



Propulsione Aerospaziale

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Termodinamica

Il fluido all'interno di un motore aerospaziale può essere modellato come un **Gas perfetto**. Le ipotesi fondamentali sono:

- Le forze intermolecolari sono piccole;
- Il volume proprio delle molecole può essere trascurato rispetto al volume occupato dal gas.

In queste ipotesi la prima equazione di stato è:

$$p = \rho RT$$

Dove R è la costante dei gas che può essere espressa in termini della costante universale dei gas:

$$R = \frac{\bar{R}}{MW} \quad \bar{R} = 8314.5 \frac{J}{\text{kmol} \cdot K}$$

La seconda equazione di stato è una fra le seguenti:

$$dh = c_p dT \quad de = c_v dT$$

Dove c_p e c_v sono i **calori specifici** a pressione e volume costante, mentre h ed e sono **l'entalpia** e **l'energia** interna ($h = e + pv$).



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In generale:

$$c_p = c_p(T) \quad c_v = c_v(T)$$

Nell'ipotesi di gas **più che perfetto** i calori specifici sono costanti e la seconda equazione di stato può essere espressa come:

$$h = h_0 + c_p T \quad e = e_0 + c_v T$$

Le tre costanti sono legate dalla relazione di Meyer:

$$c_p = c_v + R$$

Il rapporto fra i calori specifici è indicato con:

$$\gamma = \frac{c_p}{c_v}$$

Si ha anche con $k = \frac{\gamma-1}{\gamma}$:

$$c_p = \frac{\gamma}{\gamma-1} R = \frac{1}{k} R \quad c_v = \frac{1}{\gamma-1} R$$



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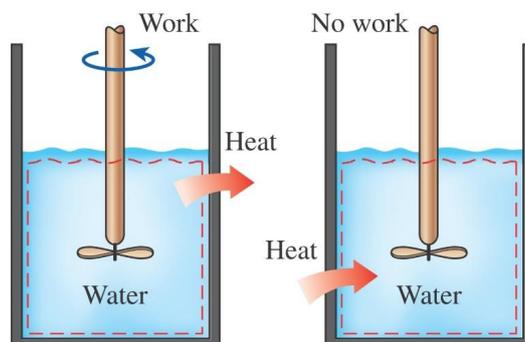
Per un sistema a massa costante, il **primo principio della termodinamica** è:

$$de = \delta q - \delta w$$

Dove per convenzione si considerano positivi il flusso di calore **entrante** δq nel sistema ed il lavoro δw fornito (**uscende**) dal sistema.

Il **secondo principio della termodinamica** introduce l'entropia ed un **verso** nelle trasformazioni termodinamiche.

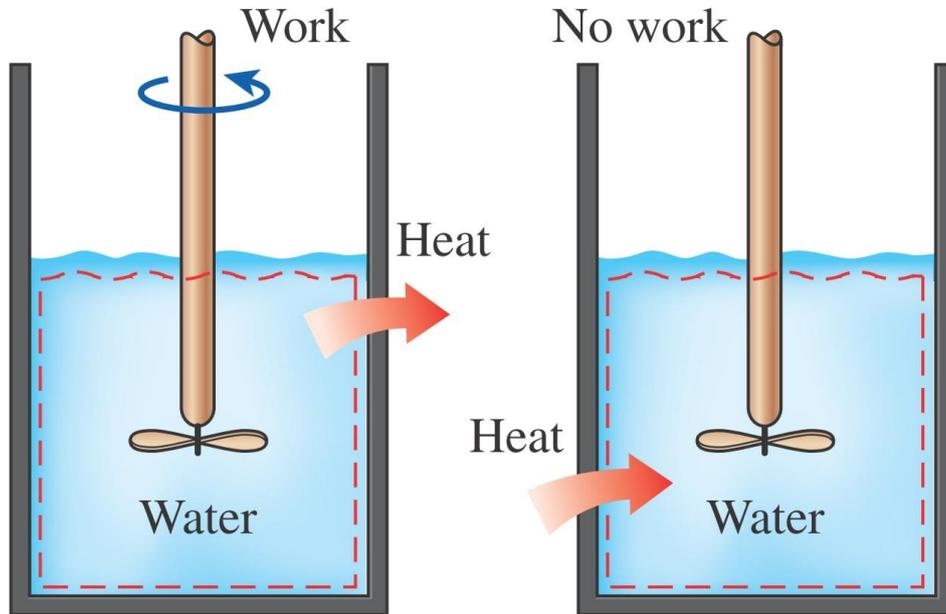
Il lavoro può essere facilmente trasformato in energia interna mentre il processo inverso è possibile solo utilizzando una macchina termica.



Termodinamica

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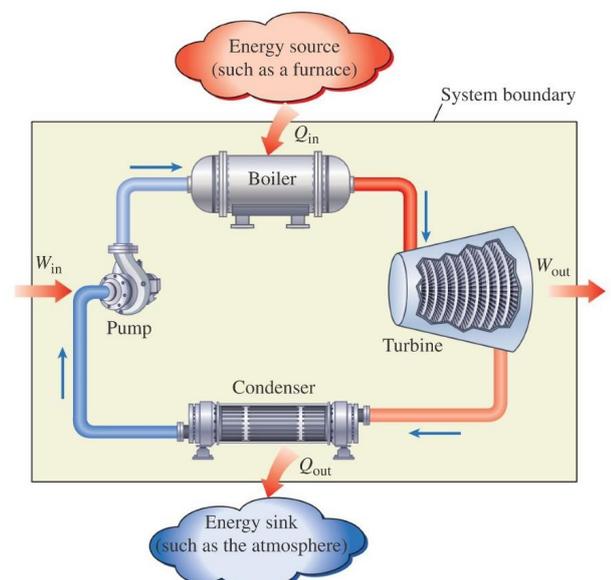
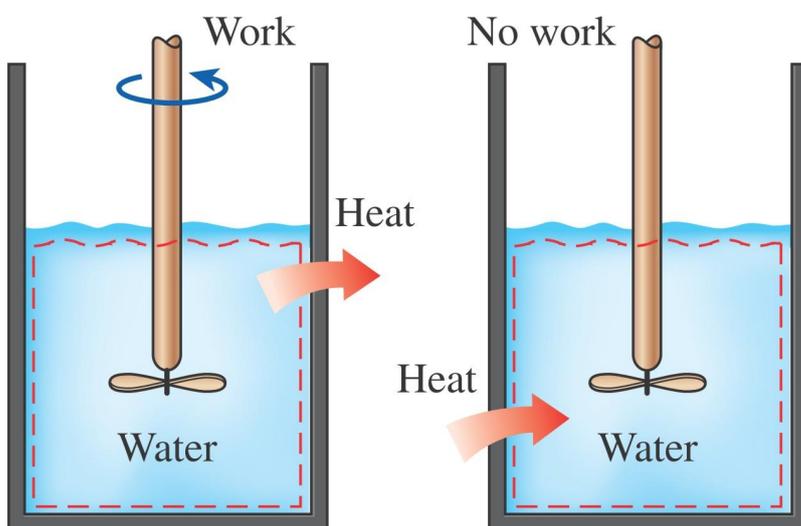


Termodinamica

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Ad esempio una centrale a vapore è schematizzata in figura.



Termodinamica

L'**evaporatore** trasforma l'acqua in vapore fornendo una certa quantità di **calore** Q_{in} .

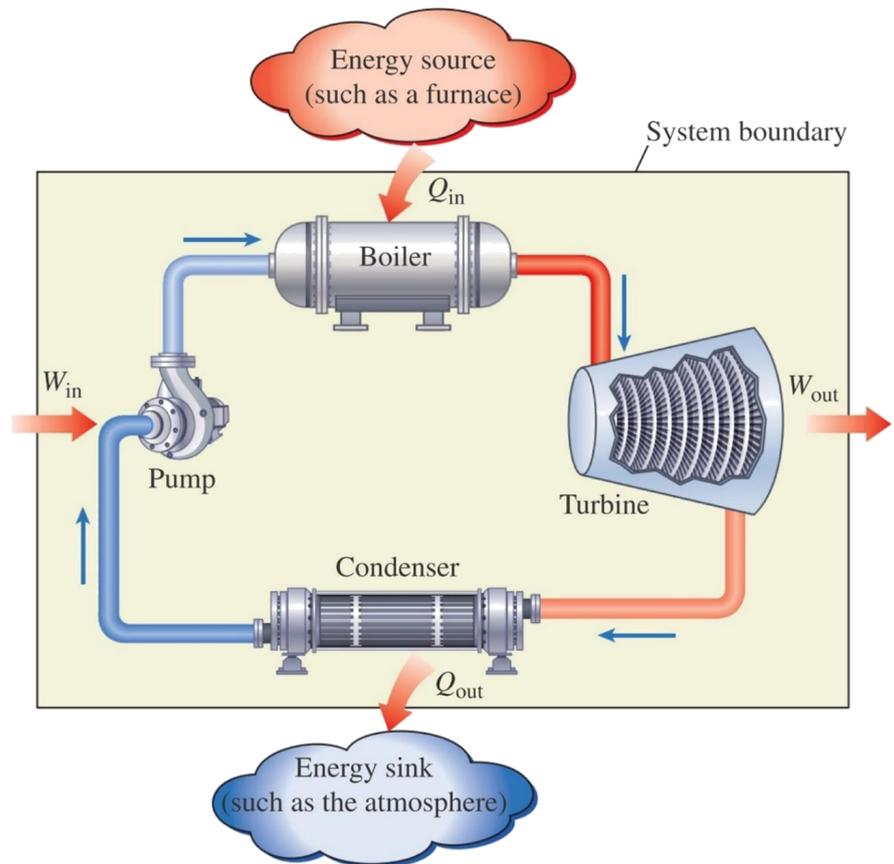
Nella **turbina** viene estratto il **lavoro** W_{out} .

Nel **condensatore** il vapore **condensa** e nel processo cede il **calore** Q_{out} .

La **pompa** comprime l'acqua fornendo al sistema il **lavoro** W_{in} .

