

Algoritmi e Strutture Dati

HeapSort II

Complessità di Heapify

$$T(n) = \max(O(1), \max(O(1), T(?) + O(1)))$$

He

$l = \text{SINISTRO}(i)$

$r = \text{DESTRO}(i)$

IF $l \notin \text{heapsize}[A]$ AND $A[l] > A[i]$

THEN $\text{maggiore} = l$

ELSE $\text{maggiore} = i$

IF $r \notin \text{heapsize}[A]$ AND $A[r] > A[\text{maggiore}]$

THEN $\text{maggiore} = r$

IF $\text{maggiore} \neq i$ } = $O(1)$

$O(1) =$ { THEN "scambia $A[i]$ e $A[\text{maggiore}]$ "

$T(?) =$ { $\text{Heapify}(A, \text{maggiore})$

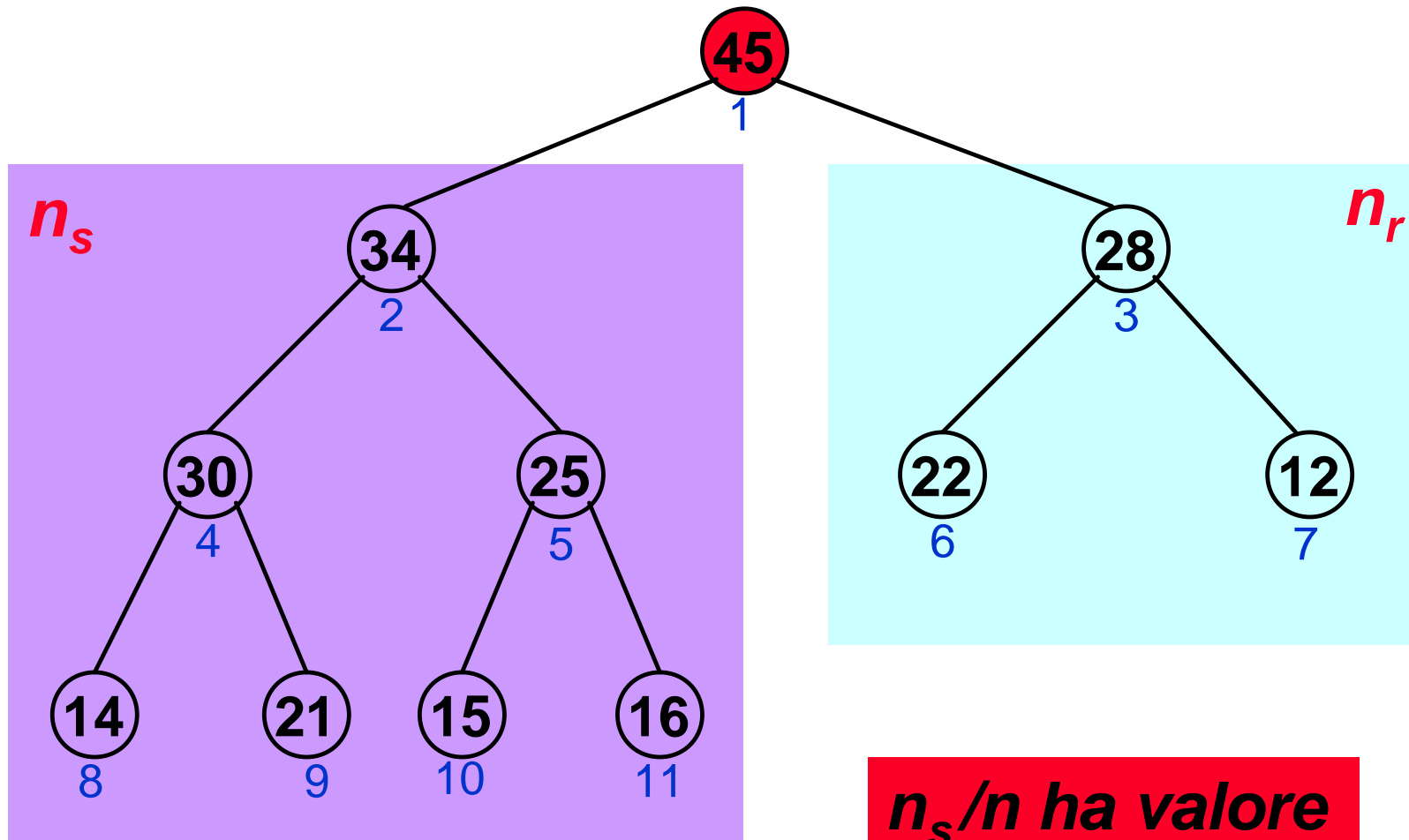
Complessità di Heapify: caso peggiore

$$T(n) = \max(O(1), \max(O(1), T(?) + O(1)))$$

Nel **caso peggiore Heapify** ad ogni chiamata ricorsiva, viene eseguito su un numero di nodi che è minore dei **2/3** del numero di nodi correnti **n** .

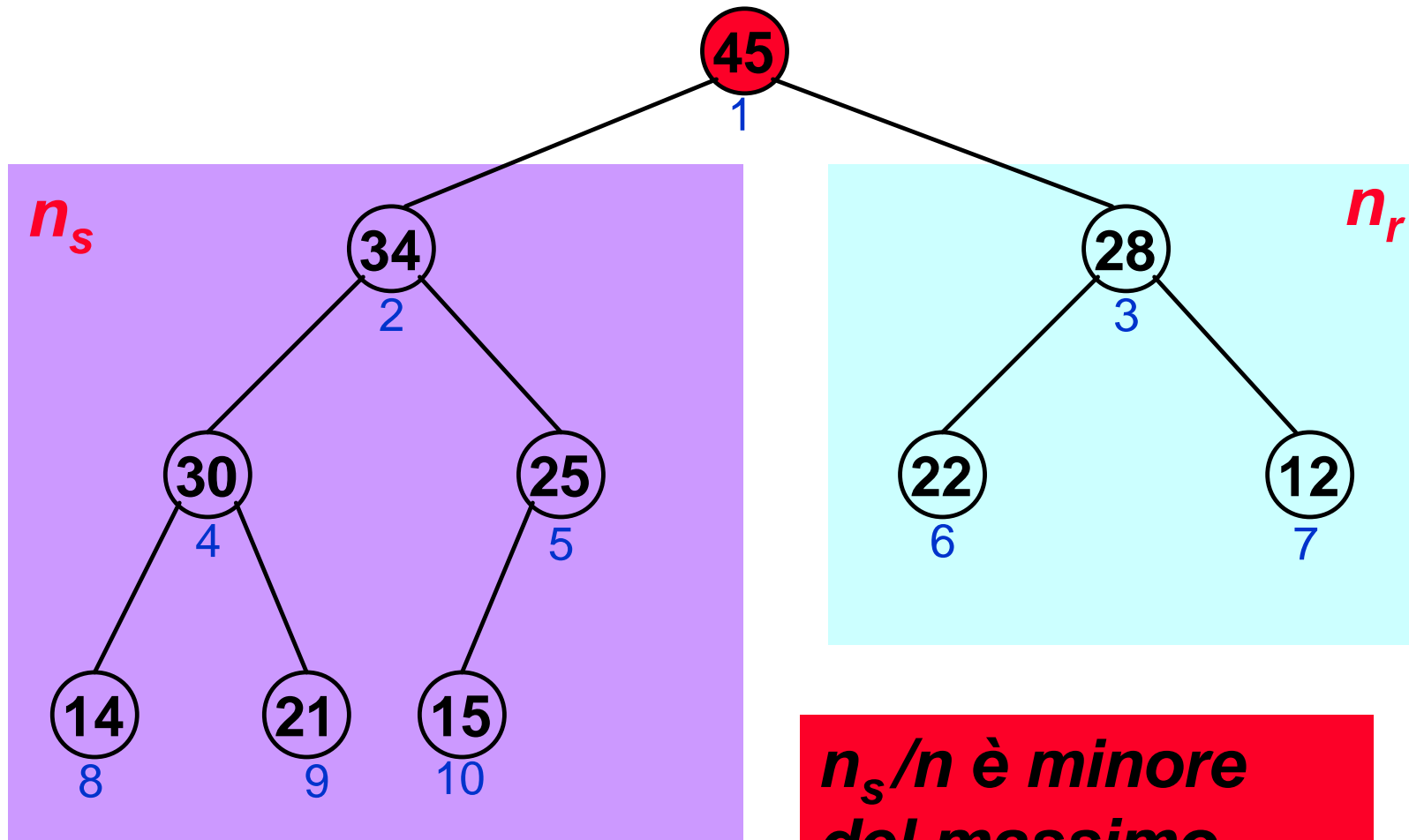
Cioè il numero di nodi **n_s** del sottoalbero su cui **Heapify** è chiamato ricorsivamente è al più **2/3 n** (o **$n_s \leq 2/3 n$**)

Complessità di Heapify: caso peggiore



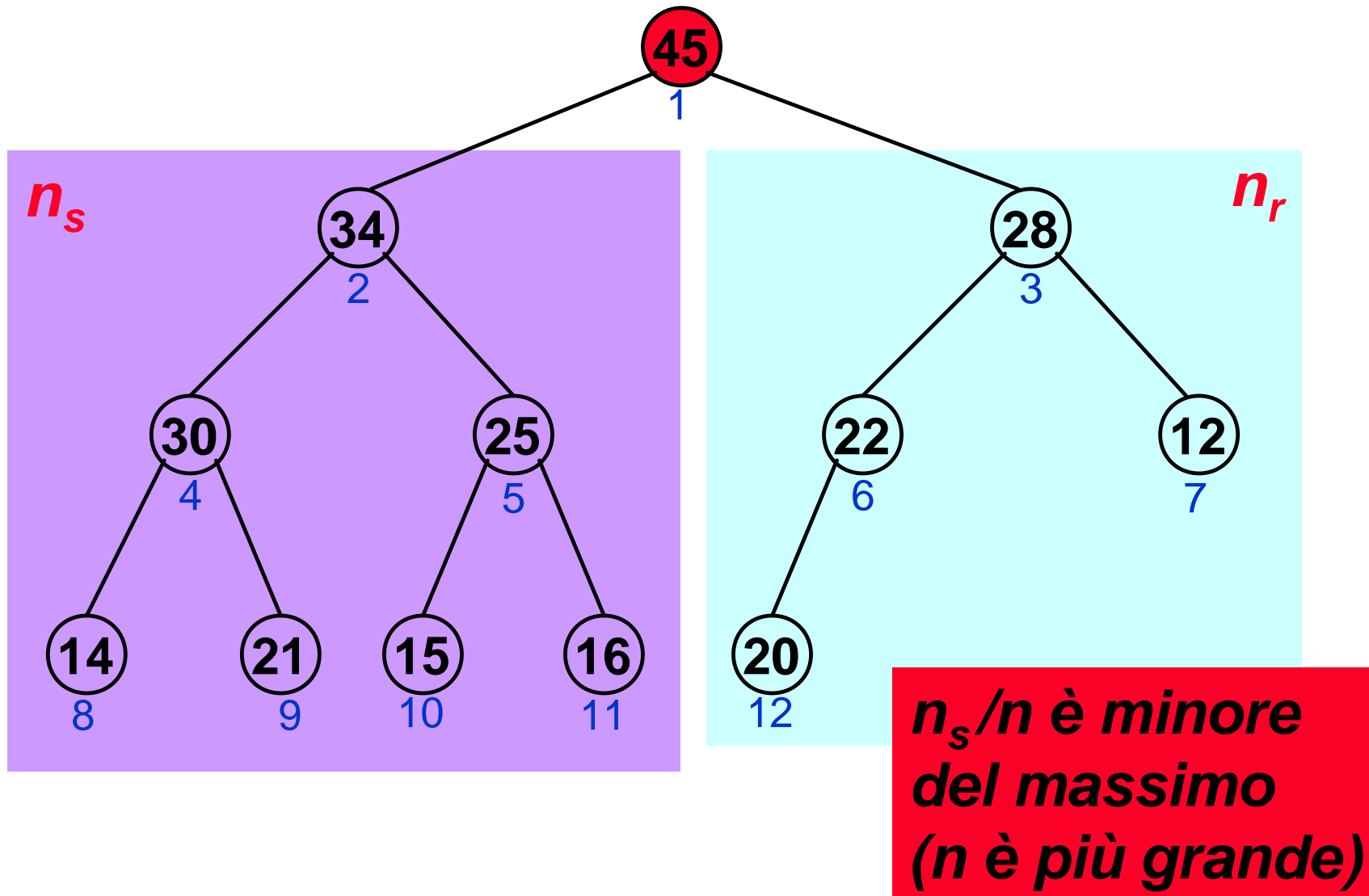
n_s/n ha valore massimo

Complessità di Heapify: caso peggiore



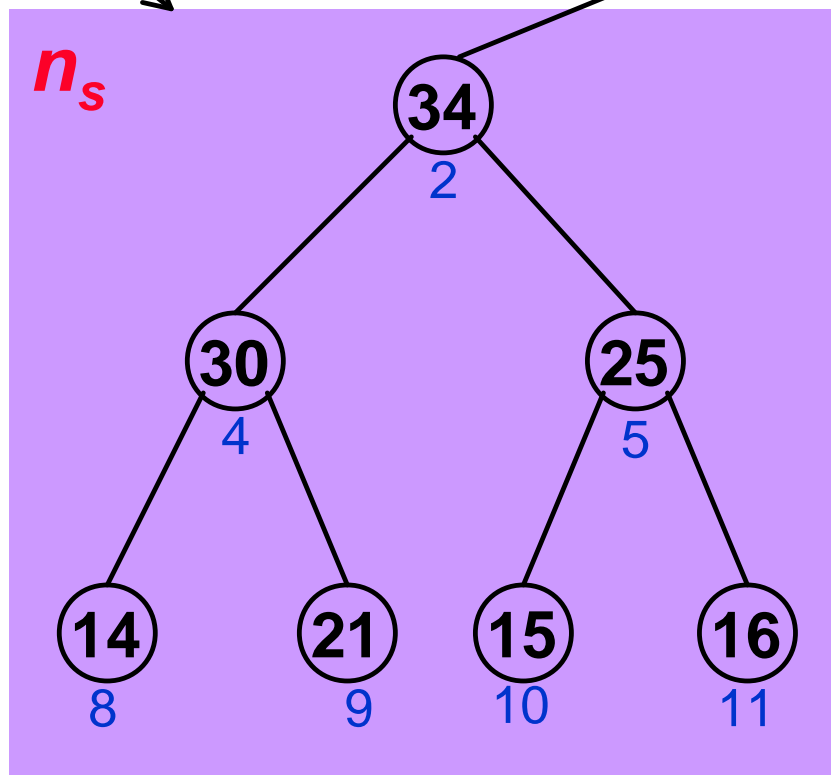
**n_s/n è minore
del massimo
(n_s è più piccolo)**

Complessità di Heapify: caso peggiore



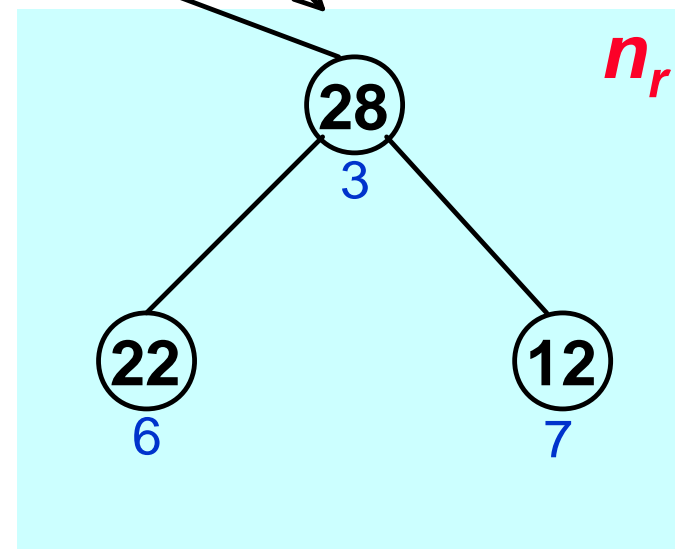
Complessità di Heapify: caso peggiore

Albero completo di altezza $h-1$



Numero di nodi $n_s = 2^{h-1} - 1$

Albero completo di altezza $h-2$

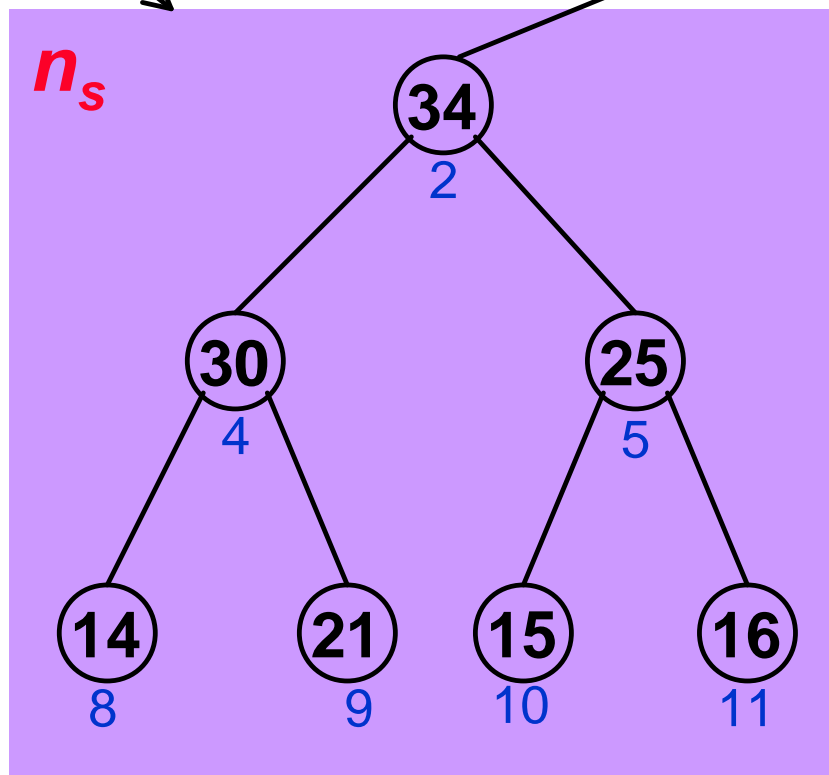


Numero di nodi $n_r = 2^{h-1} - 1$

$$n = 1 + 2^{h-1} + 2^{h-1} - 1 = 3 \cdot 2^{h-1} - 1$$

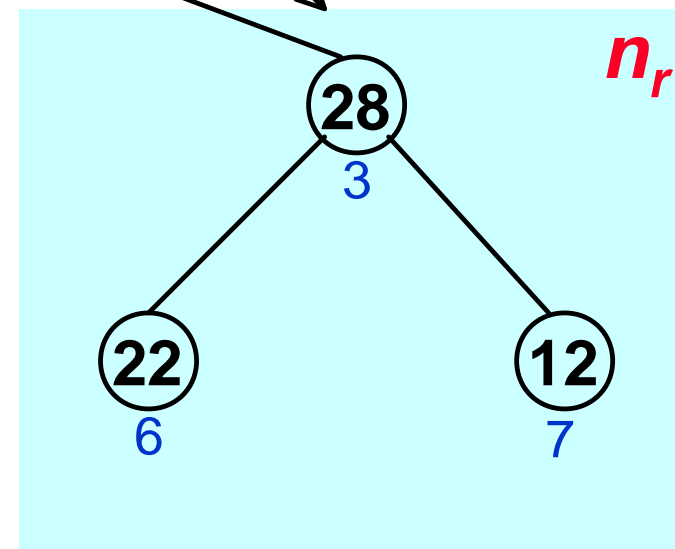
Complessità di Heapify: caso peggiore

Albero completo di altezza $h-1$



Numero di nodi $n_s = 2^{h-1} - 1$

Albero completo di altezza $h-2$



Numero di nodi $n_r = 2^{h-2} - 1$

$n_s/n = 2^{h-1}/(3 \cdot 2^{h-2} - 1) \approx 2/3$

Complessità di Heapify: caso peggiore

$$T(n) = \max(O(1), \max(O(1), T(?) + O(1)))$$

$$\text{£ } \max(O(1), \max(O(1), T(2n/3) + O(1)))$$

$$\text{£ } T(2n/3) + Q(1)$$

$$T'(n) = T'(2n/3) + Q(1)$$

Proviamo ad applicare il Metodo Iterativo!

$$T'(n) = Q(\log n)$$

Complessità di Heapify: caso peggiore

$$T(n) = \max(O(1), \max(O(1), T(?) + O(1)))$$

$$\text{£ } \max(O(1), \max(O(1), T(2n/3) + O(1)))$$

$$\text{£ } T(2n/3) + O(1)$$

Quindi

$$T(n) = O(\log n)$$

Heapify *impiega tempo proporzionale all'altezza dell'albero su cui opera !*

Complessità di *Heapify*: caso migliore

$$T(n) = T(?) + O(1)$$

Nel **caso migliore** *Heapify* ad ogni chiamata ricorsiva, viene eseguito su un numero di nodi che è maggiore di **1/3** del numero di nodi correnti **n** .

Cioè il numero di nodi **n_s** del sottoalbero su cui *Heapify* è chiamato ricorsivamente è al più **$1/3 n$** (o **$n_s \geq 1/3 n$**)

Complessità di Heapify: caso migliore

$$\begin{aligned} T(n) &= T(?) + O(1) \\ &\geq T(n/3) + Q(1) \end{aligned}$$

$$T'(n) = T'(n/3) + Q(1)$$

Applicando il Metodo Iterativo!

$$T'(n) = Q(\log n)$$

quindi

$$T(n) = W(\log n)$$

Costruisci Heap: intuizioni

Costruisci-Heap(*A*): utilizza l'algoritmo **Heapify**, per inserire ogni elemento dell'array in uno **Heap**, risistemando sul posto gli elementi:

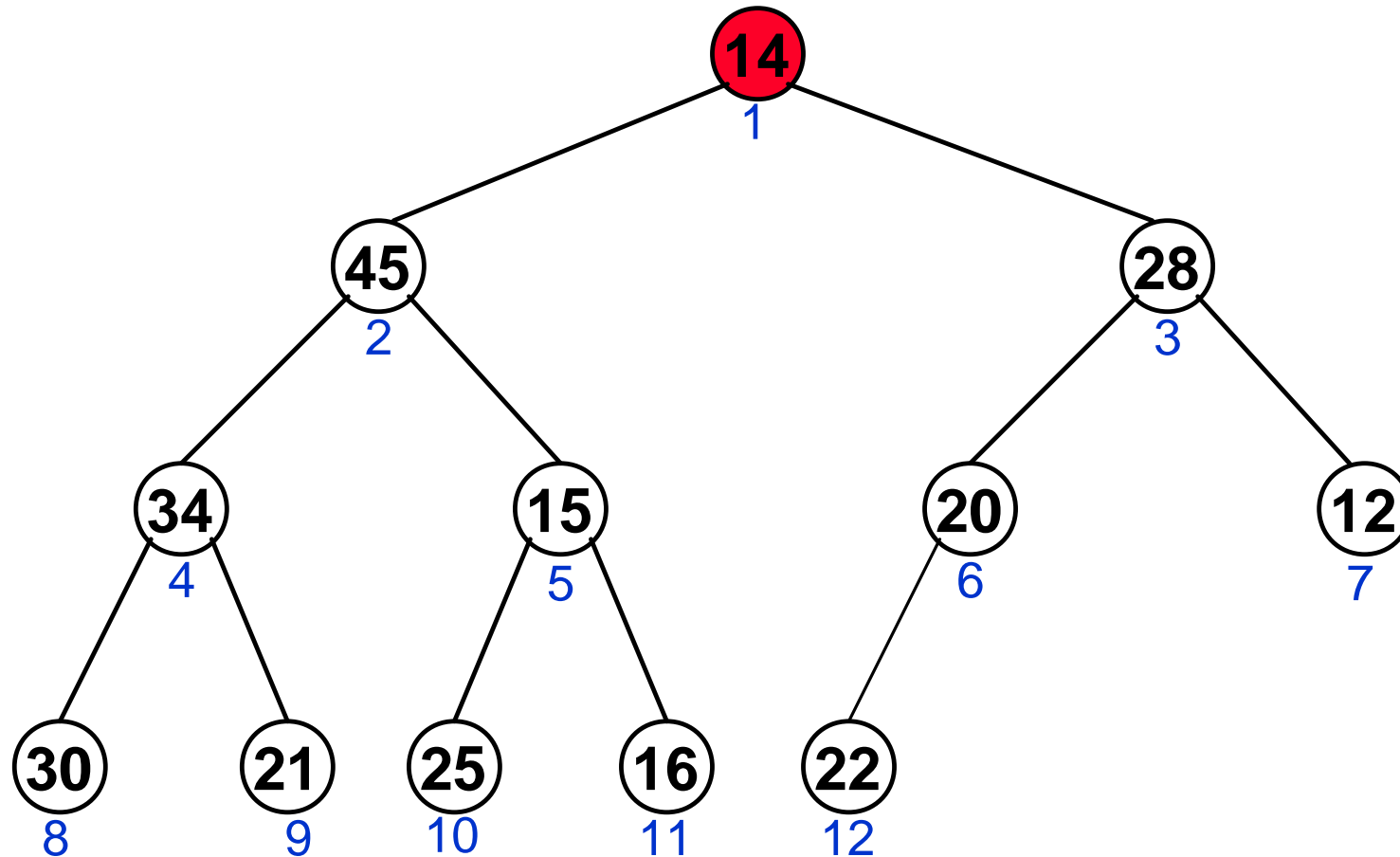
- gli ultimi $\hat{=} n/2$ elementi dell'array sono foglie, cioè radici di sottoalberi vuoti, quindi sono già degli **Heap**
- è sufficiente inserire nello **Heap** solo i primi $\hat{=} n/2$ elementi, utilizzando **Heapify** per ripristinare la proprietà **Heap** sul sottoalbero del nuovo elemento.

