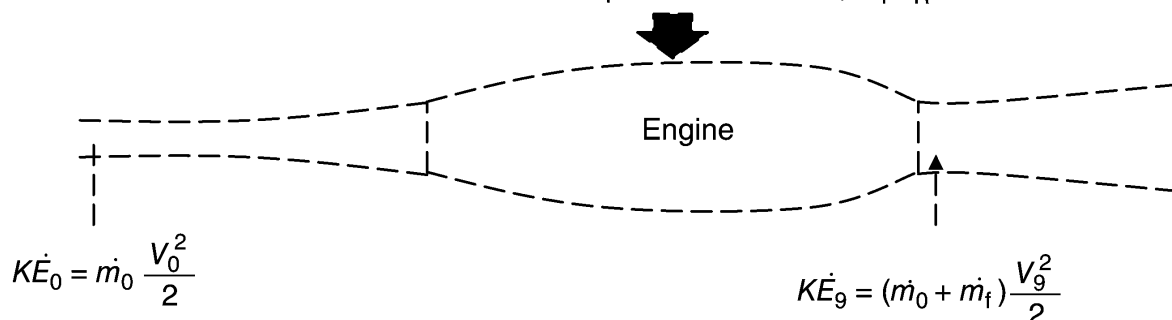
Se le seguenti ipotesi sono verificate:

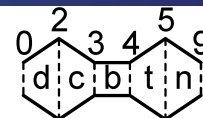
- f sia **trascurabile**;
- **componenti ideali** (reversibili);
- **espansione corretta**:

Si può dimostrare che la relazione per il rendimento termico, trovata in precedenza, coincide con la definizione termodinamica del rendimento termico:

$$\eta_{th} = \frac{\Delta K\dot{E}}{\mathcal{P}_t} = \frac{(1+f)V_9^2 - V_0^2}{2fQ_R} \rightarrow = 1 - \frac{T_0}{T_{t3}} = \eta_{th}$$

Thermal power investment, $\dot{m}_f Q_R$





Nella **progettazione preliminare** alcuni parametri sono assegnati mentre altri vengono derivati. **Una scelta possibile** (a sinistra della freccia quelli assegnati a destra quelli ricavati):

- Parametri di volo: $M_0, T_0, T_{t4}, c_p, \gamma, c_{p_t} = c_{p_9}, \gamma_t = \gamma_9 \rightarrow \tau_\lambda$;
- Presa d'aria: $\eta_d \rightarrow \pi_d, \pi_r$;
- Compressore: $e_c, \pi_c \rightarrow \tau_c, \eta_c$;
- Camera di combustione: $\eta_b, \pi_b, Q_R \rightarrow f, \tau_b$;
- Turbina: $e_t, \eta_m \rightarrow \tau_t, \pi_t, \eta_t$;
- Ugello: $\pi_n, p_0/p_9 \rightarrow \eta_n$.

Si suppone inoltre che:

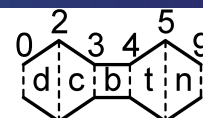
- $\tau_d = \tau_n = 1$.

Nel **caso ideale** si suppone che il gas sia sempre lo stesso e che:

- tutte le trasformazioni siano reversibili;
- $p_0 = p_9$;
- $(f + 1) \rightarrow 1, \pi_b = 1$.



Valutazione della spinta e degli indici di prestazione



Per calcolare gli **indici di prestazioni** si deve procedere a calcolare i parametri del ciclo componente per componente.

La spinta specifica è: $\frac{F_u}{\dot{m}_0} = (1 + f)V_9 - V_0 + \frac{(p_9 - p_0)A_9}{\dot{m}_0}$

dove la $\dot{m}_0$ può essere messa nella forma:

$$\dot{m}_0 = \frac{\dot{m}_9}{1 + f} = \frac{\rho_9 V_9 A_9}{1 + f} = \frac{p_9 V_9 A_9}{R_9 T_9 (1 + f)}$$

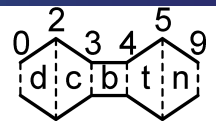
$$\frac{F_u}{\dot{m}_0} = (1 + f)V_9 - V_0 + \frac{R_9 T_9 (1 + f)}{V_9} \left(1 - \frac{p_0}{p_9} \right)$$

$$= (1 + f)V_9 \left[1 + \frac{1}{\gamma_9 M_9^2} \left(1 - \frac{p_0}{p_9} \right) \right] - V_0 \quad V_{9.e} = V_9 \left[1 + \frac{1}{\gamma_9 M_9^2} \left(1 - \frac{p_0}{p_9} \right) \right]$$

$$\frac{F_u}{\dot{m}_0 a_0} = (1 + f) \frac{V_9}{a_0} \left(1 + \frac{1 - \frac{p_0}{p_9}}{\gamma_9 M_9^2} \right) - M_0$$

dove i parametri ancora incogniti sono $\frac{V_9}{a_0}, M_9$ e f .