Il **lavoro** estratto è dato dalla **differenza** dei due:

$$W_{net} = W_{out} - W_{in}$$

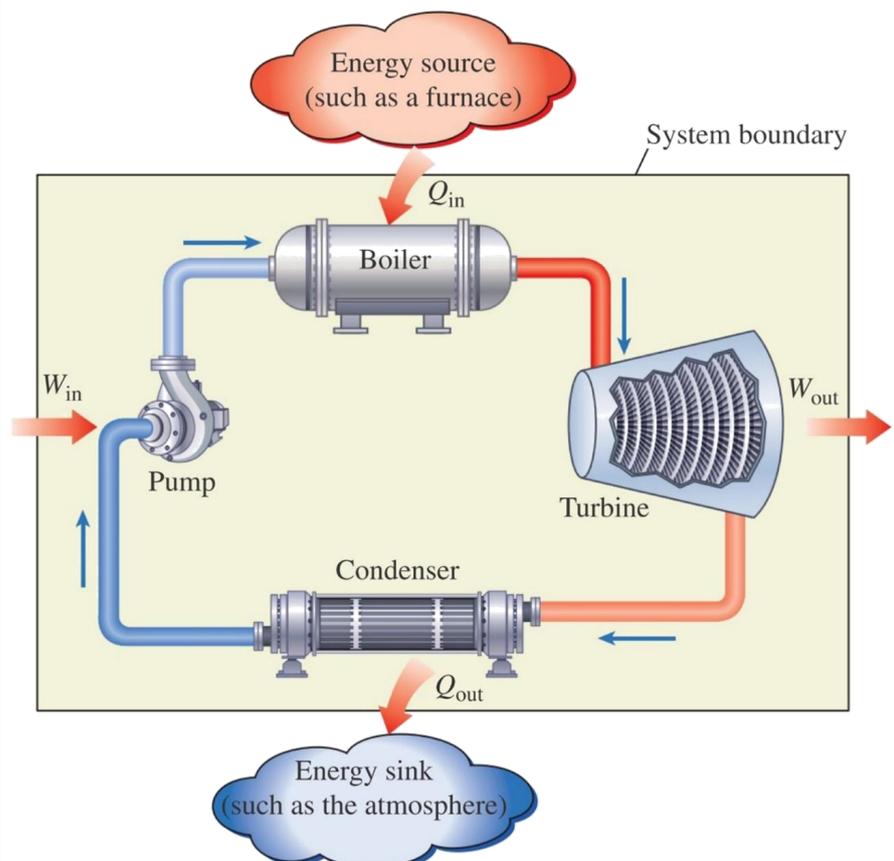


Termodinamica

Nella centrale a vapore l'acqua utilizzata nel **ciclo** è sempre la stessa quindi lo scambio netto di energia, in condizioni **stazionarie**, è nullo.

Pertanto il lavoro può essere calcolato anche come **differenza** della quantità di **calore** scambiato:

$$W_{net} = Q_{in} - Q_{out}$$



Termodinamica

Alcune cause di **irreversibilità** sono (con particolare riferimento ai motori aerospaziali):

- Attriti;
- Miscelamento di fluidi;
- Scambio di energia nel modo calore;
- Reazioni chimiche;

In assenza di queste cause di irreversibilità le trasformazioni vengono chiamate **reversibili**.

L'**entropia** è una **misura** di queste irreversibilità. Per un **sistema isolato** la **variazione** di **entropia** (in questo caso coincidente con la **produzione**) è sempre **positiva**, solo quando la **trasformazione** è **ideale** (reversibile) l'**entropia** rimane **costante**.

La **positività** della **produzione d'entropia** permette di dare un **verso** alle trasformazioni.

Nei **sistemi** non **isolati** la variazione di entropia può aumentare (o diminuire) anche a causa di **scambi di energia nel modo calore**.



Termodinamica

Se una trasformazione è **reversibile**, cioè quando la **produzione** di **entropia** è nulla, si ha:

$$dq = Tds \quad dw = pdv$$

L'equazione di **Gibbs** collega le variabili termodinamiche, in termini energetici:

$$de = Tds - pdv$$

Mentre in termini entalpici:

$$dh = Tds + vdp$$

In un processo **isentropico** si ha:

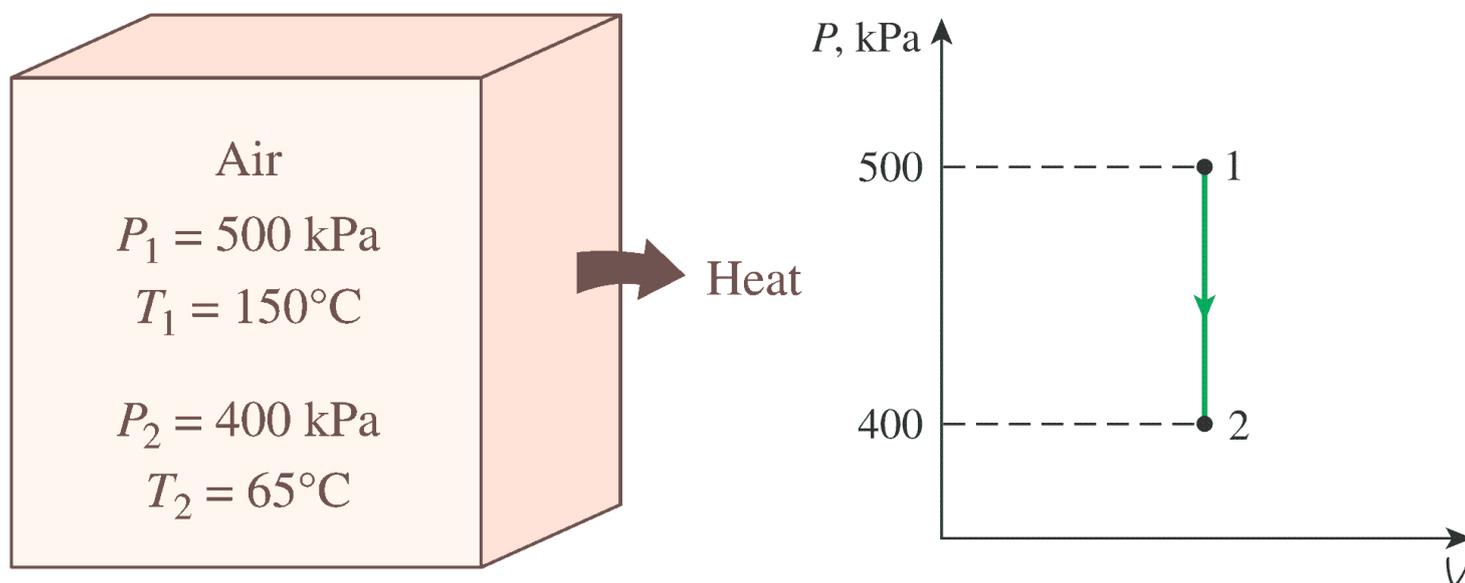
$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{c_p}{R}} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{k}} \quad \frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\frac{c_p}{c_v}} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}$$

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$$



EXAMPLE 4–1 Boundary Work for a Constant-Volume Process

A rigid tank contains air at 500 kPa and 150°C. As a result of heat transfer to the surroundings, the temperature and pressure inside the tank drop to 65°C and 400 kPa, respectively. Determine the boundary work done during this process.



Termodinamica

A rigid tank contains air at 500 kPa and 150°C. As a result of heat transfer to the surroundings, the temperature and pressure inside the tank drop to 65°C and 400 kPa, respectively. Determine the boundary work done during this process.

SOLUTION Air in a rigid tank is cooled, and both the pressure and temperature drop. The boundary work done is to be determined.

Analysis A sketch of the system and the P - V diagram of the process are shown in Fig. 4–6. The boundary work can be determined from Eq. 4–2 to be

$$W_b = \int_1^2 P dV = 0$$

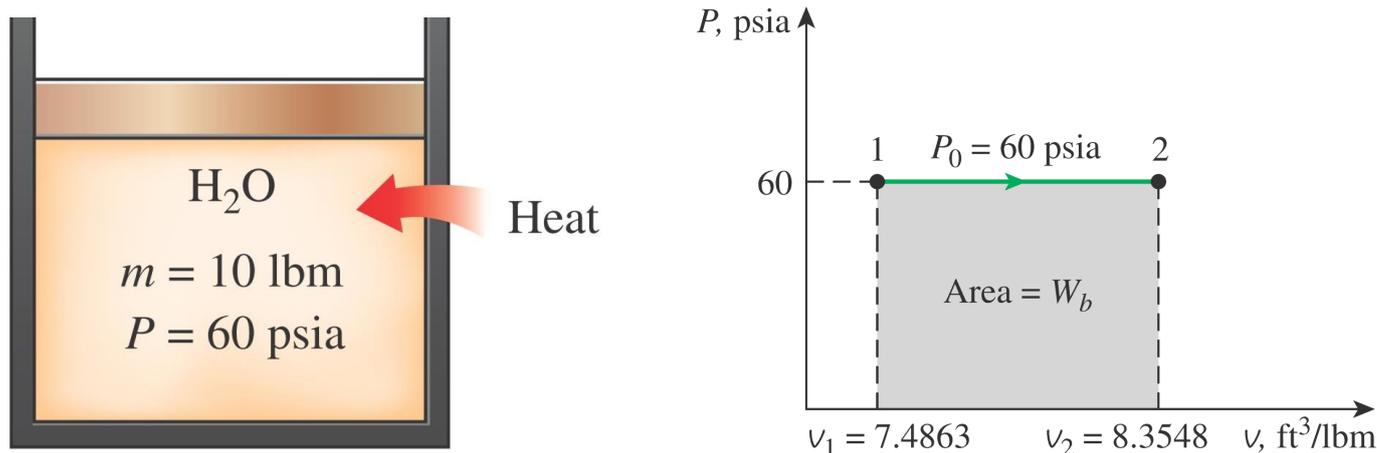
Discussion This is expected since a rigid tank has a constant volume and $dV = 0$ in this equation. Therefore, there is no boundary work done during this process. That is, the boundary work done during a constant-volume process is always zero. This is also evident from the P - V diagram of the process (the area under the process curve is zero).



