Esempio Farokhi 2.12

Consider a diffusing duct of the following geometrical and flow characteristics: A_1 is the inlet area, $M_1 = 0.70$, total pressure loss in the diffuser is 1% of the inlet total pressure, i.e., $p_{t2/}p_{t1} = 0.99$, the exit area is $A_2 = 1.237A_1$.

Assume that the flow in the diffuser is adiabatic and unseparated and the exit flow is uniform, calculate:

a) the exit Mach number M_2
b) the static pressure recovery in the diffuser C_{PR}
c) the force acting on the diffuser inner wall, i.e., $F_{x,wall}$, nondimensionalized by the inlet static pressure and area, i.e., $p_1\ A_1$

$$k = \frac{\gamma - 1}{\gamma} = \frac{0.4}{1.4} = 0.2857 \quad \frac{1}{k} = \frac{1}{0.2857} = 3.500 \quad K = \frac{\gamma + 1}{2(\gamma - 1)} = \frac{2.4}{0.4} = 3$$

$$\psi_1 = 1 + \frac{\gamma - 1}{2} M_1^2 = 1 + 0.2 \cdot 0.7^2 = 1.0980$$

Dalla:

$$\dot{m} = \frac{p_t A}{a_t} \Psi = \frac{p_{t1} A_1}{a_{t1}} \Psi_1 = \frac{p_{t2} A_2}{a_{t2}} \Psi_2 \rightarrow \Psi_2 = \Psi_1 \left(\frac{p_{t1}}{p_{t2}}\right)\left(\frac{A_1}{A_2}\right)$$

Ricordando che $\Psi(\gamma, M) = \gamma M \psi^{-K}, \psi = 1 + \frac{\gamma-1}{2} M^2,\ K = \frac{(\gamma+1)}{2(\gamma-1)}$ si ha:

$$M_2 \psi_2^{-K} = M_1 \psi_1^{-K} \left(\frac{p_{t1}}{p_{t2}}\right)\left(\frac{A_1}{A_2}\right) = \frac{0.7}{1.0980^3} \frac{1}{0.99} \frac{1}{1.237} = 0.4318$$

Supponendo che $M_2 = 0.7$ e risolvendo l'equazione precedente si ha:

$$M_{2,0} = 0.4138(1 + 0.2 \cdot 0.7^2)^3 = 0.5716 \quad \rightarrow \quad E_0 = 0.7 - 0.5716 = 0.1284$$

$$\mathrm{M}_{2,1} = 0.4138(1 + 0.2 \cdot 0.5716^2)^3 = 0.5221 \quad \rightarrow \quad \mathrm{E}_1 = 0.5716 - 0.5221 = 0.0495$$

Applicando il metodo di falsa posizione:

$$M_{2,3} = \frac{M_{2,0} E_1 - M_{2,1} E_0}{E_1 - E_0} = \frac{0.7 \cdot 0.0495 - 0.5716 \cdot 0.1284}{(0.0495 - 0.1284)} = 0.4973$$

Iterando si trova $M_2 = 0.4998\ e\ \psi_2 = 1.0500$

Il coefficiente di recupero di pressione statico è definito come:

$$C_{PR} = \frac{p_2 - p_1}{q_1} = \frac{p_1}{\frac{1}{2}\rho_1 V_1^2}\left(\frac{p_2}{p_1} - 1\right) = \frac{\gamma R T_1}{\frac{1}{2}\gamma V_1^2}\left(\frac{p_2}{p_1} - 1\right) = \frac{1}{\frac{\gamma}{2} M_1^2}\left(\frac{p_2}{p_1} - 1\right)$$

Ricordando che $p_t/\ p = \psi^{\frac{1}{k}}$ si ha

$$\frac{p_2}{p_1} = \frac{p_2}{p_{t2}} \frac{p_{t2}}{p_{t1}} \frac{p_{t1}}{p_1} = \frac{p_{t2}}{p_{t1}} \frac{\psi_1^{\frac{1}{k}}}{\psi_2^{\frac{1}{k}}} = 0.99 \left(\frac{1.0980}{1.0500}\right)^{3.5} = 1.1578$$

Da cui:

$$C_{PR} = \frac{0.1578}{0.3430} = 0.4600$$

La spinta si ricava dalla

$F_{x,wall} = I_2 - I_1$

Dove l'impulso totale può essere scritto come:

$I = A(p + \rho V^2) = Ap\left(1 + \frac{\gamma\rho}{\gamma p}V^2\right) = Ap\left(1 + \frac{\gamma}{a^2}V^2\right) = Ap(1 + \gamma M^2)$:

Quindi.

$$\frac{F_{x,wall}}{p_1\,A_1} = (1 + \gamma M_2^2)\frac{A_2}{A_1}\frac{p_2}{p_1} - (1 + \gamma M_1^2) = 1.3497\ \cdot 1.237 \cdot 1.1578 - 1.6860 = 0.2471$$

Essendo positiva significa che l'impulso uscente è maggiore di quello entrante quindi il diffusore produce una spinta positiva.