Costruisci Heap

```
Costruisci-Heap(A)
  heapsize[A] = length[A]
  FOR i = ⌊length[A]/2⌋ DOWNTO 1
    DO Heapify(A, i)
```

Costruisci Heap

1	2	3	4	5	6	7	8	9	10	11	12
14	45	28	34	15	20	12	30	21	25	16	22



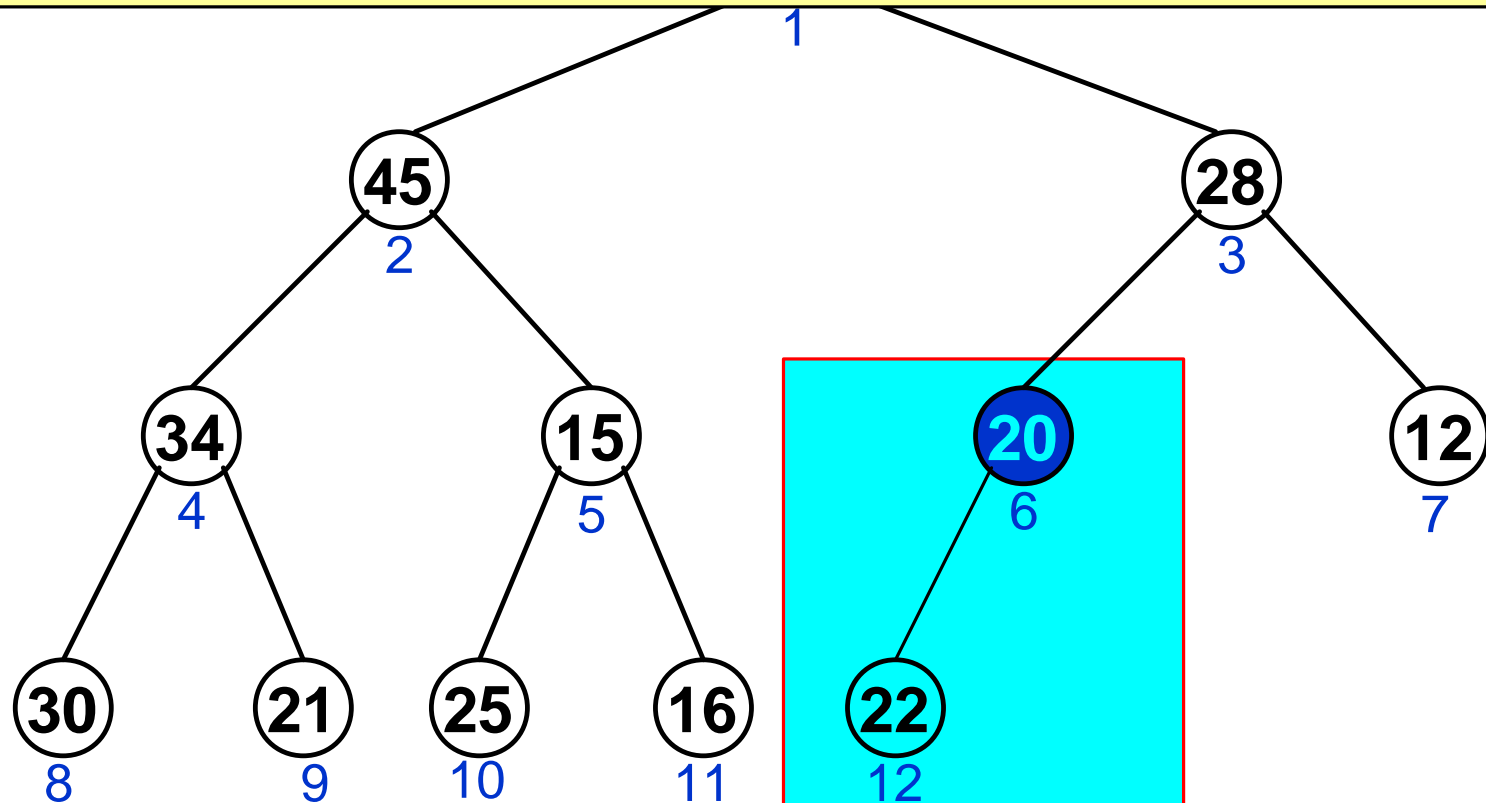
Costruisci Heap

```
Costruisci-Heap(A)
```

```
  heapsize[A] = length[A]
```

```
  FOR  $i = \lfloor \text{length}[A] / 2 \rfloor$  DOWNTO 1
```

```
    DO Heapify(A,  $i$ )
```



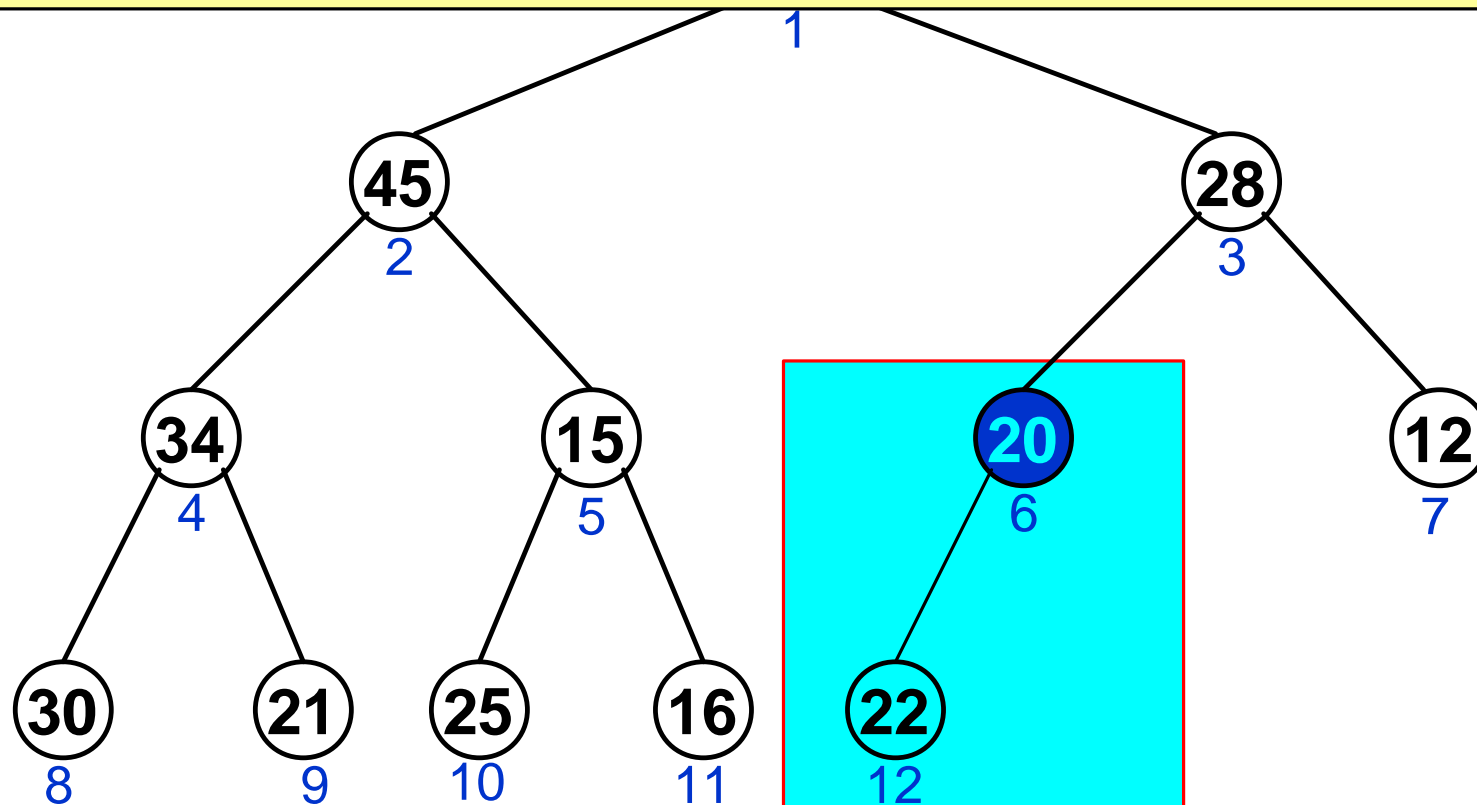
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    DO Heapify(A, i)
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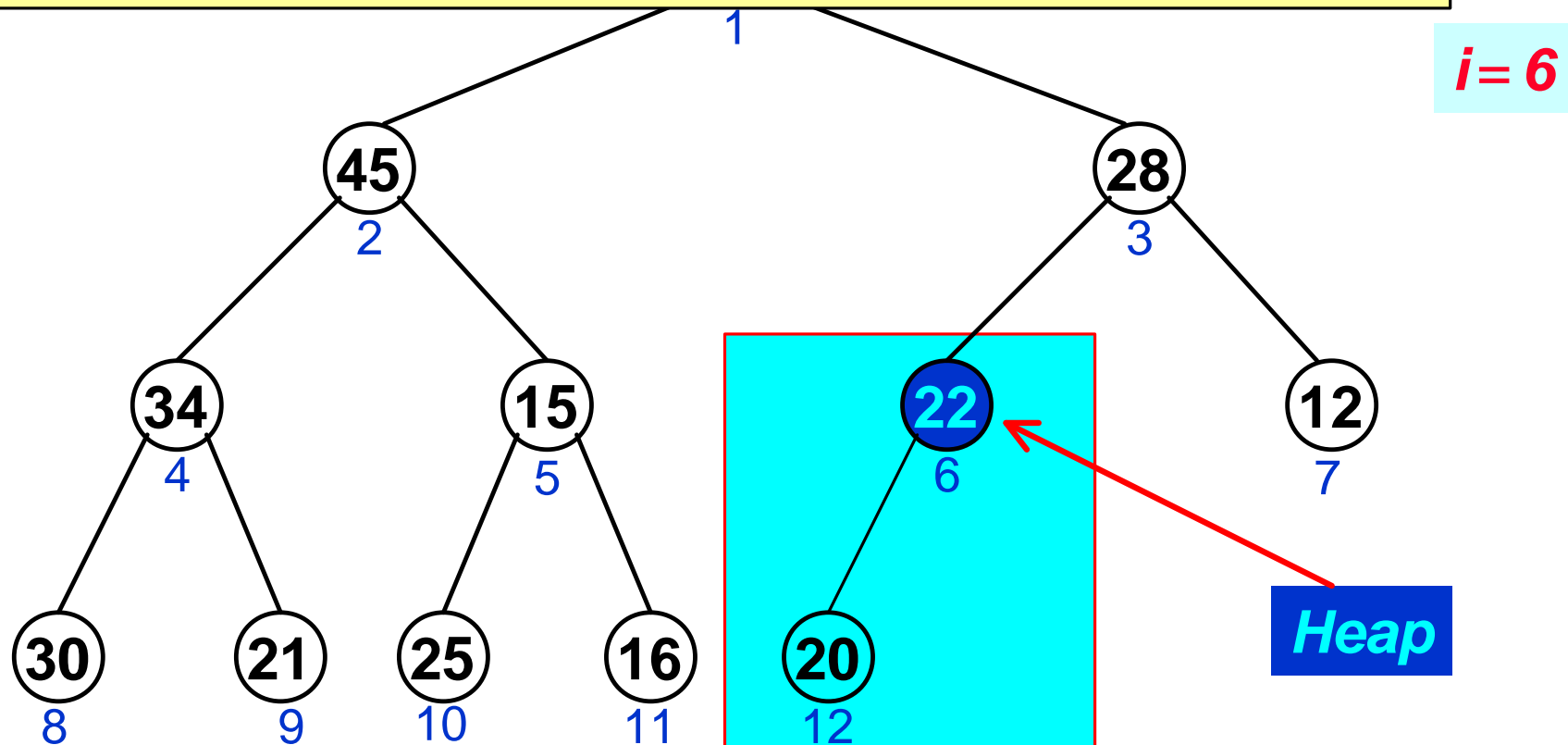
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    DO Heapify(A, i)
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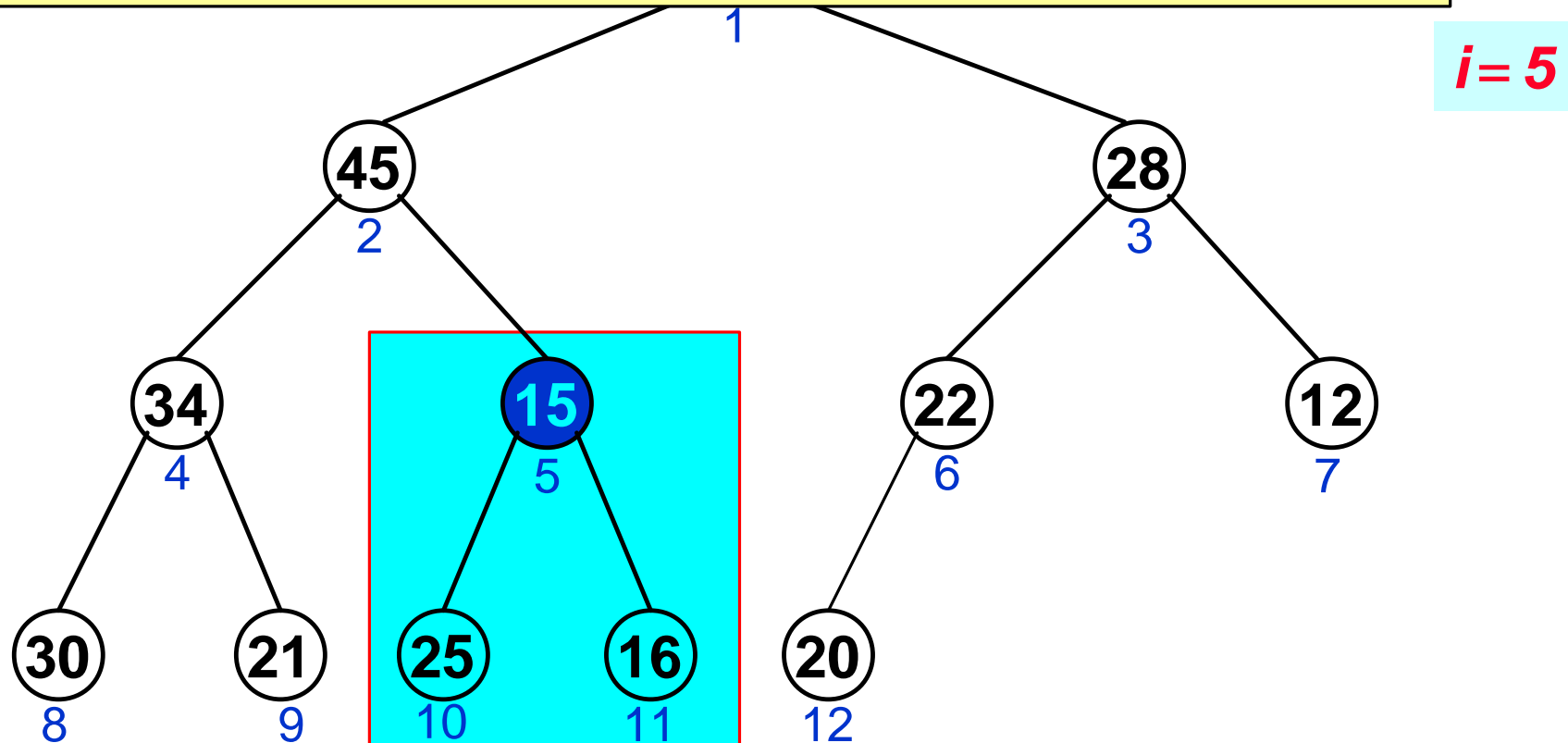
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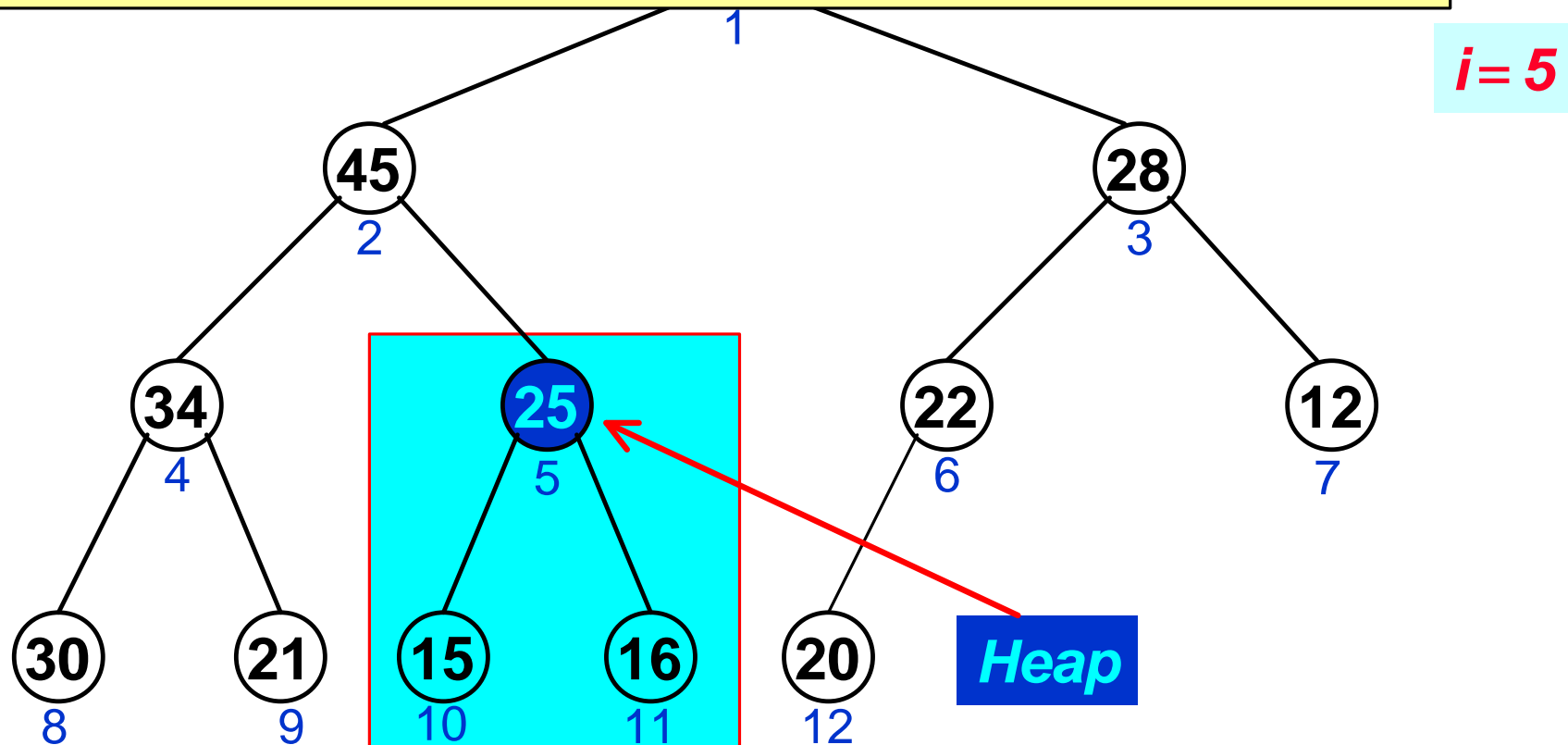
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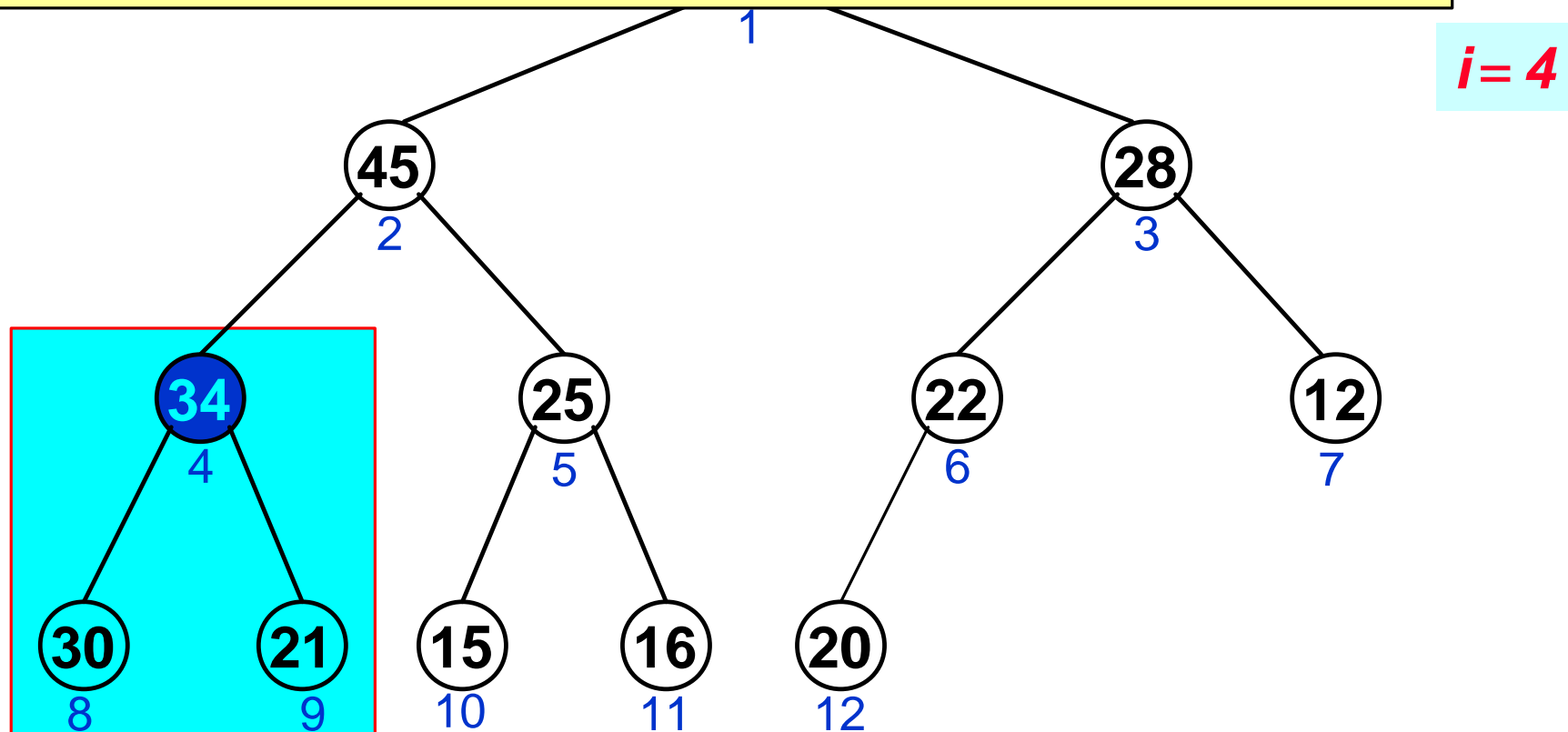
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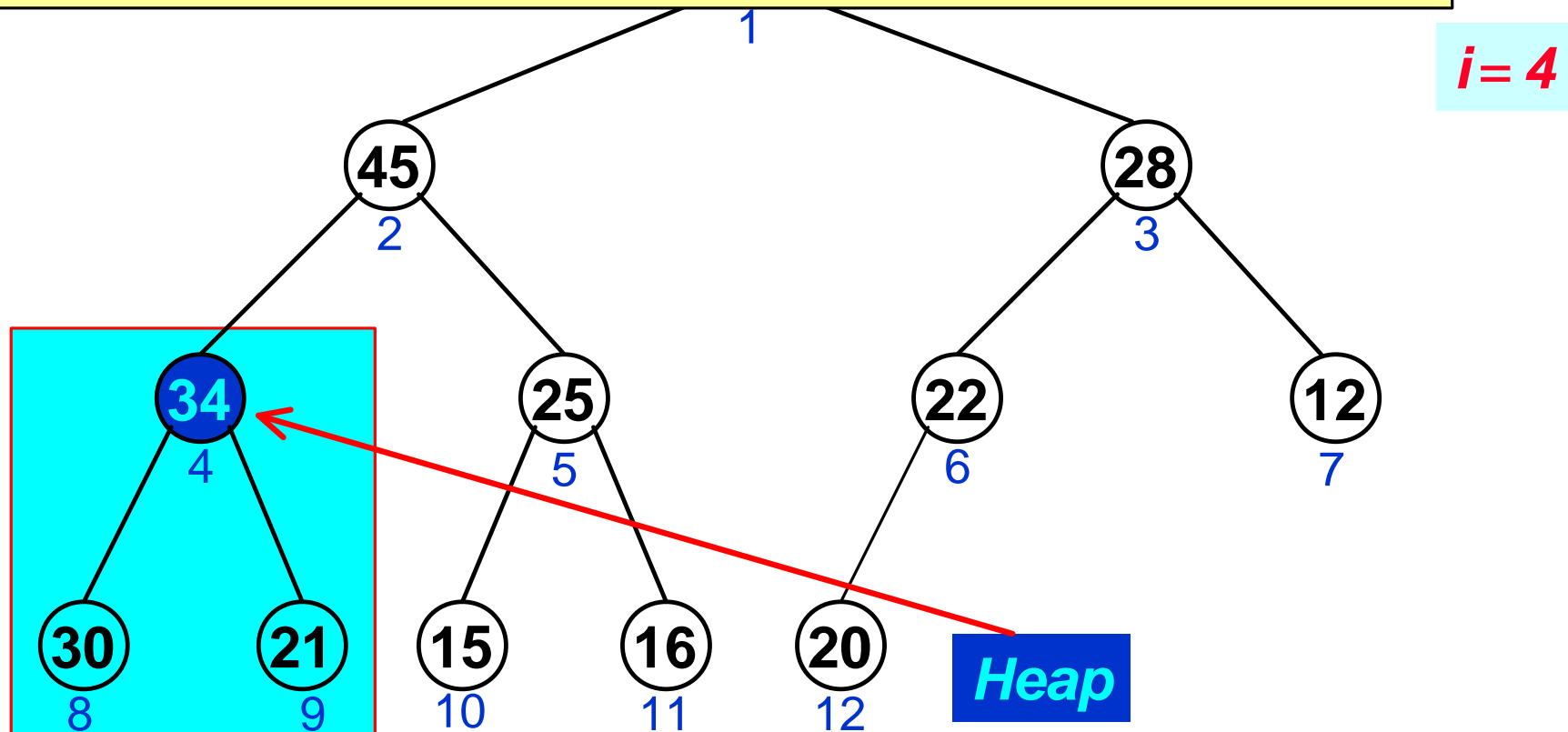
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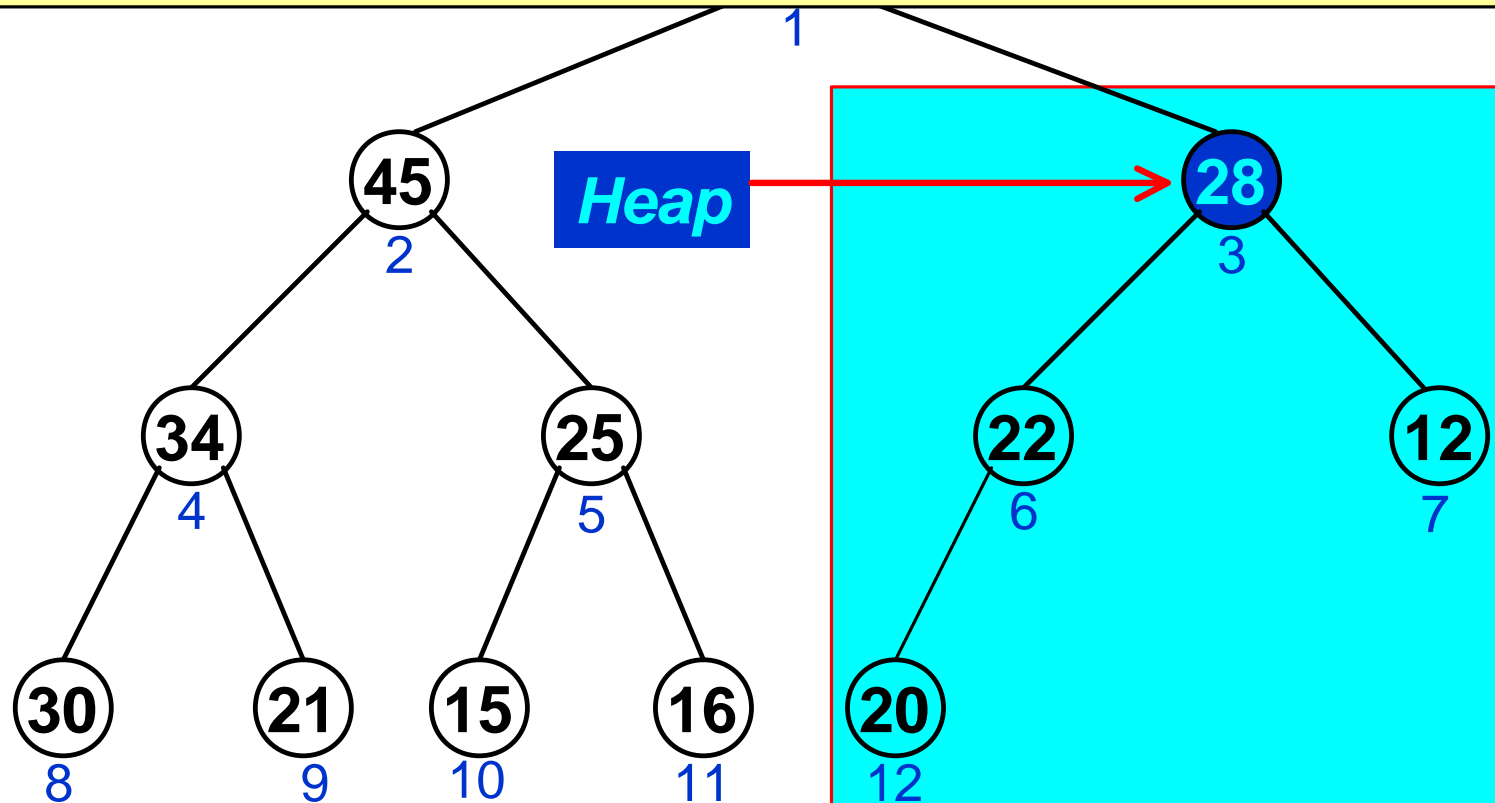
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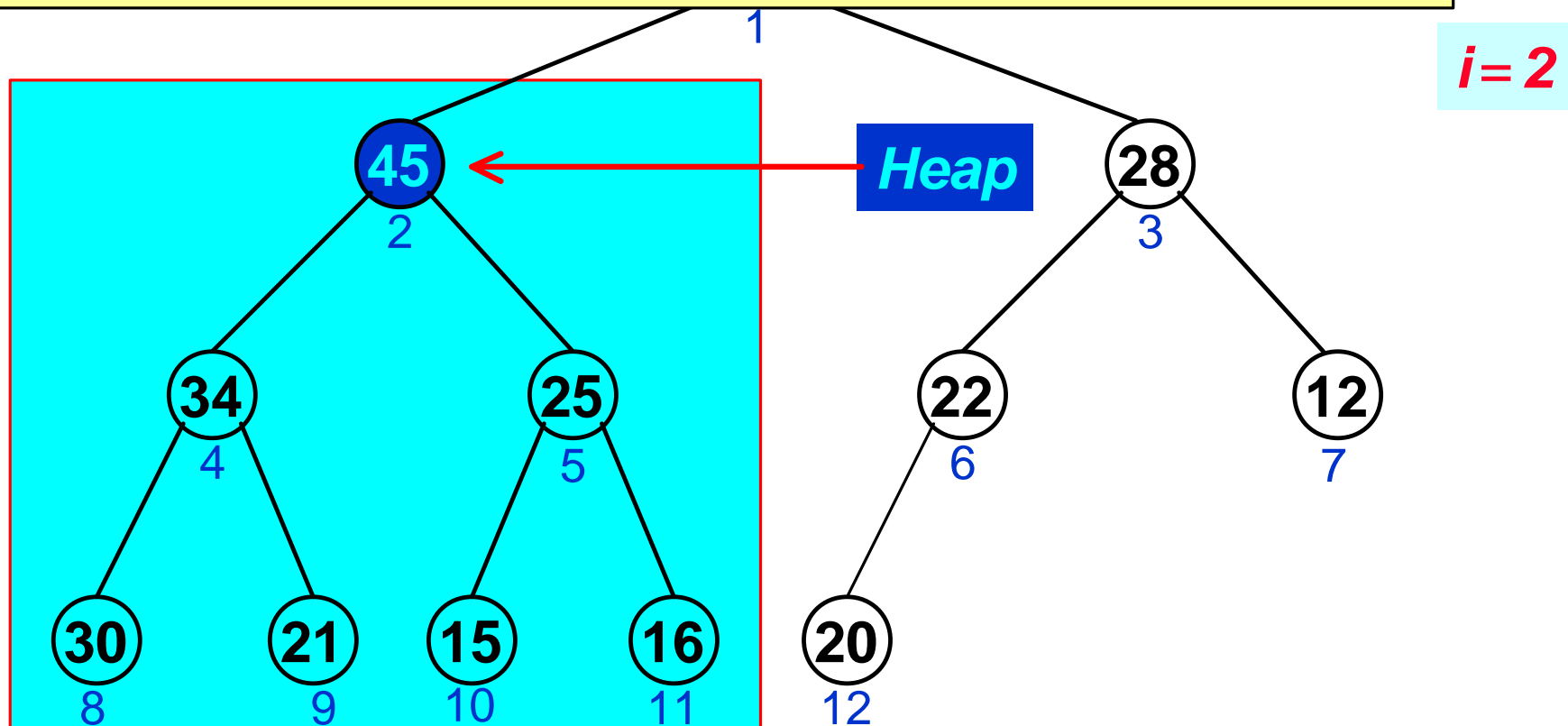
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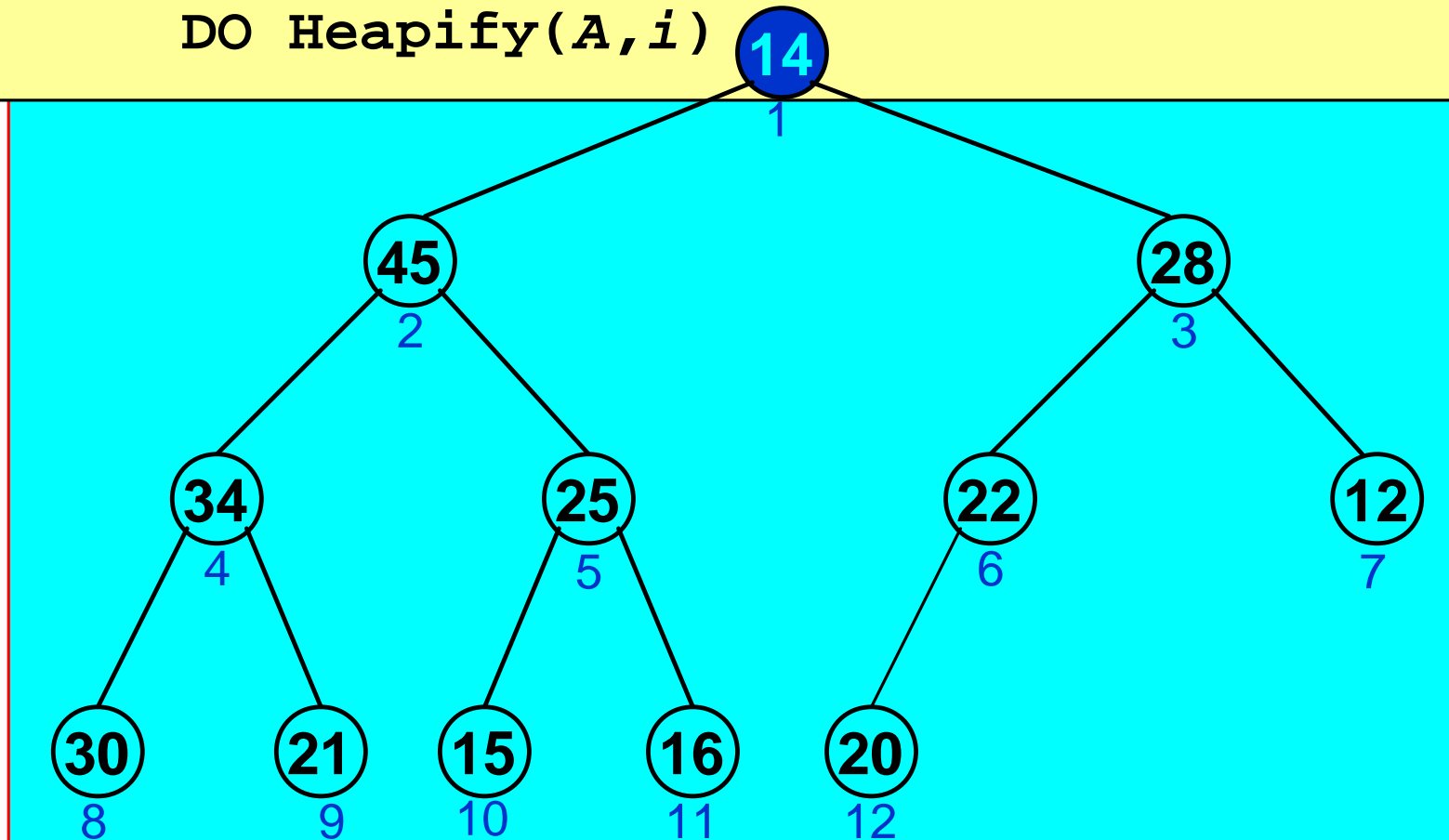
Costruisci Heap

```
Costruisci-Heap(A)
```

```
  heapsize[A] = length[A]
```

```
  FOR  $i = \lfloor \text{length}[A] / 2 \rfloor$  DOWNTO 1
```

```
    DO Heapify(A,  $i$ )
```