EXAMPLE 4–2 Boundary Work for a Constant-Pressure Process

A frictionless piston–cylinder device contains 10 lbm of steam at 60 psia and 320°F. Heat is now transferred to the steam until the temperature reaches 400°F. If the piston is not attached to a shaft and its mass is constant, determine the work done by the steam during this process.

SOLUTION Steam in a piston cylinder device is heated and the temperature rises at constant pressure. The boundary work done is to be determined.



Termodinamica

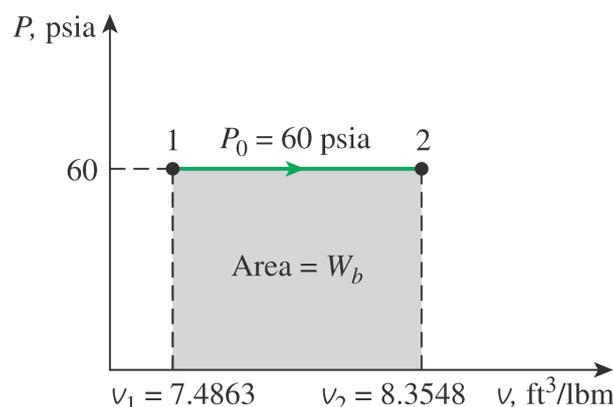
Assumption The expansion process is quasi-equilibrium.

Analysis A sketch of the system and the P - v diagram of the process are shown in Fig. 5–7. Even though it is not explicitly stated, the pressure of the steam within the cylinder remains constant during this process since both the atmospheric pressure and the weight of the piston remain constant. Therefore, this is a constant-pressure process, and, from Eq. 5–2

$$W_b = \int_1^2 P dV = P_0 \int_1^2 dV = P_0(V_2 - V_1) \quad (5-6)$$

or

$$W_b = mP_0(v_2 - v_1)$$



$$W_b = mP_0(v_2 - v_1)$$

since $V = mv$. From the superheated vapor table (Table A-6E), the specific volumes are determined to be $v_1 = 7.4863 \text{ ft}^3/\text{lbm}$ at state 1 (60 psia, 320°F) and $v_2 = 8.3548 \text{ ft}^3/\text{lbm}$ at state 2 (60 psia, 400°F). Substituting these values yields

$$W_b = (10 \text{ lbm})(60 \text{ psia})[(8.3548 - 7.4863) \text{ ft}^3/\text{lbm}] \left(\frac{1 \text{ Btu}}{5.404 \text{ psia}\cdot\text{ft}^3} \right)$$

$$= \mathbf{96.4 \text{ Btu}}$$

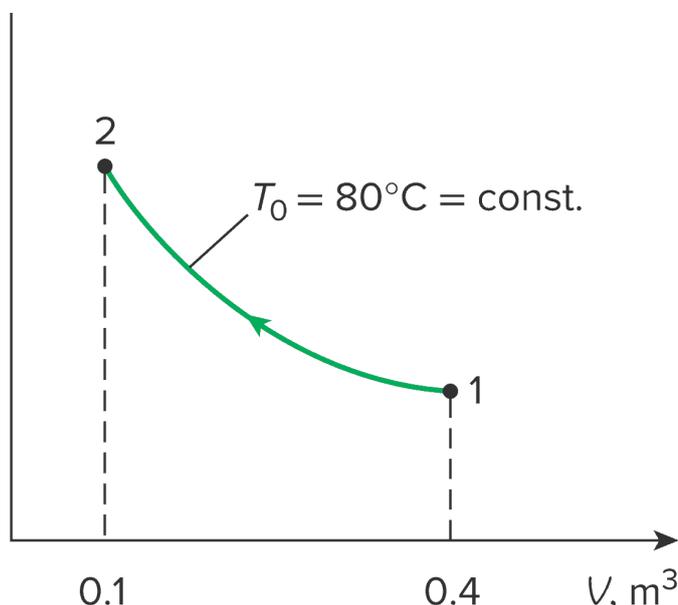
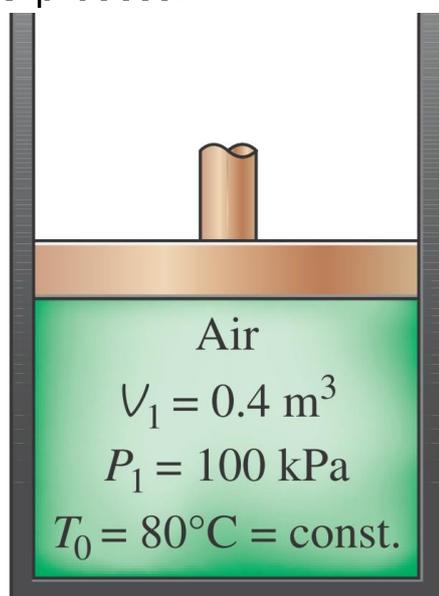
Discussion The positive sign indicates that the work is done by the system. That is, the steam used 96.4 Btu of its energy to do this work. The magnitude of this work could also be determined by calculating the area under the process curve on the P - V diagram, which is simply $P_0 \Delta V$ for this case.



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EXAMPLE 4-3 Isothermal Compression of an Ideal Gas

A piston–cylinder device initially contains 0.4 m^3 of air at 100 kPa and 80°C . The air is now compressed to 0.1 m^3 in such a way that the temperature inside the cylinder remains constant. Determine the work done during this process.



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SOLUTION Air in a piston–cylinder device is compressed isothermally. The boundary work done is to be determined.

Analysis A sketch of the system and the P - V diagram of the process are shown in Fig. 4–8.

Assumptions **1** The compression process is quasi-equilibrium. **2** At specified conditions, air can be considered to be an ideal gas since it is at a high temperature and low pressure relative to its critical-point values.

Analysis For an ideal gas at constant temperature T_0 ,

$$PV = mRT_0 = C \quad \text{or} \quad P = \frac{C}{V}$$

where C is a constant. Substituting this into Eq. 4–2, we have

$$W_b = \int_1^2 P dV = \int_1^2 \frac{C}{V} dV = C \int_1^2 \frac{dV}{V} = C \ln \frac{V_2}{V_1} = P_1 V_1 \ln \frac{V_2}{V_1} \quad (4-7)$$

In Eq. 4–7, $P_1 V_1$ can be replaced by $P_2 V_2$ or mRT_0 . Also, V_2/V_1 can be replaced by P_1/P_2 for this case since $P_1 V_1 = P_2 V_2$.



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where C is a constant. Substituting this into Eq. 4–2, we have

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Substituting the numerical values into Eq. 4–7 yields

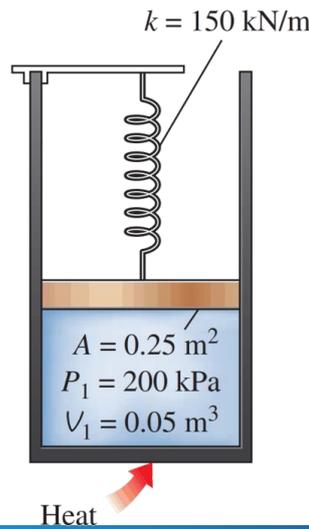
$$\begin{aligned} W_b &= (100 \text{ kPa})(0.4 \text{ m}^3) \left(\ln \frac{0.1}{0.4} \right) \left(\frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3} \right) \\ &= -55.5 \text{ kJ} \end{aligned}$$

Discussion The negative sign indicates that this work is done on the system (a work input), which is always the case for compression processes.



EXAMPLE 4–4 Expansion of a Gas against a Spring

A piston–cylinder device contains 0.05 m^3 of a gas initially at 200 kPa . At this state, a linear spring that has a spring constant of 150 kN/m is touching the piston but exerting no force on it. Now heat is transferred to the gas, causing the piston to rise and to compress the spring until the volume inside the cylinder doubles. If the cross-sectional area of the piston is 0.25 m^2 , determine (a) the final pressure inside the cylinder, (b) the total work done by the gas, and (c) the fraction of this work done against the spring to compress it.



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SOLUTION A gas in a piston–cylinder device equipped with a linear spring expands as a result of heating. The final gas pressure, the total work done, and the fraction of the work done to compress the spring are to be determined.

Assumptions **1** The expansion process is quasi-equilibrium. **2** The spring is linear in the range of interest.

Analysis A sketch of the system and the P - V diagram of the process are shown in Fig. 4–10.

(a) The enclosed volume at the final state is

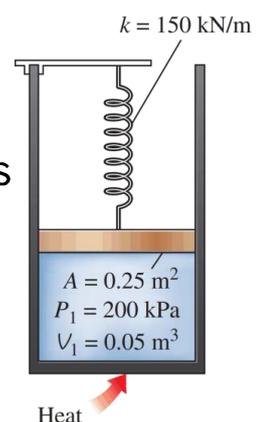
$$V_2 = 2V_1 = (2)(0.05 \text{ m}^3) = 0.1 \text{ m}^3$$

Then the displacement of the piston (and of the spring) becomes

$$x = \frac{\Delta V}{A} = \frac{(0.1 - 0.05) \text{ m}^3}{0.25 \text{ m}^2} = 0.2 \text{ m}$$

The force applied by the linear spring at the final state is

$$F = kx = (150 \text{ kN/m})(0.2 \text{ m}) = 30 \text{ kN}$$



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The additional pressure applied by the spring on the gas at this state is

$$P = \frac{F}{A} = \frac{30 \text{ kN}}{0.25 \text{ m}^2} = 120 \text{ kPa}$$

Without the spring, the pressure of the gas would remain constant at 200 kPa while the piston is rising. But under the effect of the spring, the pressure rises linearly from 200 kPa to

$$200 + 120 = \mathbf{320 \text{ kPa}}$$

at the final state.