γ	1.4	k	0.285714	RHS	0.431803	
M_1	0.7	K	3	M2	M2	E
A_2/A_1	1.237	$(\gamma$-1)/2	0.2	0.7	0.5716	0.1284
p_{t2}/p_{t1}	0.99	ψ_1	1.0980	0.571601	0.5221	0.0495
$\gamma/2\,M_1^2$	0.3430	ψ_2	1.0500	0.491055	0.4973	-0.0063
p_2/p_1	1.1578	$1+\gamma M_1^2$	1.6860	0.500128	0.4999	0.0002
C_{PR}	0.460069	$1+\gamma M_2^2$	1.3497	0.499814	0.4998	0.0000
F	0.2471			0.499813	0.4998	0.0000

Esempio Farokhi 2.13

Consider a convergent–divergent nozzle as shown in Figure 2.57. The flow conditions are:

a) inlet Mach number $M_1 = 0.5$
b) inlet nozzle total pressure is $p_{t1} = 10p_0$ where p_0 is the ambient pressure
c) total pressure loss in the convergent section of the nozzle is 1%, i.e. $(p_{t1} - p_{t,th})/p_{t1} = 0.01$
d) total pressure loss in the divergent section of the nozzle is 2%, i.e. $(p_{t,th} - p_{t2})/p_{t,th} = 0.02$
e) nozzle area expansion ratio is $A_2/A_{th} = 2.0$

In addition, we assume that the gas is perfect and its properties remain unchanged throughout the nozzle. The gas is characterized by: γ =1.4, R=287J/kgK

The flow in the nozzle is assumed to be steady and adiabatic, therefore the total enthalpy remains constant, i.e., $h_{t2} = h_{t1}$.

Calculate

(a) the exit Mach number M_2

(b) the exit static pressure in terms of ambient pressure p_2/p_0

(c) the nondimensional axial force acting on the convergent nozzle $F_{x,con.-wall}/A_{th}p_{t1}$

(d) the nondimensional axial force acting on the diver-gent nozzle $F_{x,div.-wall}/A_{th}p_{t1}$

(e) the total (nondimensional) axial force acting on the nozzle $F_{x,\text{cnozzle}}/A_{th}p_{t1}$.

$$k = \frac{\gamma - 1}{\gamma} = \frac{0.4}{1.4} = 0.2857 \quad \frac{1}{k} = \frac{1}{0.2857} = 3.500 \quad K = \frac{\gamma + 1}{2(\gamma - 1)} = \frac{2.4}{0.4} = 3$$

$$\psi_1 = 1 + \frac{\gamma - 1}{2}M_1^2 = 1 + 0.2 \cdot 0.5^2 = 1.0500$$

Dalla:

$$\dot{m} = \frac{p_t A}{a_t}\Psi = \frac{p_{t,th}A_{\text{th}}}{a_{t,th}}\Psi^* = \frac{p_{t2}A_2}{a_{t2}}\Psi_2 \rightarrow \Psi_2 = \Psi^*\left(\frac{p_{t,\text{th}}}{p_{t2}}\right)\left(\frac{A_{th}}{A_2}\right)$$

Ricordando che $\Psi(\gamma, M) = \gamma M\psi^{-K}, \psi = 1 + \frac{\gamma-1}{2}M^2,\ K = \frac{(\gamma+1)}{2(\gamma-1)}$ si ha:

$$M_2\psi_2^{-K} = M_{th}\psi_{th}^{-K}\left(\frac{p_{t,\text{th}}}{p_{t2}}\right)\left(\frac{A_{th}}{A_2}\right) = \frac{1}{1.200^3}\frac{1}{0.98}\frac{1}{2} = 0.2953$$

Supponendo che $M_2 = 2$ e risolvendo l'equazione precedente si ha:

$M_{2,0} = 0.2953(1 + 0.2 \cdot 2^2)^3 = 1.7219 \quad \rightarrow \quad E_0 = 2 - 1.7219 = 0.2781$

$\mathrm{M}_{2,1} = 0.2953(1 + 0.2 \cdot 1.7219^2)^3 = 1.1936 \ \rightarrow\ \mathrm{E}_1 = 1.7219 - 1.1936 = 0.5283$

Applicando il metodo di falsa posizione:

$$M_{2,3} = \frac{M_{2,0}E_1 - M_{2,1}E_0}{E_1 - E_0} = \frac{2 \cdot 0.5283 - 1.7219 \cdot 0.2781}{(0.5283 - 0.2781)} = 2.605$$

Iterando si trova $M_2 = 2.1743\ e\ \psi_2 = 1.9455$

Ricordando che $p_t/\ p = \psi^{\frac{1}{k}}$ si ha

$$\frac{p_2}{p_0} = \frac{p_2}{p_{t2}}\frac{p_{t2}}{p_{t,th}}\frac{p_{t,th}}{p_{t1}}\frac{p_{t1}}{p_0} = \psi_2^{-\frac{1}{k}}\frac{p_{t2}}{p_{t,th}}\frac{p_{t,th}}{p_{t1}}\frac{p_{t1}}{p_0} = \left(\frac{1.}{1.9455}\right)^{3.5} 0.98 \cdot 0.99 \cdot 10\ = 0.9445$$

Quindi l'ugello è leggermente sovra espanso.