Il primo può essere calcolato come:

$$\frac{V_9}{a_0} = \frac{M_9 a_9}{a_0} = M_9 \sqrt{\frac{\gamma_9 R_9 T_9}{\gamma R T_0}}$$

Ricordando i simboli (θ rispetto alla temperatura all'infinito a monte):

$$k = \frac{\gamma - 1}{\gamma} \quad k_9 = \frac{\gamma_9 - 1}{\gamma_9} \quad \psi = 1 + \frac{\gamma - 1}{2} M^2 = \frac{T_t}{T}$$

Si ha:

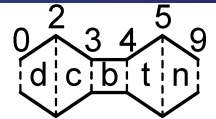
$$\frac{T_{t9}}{T_9} = \left(\frac{p_{t9}}{p_9} \right)^{k_9} = \psi_9 \quad \rightarrow \quad M_9^2 = \frac{2}{\gamma_9 - 1} \left[\left(\frac{p_{t9}}{p_9} \right)^{k_9} - 1 \right]$$

$$\frac{p_{t9}}{p_9} = \pi_n \pi_t \pi_b \pi_c \pi_d \pi_r \frac{p_0}{p_9} \quad \pi_r = \psi_0^{1/k}$$

dove $\pi_n, \pi_b, \pi_c, \pi_d$ e p_0/p_9 sono dati di input, mentre π_t si deve determinare.



Valutazione della spinta e degli indici di prestazione



Il rapporto di temperature:

$$\frac{T_9}{T_0} = \frac{T_9}{T_{t9}} \frac{T_{t9}}{T_0} = \frac{\theta_9}{\left(\frac{p_{t9}}{p_9} \right)^{k_t}} \quad \theta_9 = \cancel{\frac{T_{t9}}{T_{t5}}} \tau_t \frac{T_{t4}}{T_0} = \tau_t \tau_\lambda \frac{c_p}{c_{pt}}$$

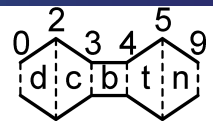
Come già detto nella CC:

$$f = \frac{\tau_\lambda - \tau_c \tau_r}{Q_R \eta_b / (c_p T_0) - \tau_\lambda} \quad \tau_b = \frac{\tau_\lambda}{\tau_c \tau_r}$$

Mentre dalla turbina e nel compressore si ha:

$$\tau_t = 1 - \frac{(\tau_c - 1) \tau_r}{\eta_m (1 + f) \tau_\lambda} \quad \pi_t = \tau_t^{\frac{1}{k_t e_t}} \quad \tau_c = \pi_c^{\frac{k}{e_c}}$$





Infine nella presa d'aria:

$$\tau_r = \theta_0 = \frac{T_{t0}}{T_0} = \psi_0 \quad \pi_d = \left(\frac{1 + \eta_d \frac{\gamma - 1}{2} M_0^2}{1 + \frac{\gamma - 1}{2} M_0^2} \right)^{1/k}$$

Che completa la ricerca dei parametri incogniti.

Se necessario si possono calcolare anche i rendimenti:

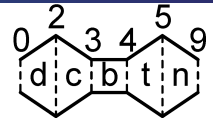
$$\eta_t = \frac{1 - \tau_t}{1 - \tau_\tau^{e_t}} \quad \eta_c = \frac{\pi_c^k - 1}{\tau_c - 1} \quad TSFC = \frac{f}{F_u/\dot{m}_0}$$

$$\eta_{th} = \frac{\Delta K \dot{E}}{\mathcal{P}_t} = \frac{(1 + f)V_9^2 - V_0^2}{2fQ_R} = \frac{a_0^2[(1 + f)(V_9/a_0)^2 - M_0^2]}{2fQ_R}$$

$$\eta_p = \frac{F_i V_0}{\Delta K \dot{E}} \approx \frac{2F_u V_0/\dot{m}_0}{a_0^2[(1 + f)(V_9/a_0)^2 - M_0^2]}$$



Valutazione della spinta e degli indici di prestazione

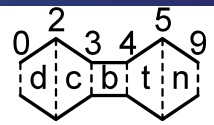


Per tenere in conto di una eventuale non corretta espansione nell'ugello si deve utilizzare la **velocità effettiva** nel calcolo dei rendimenti:

$$V_{9,e} = V_9 \left[1 + \frac{1 - \frac{p_0}{p_9}}{\gamma_9 M_9^2} \right]$$

$$\frac{F_u}{\dot{m}_0 a_0} = (1 + f) \frac{V_9}{a_0} \left(1 + \frac{1 - \frac{p_0}{p_9}}{\gamma_9 M_9^2} \right) - M_0 = (1 + f) \frac{V_{9,e}}{a_0} - M_0$$