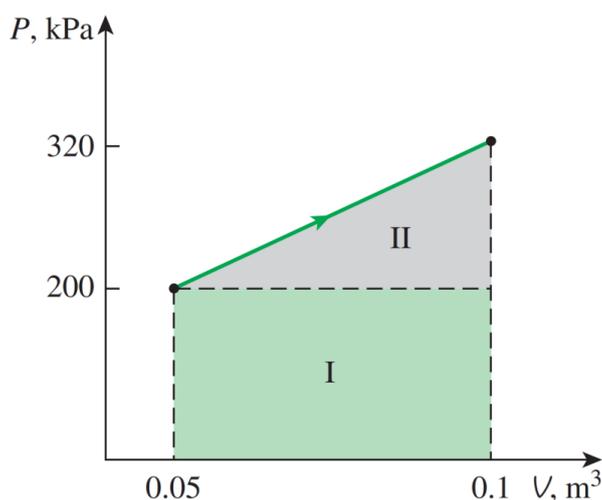


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(b) An easy way of finding the work done is to plot the process on a P - V diagram and find the area under the process curve. From Fig. 4–10 the area under the process curve (a trapezoid) is determined to be

$$W = \text{area} = \frac{(200 + 320) \text{ kPa}}{2} [(0.1 - 0.05) \text{ m}^3] \left(\frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3} \right) = \mathbf{13 \text{ kJ}}$$

Note that the work is done by the system.



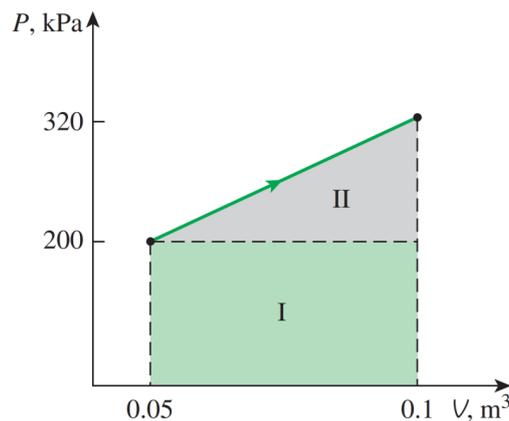
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(c) The work represented by the rectangular area (region I) is done against the piston and the atmosphere, and the work represented by the triangular area (region II) is done against the spring. Thus,

$$W_{\text{spring}} = \frac{1}{2}[(320 - 200) \text{ kPa}](0.05 \text{ m}^3) \left(\frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3} \right) = \mathbf{3 \text{ kJ}}$$

Discussion This result could also be obtained from

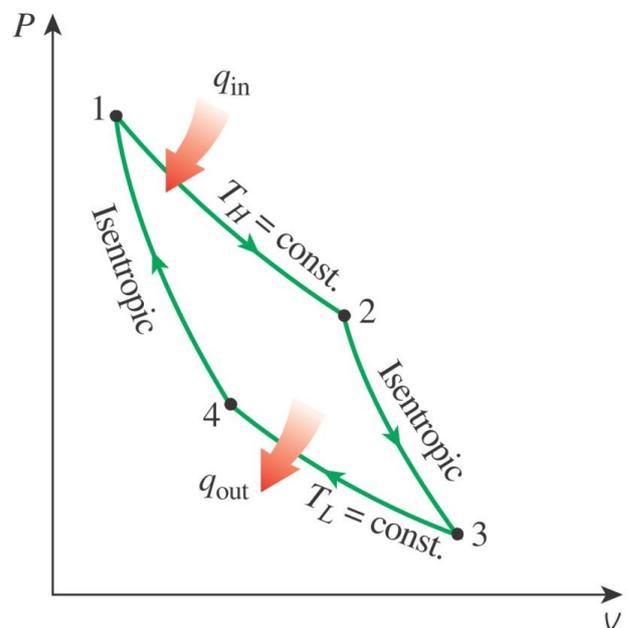
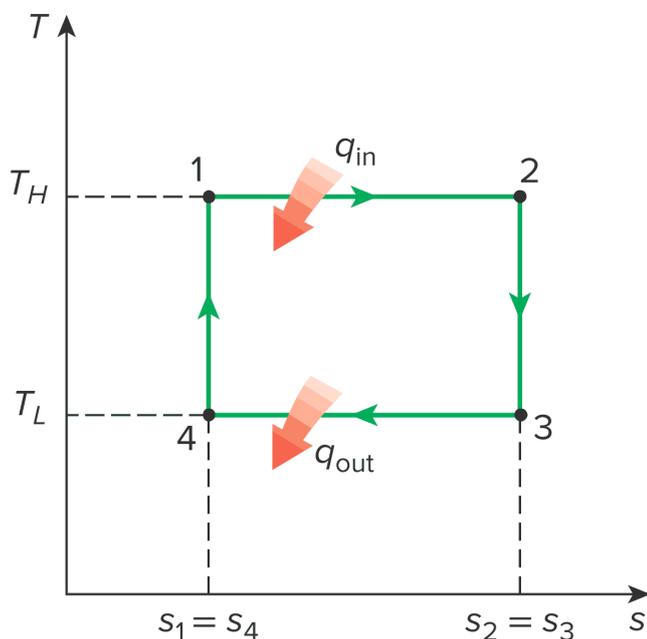
$$W_{\text{spring}} = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}(150 \text{ kN/m})[(0.2 \text{ m})^2 - 0^2] \left(\frac{1 \text{ kJ}}{1 \text{ kN} \cdot \text{m}} \right) = 3 \text{ kJ}$$



Termodinamica

EXAMPLE 9–1 Derivation of the Efficiency of the Carnot Cycle

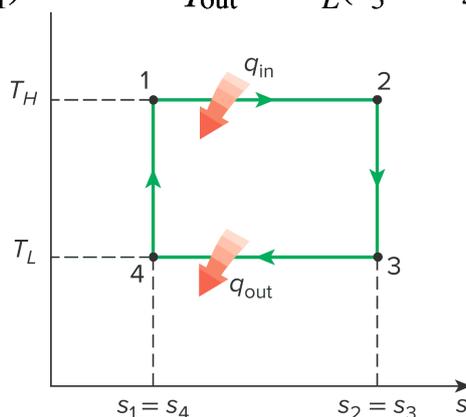
Show that the thermal efficiency of a Carnot cycle operating between the temperature limits of T_H and T_L is solely a function of these two temperatures and is given by Eq. 9–2.



SOLUTION It is to be shown that the efficiency of a Carnot cycle depends on the source and sink temperatures alone.

Analysis The T - s diagram of a Carnot cycle is redrawn in Fig. 9–7. All four processes that comprise the Carnot cycle are reversible, and thus the area under each process curve represents the heat transfer for that process. Heat is transferred to the system during process 1-2 and rejected during process 3-4. Therefore, the amount of heat input and heat output for the cycle can be expressed as

$$q_{in} = T_H(s_2 - s_1) \quad \text{and} \quad q_{out} = T_L(s_3 - s_4) = T_L(s_2 - s_1)$$



Termodinamica

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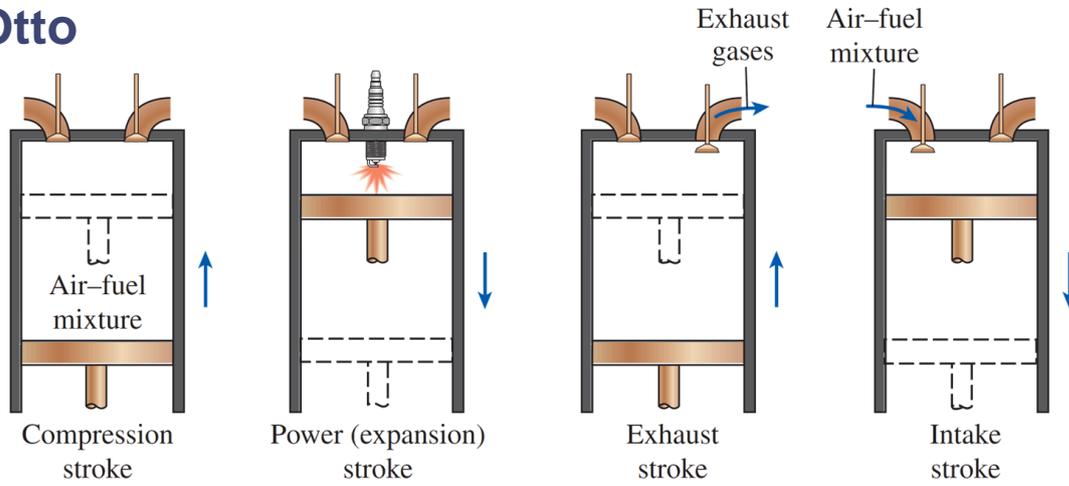
$$q_{in} = T_H(s_2 - s_1) \quad \text{and} \quad q_{out} = T_L(s_3 - s_4) = T_L(s_2 - s_1)$$

since processes 2-3 and 4-1 are isentropic, and thus $s_2 = s_3$ and $s_4 = s_1$. Substituting these into Eq. 9–1, we see that the thermal efficiency of a Carnot cycle is

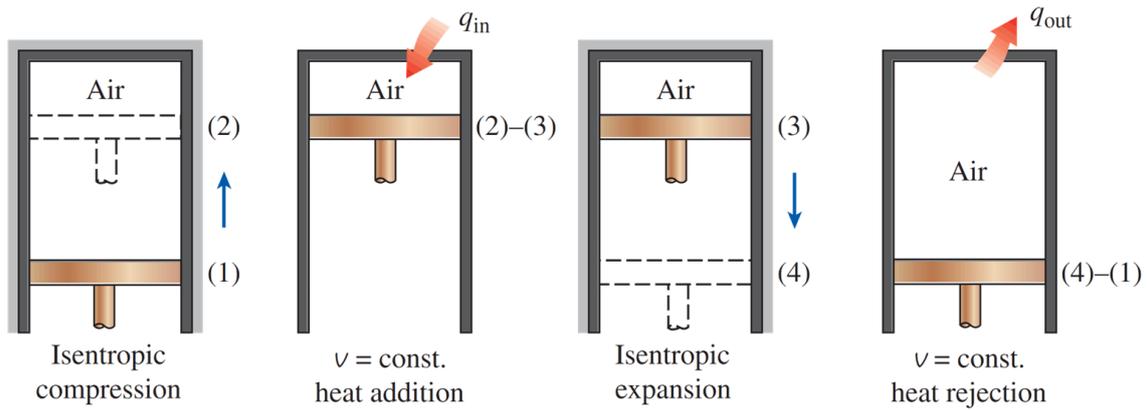
$$\eta_{th} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_L(s_2 - s_1)}{T_H(s_2 - s_1)} = 1 - \frac{T_L}{T_H}$$