$i = 1$

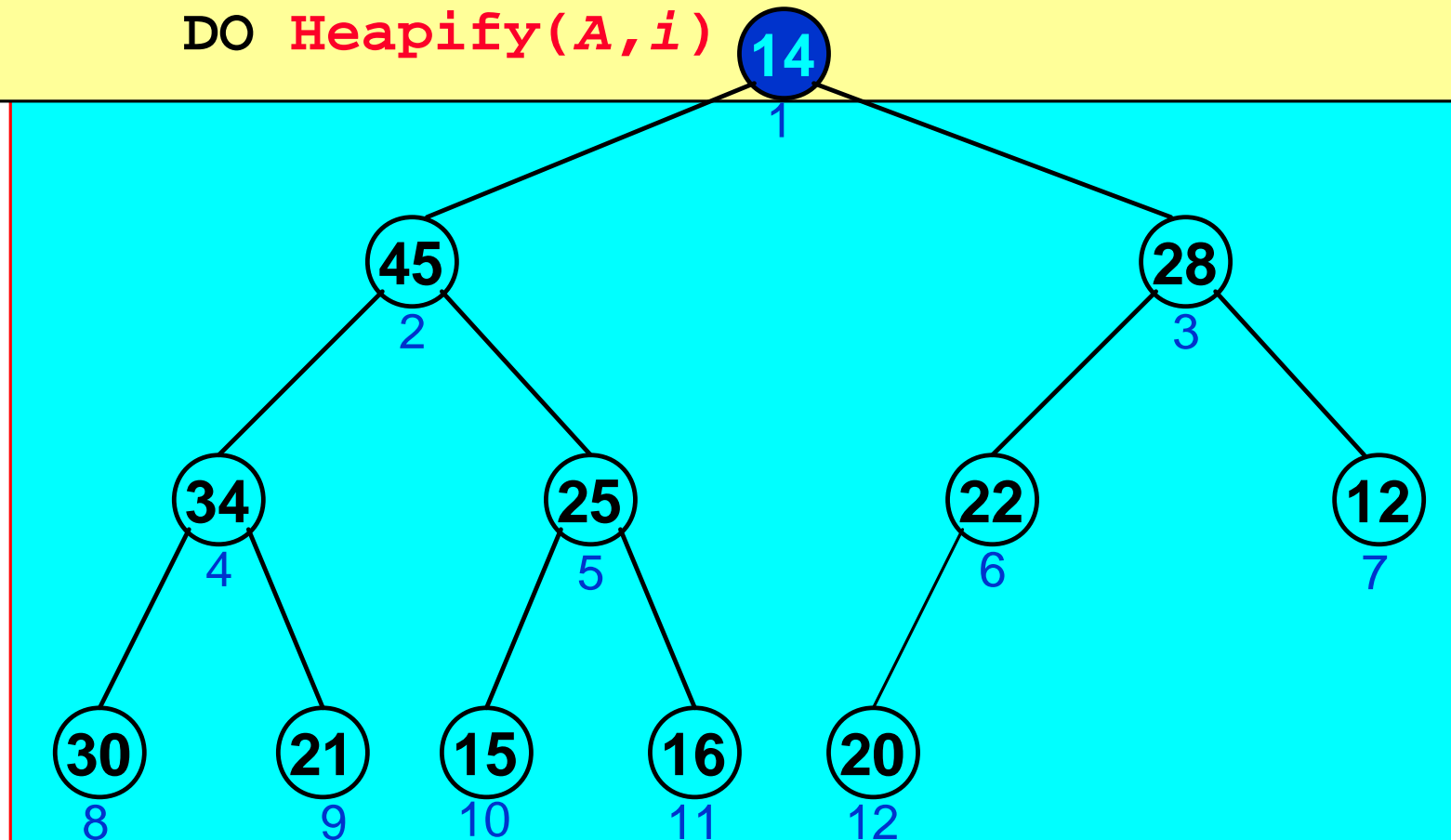
Costruisci Heap

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Costruisci-Heap(A)
```

```
  heapsize[A] = length[A]
```

```
  FOR i = ⌊length[A]/2⌋ DOWNTO 1
```

```
    DO Heapify(A, i)
```



i = 1

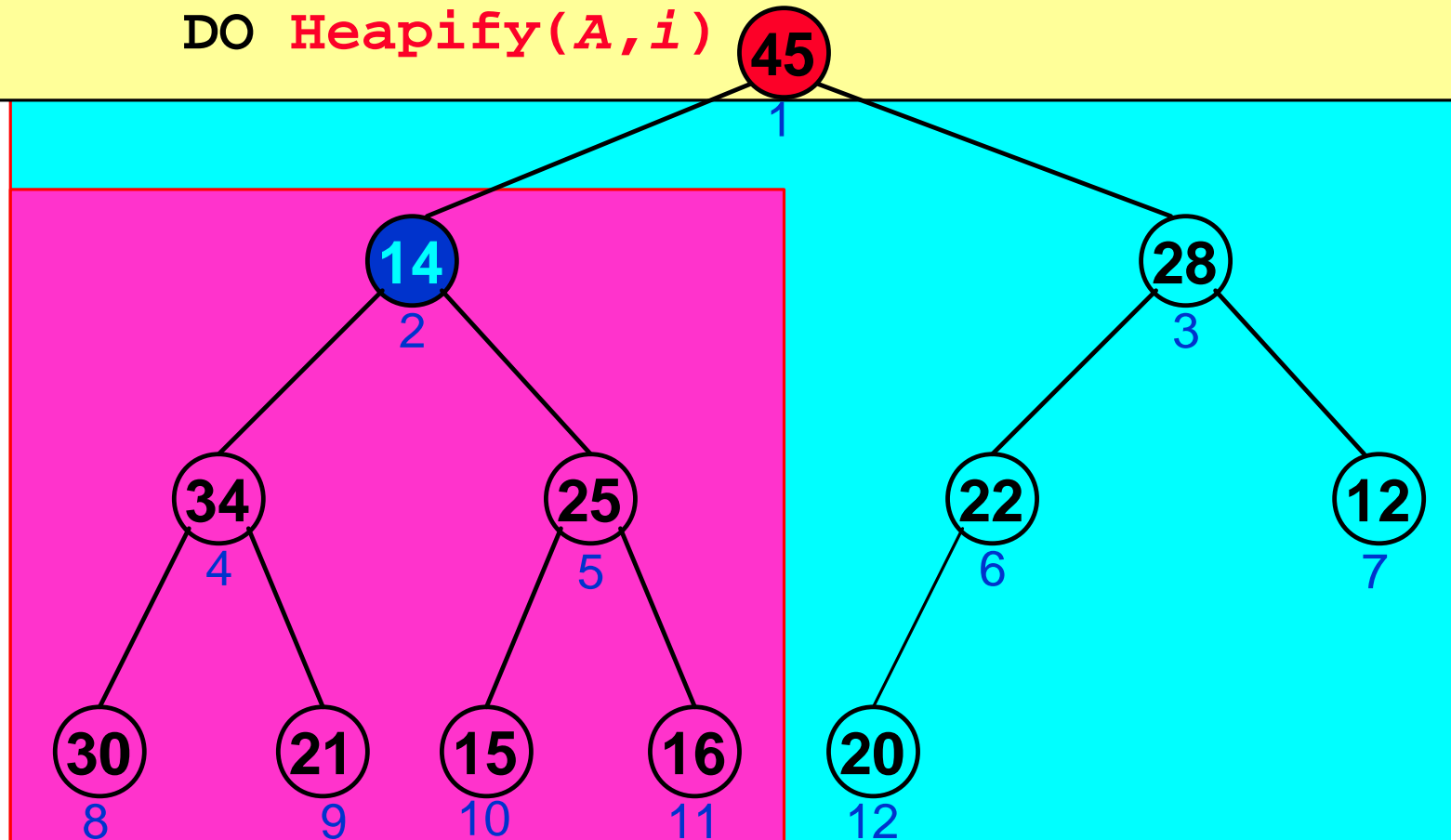
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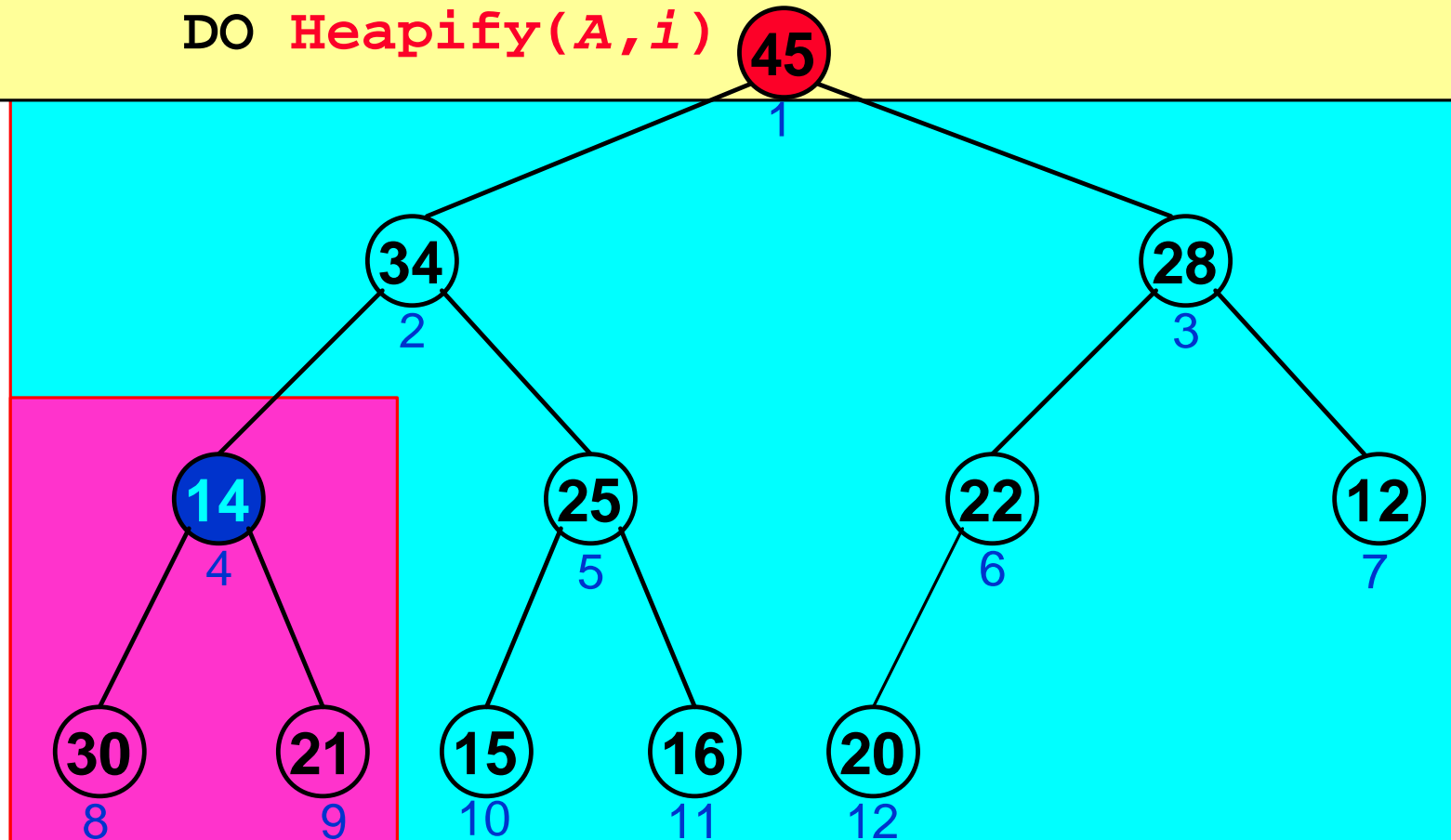
Costruisci Heap

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Costruisci-Heap(A)
```

```
  heapsize[A] = length[A]
```

```
  FOR i = ⌊length[A]/2⌋ DOWNTO 1
```

```
    DO Heapify(A, i)
```



i = 1

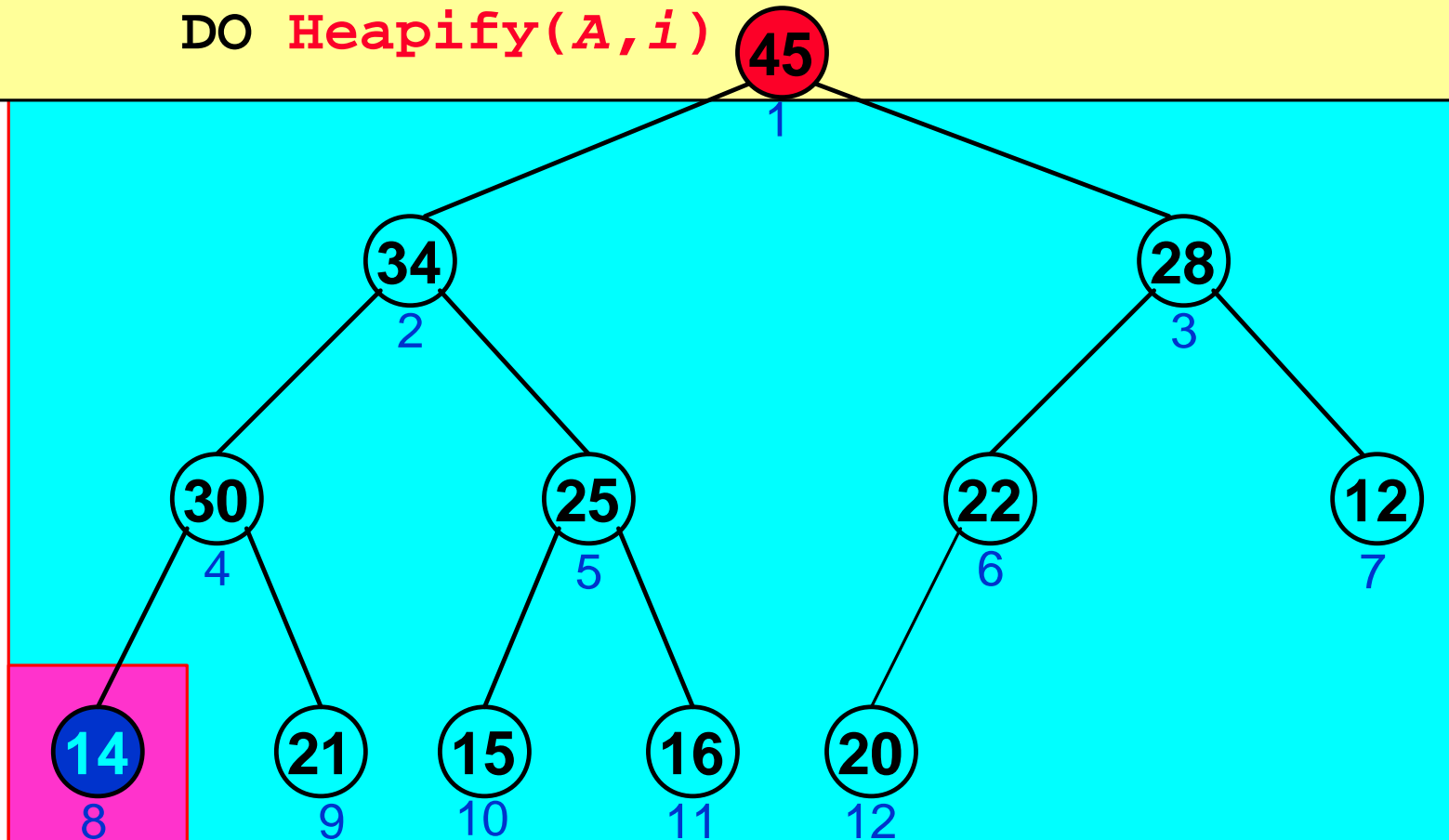
Costruisci Heap

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Costruisci-Heap(A)
```

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```