Nell'ipotesi di ciclo ideale si ha:

$$\gamma_9 = \gamma \quad \tau_t = \pi_t^{k_t} \quad \tau_c = \pi_c^k \quad \pi_d = \pi_n = \pi_b = 1$$

$$\dot{m}_0 \approx \dot{m}_9 \quad \rightarrow \quad 1 + f \rightarrow 1 \quad p_9 = p_0$$

Quindi i parametri sono:

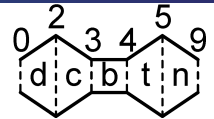
- Parametri di volo: $M_0, T_0, T_{t4}, c_p, \gamma \rightarrow \tau_\lambda$;
- Presa d'aria: $\rightarrow \pi_r$;
- Compressore: $\pi_c \rightarrow \tau_c$;
- Camera di combustione: $Q_R \rightarrow f, \tau_b$;
- Turbina: $\rightarrow \tau_t, \pi_t$;

La spinta è:

$$\frac{F_u}{\dot{m}_0 a_0} = (1 + f) \frac{V_9}{a_0} \left(1 + \frac{1 - \frac{p_0}{p_9}}{\gamma_9 M_9^2} \right) - M_0 = \frac{V_9}{a_0} - M_0$$



Valutazione della spinta caso ideale



$$\frac{F_u}{\dot{m}_0 a_0} = \frac{V_9}{a_0} - M_0$$

Riprendendo le equazioni precedenti::

$$\frac{V_9}{a_0} = \frac{M_9 a_9}{a_0} = M_9 \sqrt{\frac{T_9}{T_0}} \quad \frac{p_{t9}}{p_9} = \cancel{\pi_n}^1 \cancel{\pi_t}^1 \cancel{\pi_b}^1 \cancel{\pi_c}^1 \cancel{\pi_d}^1 \cancel{\pi_r}^1 \frac{p_9}{p_9} = \pi_t \pi_c \pi_r$$

$$M_9^2 = \frac{2}{\gamma - 1} \left[\left(\frac{p_{t9}}{p_9} \right)^k - 1 \right] = \frac{2}{\gamma - 1} [(\pi_t \pi_c \pi_r)^k - 1] = \frac{2}{\gamma - 1} (\tau_t \tau_c \tau_r - 1)$$

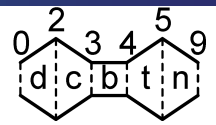
$$\frac{T_9}{T_0} = \frac{T_9}{T_{t9}} \frac{T_{t9}}{T_0} = \frac{\theta_9}{\left(\frac{p_{t9}}{p_9} \right)^k} = \frac{\theta_9}{\tau_t \tau_c \tau_r} = \frac{\tau_\lambda = \theta_9 / \tau_t}{\overbrace{T_{t9} T_{t4}}^{T_0 T_{t5}}} \frac{1}{\tau_c \tau_r} = \frac{\tau_\lambda}{\tau_c \tau_r} = \tau_b$$

$$\frac{F_u}{\dot{m}_0 a_0} = \sqrt{\frac{2}{\gamma - 1} \frac{\tau_\lambda}{\tau_c \tau_r} (\tau_t \tau_c \tau_r - 1)} - M_0$$



Valutazione della spinta caso ideale

$$\tau_b = \frac{\tau_\lambda}{\tau_c \tau_r}$$



$$\frac{F_u}{\dot{m}_0 a_0} = \sqrt{\frac{2}{\gamma - 1} \frac{\tau_\lambda}{\tau_c \tau_r} (\tau_t \tau_c \tau_r - 1) - M_0^2}$$

inoltre:

$$\tau_t = 1 - \frac{(\tau_c - 1)\tau_r}{\eta_m(1 + f)\tau_\lambda} = 1 - \frac{(\tau_c - 1)\tau_r}{\tau_\lambda}$$

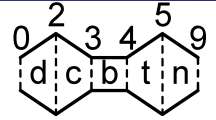
La spinta può essere manipolata:

$$\begin{aligned} \frac{\tau_\lambda}{\tau_c \tau_r} (\tau_t \tau_c \tau_r - 1) &= \frac{\tau_\lambda}{\tau_c \tau_r} \left(\left(1 - \frac{(\tau_c - 1)\tau_r}{\tau_\lambda} \right) \tau_c \tau_r - 1 \right) = \\ &= \tau_\lambda - \tau_c \tau_r + \tau_r + \left(\frac{\tau_\lambda}{\tau_c} - \frac{\tau_\lambda}{\tau_c} \right) - \frac{\tau_\lambda}{\tau_c \tau_r} = \frac{\tau_\lambda}{\tau_c} (\tau_c - 1) - \tau_r (\tau_c - 1) + \tau_b (\tau_r - 1) \\ &= \tau_r (\tau_b - 1) (\tau_c - 1) + \tau_b (\tau_r - 1) \end{aligned}$$