Ciclo Otto



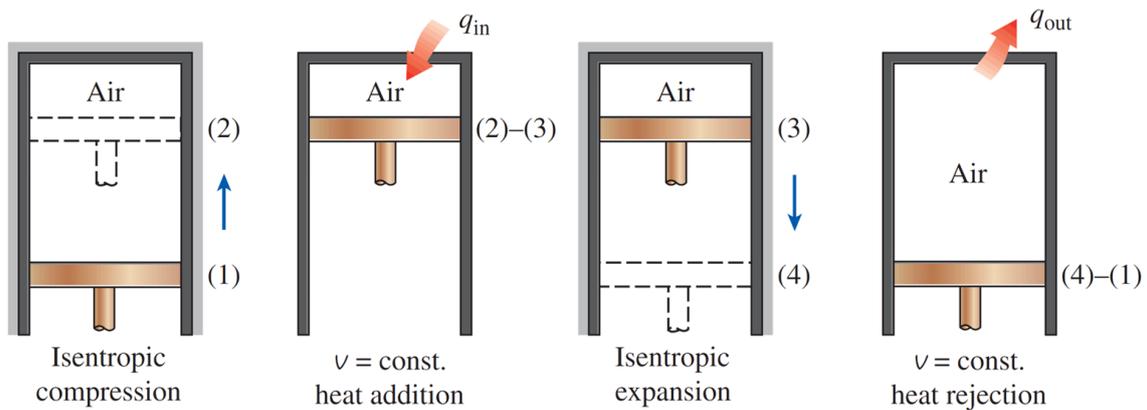
(a) Actual four-stroke spark-ignition engine



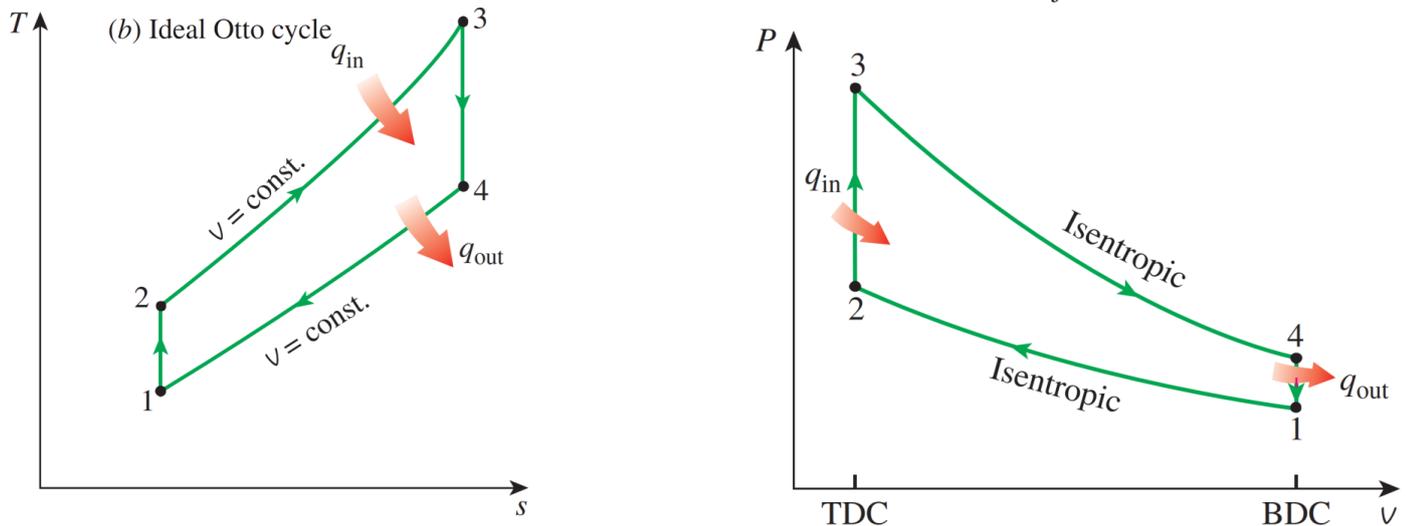
(b) Ideal Otto cycle



Ciclo Otto



(b) Ideal Otto cycle



Rendimento del Ciclo Otto

Il **rendimento** del ciclo si valuta come:

$$\eta_{th} = \frac{W_{net}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} =$$

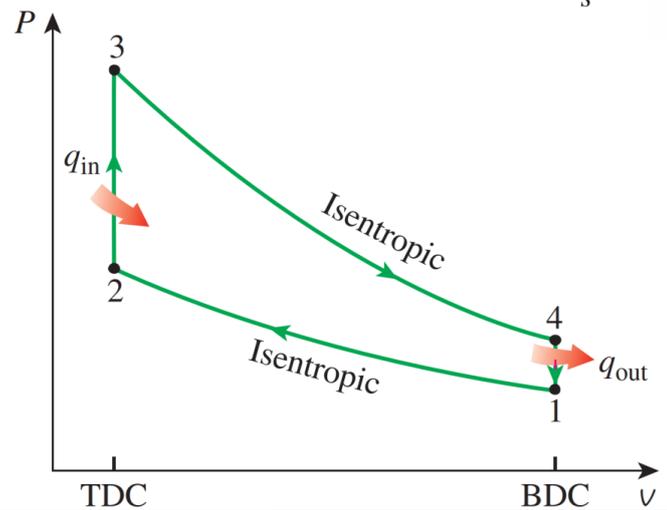
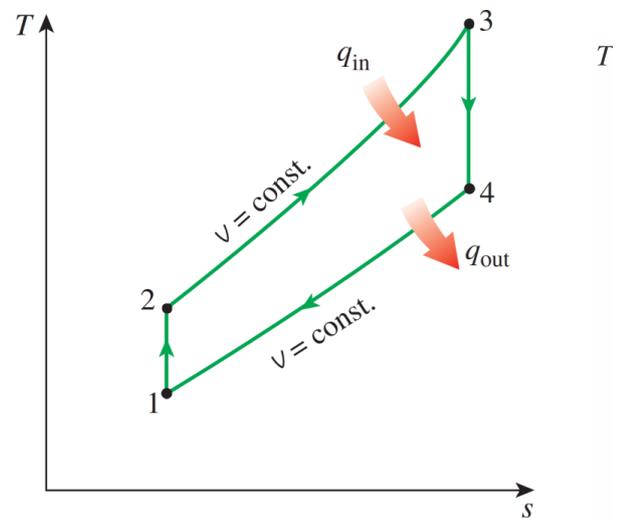
$$\eta_{th} = 1 - \frac{c_p(T_4 - T_1)}{c_p(T_3 - T_2)} = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

ma:

$$\frac{T_2}{T_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma-1} = \left(\frac{\rho_3}{\rho_4}\right)^{\gamma-1} = \frac{T_3}{T_4}$$

$$\frac{T_3}{T_2} = \frac{T_4}{T_1}$$

$$\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{\frac{T_2}{T_1}}$$



Equazioni di bilancio

Le **equazioni di bilancio** possono essere scritte in modo semplificato nelle ipotesi di moto **unidimensionale** e **stazionario**. In dettaglio la conservazione della massa è:

$$\dot{m} = \rho VA = cost$$

Dove \dot{m} è la **portata** che attraversa la generica sezione di area frontale A , V è la **velocità** media e ρ è la **densità** del fluido.

L'estensione al caso in cui ci possano essere più ingressi e/o uscite è banale. Per esempio nel sistema mostrato in figura si ha:

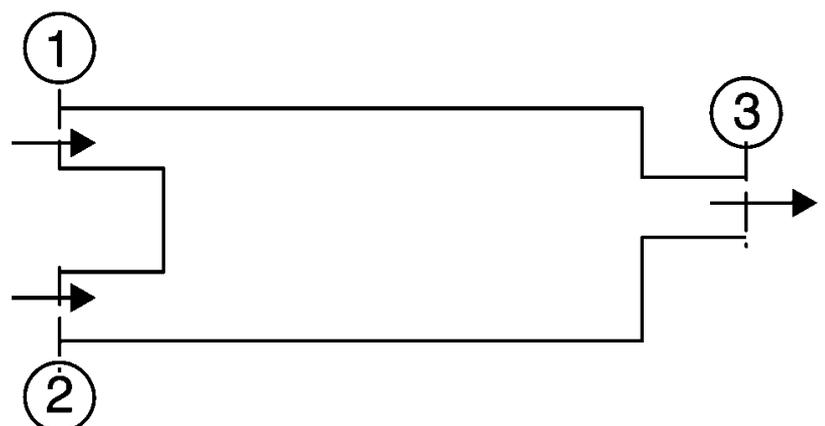
$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

Dove:

$$\dot{m}_1 = \rho_1 V_1 A_1$$

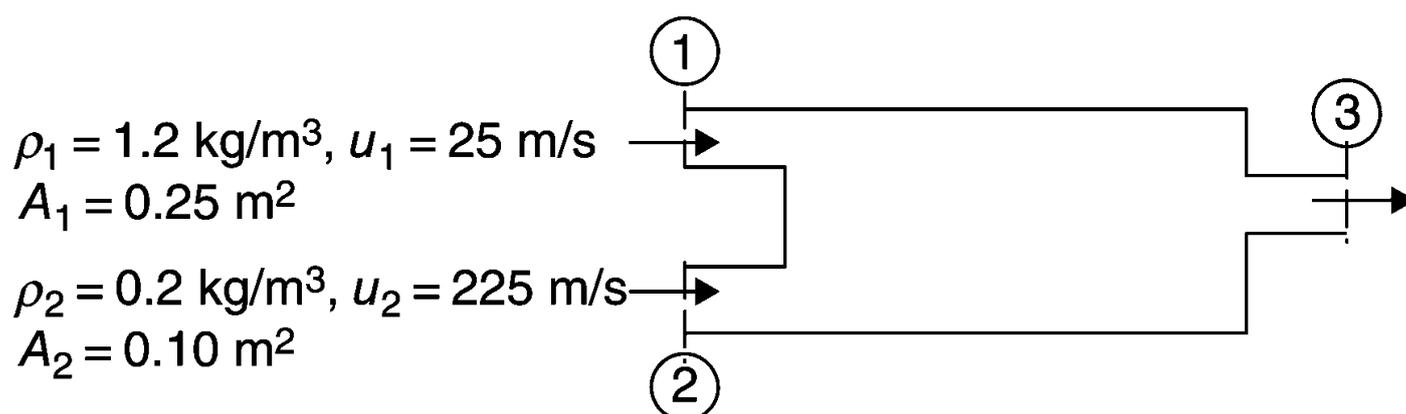
$$\dot{m}_2 = \rho_2 V_2 A_2$$

$$\dot{m}_3 = \rho_3 V_3 A_3$$



Equazioni di bilancio

Consider the control volume shown. There are two inlet boundaries and one exit boundary where flow crosses the control volume. Assuming the flow is steady and uniform, calculate the exit flow rate from the known inlet conditions.



