Esercizio Mattingly 9.35

m	40	kg/s	Cz2=Cz3	
ω	1000	rad/s	M1	0.3
Ro	0.4	m	M2	1.15
Tt1	1780	K	$\alpha 1,3$	0
pt1	1,400,000	Pa	ω_r	0.08
ΔTt	230	K	ω_s	0.04
R	287	J/kgK	σ_r	1
cp	1243.6667		γ	1.3
Ψ^*	0.7607961		k	0.2308

$$k = \frac{\gamma - 1}{\gamma} = \frac{0.3}{1.3} = 0.2308 \qquad K = \frac{\gamma + 1}{2(\gamma - 1)} = \frac{2.3}{0.6} = 3.833$$

$$\psi_1 = 1 + \frac{\gamma - 1}{2} M_1^2 = 1 + 0.15 \cdot 0.3^2 = 1.0135$$

$$T_1 = \frac{T_{t1}}{\psi_1} = \frac{1780}{1.0135} = 1756.3K$$

$$p_1 = \frac{p_{t1}}{\psi_1^{\frac{1}{k}}} = \frac{1400}{1.0135^{\frac{1}{0.2308}}} = 1321.0kPa$$

$$a_{t1} = \sqrt{\gamma R T_{t1}} = 1.3 \cdot 287 \cdot 1780 = 814.9m/s$$

$$a_1 = \sqrt{\gamma R T_1} = 1.3 \cdot 287 \cdot 1756 = 809.5m/s$$

$$C_1 = M_1 \cdot a_1 = 0.3 \cdot 809.5 = 242.8m/s$$

$$C_{\theta 1} = C_1 \cdot \sin\alpha_1 = 0m/s \qquad C_{z1} = C_1 \cdot \cos\alpha_1 = 242.8m/s$$

$$U = \omega r_0 = 1000 \cdot 0.4 = 400\frac{m}{s}$$

$$\dot{m} = \frac{p_{t1}\frac{A^*}{A_{1R}} A_{1R} \psi^*}{a_{t1}}$$

$$A_1 = \frac{A_{1R}}{\cos\alpha} = \frac{\dot{m} a_{t1}}{\cos\alpha\, p_{t1}} \frac{A_{1R}}{\psi^* A^*} = \frac{\dot{m} a_{t1}}{\cos\alpha\, p_{t1}} \frac{1}{\Psi_1} = \frac{\dot{m} a_{t1}}{\cos\alpha\, p_{t1}} \frac{\psi_1^K}{\gamma M_1}$$

$$= \frac{40 \cdot 814.9}{1400 \cdot 10^3} \frac{1.0135\text{^}3.833}{1.3 \cdot 0.3} = 0.06285m^2$$

$$T_{t2} = T_{t1} = 1780K$$

$$\psi_2 = 1 + \frac{\gamma - 1}{2} M_2^2 = 1 + 0.15 \cdot 1.15^2 = 1.1984$$

$$T_2 = \frac{T_{t2}}{\psi_2} = \frac{1780}{1.1984} = 1485K$$

$$\omega_s = \frac{p_{t1} - p_{t2}}{p_{t2} - p_2} = \omega_s = \frac{1 - \frac{p_{t2}}{p_{t1}}}{\frac{p_{t2}}{p_{t1}}\left(1 - \frac{p_2}{p_{t2}}\right)} \rightarrow$$

$$p_{t2} = \frac{p_{t1}}{1 + \omega_s\left(1 - \psi_2^{-\frac{1}{k}}\right)} = \frac{1400 \cdot 10^3}{1 + 0.04\left(1 - 1.1984^{\frac{-1}{0.2308}}\right)} = 1{,}370KPa$$

$$p_2 = p_{t2}\psi_2^{\frac{-1}{k}} = 1370 \cdot 1.1984^{\frac{-1}{0.2308}} = 625.5kPa$$

$$a_{t2} = \sqrt{\gamma R T_{t2}} = 1.3 \cdot 287 \cdot 1780 = 814.9m/s$$

$$a_2 = \sqrt{\gamma R T_2} = 1.3 \cdot 287 \cdot 1485 = 744.4m/s$$

$$C_2 = M_2 \cdot a_2 = 1.15 \cdot 744.4 = 856.1m/s$$

Dall'equazione di Eulero:

$$C_{\theta 2} = \frac{\Delta h_t}{U} = \frac{c_p \Delta T_t}{U} = 1243.7 \cdot \frac{230}{400} = 715.1m/s$$

$$\alpha_2 = \operatorname{asin}\frac{C_{\theta 2}}{C_2} = \operatorname{asin}\frac{715.1}{856.1} = 56.65°$$

$$A_2 = \frac{A_{2R}}{\cos\alpha_2} = \frac{\dot{m} a_{t2}}{\cos\alpha_2\, p_{t2}} \frac{\psi_2^K}{\gamma M_2} = \frac{40 \cdot 814.9}{0.5498 \cdot 1{,}370 \cdot 10^3} \frac{1.1984^{3.833}}{1.3 \cdot 1.15} = 0.05792m^2$$

$$C_{z2} = C_2 \cdot \cos\alpha_2 = 470.7m/s$$

$$W_{\theta 2} = C_{\theta 2} - U = 715.1 - 400 = 315.1\frac{m}{s}$$

$$W_2 = \sqrt{w_{\theta 2}^2 + W_{z2}^2} = \sqrt{470.7^2 + 315.1^2} = 566.4\frac{m}{s}$$