Valutazione della spinta caso ideale

$$\tau_b = \frac{\tau_\lambda}{\tau_c \tau_r}$$



$$\frac{\tau_\lambda}{\tau_c \tau_r} (\tau_t \tau_c \tau_r - 1) = \tau_r (\tau_b - 1) (\tau_c - 1) + \tau_b (\tau_r - 1)$$

ma:

$$\tau_r = \psi_0 = 1 + \frac{\gamma - 1}{2} M_0^2 \quad \rightarrow \quad (\tau_r - 1) = \frac{\gamma - 1}{2} M_0^2$$

La spinta diventa:

$$\frac{F_u}{\dot{m}_0 a_0} = \sqrt{\frac{2}{\gamma - 1} \tau_r (\tau_b - 1) (\tau_c - 1) + \tau_b M_0^2 - M_0^2}$$

In questa forma è chiaro che:

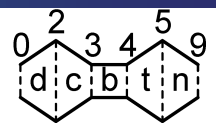
- se $\tau_b \rightarrow 1$, ovvero non c'è combustione, la spinta è nulla;
- se $\tau_c \rightarrow 1$ si ricade nel caso del RamJet:

$$\frac{F_u}{\dot{m}_0 a_0} = M_0 (\sqrt{\tau_b} - 1);$$



Valutazione della spinta caso ideale

$$\tau_b = \frac{\tau_\lambda}{\tau_c \tau_r}$$



$$\frac{F_u}{\dot{m}_0 a_0} = \sqrt{\frac{2}{\gamma - 1} \tau_r (\tau_b - 1) (\tau_c - 1) + \tau_b M_0^2 - M_0}$$

Inoltre:

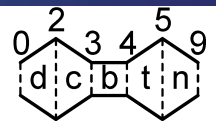
$$f = \frac{\tau_\lambda - \tau_c \tau_r}{Q_R \eta_b / (c_p T_0) - \tau_\lambda} = \frac{\tau_\lambda - \tau_c \tau_r}{Q_R / (c_p T_0)} = \frac{c_p T_0}{Q_R} (\tau_\lambda - \tau_c \tau_r)$$

dove si è trascurato $f \tau_\lambda$ rispetto a τ_λ . Quindi:

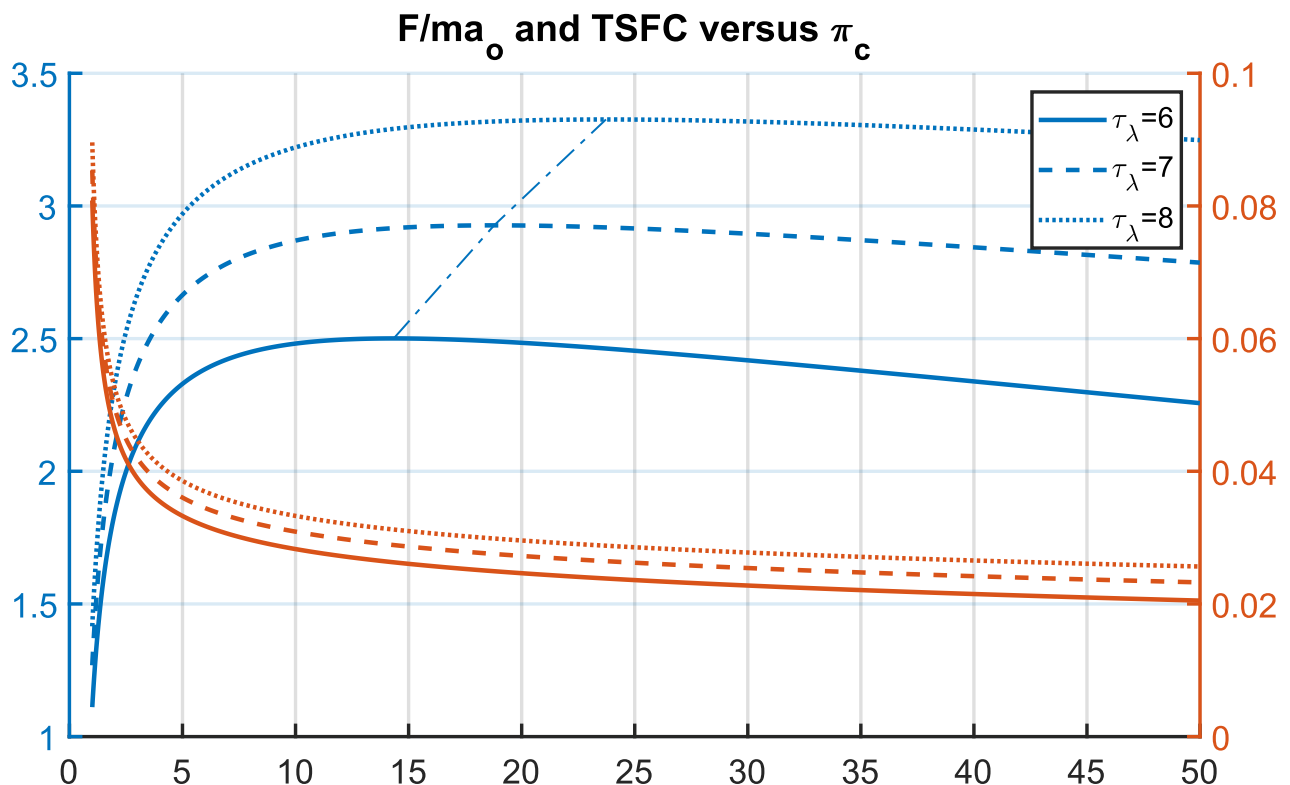
$$TSFC = \frac{f}{F_u / \dot{m}_0} = \frac{\frac{c_p T_0}{a_0 Q_R} (\tau_\lambda - \tau_c \tau_r)}{\sqrt{\frac{2}{\gamma - 1} \tau_r (\tau_b - 1) (\tau_c - 1) + \tau_b M_0^2 - M_0}}$$