Equazioni di bilancio

The mass flow rate entering boundary 1, is

$$\dot{m}_1 = \rho_1 A_1 u_1 = (1.2 \text{ kg/m}^3)(0.25 \text{ m}^2)(25 \text{ m/s}) = 7.5 \text{ kg/s}$$

The mass flow rate entering boundary 2, is

$$\dot{m}_2 = \rho_2 A_2 u_2 = (0.2 \text{ kg/m}^3)(0.10 \text{ m}^2)(225 \text{ m/s}) = 4.5 \text{ kg/s}$$

Therefore, the mass flow rate out of boundary 3 is the sum of the mass flow rate into the box, i.e.,

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2 = (7.5 + 4.5) \text{ kg/s} = 12 \text{ kg/s}$$



Equazioni di bilancio

L'equazione di **bilancio** della **quantità di moto** permette di valutare le forze; trascurando gli effetti gravitazionali e supponendo che il vettore velocità sia allineato con la **normale uscente** si ha:

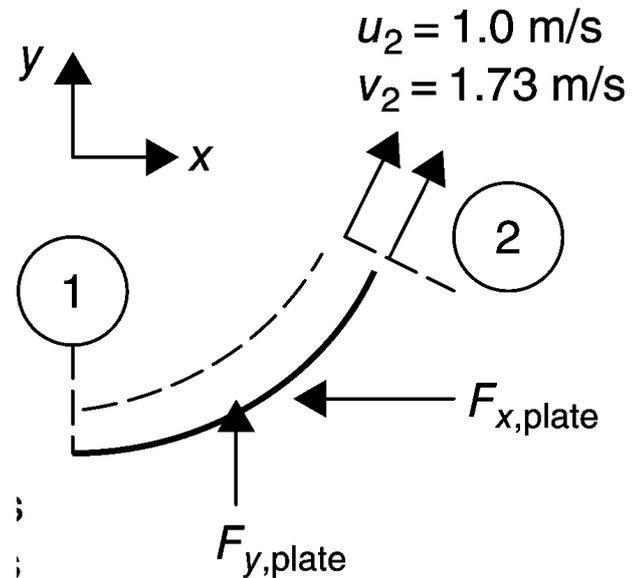
$$\underline{F} = (\dot{m}_2 V_2 + p_2 A_2) \underline{n}_2 - (\dot{m}_1 V_1 + p_1 A_1) \underline{n}_1 = I_2 \underline{n}_2 - I_1 \underline{n}_1$$

Dove si è introdotto l'**impulso totale**:

$$I = \rho V^2 A + pA$$

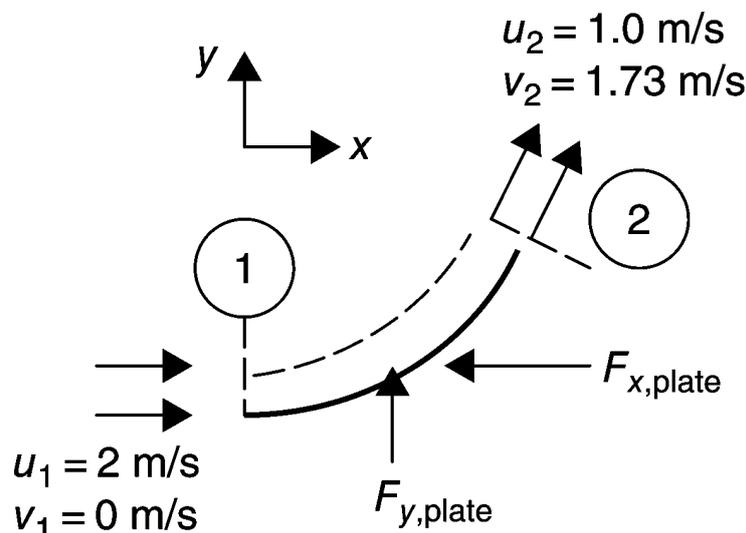
Evidentemente l'equazione di bilancio della QM deve essere **proiettata** sugli assi per valutare le **componenti** del **vettore** forza.

Spesso è utile trascurare l'effetto della **pressione atmosferica**, che agisce ovunque e si bilancia, semplicemente considerando le **pressioni relative** e le cosiddette **forze nette**.



Equazi In placing a curved plate in front of a garden hose with a water flow rate of 0.1 kg/s, we have to exert an axial force on the plate, as shown, to hold it in place. Estimate the axial and lateral forces, F_x and F_y , respectively, that are needed to support the plate.

(Note: u and v are x - and y -component of velocity, respectively.)



Equazioni di bilancio

$$\begin{aligned} F_{x,\text{fluid}} &= \dot{m}_2 u_2 - \dot{m}_1 u_1 = \dot{m}(u_2 - u_1) \\ &= 0.1 \text{ kg/s}(-1 \text{ m/s}) = -0.1 \text{ N} \end{aligned}$$

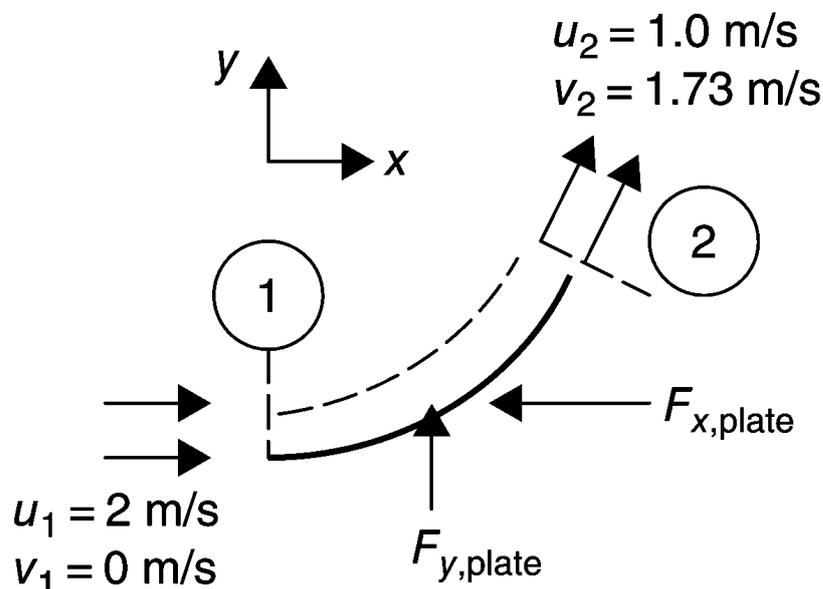
The axial force acting on the curved plate is equal and opposite to $F_{x,\text{fluid}}$. Therefore, the external force needed to hold the plate horizontally is -0.1 N (that is in the negative x -direction, as shown). Also, the lateral force on the fluid is calculated from the lateral momentum balance, i.e.,

$$\begin{aligned} F_{y,\text{fluid}} &= \dot{m}_2 v_2 - \dot{m}_1 v_1 = \dot{m}(v_2 - v_1) \\ &= 0.1 \text{ kg/s}(1.73 \text{ m/s}) = 0.173 \text{ N} \end{aligned}$$



Equazioni di bilancio

The lateral force on the plate is in the opposite direction (i.e., in the $-y$ direction), and thus the external force to hold the plate laterally is $+0.173 \text{ N}$, as shown.



Equazioni di bilancio

L'equazione di **conservazione** dell'**energia** per sistemi **aperti** può essere scritta come:

$$\dot{m}\Delta h_t = \dot{Q} - \dot{W}$$

Dove \dot{Q} e \dot{W} sono le quantità di energia scambiate nel modo calore e lavoro. È stata inoltre introdotta l'**entalpia totale** o di **ristagno**:

$$h_t = h + \frac{V^2}{2}$$

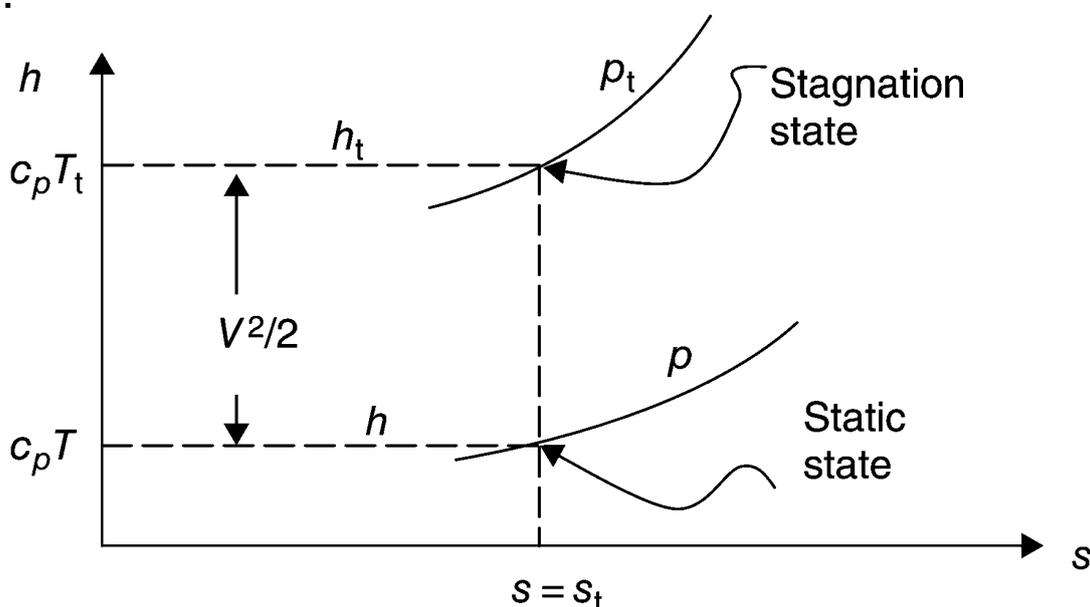
La **condizione di ristagno** (indicate con il pedice t) è quella che una particella di fluido raggiungerebbe qualora venisse rallentata fino a velocità nulla con una trasformazione **adiabatica**, **anergodica** e **isoentropica** (omoenergetica e isoentropica).