```
  FOR i = ⌊length[A]/2⌋ DOWNTO 1
```

```
    DO Heapify(A, i)
```



i = 1

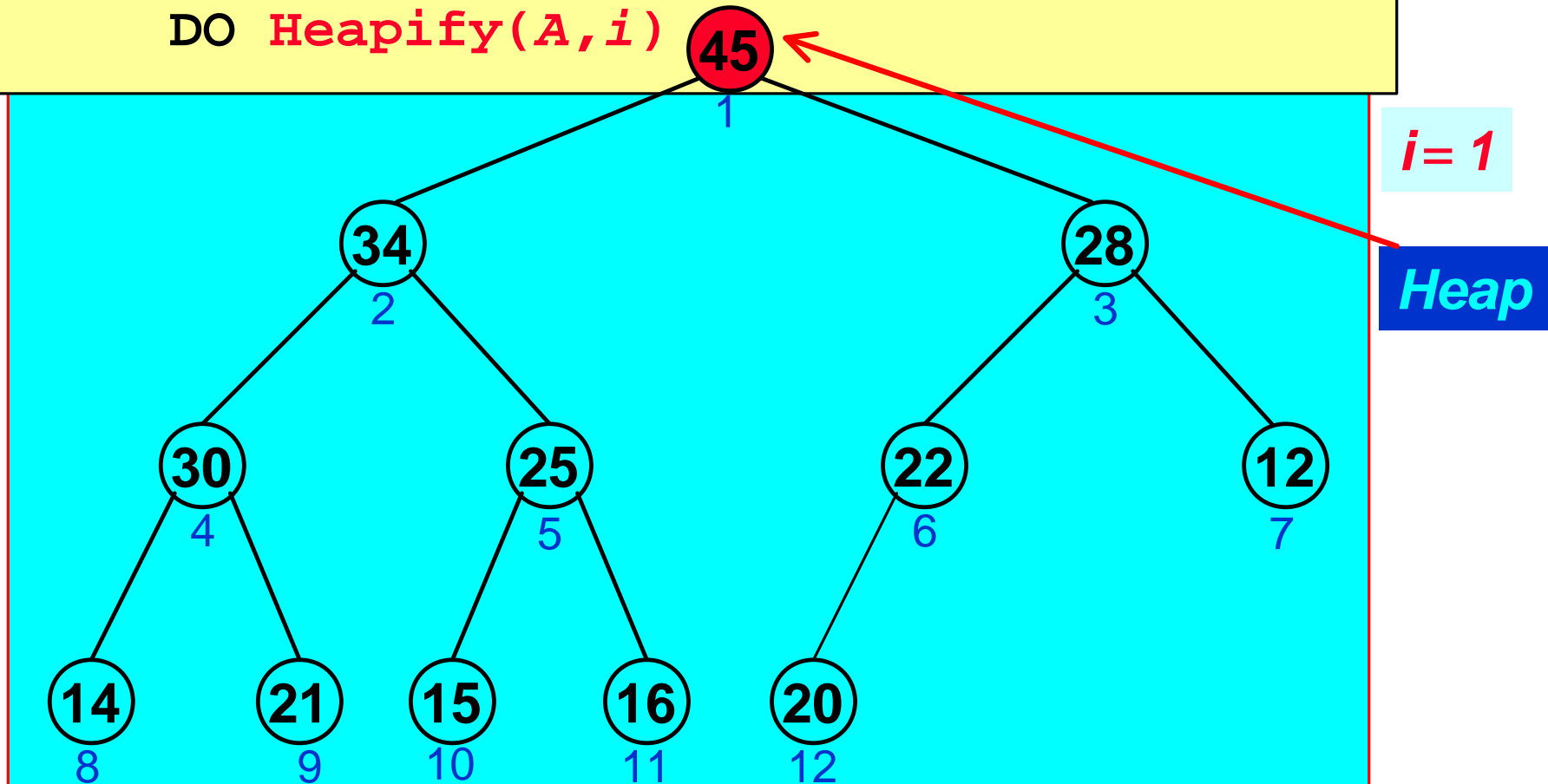
Costruisci Heap

```
Costruisci-Heap(A)
```

```
  heapsize[A] = length[A]
```

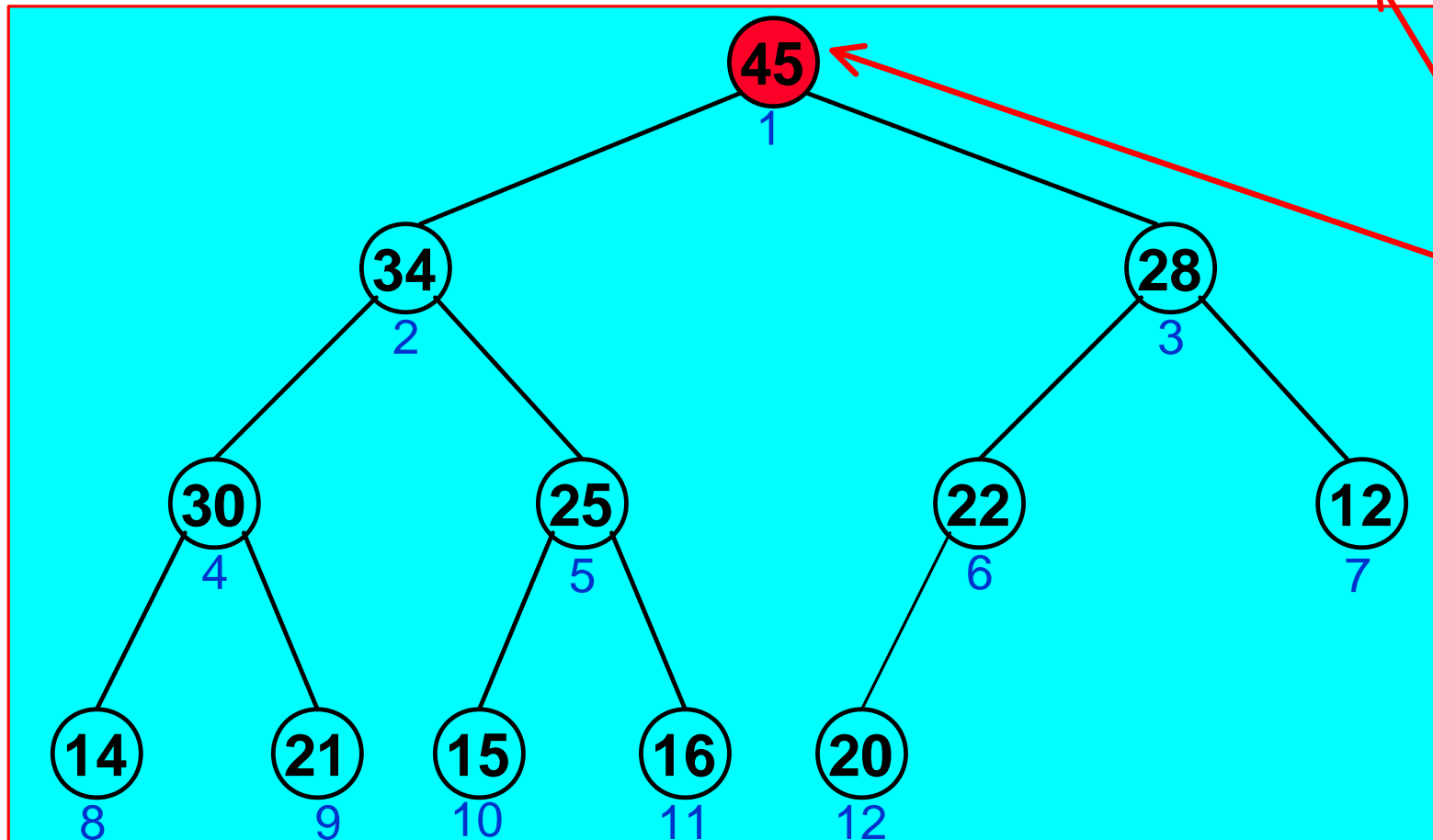
```
  FOR i = ⌊length[A]/2⌋ DOWNTO 1
```

```
    DO Heapify(A, i)
```



Costruisci Heap

1	2	3	4	5	6	7	8	9	10	11	12
45	34	28	30	25	22	12	14	21	15	16	20



Heap

Complessità di Costruisci Heap

```
Costruisci-Heap(A)
```

```
    heapsize[A] = length[A] } =  $O(1)$ 
```

```
    FOR i = ⌊length[A]/2⌋ DOWNTO 1 } =  $O(?)$   
        DO Heapify(A, i)
```


Complessità di Costruisci Heap

$$T(n) = \max(O(1), O(?)) = \max(O(1), O(f(n)))$$

Poiché *Heapify* viene chiamata $n/2$ volte si potrebbe ipotizzare

$$f(n) = O(n \log n)$$

e quindi

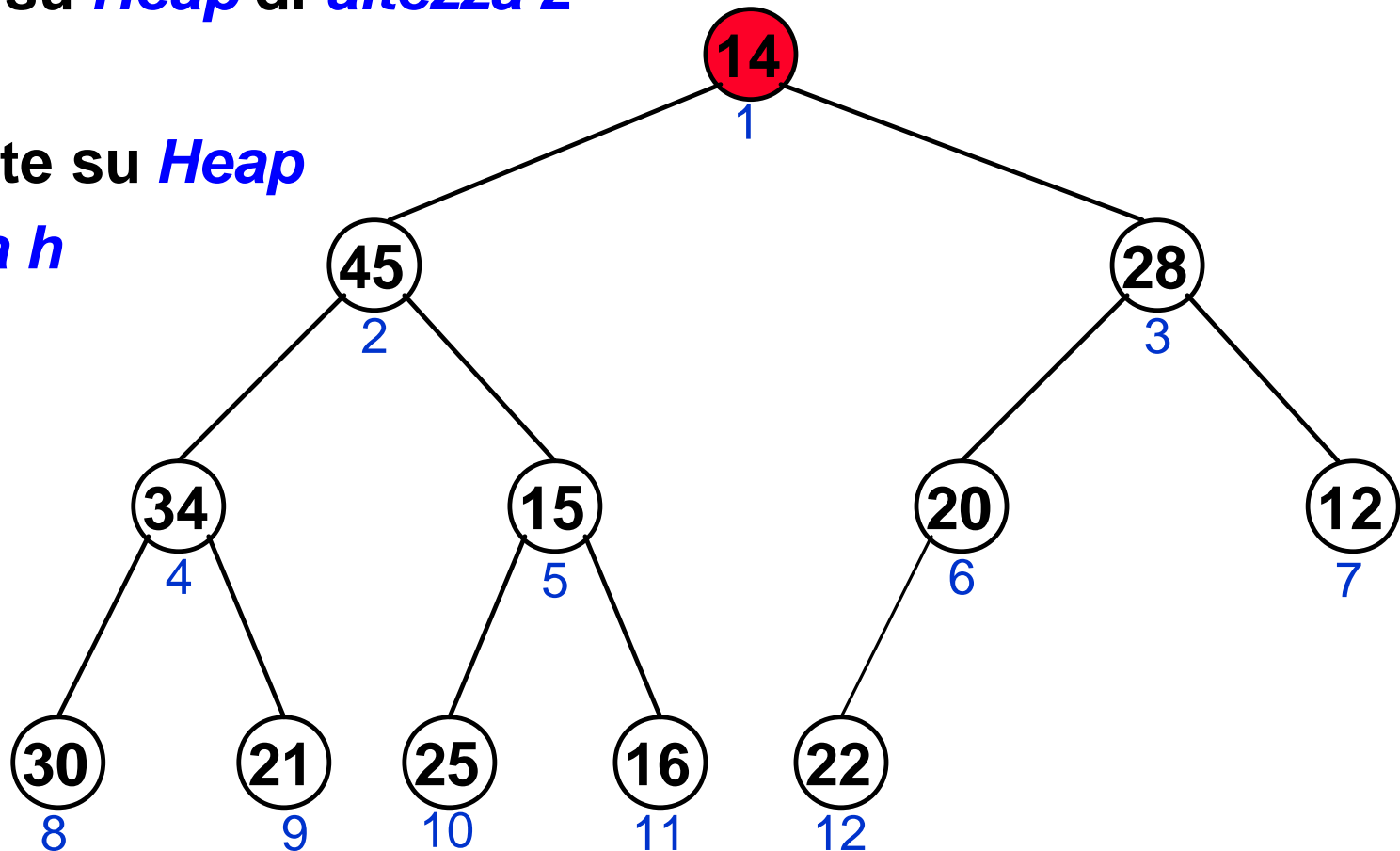
$$T(n) = \max(O(1), O(n \log n)) = O(n \log n)$$

ma....

Complessità di Costruisci Heap

Costruisci-Heap chiama *Heapify*

- $n/2$ volte su *Heap* di *altezza 0* (non eseguito)
- $n/4$ volte su *Heap* di *altezza 1*
- $n/8$ volte su *Heap* di *altezza 2*
- ...
- $n/2^{h+1}$ volte su *Heap* di *altezza h*



Complessità di Costruisci Heap

$$T(n) = \max(O(1), O(?)) = \max(O(1), O(f(n)))$$

$$f(n) = \sum_{h=0}^{\log n} n / 2^{h+1} O(h)$$

Costruisci-Heap chiama *Heapify*

- $n/2$ volte su *Heap* di *altezza 0* (in realtà non eseguito)
- $n/4$ volte su *Heap* di *altezza 1*
- $n/8$ volte su *Heap* di *altezza 2*
- ...
- $n/2^{h+1}$ volte su *Heap* di *altezza h*

Complessità di Costruisci Heap

$$T(n) = \max(O(1), O(?)) = \max(O(1), O(f(n)))$$

$$\begin{aligned} f(n) &= \sum_{h=0}^{\lfloor \log n \rfloor} n / 2^{h+1} O(h) \\ &= O\left(\frac{n}{2} \sum_{h=0}^{\lfloor \log n \rfloor} h / 2^h\right) \end{aligned}$$

Complessità di Costruisci Heap

$$T(n) = \max(O(1), O(?)) = \max(O(1), O(f(n)))$$

$$f(n) = \sum_{h=0}^{\lfloor \log n \rfloor} \frac{n}{2^{h+1}} O(h)$$

$$= O\left(\frac{n}{2} \sum_{h=0}^{\lfloor \log n \rfloor} h / 2^h\right)$$

$$= O\left(\frac{n}{2} \sum_{h=0}^{\infty} h / 2^h\right)$$

Complessità di Costruisci Heap

$$T(n) = \max(O(1), O(?)) = \max(O(1), O(f(n)))$$

$$f(n) = \sum_{h=0}^{\lfloor \log n \rfloor} \left(\frac{n}{2^{h+1}} \right) O(h)$$

$$= O\left(\frac{n}{2} \sum_{h=0}^{\lfloor \log n \rfloor} h / 2^h \right)$$

$$= O\left(\frac{n}{2} \sum_{h=0}^{\infty} h / 2^h \right)$$

$$= O(2n/2)$$

$$\sum_{h=0}^{\infty} hx^h = \frac{x}{(1-x)^2}$$

$$\frac{x}{(1-x)^2} = 2$$

$$x = 1/2 \text{ \& } 1$$

Complessità di Costruisci Heap

$$T(n) = \max(O(1), O(?)) = \max(O(1), O(f(n)))$$

$$f(n) = \sum_{h=0}^{\lfloor \log n \rfloor} \frac{n}{2^{h+1}} O(h)$$

$$= O\left(\frac{n}{2} \sum_{h=0}^{\lfloor \log n \rfloor} h / 2^h\right)$$

$$= O\left(\frac{n}{2} \sum_{h=0}^{\infty} h / 2^h\right)$$

$$= O(n)$$

Complessità di Costruisci Heap

$$T(n) = \max(O(1), O(?)) = \max(O(1), O(f(n)))$$

$$f(n) = O(n)$$

$$T(n) = \max(O(1), O(n)) = O(n)$$

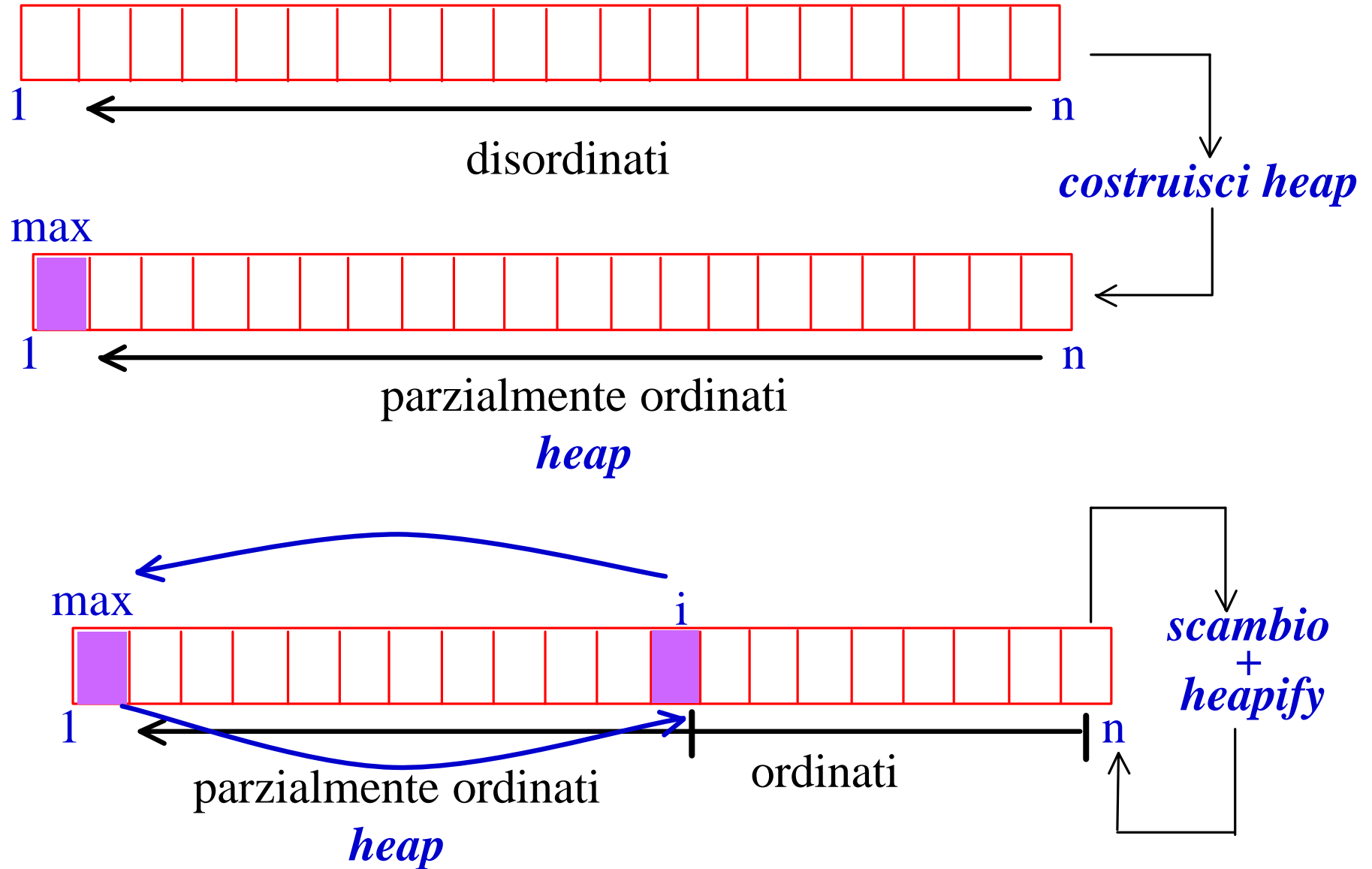
Costruire uno **Heap** di **n** elementi è poco costoso, al più costa **$O(n)$** !

Heap Sort: intuizioni

Heap-Sort: è una variazione di **select-sort** in cui la ricerca dell'elemento massimo è facilitata dal mantenimento della sequenza in uno heap:

- si costruisce uno **Heap** a partire dall'array non ordinato in input.
- viene sfruttata la proprietà degli **Heap** per cui la radice **A[1]** dello **Heap** è sempre il massimo:
 - scandisce tutti gli elementi dell'array a partire dall'ultimo e ad ogni iterazione
 - la radice **A[1]** viene scambiata con l'elemento nell'ultima posizione corrente dello **Heap**
 - viene ridotta la dimensione dello **Heap** e
 - ripristinato lo **Heap** con **Heapify**

Heap Sort: intuizioni



Heap Sort

```
Select-Sort(A)
```

```
  FOR i = length[A] DOWNTO 2
```

```
    DO max = Findmax(A,i)
```

```
      "scambia A[max] e A[i]"
```

```
Heap-Sort(A)
```

```
  Costruisci-Heap(A)
```

```
  FOR i = length[A] DOWNTO 2
```

```
    DO /* elemento massimo in A[1] */
```

```
      "scambia A[1] e A[i]"
```

```
      /* ripristina lo heap */
```

```
      heapsize[A] = heapsize[A]-1
```

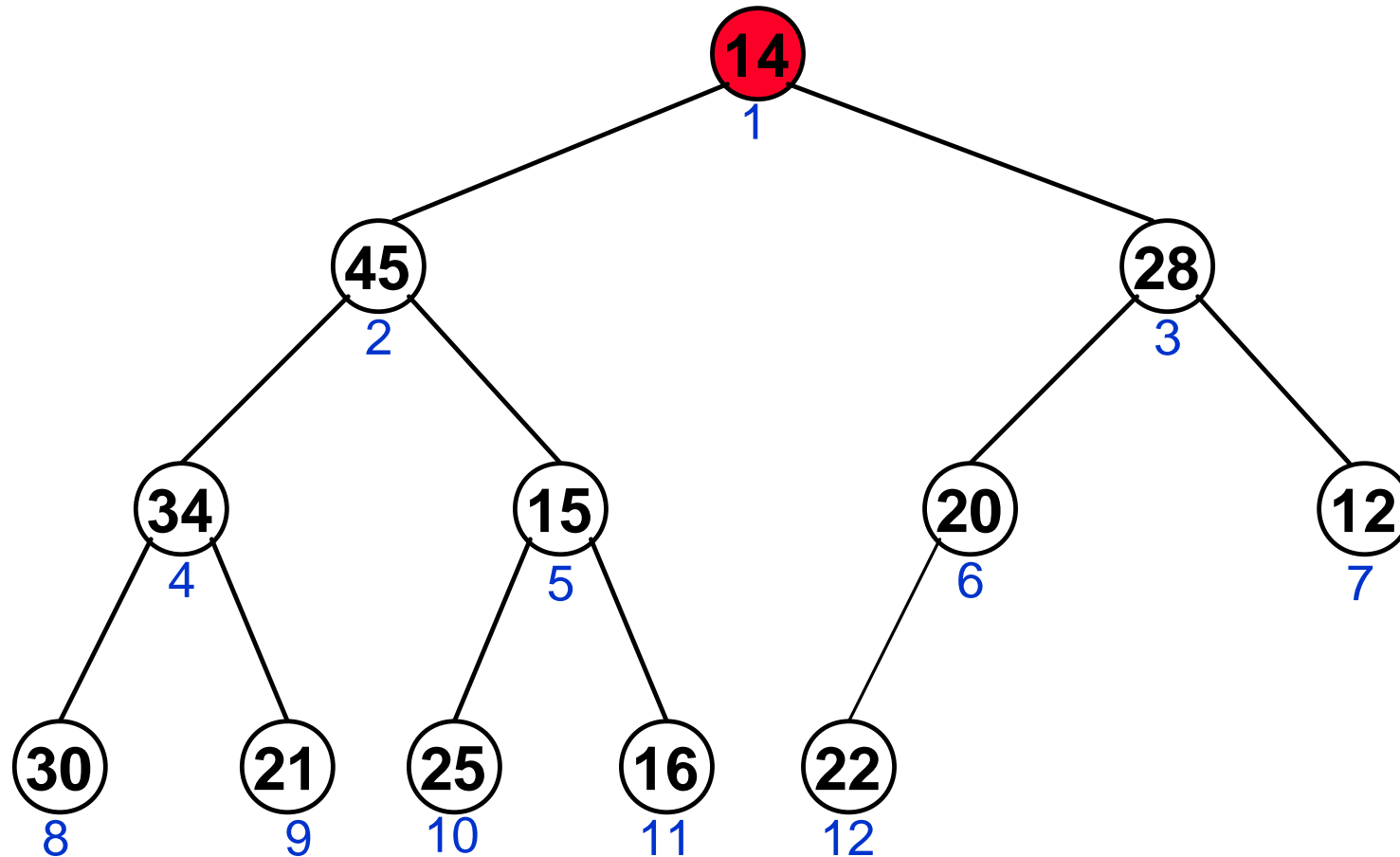
```
      Heapify(A,1)
```

Heap Sort

Heap-Sort (A)

Costruisci-Heap(A)

...

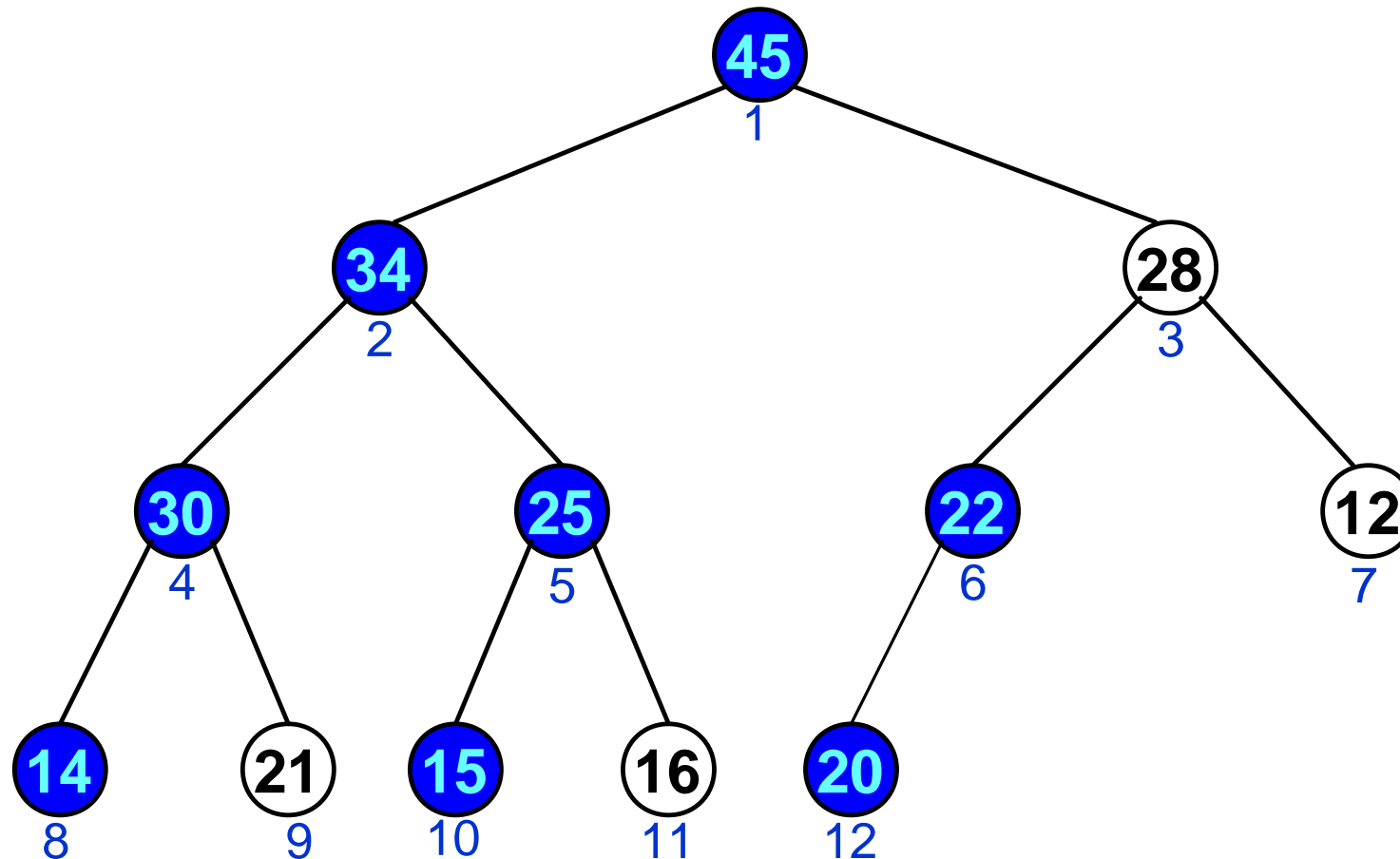


Heap Sort

Heap-Sort (A)

Costruisci-Heap(A)

...



Heap Sort

Heap-Sort(A)

...

FOR $i = \text{length}[A]$ DOWNTO 2

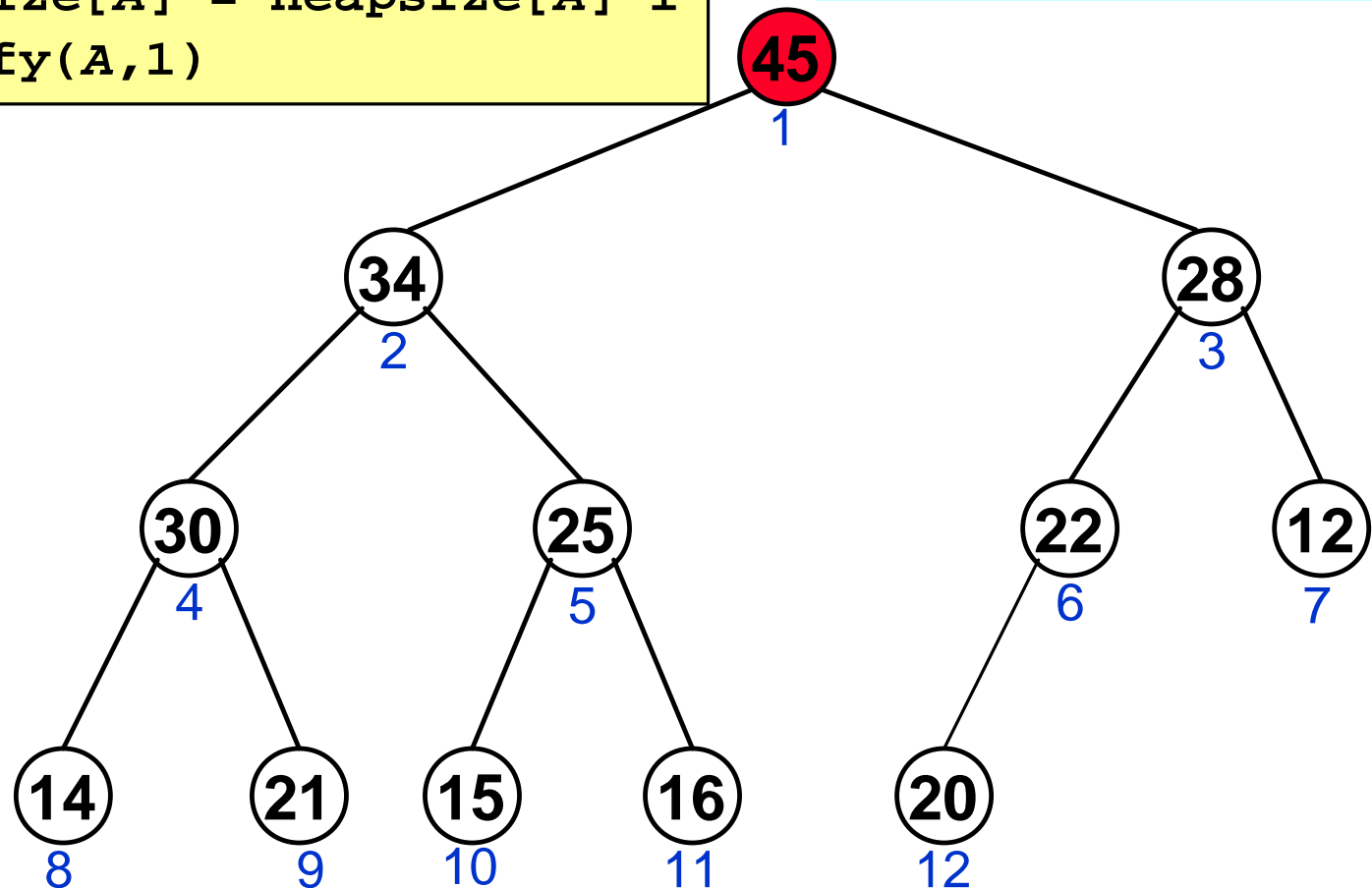
DO "scambia $A[1]$ e $A[i]$ "

$\text{heapsize}[A] = \text{heapsize}[A] - 1$

 Heapify(A,1)

$i = 12$

$\text{heapsize}[A] = 12$



Heap Sort