$$M_{2R} = \frac{W_2}{a_2} = \frac{566.4}{744.4} = 0.7609$$

$$\psi_{2R} = 1 + \frac{\gamma - 1}{2} M_{2R}^2 = 1 + 0.15 \cdot 0.7609^2 = 1.0868$$

$$T_{t2R} = T_2 \psi_{2R} = 1485 \cdot 1.0868 = 1614K$$

$$p_{t2R} = p_2 \psi_{2R}^{\frac{1}{k}} = 625.5 \cdot 1.0868^{\frac{1}{0.2308}} = 897{,}3kPa$$

$$\beta_2 = \operatorname{atan}\frac{W_{\theta 2}}{W_{z2}} = \operatorname{atan}\frac{315.1}{470.7} = 33.80°$$

$$C_{z3} = C_{z2} = 470.7m/s \qquad \alpha = 0 \rightarrow C_3 = C_{z3} = 470.7m/s \qquad C_{\theta_3} = 0m/s$$

$$T_{t3} = T_{t2} - \Delta T_t = 1780 - 230 = 1550K$$

$$T_3 = T_{t3} - \frac{C_3^3}{2c_p} = 1550 - \frac{470.7^2}{2 \cdot 1244} = 1460.9K$$

$$\psi_3 = \frac{T_{t3}}{T_3} = \frac{1550}{1460.9} = 1.0610$$

$$a_3 = \sqrt{\gamma R T_3} = 1.3 \cdot 287 \cdot 1460.9 = 738.3 m/s$$

$$a_{t3} = \sqrt{\gamma R T_{t3}} = 1.3 \cdot 287 \cdot 1550 = 760.5 m/s$$

$$M_3 = \frac{C_3}{a_3} = \frac{470.7}{738.3} = 0.6375$$

$$W_{\theta 3} = C_{\theta 3} - U = -400 = -400 \frac{m}{s}$$

$$W_3 = \sqrt{w_{\theta 3}^2 + W_{z3}^2} = \sqrt{400^2 + 470.7^2} = 617.7 \frac{m}{s}$$

$$M_{3R} = \frac{W_3}{a_3} = \frac{617.7}{738.3} = 0.8366$$

$$\psi_{3R} = 1 + \frac{\gamma - 1}{2} M_{3R}^2 = 1 + 0.15 \cdot 0.8366^2 = 1.1050$$

$$T_{t3R} = T_{t2R} = 1614K$$

$$p_{t3r} = \frac{p_{t2r}}{1 + \omega_r \left(1 - \psi_{3R}^{-\frac{1}{k}}\right)} = \frac{897{,}3 \cdot 10^3}{1 + 0.08 \left(1 - 1.1050^{\frac{-1}{0.2308}}\right)} = 872{,}8 kPa$$

$$p_3 = p_{t3R} \psi_{3R}^{\frac{-1}{k}} = 872{,}8 \cdot 1.1050^{\frac{-1}{0.2308}} = 566{,}2 kPa$$

$$p_{t3} = p_3 \psi_3^{\frac{1}{k}} = 566{,}2 \cdot 1.0610^{\frac{1}{0.2308}} = 731{,}8 kPa$$

$$\beta_3 = \operatorname{atan} \frac{W_{\theta 3}}{W_{z3}} = \operatorname{atan} \frac{-400}{470.7} = -40.36°$$

$$A_3 = \frac{A_{3R}}{\cos \alpha_3} = \frac{\dot{m} a_{t3}}{\cos \alpha_3 \, p_{t3}} \frac{\psi_3^K}{\gamma M_3} = \frac{40 \cdot 760.5}{731{,}8 \cdot 10^3} \frac{1.0610^{3.833}}{1.3 \cdot 0.6375} = 0.06293 m^2$$

$$°R = \frac{h_2 - h_3}{h_1 - h_3} = \frac{1485.3 - 1460.9}{1756.3 - 1460.9} = 0.0826$$

$$\eta = \frac{1 - \tau_t}{1 - \pi_t^k} = \frac{1 - \frac{1550}{1780}}{1 - \left(\frac{1400}{731{,}8}\right)^{0.2308}} = 0.9293$$

$$e_c = \frac{\ln \tau_t}{k \ln \pi_t} = \frac{\ln \frac{1550}{1780}}{0.2308 \ln \frac{1400}{731{,}8}} = 0.9241$$

$$\Psi_c = \frac{c_p \Delta T_c}{U^2} = \frac{1244 \cdot 230}{400^2} = 1.788$$

	1	2	2R	3R	3
Tt	**1780**	1780.0	1614.3	1614.3	**1550.0**
T	1756.3	1485.3	1485.3	1460.9	1460.9
pt	**1,400,000**	1,370,211	897,283	872,762	731,764
p	1,320,966	625,488	625487.9	566,246	470.7
M	**0.3**	**1.1500**	0.7609	0.8366	0.6375
C/W	242.8	856.1	566.4	617.7	470.7
W/C_z	242.8	470.7	470.7	470.7	470.7
W/C_θ	0.0	715.1	315.1	-400.0	0.0
α	**0**	56.65			**0**
β			33.80	-40.36	