La condizione **statica** di una corrente è invece quella misurata con uno **strumento** che si **muove alla velocità** del **fluido**, cioè con uno strumento rispetto al quale il fluido è fermo.



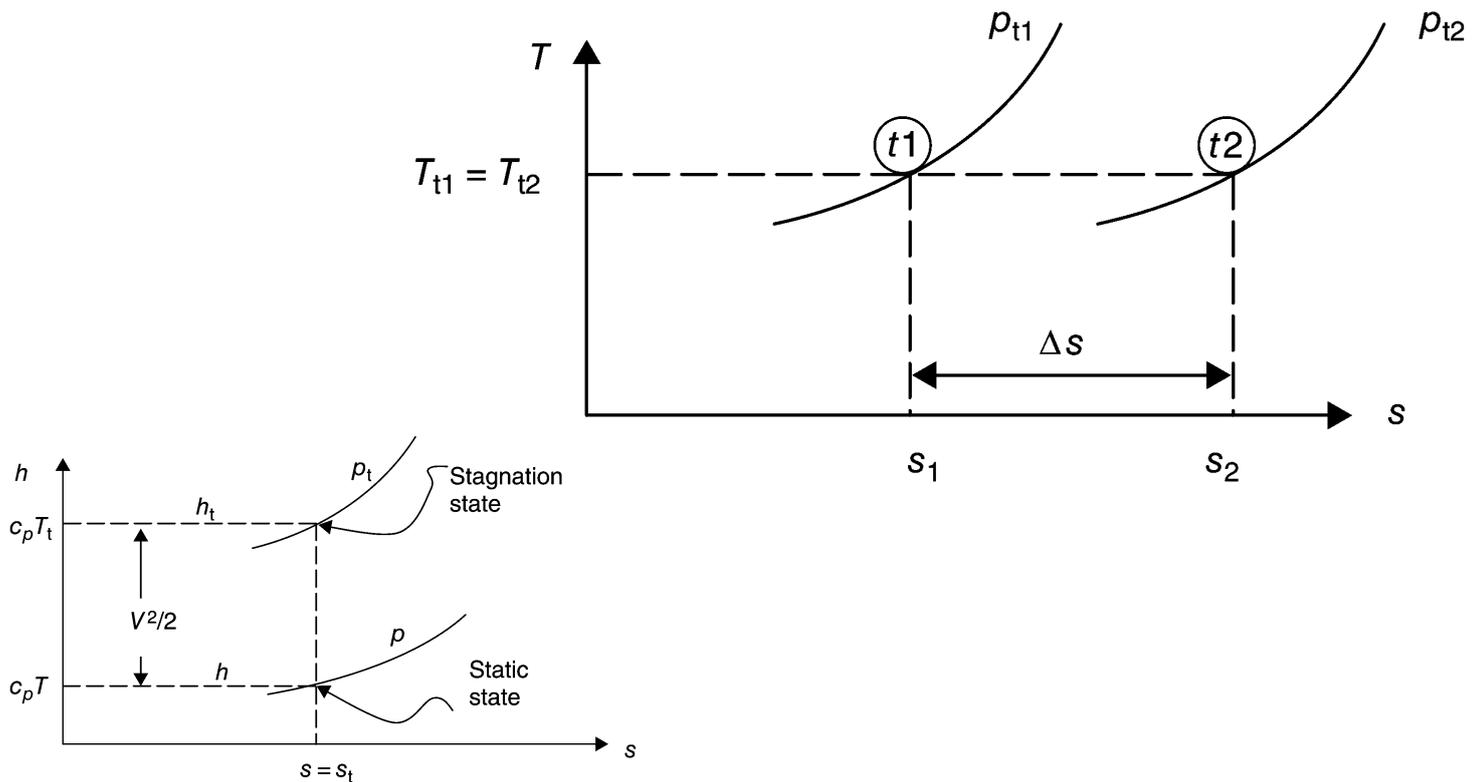
Equazioni di bilancio

Questi concetti sono più chiari in un piano h - s (o T - s). Quando una corrente avente un'entalpia specifica h ed una velocità V viene **rallentata** fino a **velocità nulla** mediante una trasformazione **adiabatica** e **anergodica**, l'energia cinetica specifica (per unità di massa $V^2/2$) viene convertita in energia interna aumentando l'entalpia specifica.



Equazioni di bilancio

La condizione di **isentropicità** è necessaria per valutare le altre grandezze di ristagno, e.g. pressione, densità.



Equazioni di bilancio

$$k = \frac{\gamma - 1}{\gamma}$$

Nel caso di **gas più che perfetto** ricordando la definizione del numero di Mach si ha:

$$\begin{aligned} h_t &= h + \frac{V^2}{2} \quad \rightarrow \quad T_t = T \left(1 + \frac{V^2}{2c_p T} \right) = T \left(1 + \frac{V^2}{2 \frac{c_p}{R} RT} \right) \\ &= T \left(1 + \frac{V^2}{2 \frac{\gamma}{\gamma - 1} RT} \right) = T \left(1 + \frac{\gamma - 1}{2} \frac{V^2}{\gamma RT} \right) = T \left(1 + \frac{\gamma - 1}{2} M^2 \right) \end{aligned}$$

Da cui:

$$\frac{T_t}{T} = 1 + \frac{\gamma - 1}{2} M^2 = \psi$$

Dove si è introdotta la quantità:

$$\psi = 1 + \frac{\gamma - 1}{2} M^2$$



Equazioni di bilancio

$$k = \frac{\gamma - 1}{\gamma}; \quad \psi = 1 + \frac{\gamma - 1}{2} M^2$$

$$T_t = T + \frac{V^2}{2c_p} = T \left(1 + \frac{\gamma - 1}{2} M^2 \right) \quad \rightarrow \quad \frac{T_t}{T} = 1 + \frac{\gamma - 1}{2} M^2 = \psi$$

Nell'ipotesi di moto **isentropico**:

$$\frac{p_t}{p} = \left(\frac{T_t}{T} \right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{T_t}{T} \right)^{\frac{1}{k}} = \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{\gamma}{\gamma-1}} = \psi^{\frac{1}{k}}$$

$$\frac{\rho_t}{\rho} = \left(\frac{T_t}{T} \right)^{\frac{1}{\gamma-1}} = \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{1}{\gamma-1}} = \psi^{\frac{1}{\gamma-1}}$$

Si ottengono la **densità** e **pressione** di **ristagno**, o **totale**:

$$\rho_t = \rho \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{1}{\gamma-1}}; \quad p_t = p \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{\gamma}{\gamma-1}} = p \psi^{\frac{1}{k}}$$

Anche la pressione (e idealmente la densità) **statica** è quella misurata da un manometro che **viaggia alla stessa velocità della corrente**



Equazioni di bilancio

$$\frac{T_t}{T} = 1 + \frac{\gamma - 1}{2} M^2 = \psi \quad \frac{p_t}{p} = \left(\frac{T_t}{T} \right)^{\frac{1}{k}} = \psi^{\frac{1}{k}} \quad \frac{\rho_t}{\rho} = \left(\frac{T_t}{T} \right)^{\frac{1}{\gamma-1}} = \psi^{\frac{1}{\gamma-1}}$$