```
Heap-Sort(A)
```

```
...
```

```
FOR i = length[A] DOWNTO 2
```

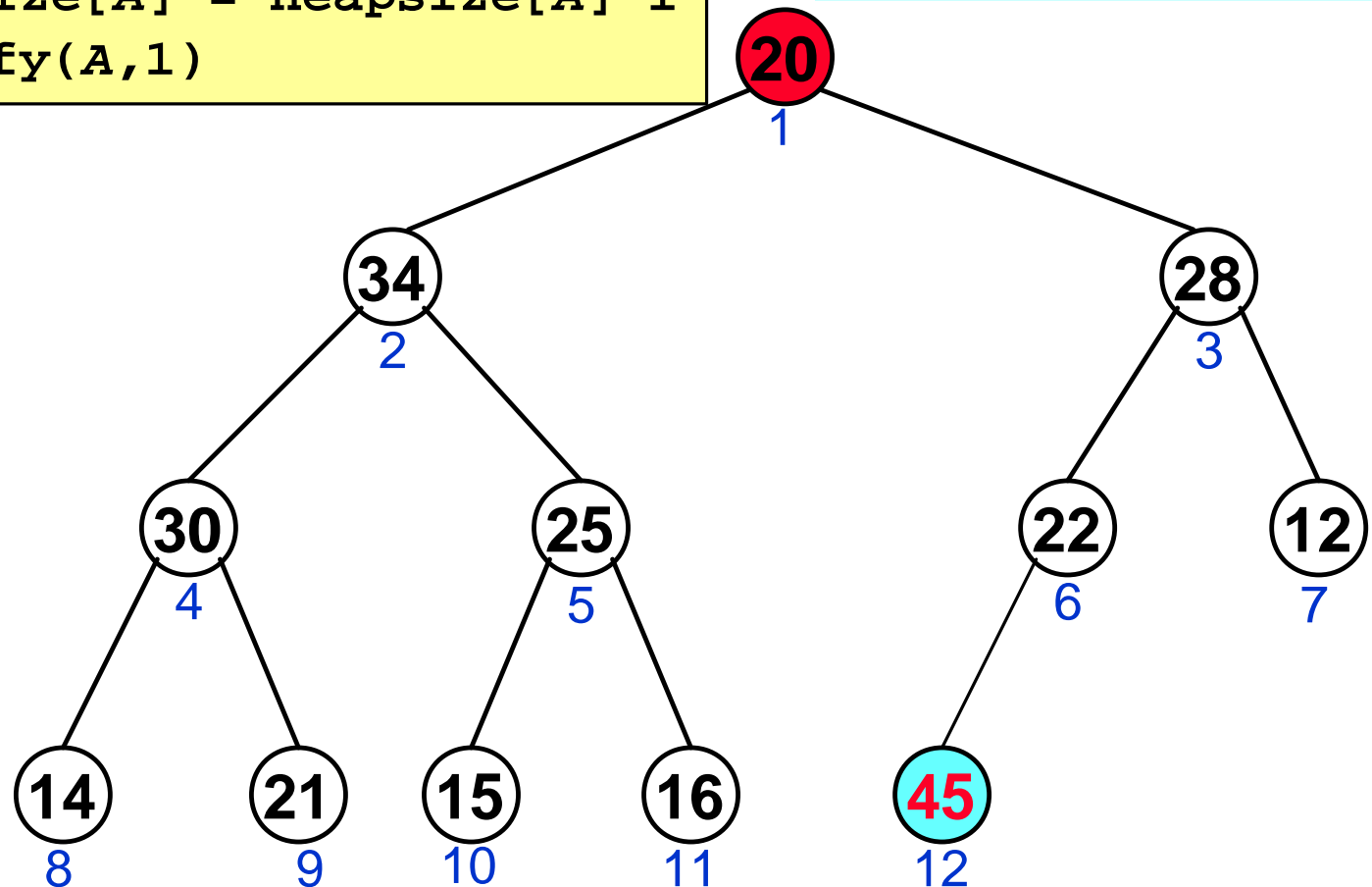
```
DO "scambia A[1] e A[i]"
```

```
    heapsize[A] = heapsize[A]-1
```

```
    Heapify(A,1)
```

$i = 12$

heapsize[A]=12



Heap Sort

```
Heap-Sort(A)
```

```
...
```

```
FOR i = length[A] DOWNTO 2
```

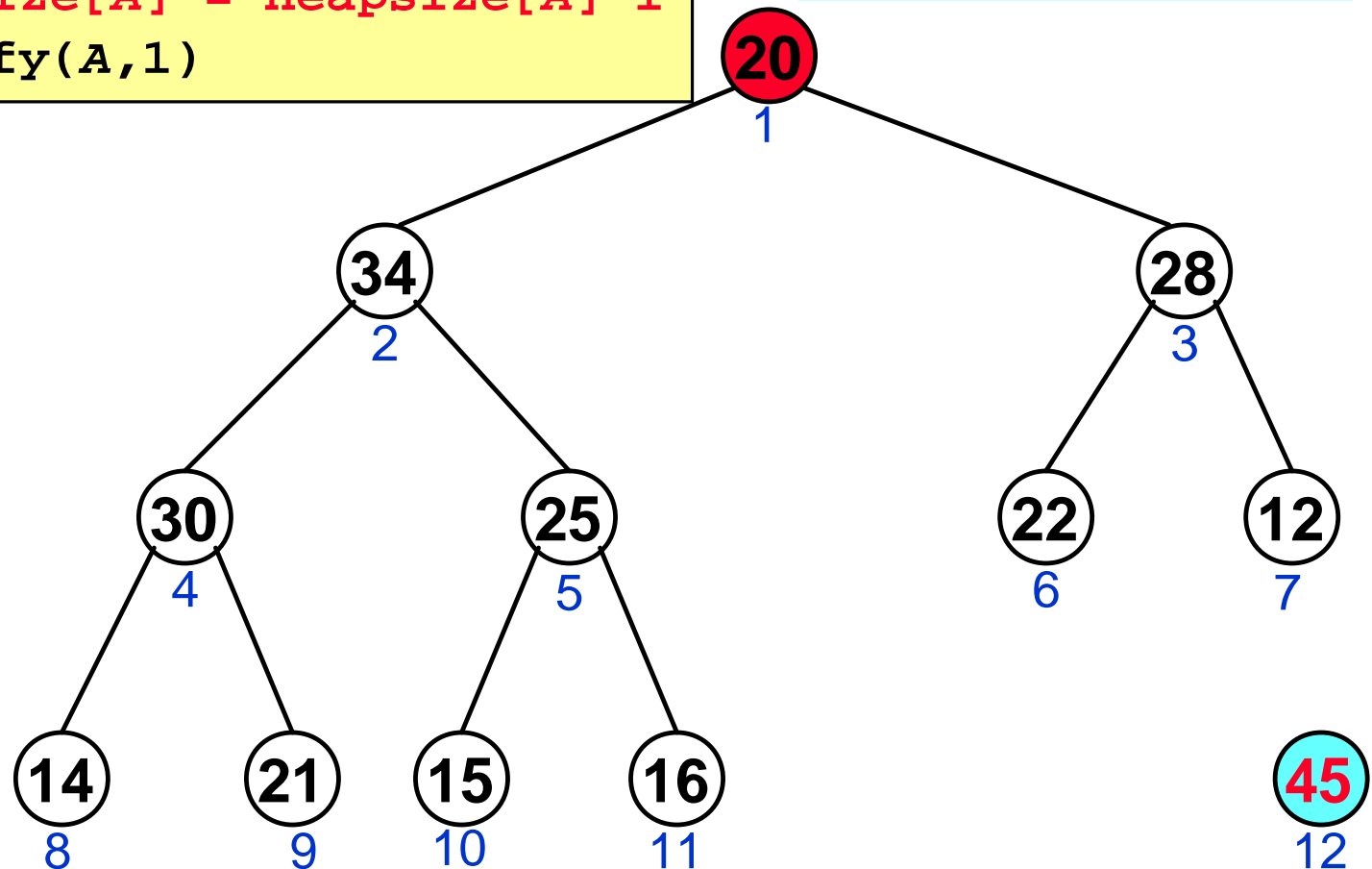
```
DO "scambia A[1] e A[i]"
```

```
    heapsize[A] = heapsize[A]-1
```

```
    Heapify(A,1)
```

$i = 12$

heapsize[A] = 11



Heap Sort

Heap-Sort(A)

...

FOR $i = \text{length}[A]$ DOWNTO 2

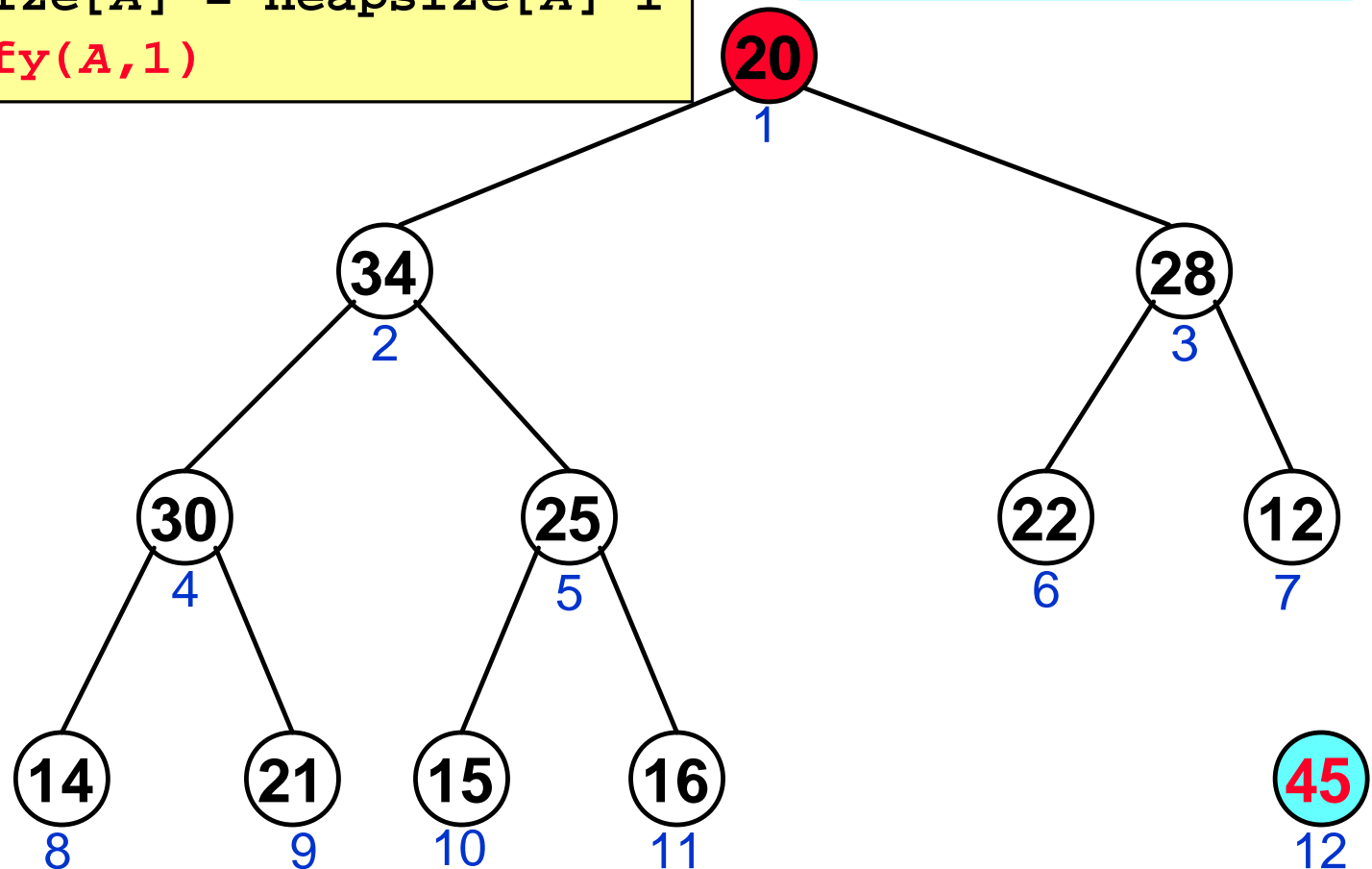
DO "scambia $A[1]$ e $A[i]$ "

$\text{heapsize}[A] = \text{heapsize}[A] - 1$

 Heapify(A, 1)

$i = 12$

$\text{heapsize}[A] = 11$



Heap Sort

Heap-Sort(A)

...

FOR $i = \text{length}[A]$ DOWNTO 2

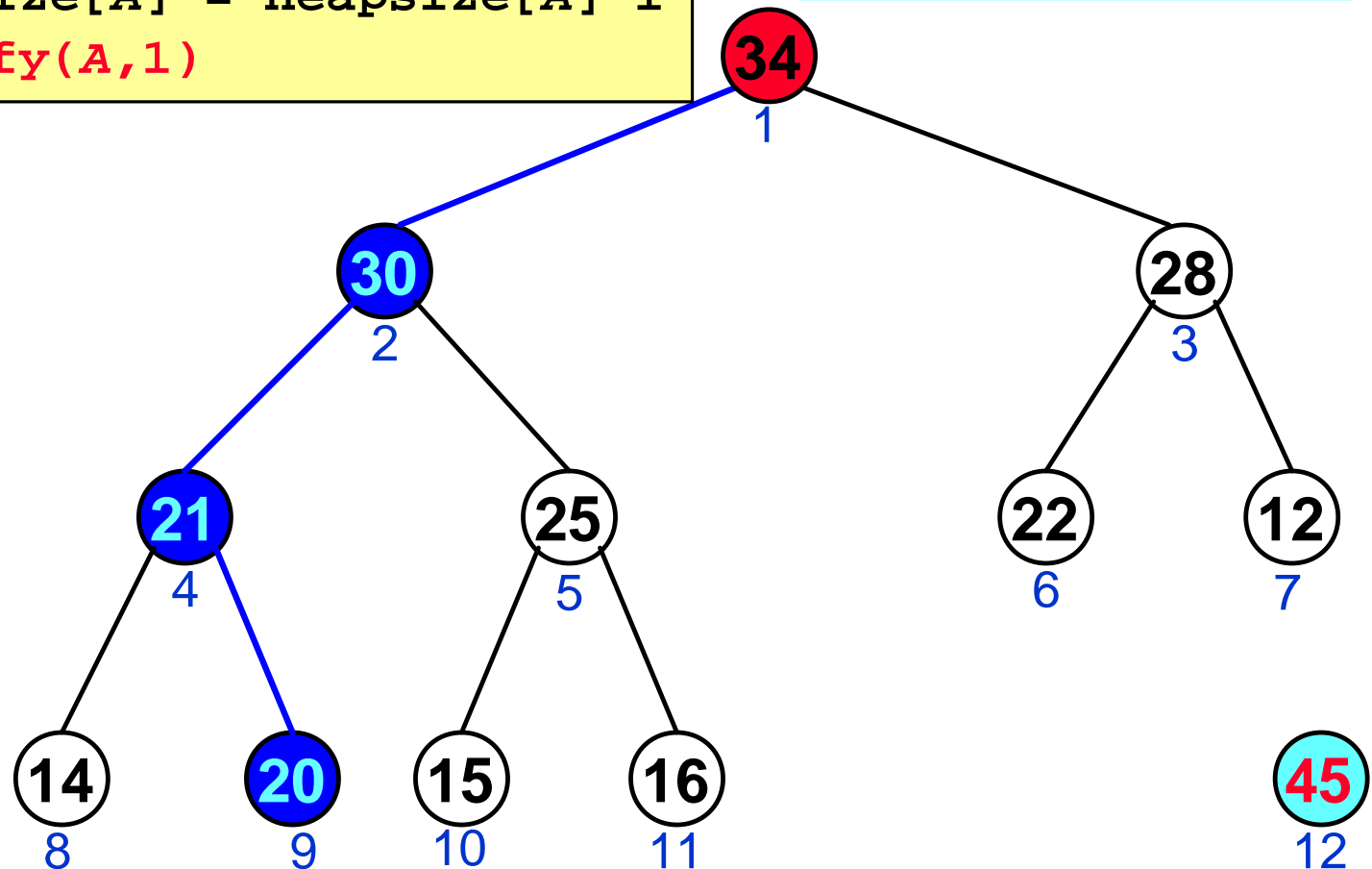
DO "scambia $A[1]$ e $A[i]$ "

$\text{heapsize}[A] = \text{heapsize}[A] - 1$

 Heapify(A, 1)

$i = 12$

$\text{heapsize}[A] = 11$



Heap Sort

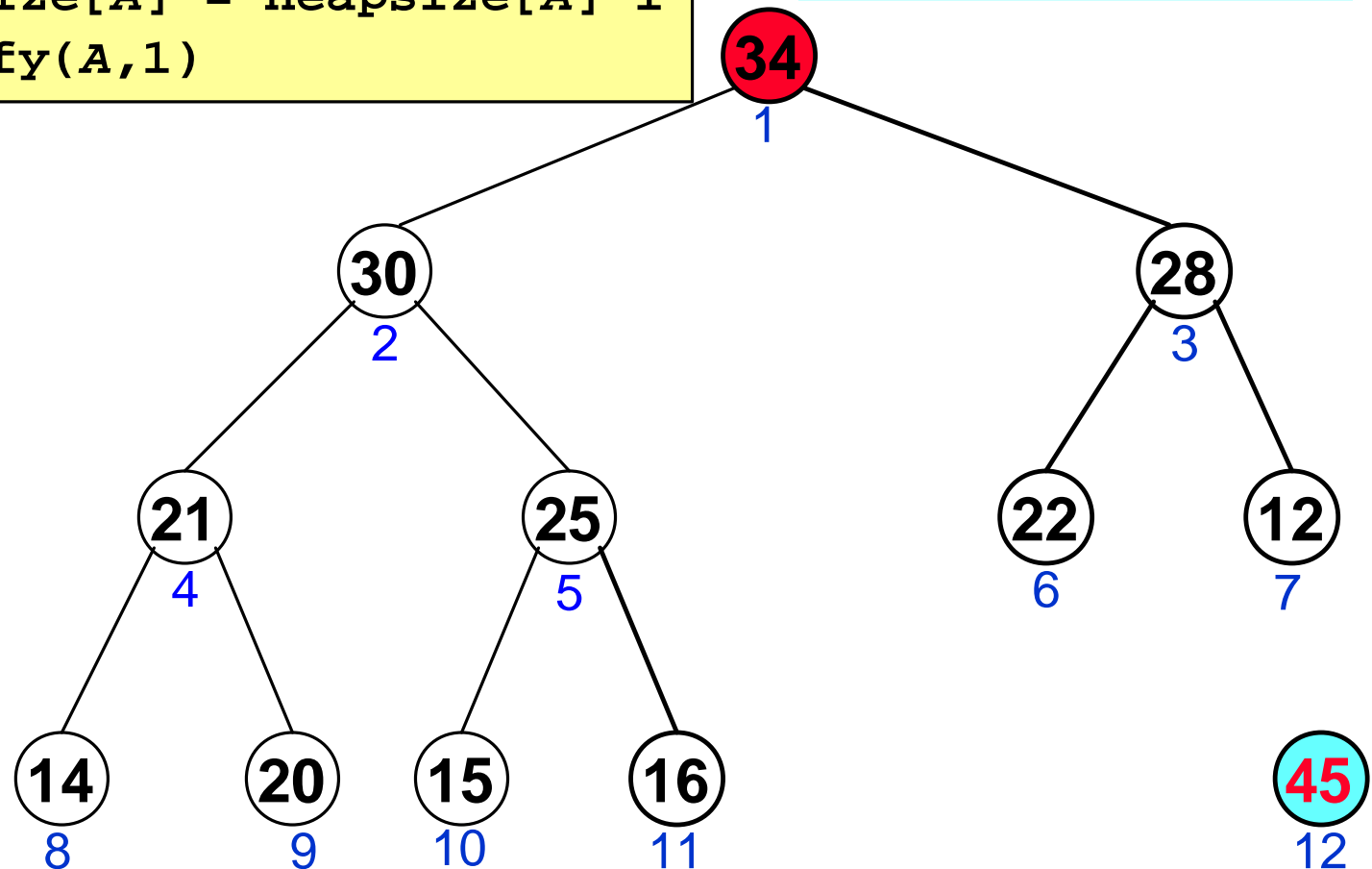
Heap-Sort(A)

...