Valutazione della spinta caso ideale

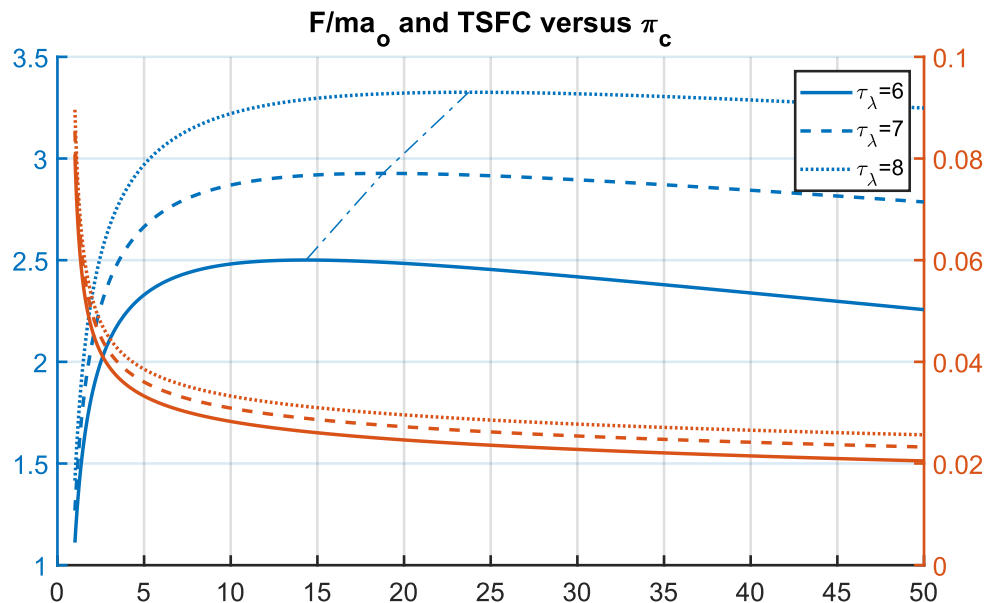


$$M_0 = 0.85, T_0 = 250K, Q_R = 42,800 kJ/kg$$

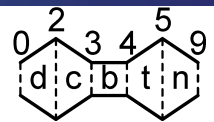


Dalla figura si nota:

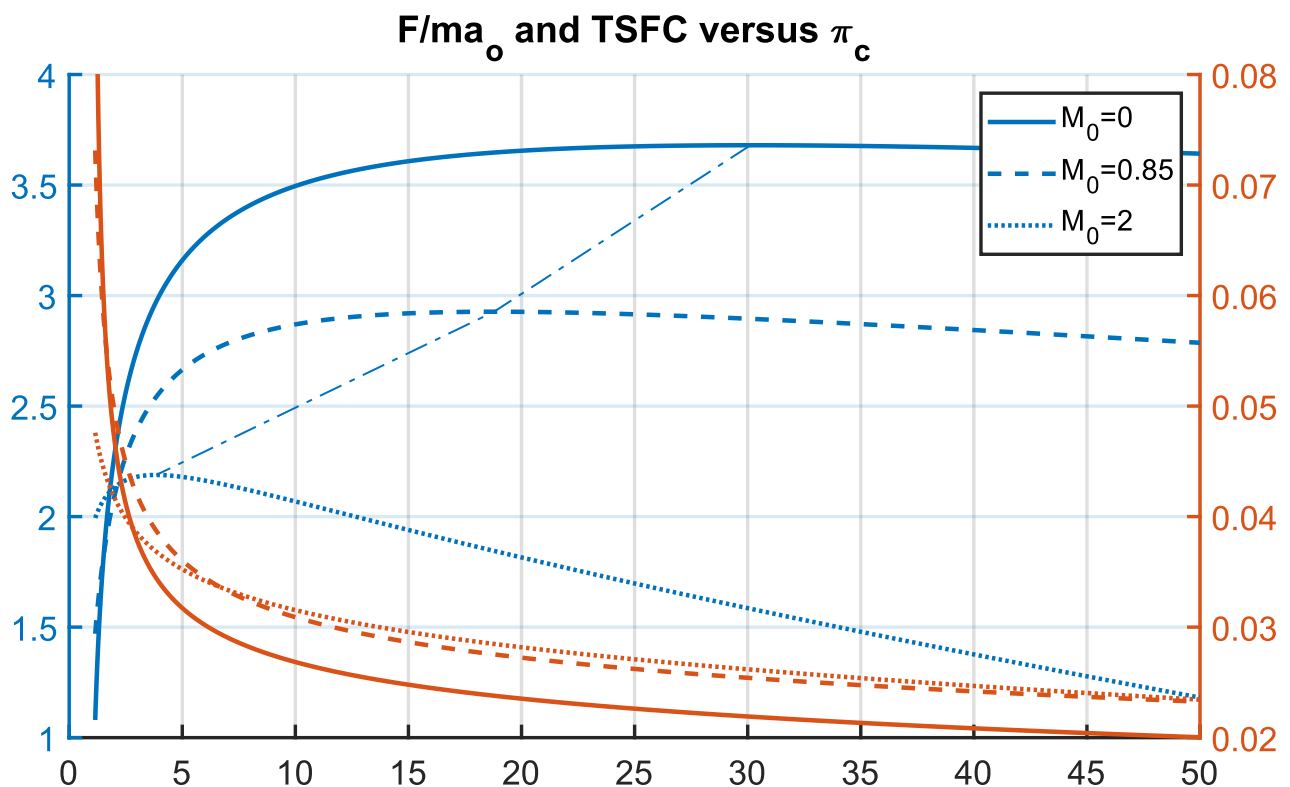
- All'aumentare di τ_λ aumentano sia la spinta specifica che il consumo specifico;
- Come già visto la curva della spinta ha un massimo, viceversa il consumo specifico ha un minimo per π_c maggiore;



Valutazione della spinta caso ideale

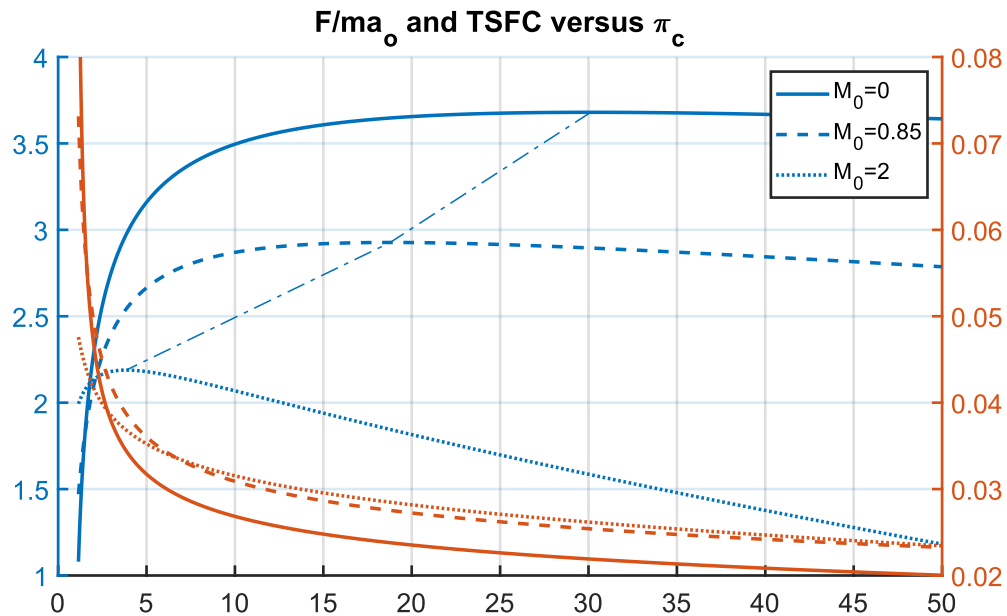


$$T_{t4} = 1750K, T_0 = 250K, Q_R = 42,800kJ/kg$$

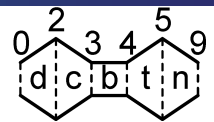


Dalla figura si nota:

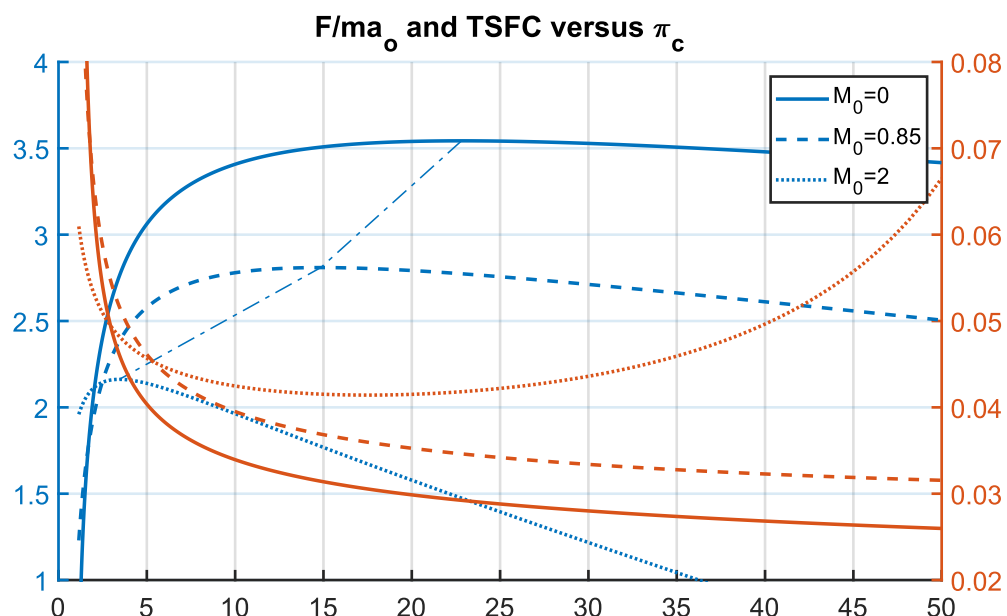
- All'aumentare del numero di Mach di volo il massimo della spinta specifica si sposta a sinistra;
- La spinta tende a zero per alti valori di π_c ma nel caso ideale il consumo specifico non diverge (f tende a zero).



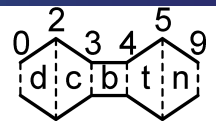
Valutazione della spinta



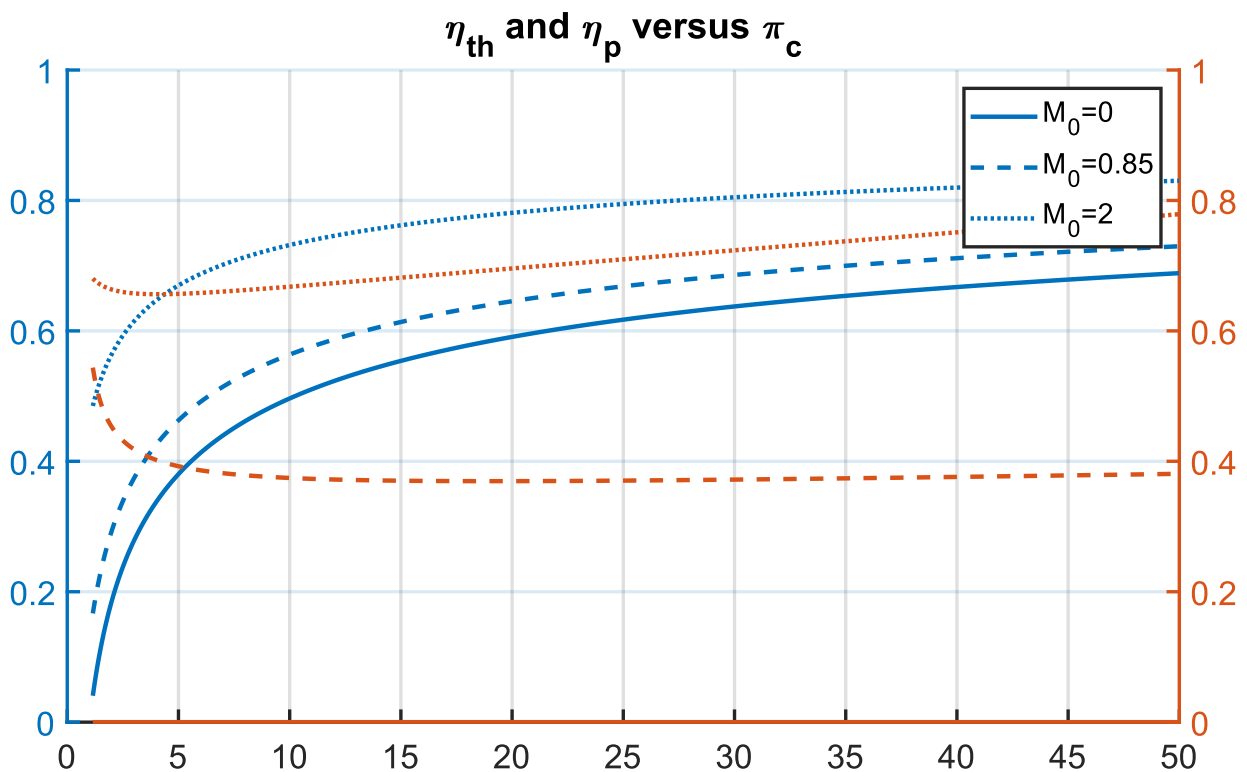
Nel **caso reale**, l'introduzione delle perdite, comportare una contenuta diminuzione della spinta ed un aumento del consumo specifico (ed una divergenza per alti π_c).