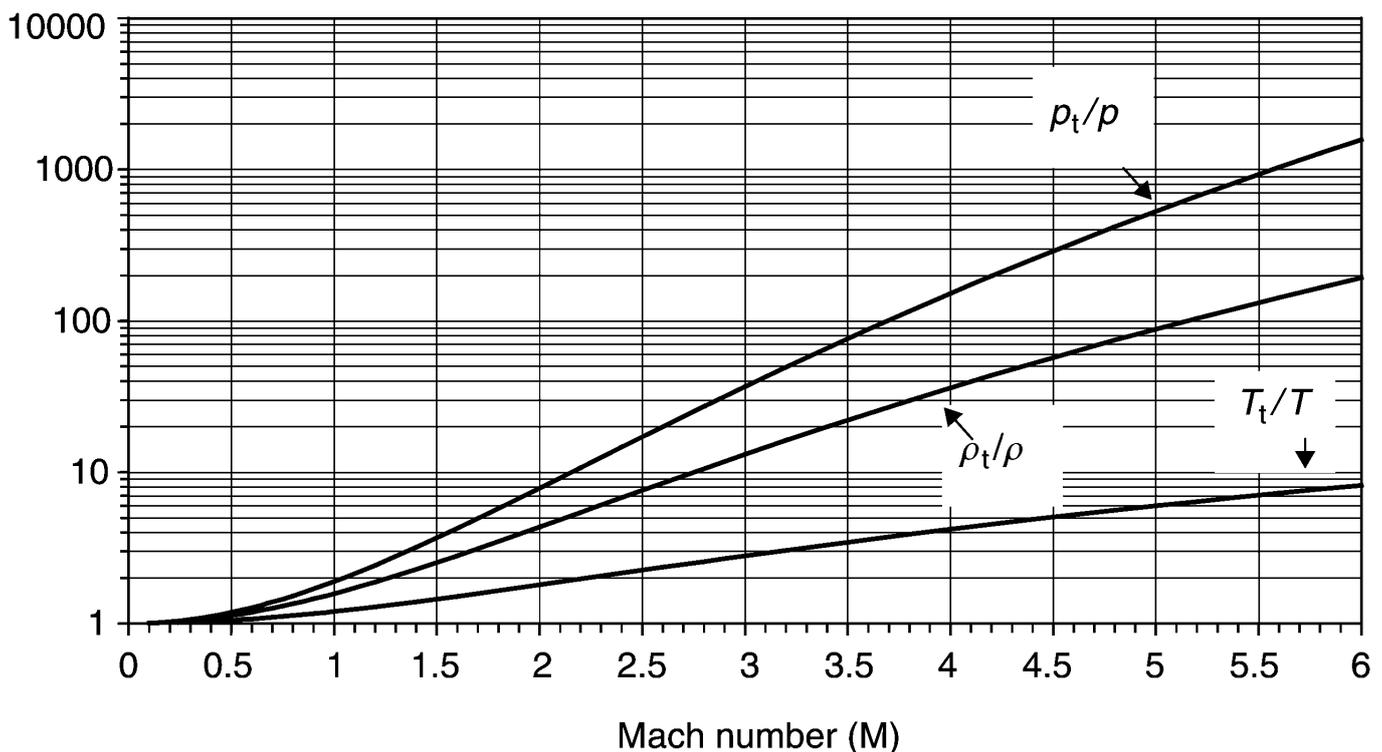


Tabelle – GasTab - Gas dynamics Tables

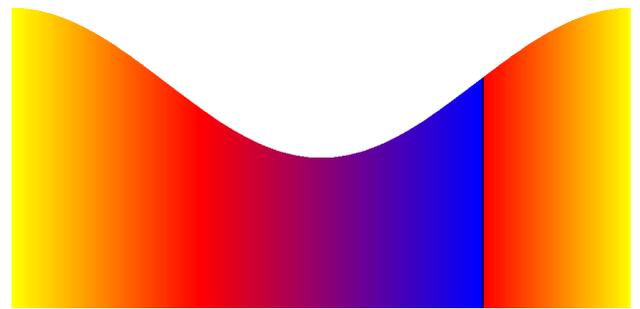
$$\frac{T_t}{T} = 1 + \frac{\gamma - 1}{2} M^2 = \psi \quad ; \quad \frac{p_t}{p} = \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{\gamma}{\gamma - 1}} = \psi^{\frac{1}{k}}$$

$$\frac{\rho_t}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{1}{\gamma - 1}} = \psi^{\frac{1}{\gamma - 1}}$$

Queste funzioni sono tabellate:

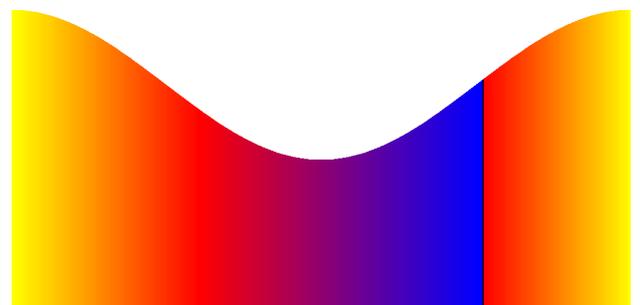
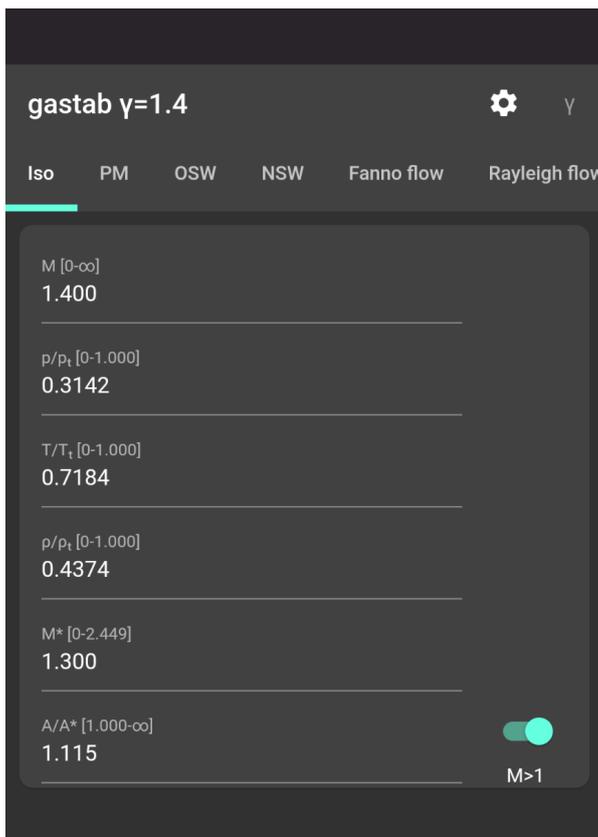
Moto Isentropico ($\gamma=1.4$)							Prandtl e Meyer		Onde d'urto normali ($\gamma=1.4$)					
M	p/p_o	ρ/ρ_o	T/T_o	$\rho V^2/2p_o$	A/A^*	M^*	$\nu(^{\circ})$	$\mu(^{\circ})$	M_2	p_2/p_1	ρ_2/ρ_1	T_2/T_1	p_{o2}/p_{o1}	p_1/p_{o2}
1.00	.5283	.6339	.8333	.3698	1.000	1.000	.0000	9.000 +1	1.000	1.000	1.000	1.000	1.000	.5283
1.01	.5221	.6287	.8306	.3728	1.000	1.008	.4472 -1	8.193 +1	.9901	1.023	1.017	1.007	1.000	.5221
1.02	.5160	.6234	.8278	.3758	1.000	1.017	.1257	7.864 +1	.9805	1.047	1.033	1.013	1.000	.5160
1.03	.5099	.6181	.8250	.3787	1.001	1.025	.2294	7.614 +1	.9712	1.071	1.050	1.020	1.000	.5100
1.04	.5039	.6129	.8222	.3815	1.001	1.033	.3510	7.406 +1	.9620	1.095	1.067	1.026	.9999	.5039

Moto Isentropico			
M	p/p_o	ρ/ρ_o	T/T_o
1.00	.5283	.6339	.8333
1.01	.5221	.6287	.8306
1.02	.5160	.6234	.8278
1.03	.5099	.6181	.8250
1.04	.5039	.6129	.8222



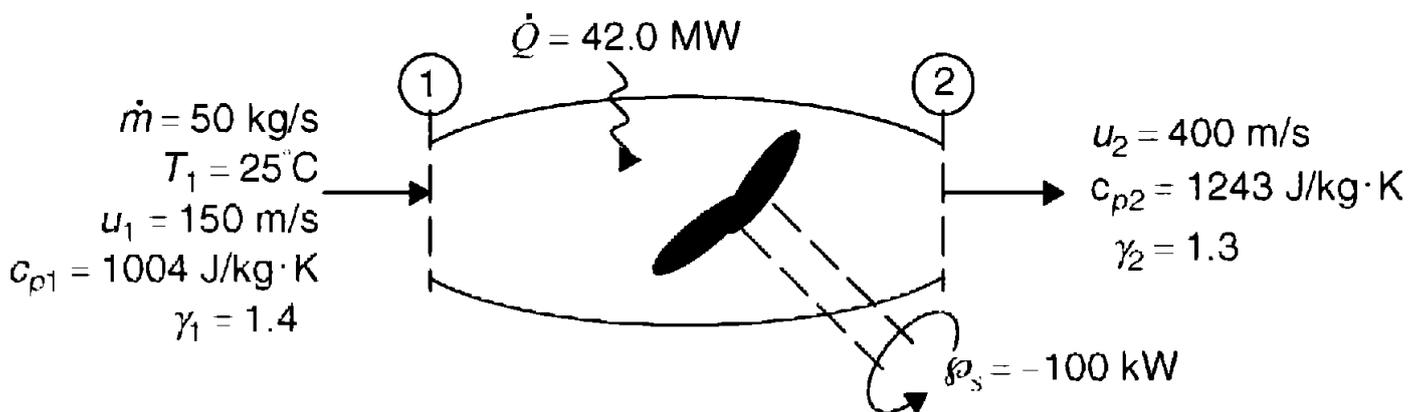
GasTab - Gas dynamics Tables

GasTab - Gas dynamics Tables



Equazioni di bilancio

Let us consider a control volume with heat transfer rate and mechanical power (i.e., shaft power) exchange specified at its boundaries. There is a single inlet and a single outlet where mass crosses the boundary in a steady and uniform flow, as shown. Calculate the exit total and static temperatures T_{t2} and T_2 , respectively (note that the gas is not calorically perfect).



Equazioni di bilancio

First, we conclude that the exit mass flow rate is 50 kg/s, to satisfy the continuity equation for steady flow. Second, we cast the inlet static temperature in the absolute scale, i.e.,

$$T_1 = 25 + 273 = 298 \text{ K}$$

Third, we apply the conservation of energy equation (2.53) to this problem, which requires the knowledge of total enthalpy at the inlet. We use the definition of total enthalpy:

$$\begin{aligned} h_{t1} &= h_1 + u_1^2/2 = c_{p1} T_1 + u_1^2/2 = (1004 \text{ J/kg}\cdot\text{K})(298 \text{ K}) \\ &\quad + (150)^2/2 \text{ J/kg} \\ &= 310,442 \text{ J/kg} \end{aligned}$$



Equazioni di bilancio

Now, Equation 2.53 gives the exit total enthalpy as

$$h_{t2} = h_{t1} + \left(\frac{\dot{Q} - \dot{\varphi}_s}{\dot{m}} \right) = 310.442 \text{ kJ/kg}$$
$$+ \left(\frac{(42,000 + 100) \times 1000 \text{ W}}{50 \text{ kg/s}} \right) \cong 1,152.4 \text{ kJ/kg}$$

$$T_{t2} = \frac{h_{t2}}{c_{p2}} = \frac{1,152.4 \text{ kJ/kg}}{1.243 \text{ kJ/kg} \cdot \text{K}} \cong 927 \text{ K}$$

$$T_2 = T_{t2} - u_2^2 / 2c_{p2}$$
$$= 927 \text{ K} - (400)^2 / (2 \times 1243 \text{ J/kg} \cdot \text{K}) \cong 863 \text{ K}$$