```
FOR i = length[A] DOWNTO 2  
  DO "scambia A[1] e A[i]"  
    heapsize[A] = heapsize[A]-1  
    Heapify(A,1)
```

$i = 11$

heapsize[A]=11



Heap Sort

Heap-Sort(A)

...

FOR $i = \text{length}[A]$ DOWNTO 2

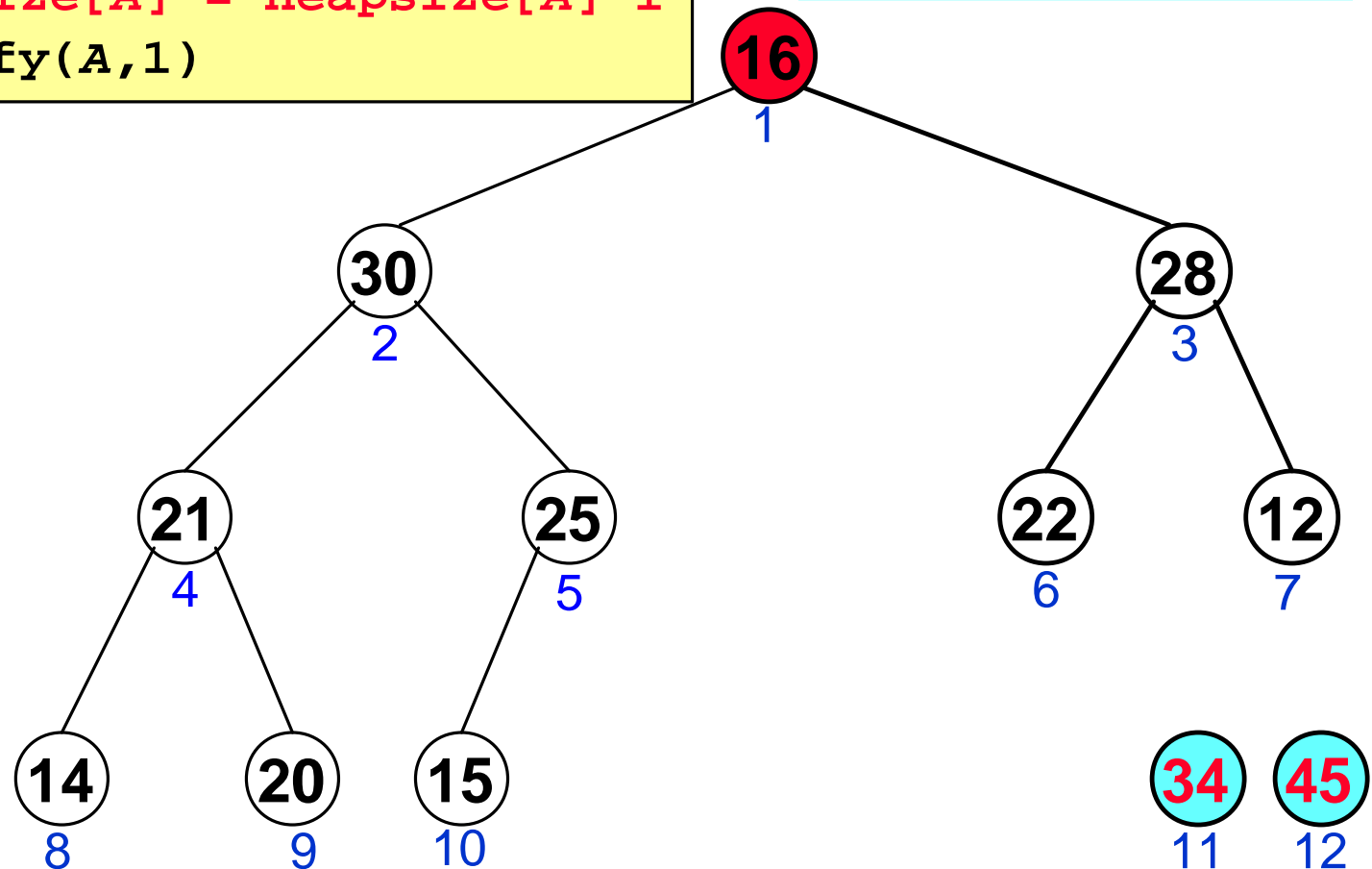
DO "scambia $A[1]$ e $A[i]$ "

$\text{heapsize}[A] = \text{heapsize}[A] - 1$

Heapify(A,1)

$i = 11$

$\text{heapsize}[A] = 10$



Heap Sort

Heap-Sort(A)

...

FOR $i = \text{length}[A]$ DOWNTO 2

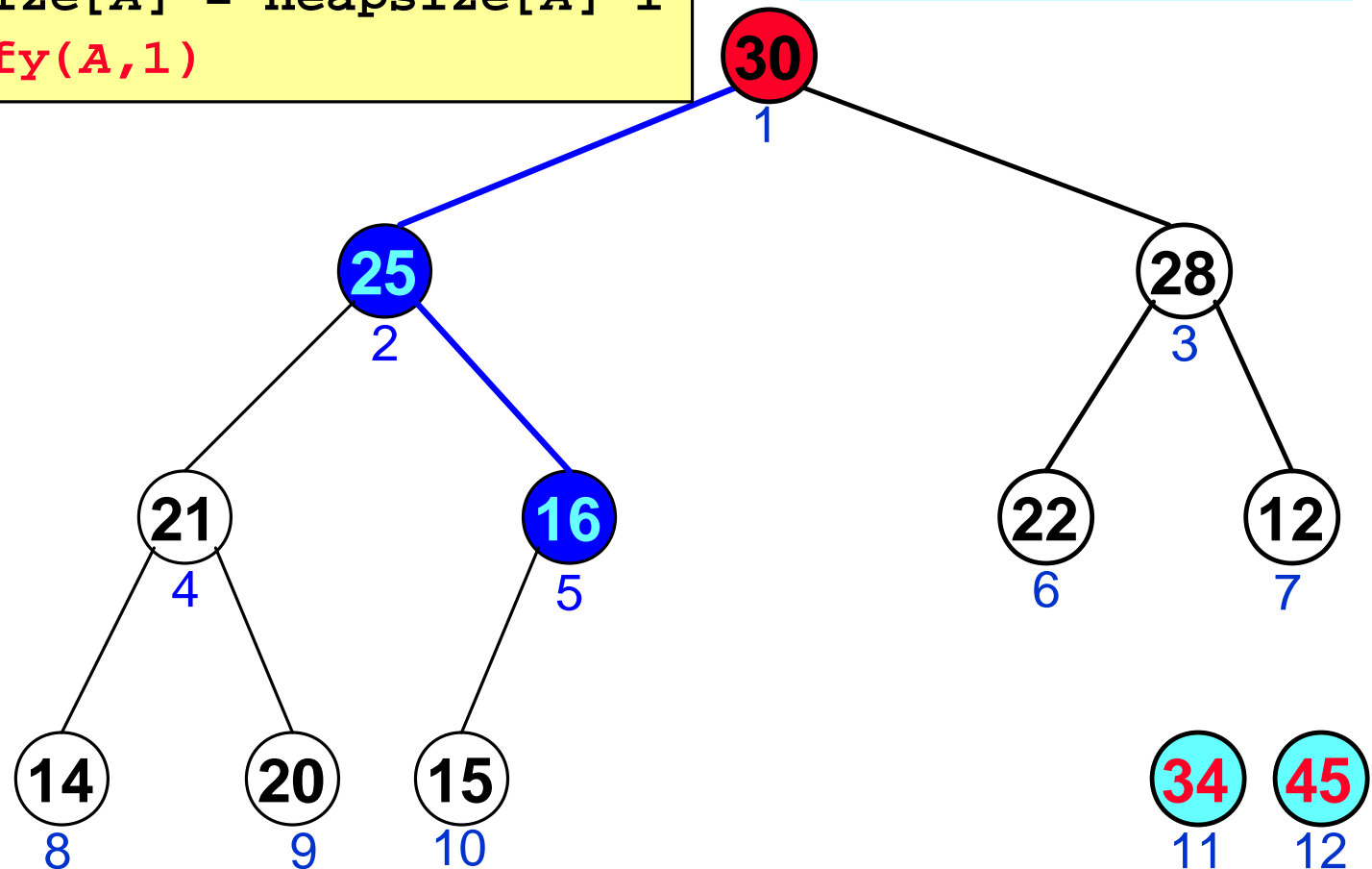
DO "scambia $A[1]$ e $A[i]$ "

$\text{heapsize}[A] = \text{heapsize}[A] - 1$

 Heapify(A, 1)

$i = 11$

$\text{heapsize}[A] = 10$



Heap Sort

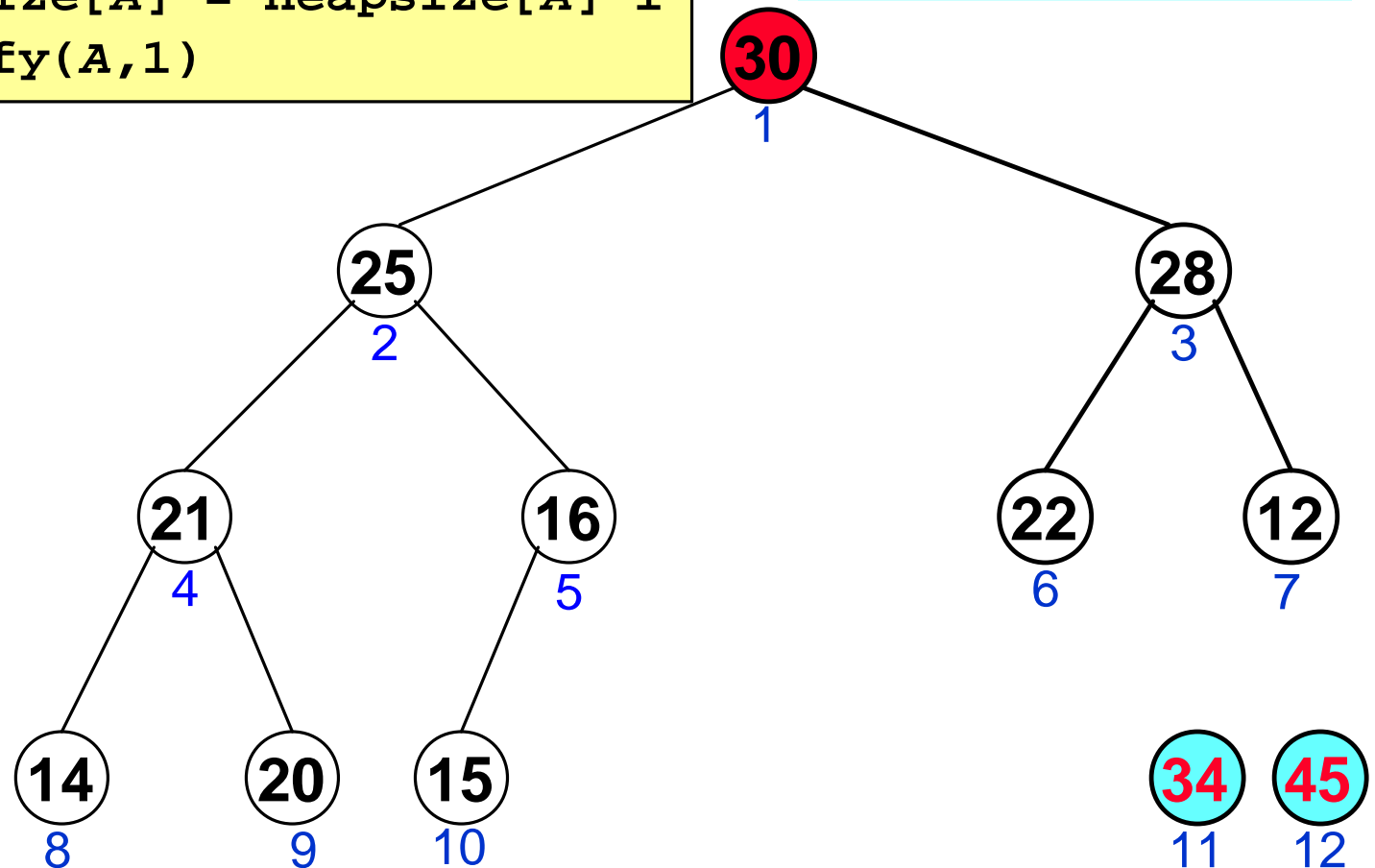
Heap-Sort(A)

...