La spinta si ricava dalla

$$F_{x,wall} = I_2 - I_1$$

Dove l'impulso totale può essere scritto come:

$$I = A(p + \rho V^2) = Ap\left(1 + \frac{\gamma\rho}{\gamma p}v^2\right) = Ap\left(1 + \frac{\gamma}{a^2}v^2\right) = Ap(1 + \gamma M^2):$$

La spinta è

$$\frac{F_{x,con.-wall}}{A_{th}p_{t1}} = (1 + \gamma M_{th}^2)\frac{p_{th}}{p_{t1}} - (1 + \gamma M_1^2)\frac{A_1}{A_{th}}\frac{p_1}{p_{t1}}$$

Dove, come già visto:

$$\dot{m} = \frac{p_t A}{a_t}\Psi = \frac{p_{t,th}A_{th}}{a_{t,th}}\Psi^* = \frac{p_{t1}A_1}{a_{t1}}\Psi_1 \rightarrow \frac{A_1}{A_{th}} = \frac{\Psi^*}{\Psi_1}\frac{p_{t,th}}{p_{t1}}$$

$$\frac{A_1}{A_{th}} = \frac{0.8102}{\Psi_1}\frac{p_{t,th}}{p_{t1}}$$

Inoltre $\frac{p_1}{p_{t1}} = \psi_1^{\frac{-1}{k}} = 0.8430$ e $\frac{p_{th}}{p_{t,th}} = \psi_{th}^{\frac{-1}{k}} = 0.5283$

Quindi:

$$\frac{F_{x,con.-wall}}{A_{th}p_{t1}} = (1 + \gamma M_{th}^2)\frac{p_{th}}{p_{t,th}}\frac{p_{t,th}}{p_{t1}} - (1 + \gamma M_1^2)\frac{A_1}{A_{th}}\frac{p_1}{p_{t1}}$$
$$= 2.400 \cdot 0.5283 \cdot 0.99 - 1.3500 \cdot 1.3264 \cdot 0.8430 = -0.2544$$

Essendo negativa significa che l'impulso uscente è minore di quello entrante quindi il convergente produce una spinta negativa.

Nel divergente si ha:

$$\frac{F_{x,div.-wall}}{A_{th}p_{t1}} = (1+\gamma M_2^2)\frac{A_2}{A_{th}}\frac{p_2}{p_0}\frac{p_0}{p_{t1}} - (1+\gamma M_{th}^2)\frac{p_{th}}{p_{t,th}}\frac{p_{t,th}}{p_{t1}}$$
$$= 7.6188 \cdot 2 \cdot 0.9445/10\ - 2.400 \cdot 0.5283 \cdot 0.99 = 0.1840$$

Essendo positiva significa che l'impulso uscente è maggiore di quello entrante quindi il divergente produce una spinta positiva.

La forza complessiva è la somma delle due forze calcolate in precedenza:

$$\frac{F_{x,\mathrm{cnozzle}}}{A_{th}p_{t1}} = \frac{F_{x,con.-wall}}{A_{th}p_{t1}} + \frac{F_{x,div.-wall}}{A_{th}p_{t1}} = -0.544 + 0.1840 = -0.0704$$

Anche questa è una resistenza.

γ	1.4	k	0.285714			RHS	0.2953	
M_1	0.5	K	3			M2	M2	E
A_2/A_{th}	2	$(\gamma-1)/2$	0.2			2	1.7219	0.2781
M_{th}	1	ψ_1	1.0500	Ψ_1	0.6047	1.7219	1.1936	0.5283
$p_{t,th}/p_{t1}$	0.99	ψ_{th}	1.2000	Ψth	0.8102	2.308931	2.6046	-0.2957
$p_{t2}/p_{t,t1}$	0.98	ψ_2	1.9455	Ψ_2	0.4134	2.098316	1.9637	0.1346
p_1/p_0	10	$1+\gamma M_1^2$	1.3500	p_1/p_{t1}	0.843019	2.164203	2.1450	0.0192
$p_2/p0$	0.9445	$1+\gamma M_{th}^2$	2.4000	$p_{th}/p_{t,th}$	0.528282	2.175178	2.1768	-0.0016
A_1/A_{th}	1.3264	$1+\gamma M_2^2$	7.6188	p_2/p_{t2}	0.097354	2.17433	2.1743	0.0000
F_{conv}	-0.2544	F_{tot}	-0.07036			2.174339	2.1743	0.0000
F_{div}	0.18404					2.174339	2.1743	0.0000