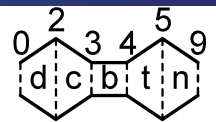
Valutazione della spinta caso ideale



$$T_{t4} = 1750K, T_0 = 250K, Q_R = 42,800kJ/kg$$

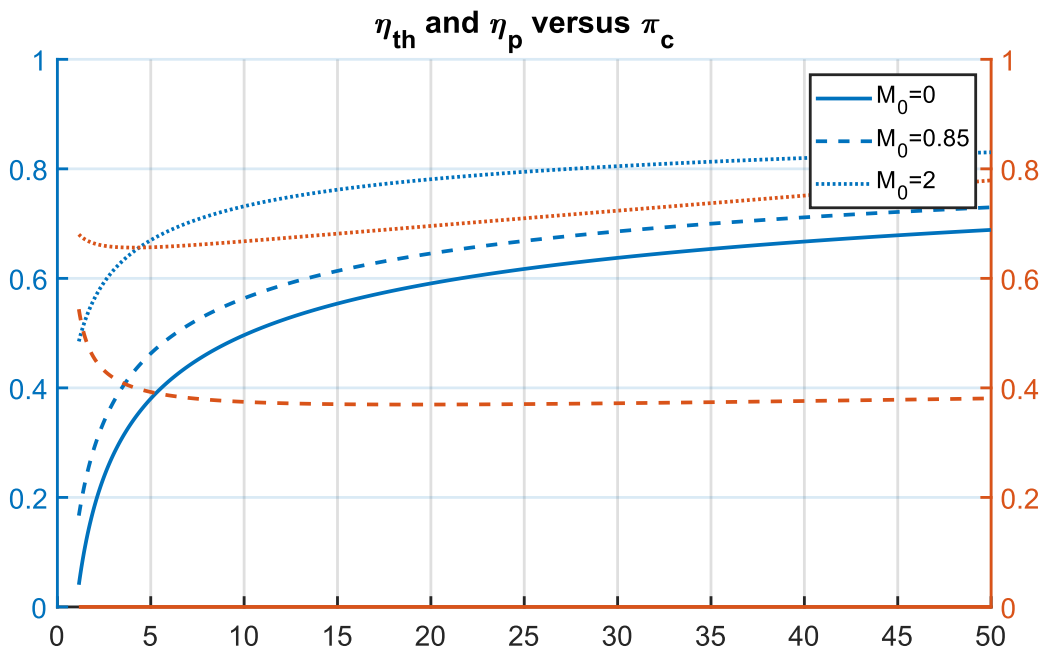


Valutazione della spinta caso ideale

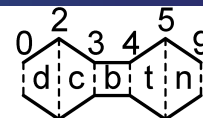


Il rendimento termico aumenta con il rapporto di compressione mentre quello propulsivo ha un minimo. Il rendimento complessivo aumenta.

$$\eta_p = \frac{F_i V_0}{\Delta K \dot{E}} \cong \frac{2V_0}{V_9 + V_0}$$



Valutazione della spinta



Chiaramente nel **caso reale** il rendimento termico peggiora significativamente.