```
FOR i = length[A] DOWNTO 2  
  DO "scambia A[1] e A[i]"  
    heapsize[A] = heapsize[A]-1  
    Heapify(A,1)
```

$i = 10$

heapsize[A]=10



Heap Sort

Heap-Sort(A)

...

FOR $i = \text{length}[A]$ DOWNTO 2

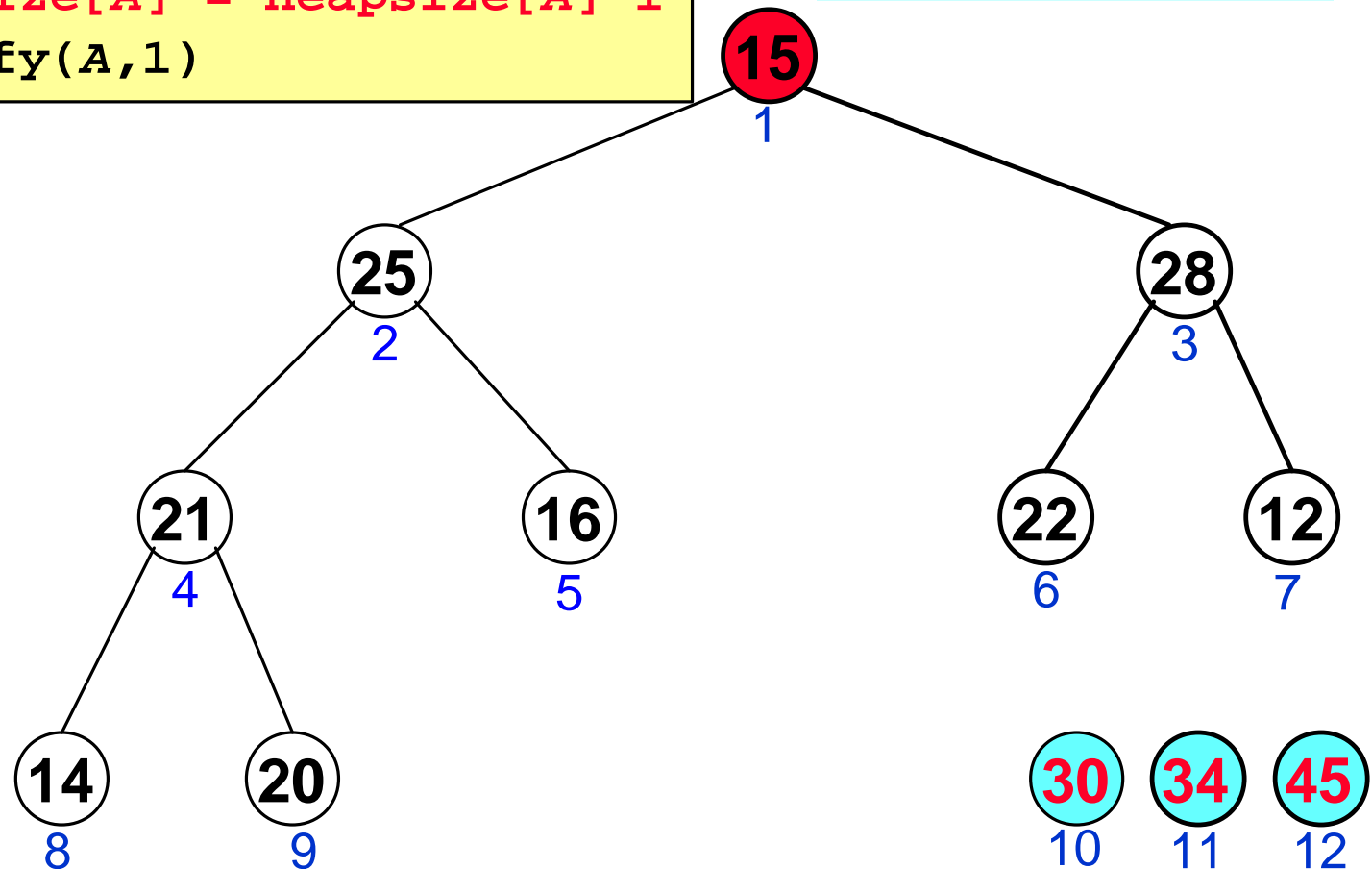
DO "scambia $A[1]$ e $A[i]$ "

$\text{heapsize}[A] = \text{heapsize}[A] - 1$

Heapify(A,1)

$i = 10$

$\text{heapsize}[A] = 9$



Heap Sort

Heap-Sort(A)

...

FOR $i = \text{length}[A]$ DOWNTO 2

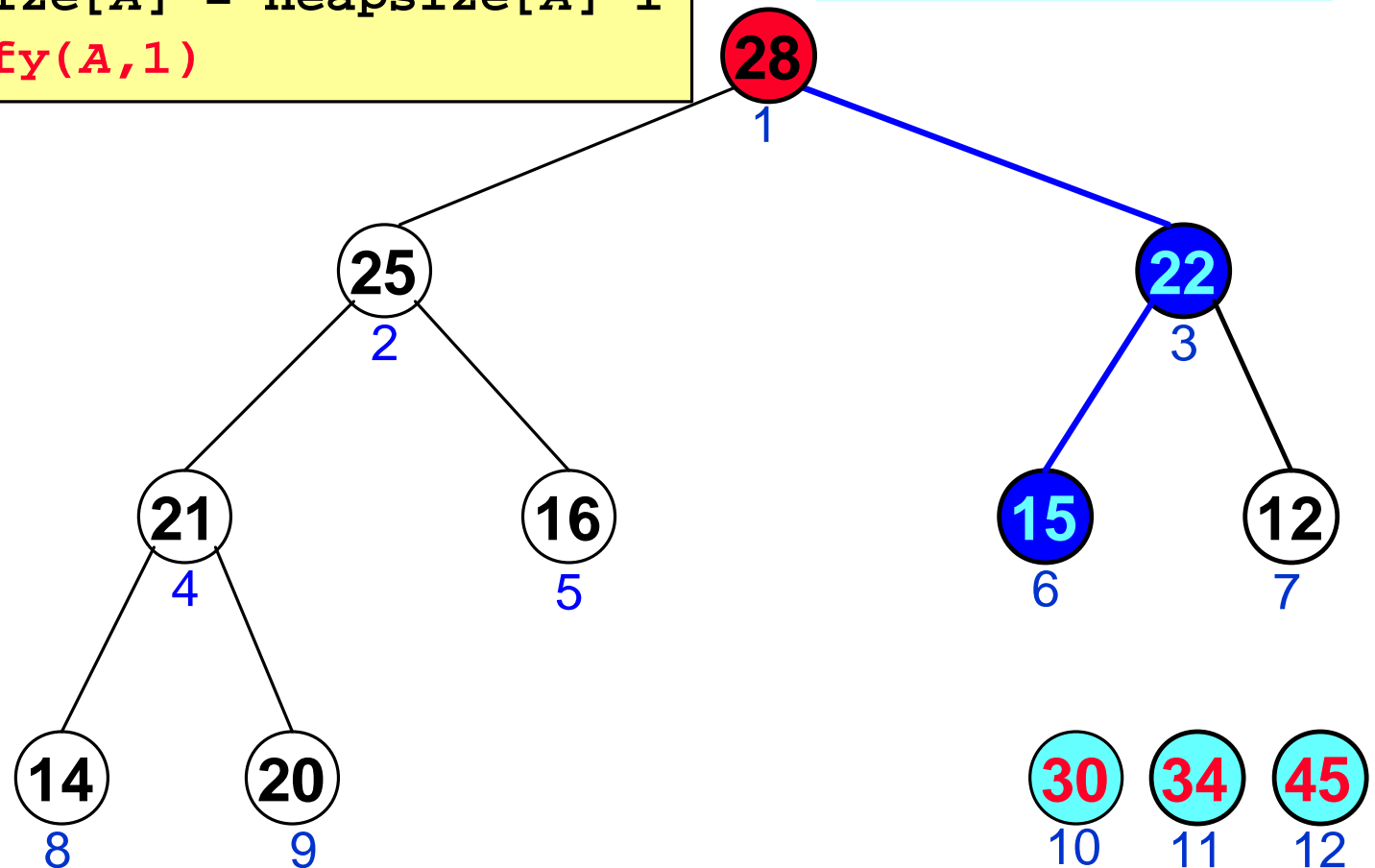
DO "scambia $A[1]$ e $A[i]$ "

$\text{heapsize}[A] = \text{heapsize}[A] - 1$

 Heapify(A, 1)

$i = 10$

$\text{heapsize}[A] = 9$



Heap Sort

Heap-Sort(A)

...

FOR $i = \text{length}[A]$ DOWNTO 2

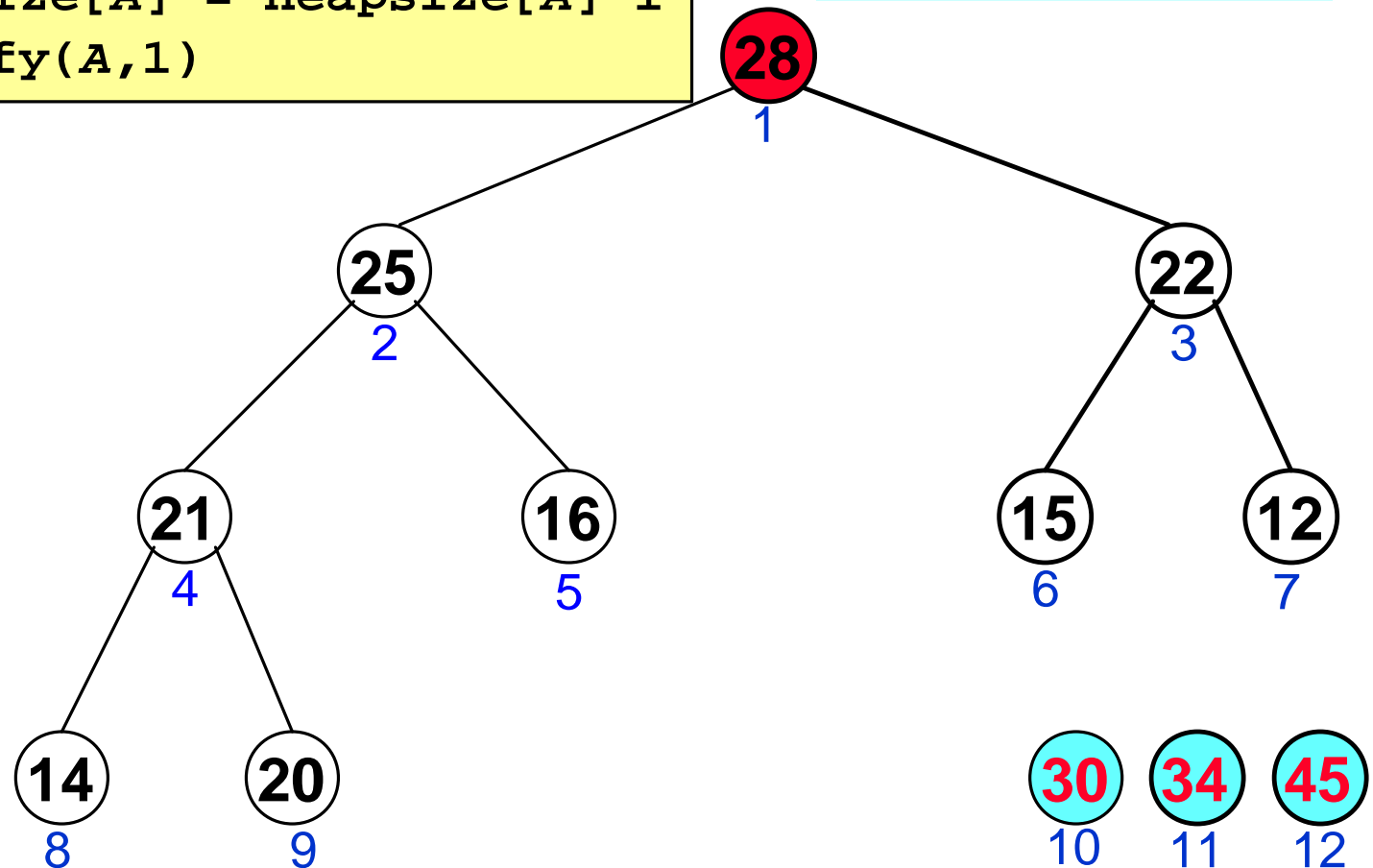
DO "scambia $A[1]$ e $A[i]$ "

$\text{heapsize}[A] = \text{heapsize}[A] - 1$

 Heapify(A,1)

$i = 9$

$\text{heapsize}[A] = 9$



Heap Sort

Heap-Sort(A)

...

FOR $i = \text{length}[A]$ DOWNTO 2

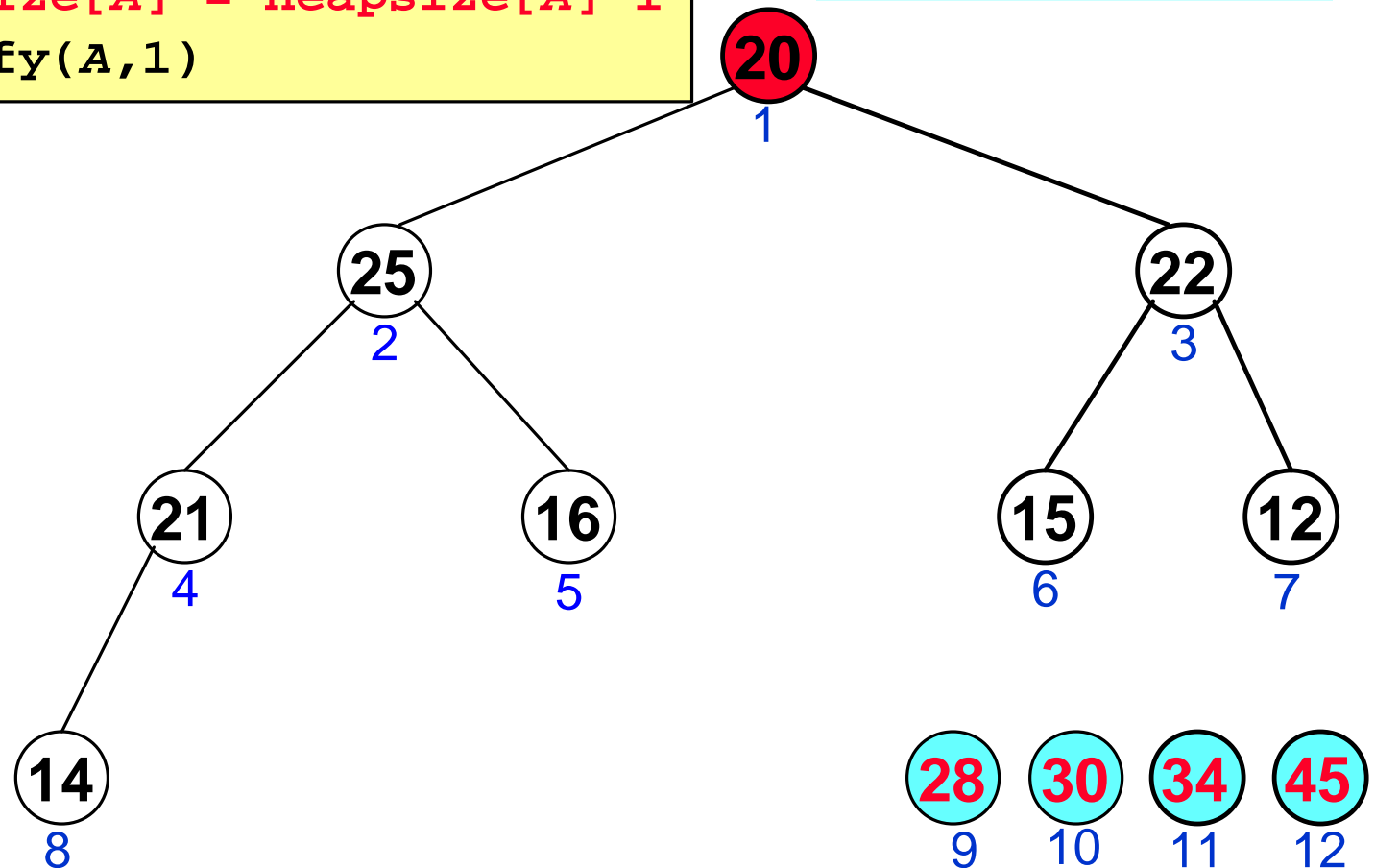
DO "scambia $A[1]$ e $A[i]$ "

$\text{heapsize}[A] = \text{heapsize}[A] - 1$

Heapify(A,1)

$i = 9$

$\text{heapsize}[A] = 8$



Heap Sort

Heap-Sort(A)

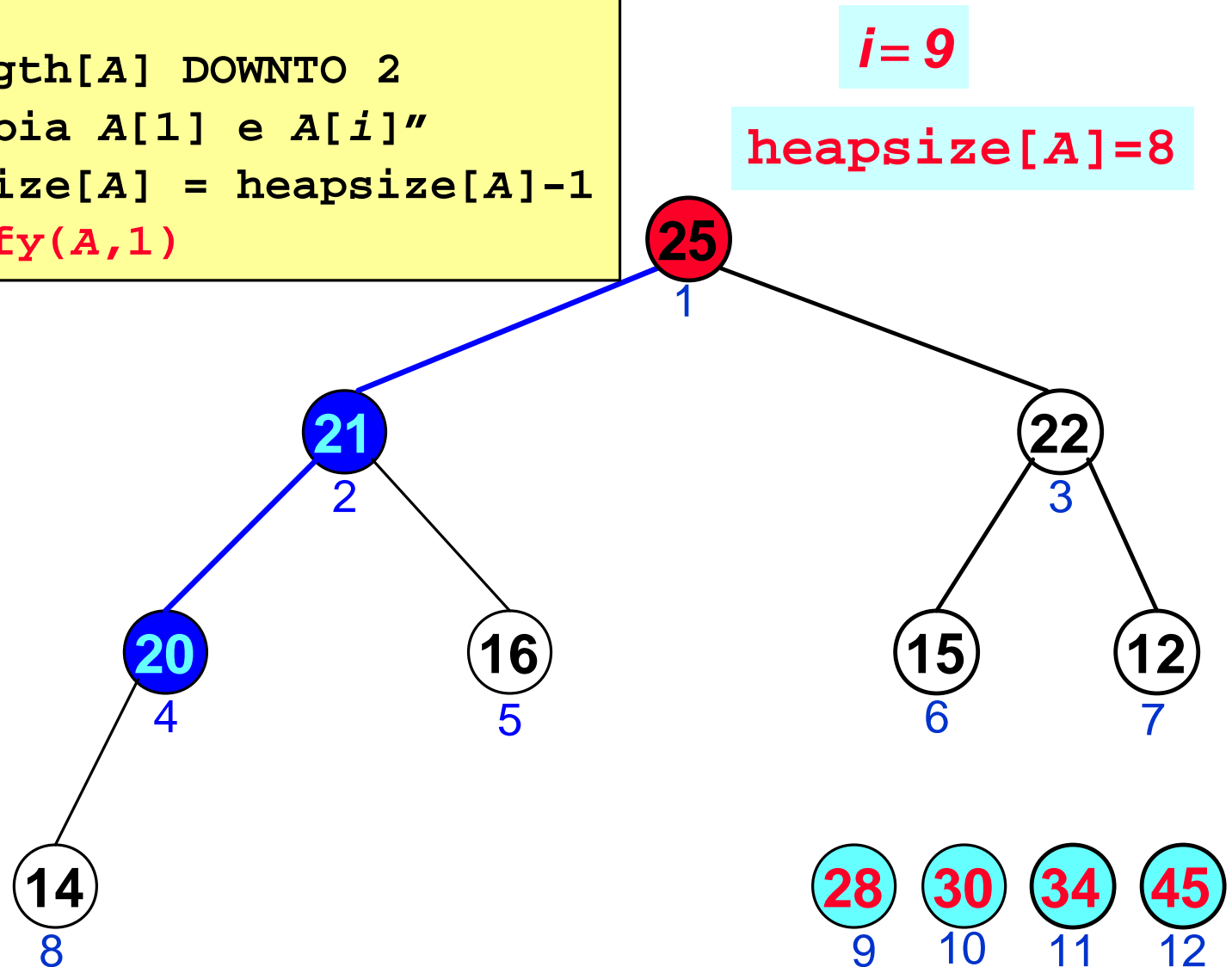
...

FOR $i = \text{length}[A]$ DOWNTO 2

DO "scambia $A[1]$ e $A[i]$ "

$\text{heapsize}[A] = \text{heapsize}[A] - 1$

 Heapify(A, 1)



Heap Sort

Heap-Sort(A)

...

FOR $i = \text{length}[A]$ DOWNTO 2

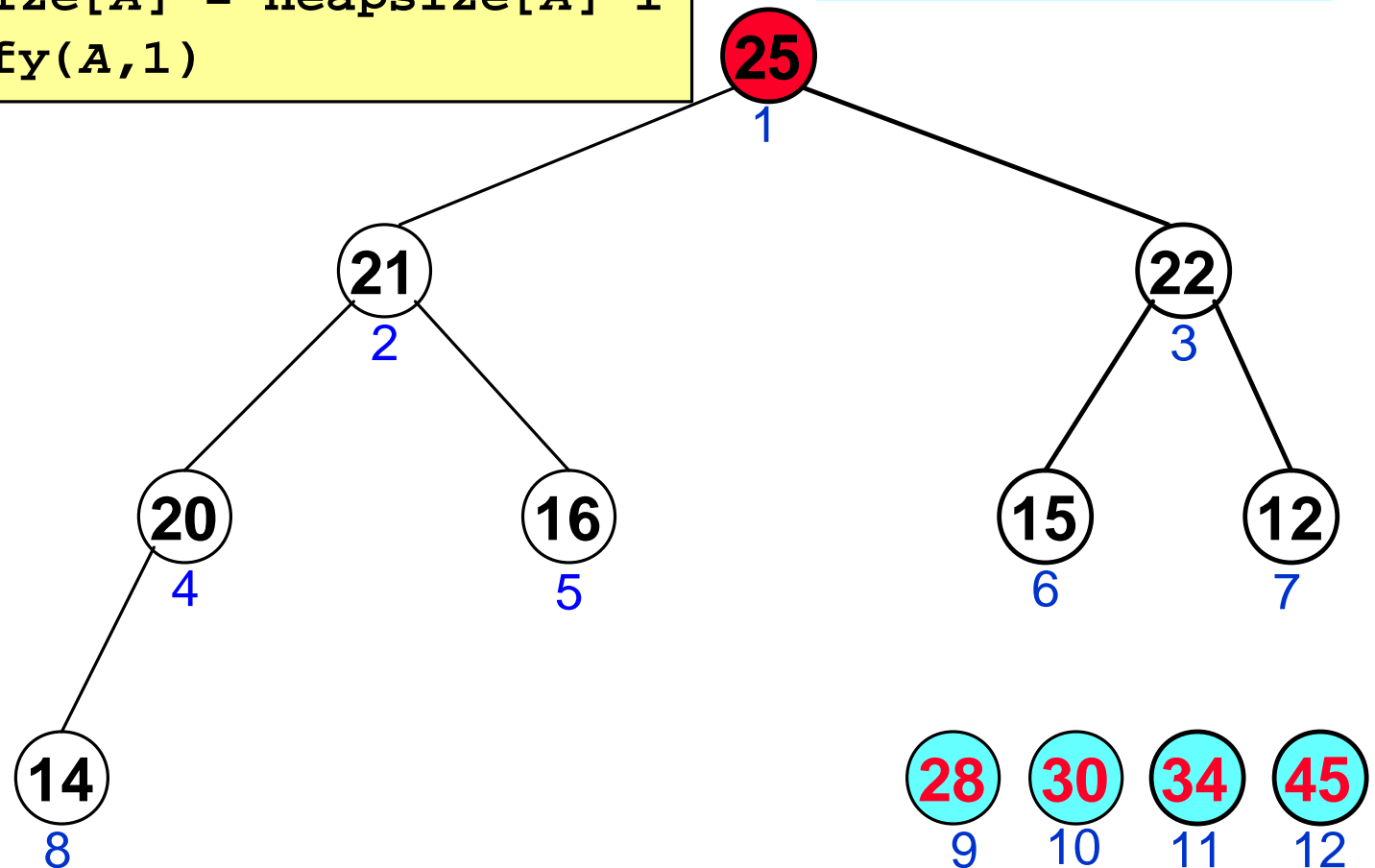
DO "scambia $A[1]$ e $A[i]$ "

$\text{heapsize}[A] = \text{heapsize}[A] - 1$

 Heapify(A,1)

$i = 8$

$\text{heapsize}[A] = 8$



Heap Sort

Heap-Sort(A)

...

FOR $i = \text{length}[A]$ DOWNTO 2

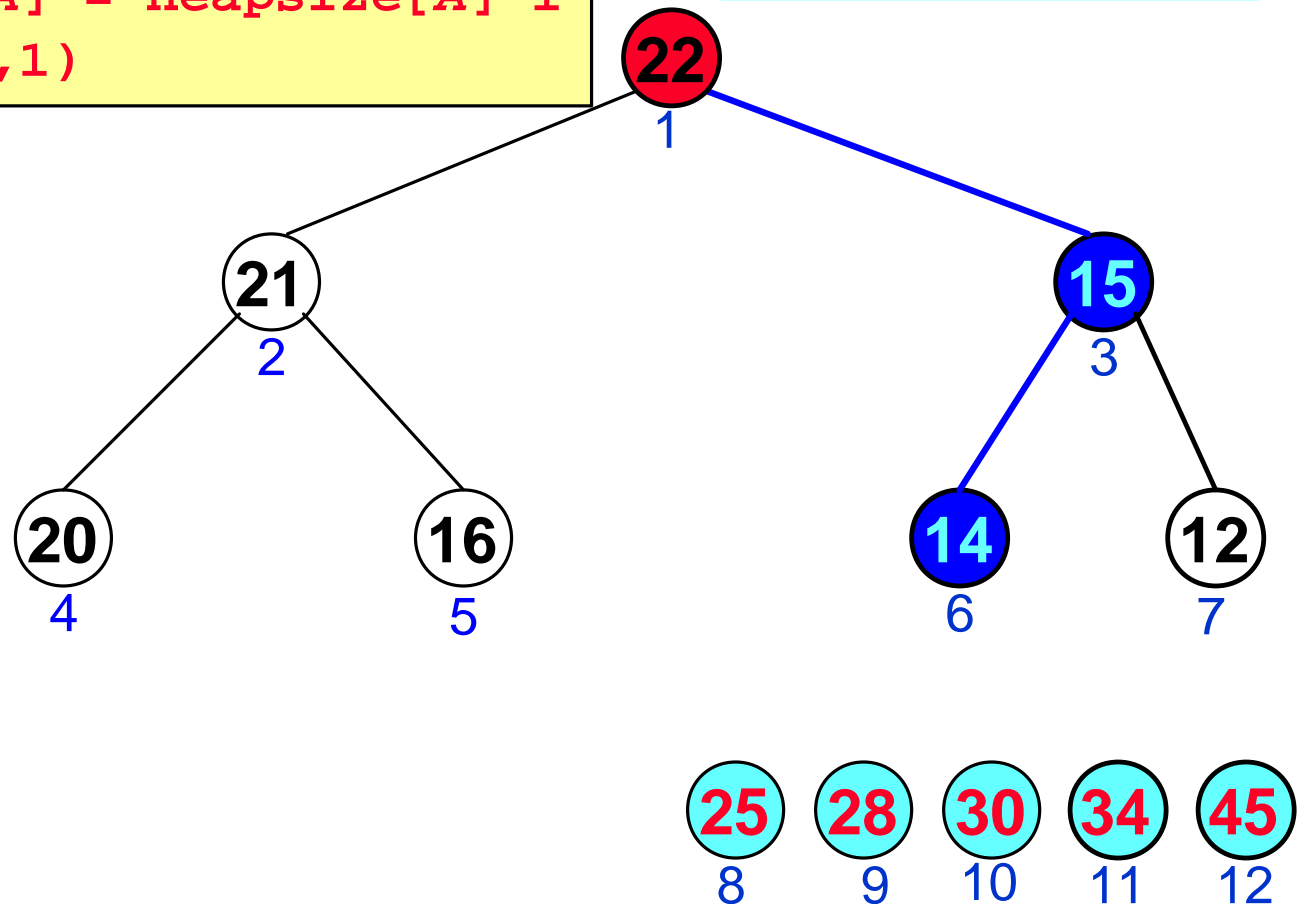
DO "scambia $A[1]$ e $A[i]$ "

$\text{heapsize}[A] = \text{heapsize}[A] - 1$

 Heapify(A,1)

$i = 8$

$\text{heapsize}[A] = 7$



Heap Sort

Heap-Sort(A)

...

FOR $i = \text{length}[A]$ DOWNTO 2

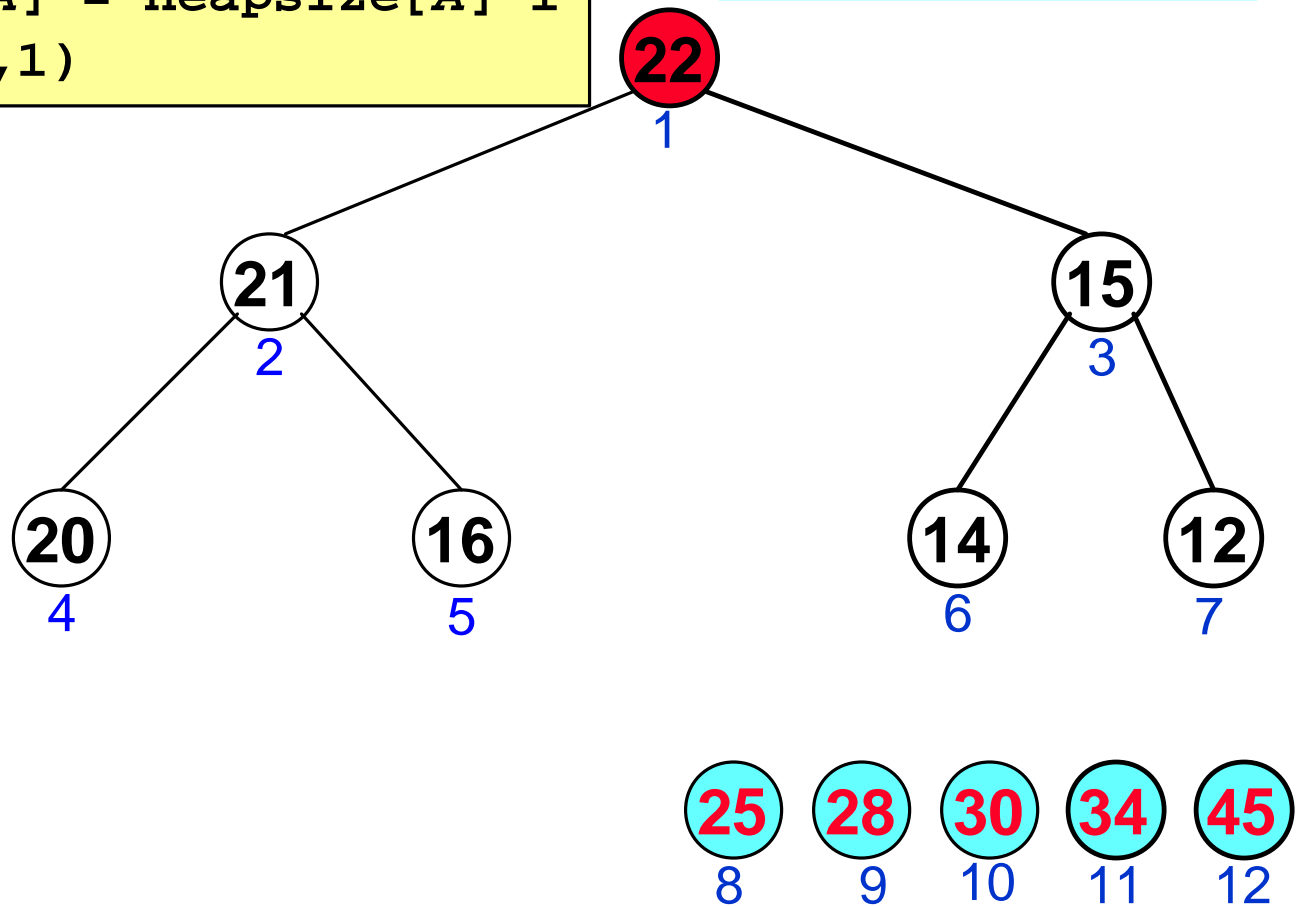
DO "scambia $A[1]$ e $A[i]$ "

$\text{heapsize}[A] = \text{heapsize}[A] - 1$

 Heapify(A,1)

$i = 7$

$\text{heapsize}[A] = 7$



Heap Sort

Heap-Sort(A)

...

FOR $i = \text{length}[A]$ DOWNTO 2

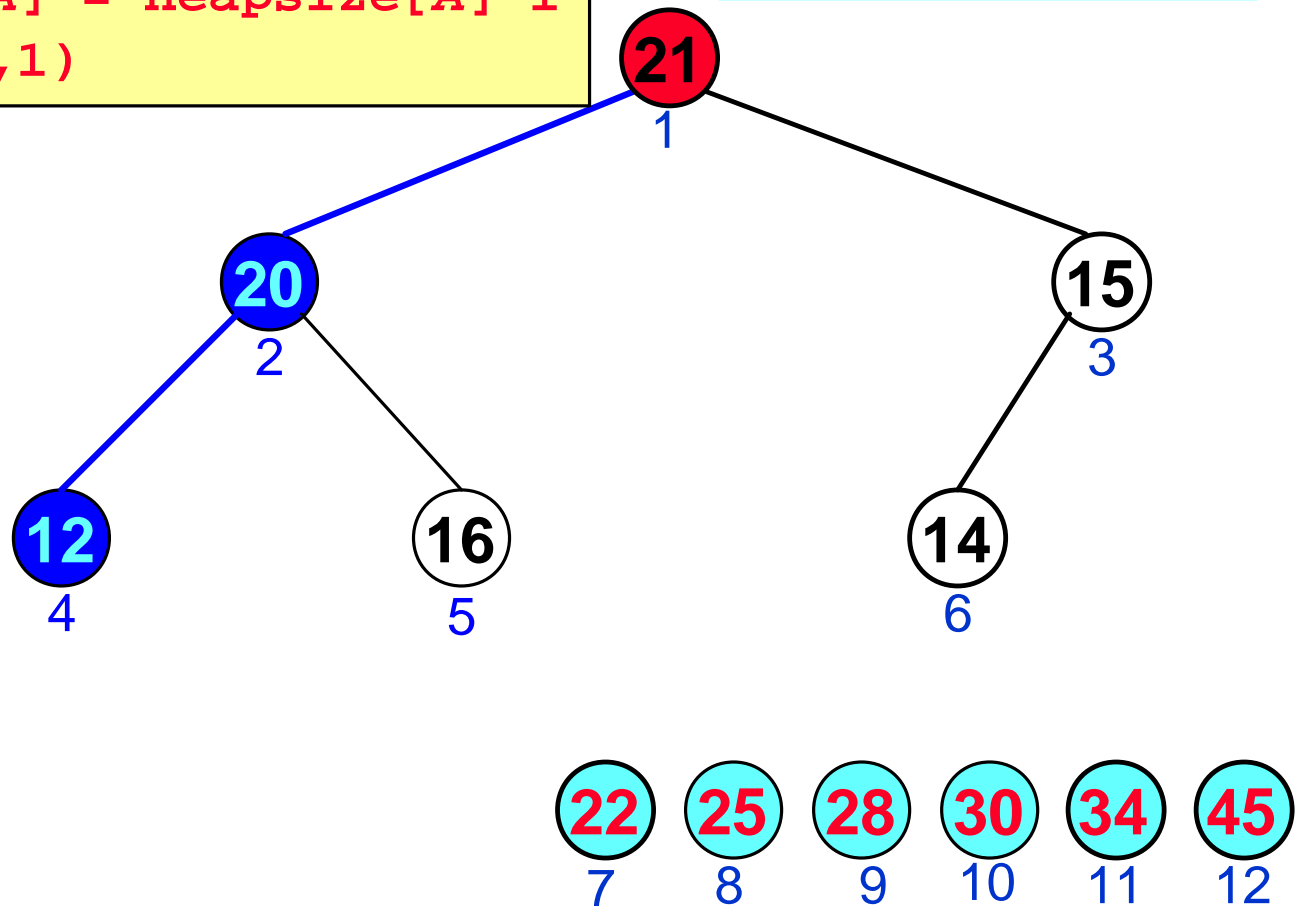
DO "scambia $A[1]$ e $A[i]$ "

$\text{heapsize}[A] = \text{heapsize}[A] - 1$

 Heapify(A,1)

$i = 7$

$\text{heapsize}[A] = 6$



Heap Sort

Heap-Sort(A)

...

FOR $i = \text{length}[A]$ DOWNTO 2

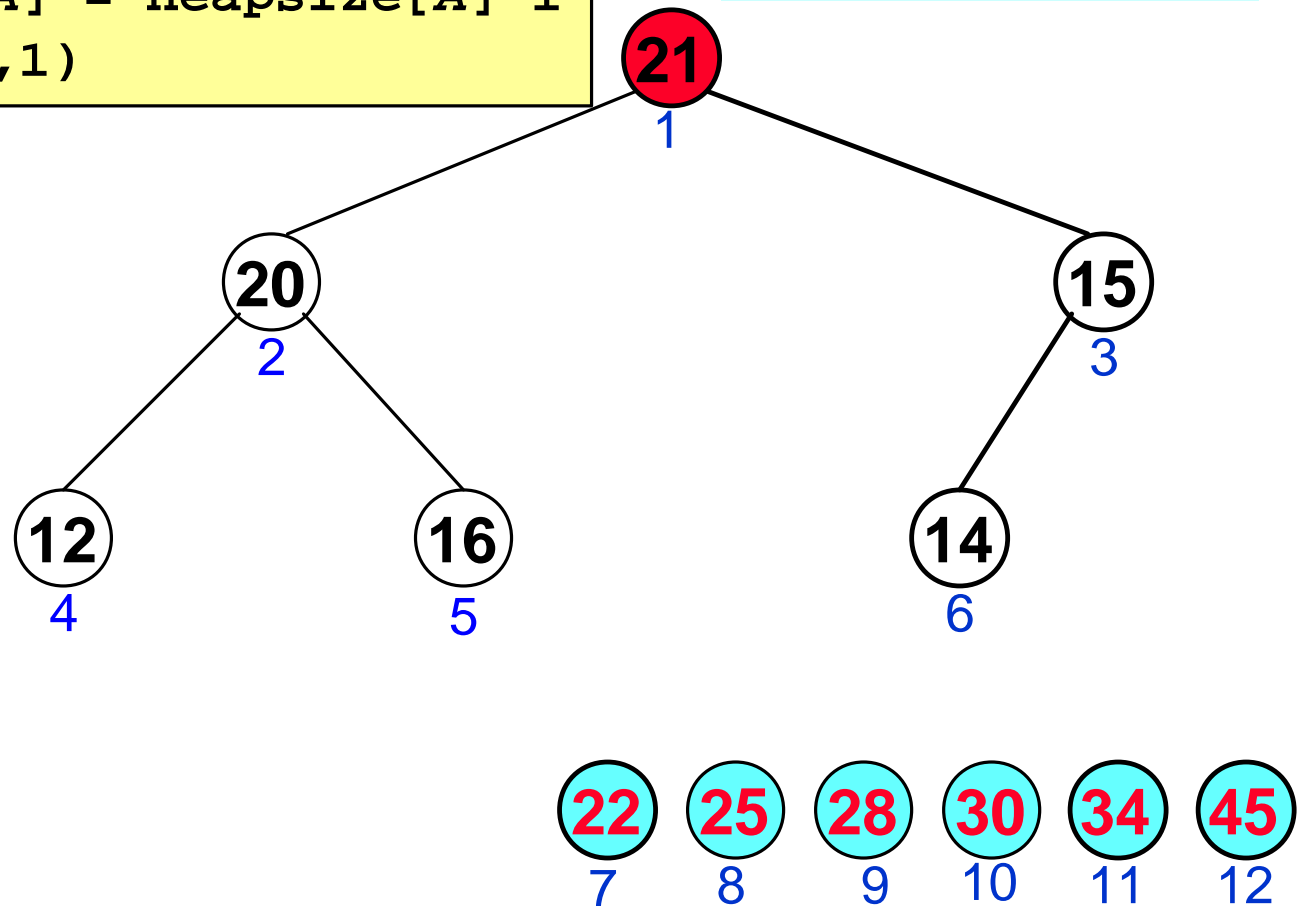
DO "scambia $A[1]$ e $A[i]$ "

$\text{heapsize}[A] = \text{heapsize}[A] - 1$

 Heapify(A,1)

$i = 6$

$\text{heapsize}[A] = 6$



Heap Sort

Heap-Sort(A)

...

FOR $i = \text{length}[A]$ DOWNTO 2

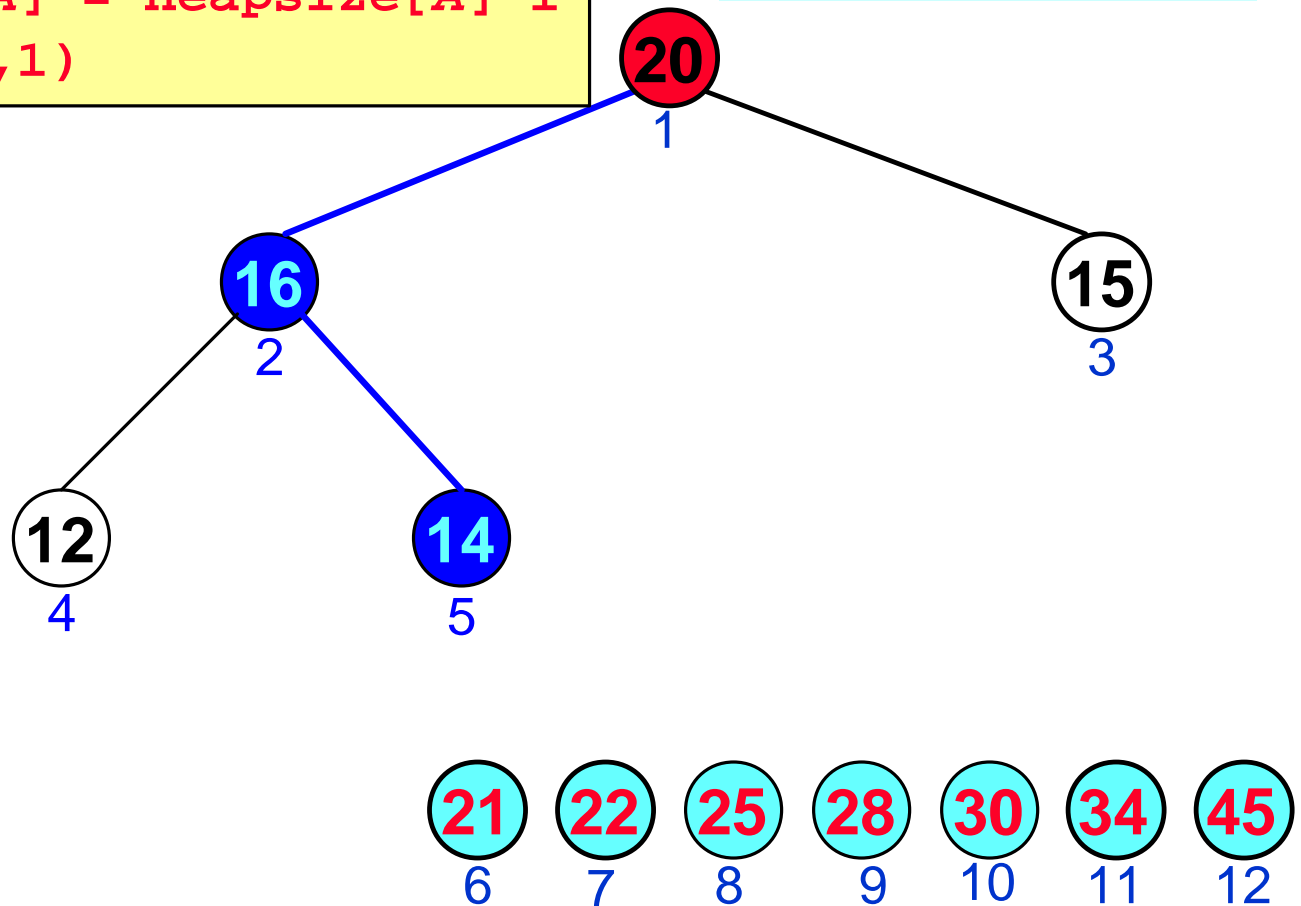
DO "scambia $A[1]$ e $A[i]$ "

$\text{heapsize}[A] = \text{heapsize}[A] - 1$

 Heapify(A,1)

$i = 6$

$\text{heapsize}[A] = 5$



Heap Sort

Heap-Sort(A)

...

FOR $i = \text{length}[A]$ DOWNTO 2

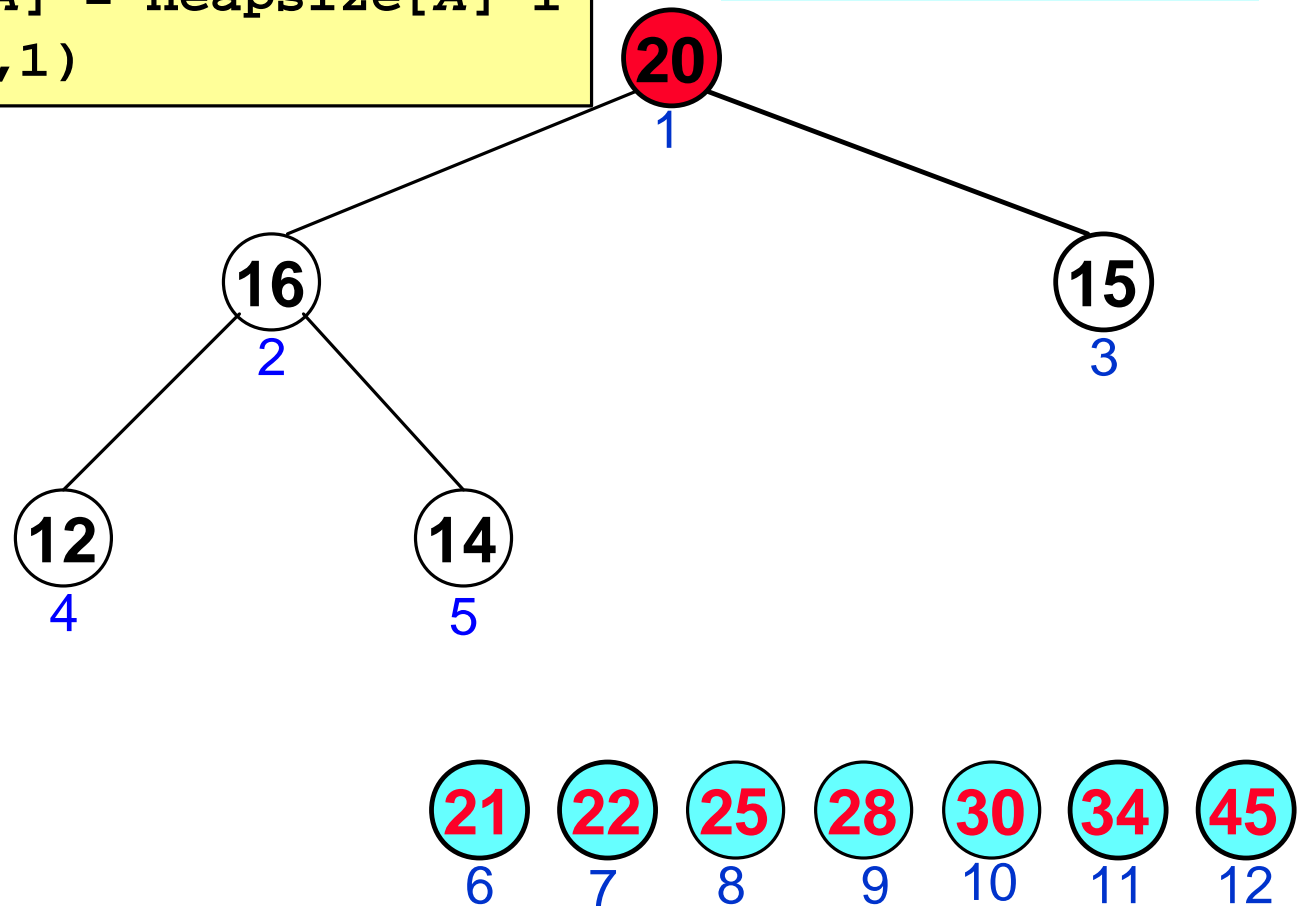
DO "scambia $A[1]$ e $A[i]$ "

$\text{heapsize}[A] = \text{heapsize}[A] - 1$

 Heapify(A,1)

$i = 5$

$\text{heapsize}[A] = 5$



Heap Sort

Heap-Sort(A)

...

FOR $i = \text{length}[A]$ DOWNTO 2

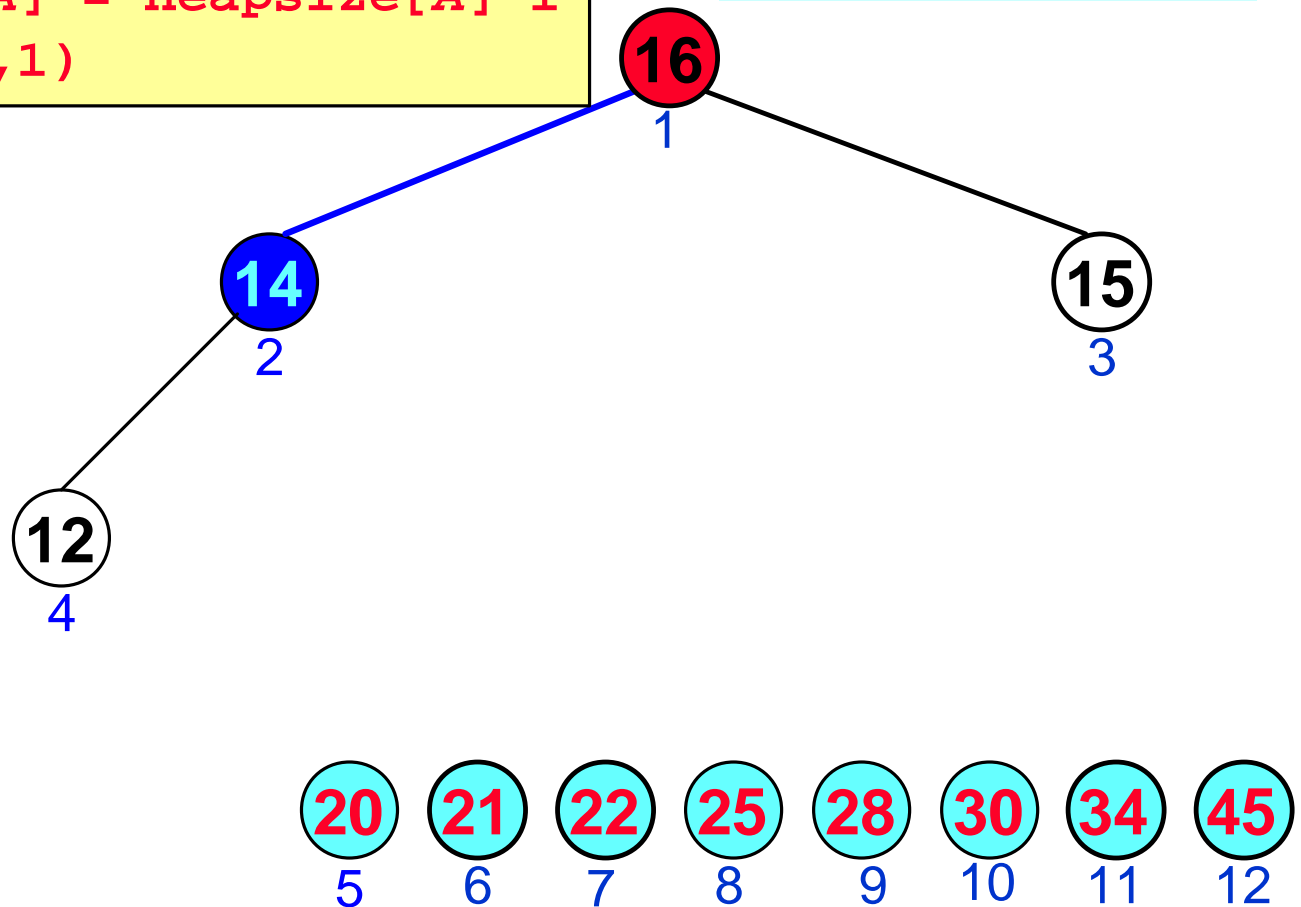
DO "scambia $A[1]$ e $A[i]$ "

$\text{heapsize}[A] = \text{heapsize}[A] - 1$

 Heapify(A,1)

$i = 5$

$\text{heapsize}[A] = 4$



Heap Sort

Heap-Sort(A)

...

FOR $i = \text{length}[A]$ DOWNTO 2

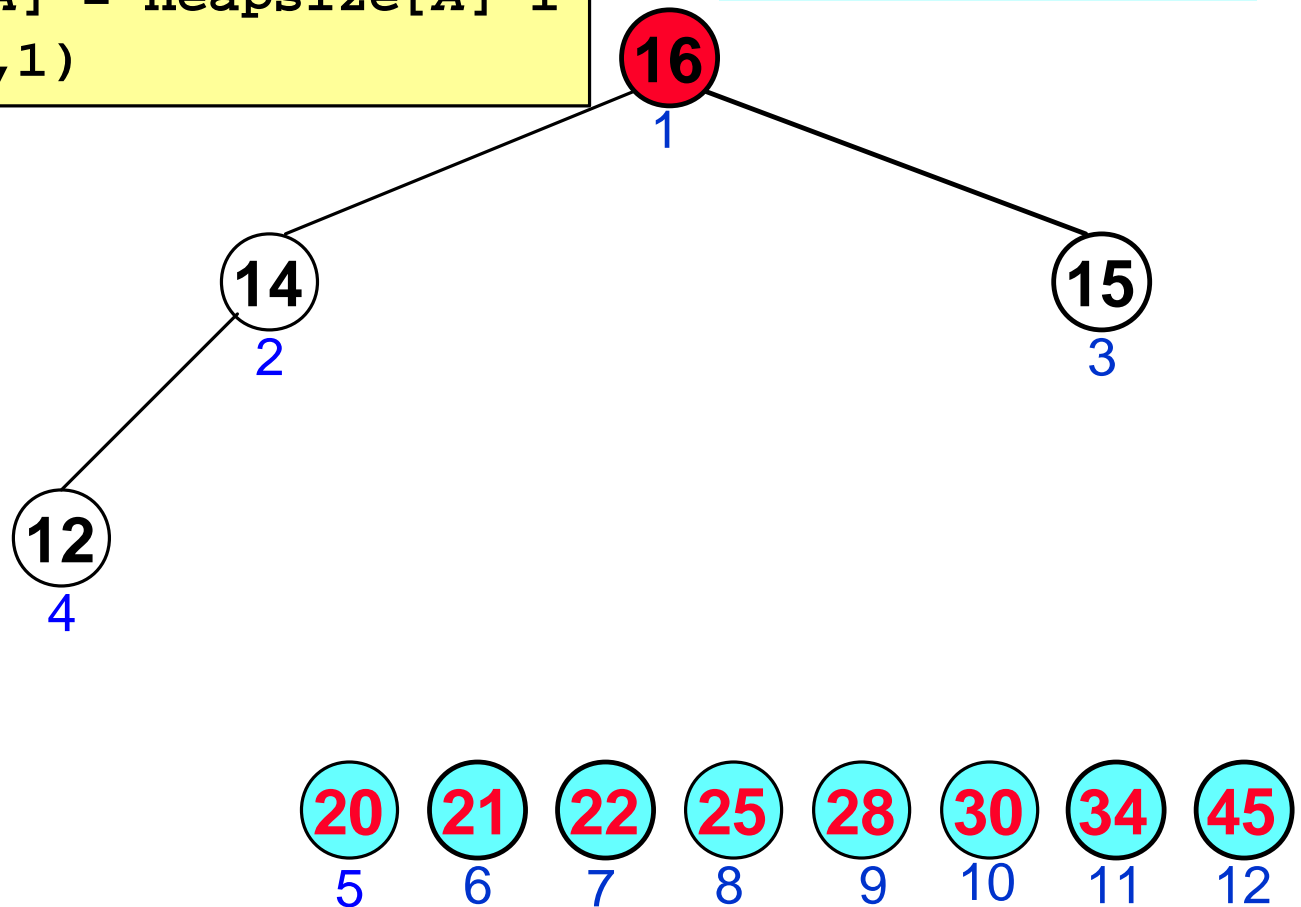
DO "scambia $A[1]$ e $A[i]$ "

$\text{heapsize}[A] = \text{heapsize}[A] - 1$

 Heapify(A,1)

$i = 4$

$\text{heapsize}[A] = 4$



Heap Sort

Heap-Sort(A)

...

FOR $i = \text{length}[A]$ DOWNTO 2

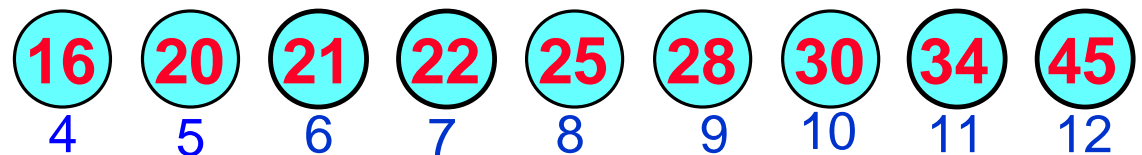
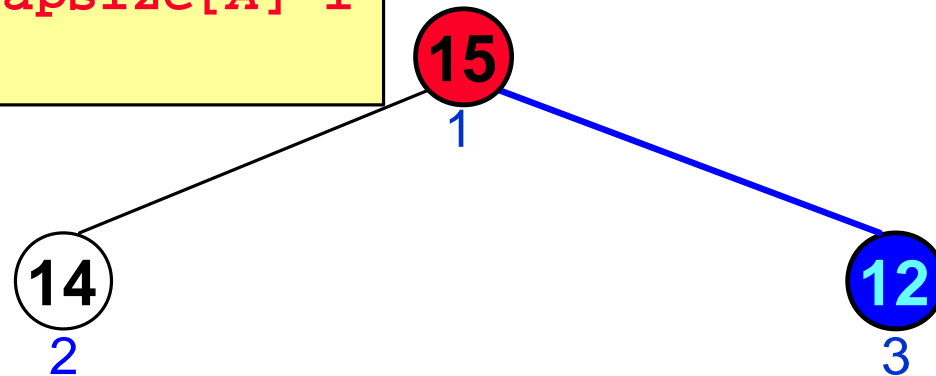
DO "scambia $A[1]$ e $A[i]$ "

$\text{heapsize}[A] = \text{heapsize}[A] - 1$

 Heapify(A, 1)

$i = 4$

$\text{heapsize}[A] = 3$



Heap Sort

Heap-Sort(A)

...

FOR $i = \text{length}[A]$ DOWNTO 2

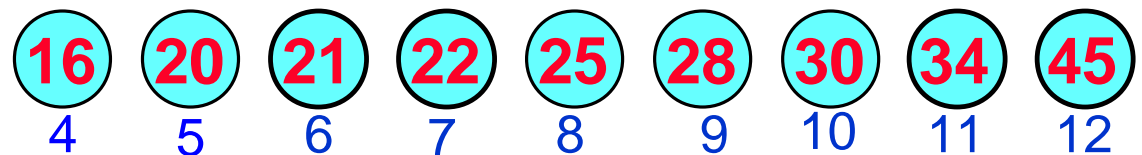
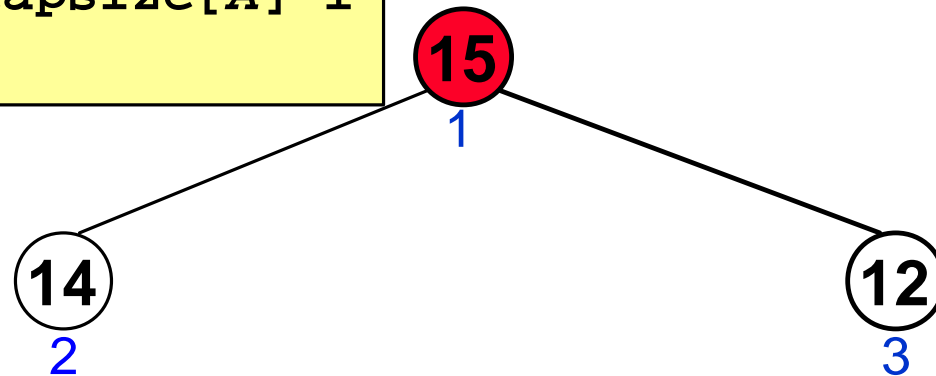
DO "scambia $A[1]$ e $A[i]$ "

$\text{heapsize}[A] = \text{heapsize}[A] - 1$

 Heapify(A,1)

$i = 3$

$\text{heapsize}[A] = 3$



Heap Sort

Heap-Sort(A)

...

FOR $i = \text{length}[A]$ DOWNTO 2

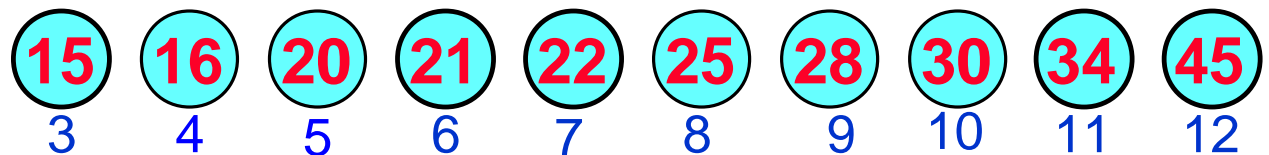
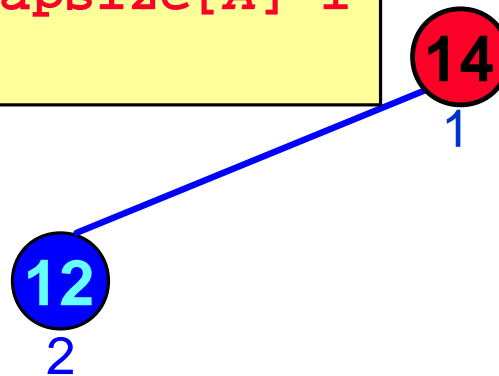
DO "scambia $A[1]$ e $A[i]$ "

$\text{heapsize}[A] = \text{heapsize}[A] - 1$

 Heapify(A,1)

$i = 3$

$\text{heapsize}[A] = 2$



Heap Sort

Heap-Sort(A)

...

FOR $i = \text{length}[A]$ DOWNTO 2

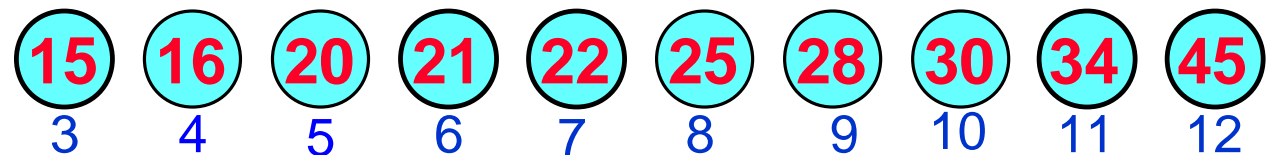
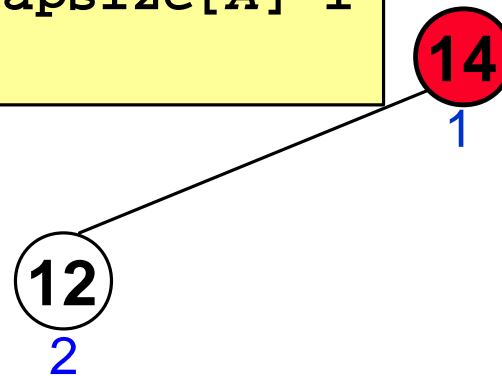
DO "scambia $A[1]$ e $A[i]$ "

$\text{heapsize}[A] = \text{heapsize}[A] - 1$

 Heapify(A,1)

$i = 2$

$\text{heapsize}[A] = 2$



Heap Sort

```
Heap-Sort(A)
```

```
...
```

```
FOR i = length[A] DOWNTO 2
```

```
DO "scambia A[1] e A[i]"
```

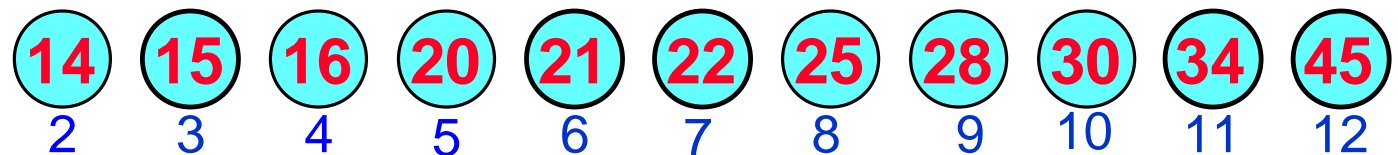
```
    heapsize[A] = heapsize[A]-1
```

```
    Heapify(A,1)
```

$i=2$

heapsize[A]=1

12
1



Heap Sort

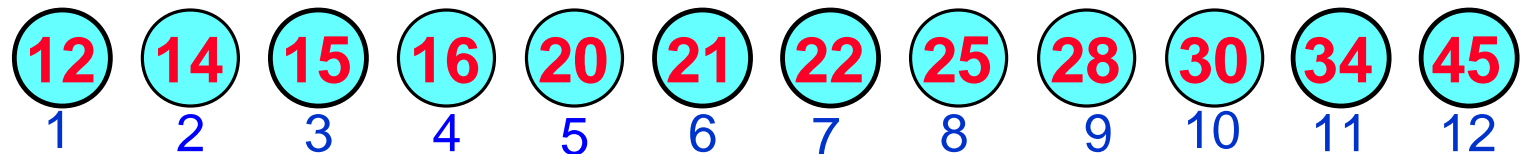
Heap-Sort(A)

...

```
FOR  $i = \text{length}[A]$  DOWNTO 2  
  DO "scambia  $A[1]$  e  $A[i]$ "  
    heapsize[A] = heapsize[A]-1  
    Heapify(A,1)
```

$i = 1$

heapsize[A]=1



Heap Sort

```
Heap-Sort(A)
```

```
...
```

```
FOR i = length[A] DOWNTO 2
```

```
    DO "scambia A[1] e A[i]"
```

```
        heapsize[A] = heapsize[A]-1
```

```
        Heapify(A,1)
```

$i = 1$

heapsize[A]=1

L'array A ora è ordinato!

1	2	3	4	5	6	7	8	9	10	11	12
12	14	15	16	20	21	22	25	28	30	34	45

Heap Sort

Heap-Sort(*A*)

Costruisci-Heap(*A*) } = $O(n)$

FOR *i* = length[*A*] DOWNTO 2

DO "scambia *A*[1] e *A*[*i*]"
 heapsize[*A*] = heapsize[*A*] - 1 } = $O(1)$

 Heapify(*A*, 1) } = $O(\log n)$

Complessità di Heap Sort

Nel caso peggiore *Heap-Sort* chiama

- *una volta Costruisci-Heap*;
- *n-1* volte *Heapify* sullo *Heap* corrente

$$T(n) = \max(O(n), (n-1) \cdot \max(O(1), T(\text{Heapify})))$$

Complessità di Heap Sort

Nel caso peggiore *Heap-Sort* chiama

- *una volta Costruisci-Heap*;
- *n-1* volte *Heapify* sull'intero *Heap* .

$$\begin{aligned} T(n) &= \max(O(n), (n-1) \cdot \max(O(1), T(\text{Heapify}))) \\ &= \max(O(n), \max(O(n), O(n \log n))) \end{aligned}$$

$$T(n) = O(n \log n)$$

HeapSort: conclusioni

HeapSort

- Algoritmo di ordinamento *in loco* per confronto che impiega tempo $O(n \log n)$.
- Algoritmo non immediato nè ovvio.
- Sfrutta le proprietà della struttura dati astratta *Heap*.

HeapSort: conclusioni

HeapSort dimostra che:

- scegliere una buona rappresentazione per i dati spesso facilita la progettazione di buoni algoritmi;
- importante pensare a quale può essere una buona rappresentazione dei dati prima di implementare una soluzione.