Tecniche di Specifica e di Verifica

Linear Time Temporal Logic I

Temporal Logics: The context

- ***Kripke Structures*** model systems.
- ***Temporal logics*** model dynamic behavioral properties of systems.
  - **Linear Time**
  - Branching Time
- ***Model checking*** can be used to determine if a system has the desired behavioral property.

Linear time temporal logics.

- ***LTL (Linear Time Temporal Logic)***
  - **Syntax**
  - **Semantics**
  - The Model Checking Problem.
  - Its solution.

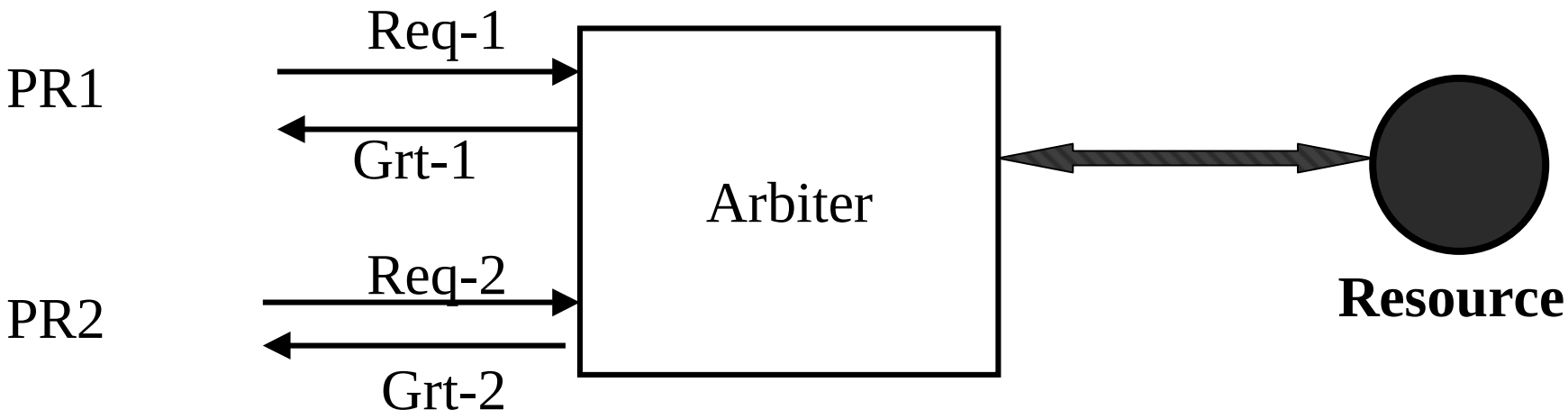
The Application

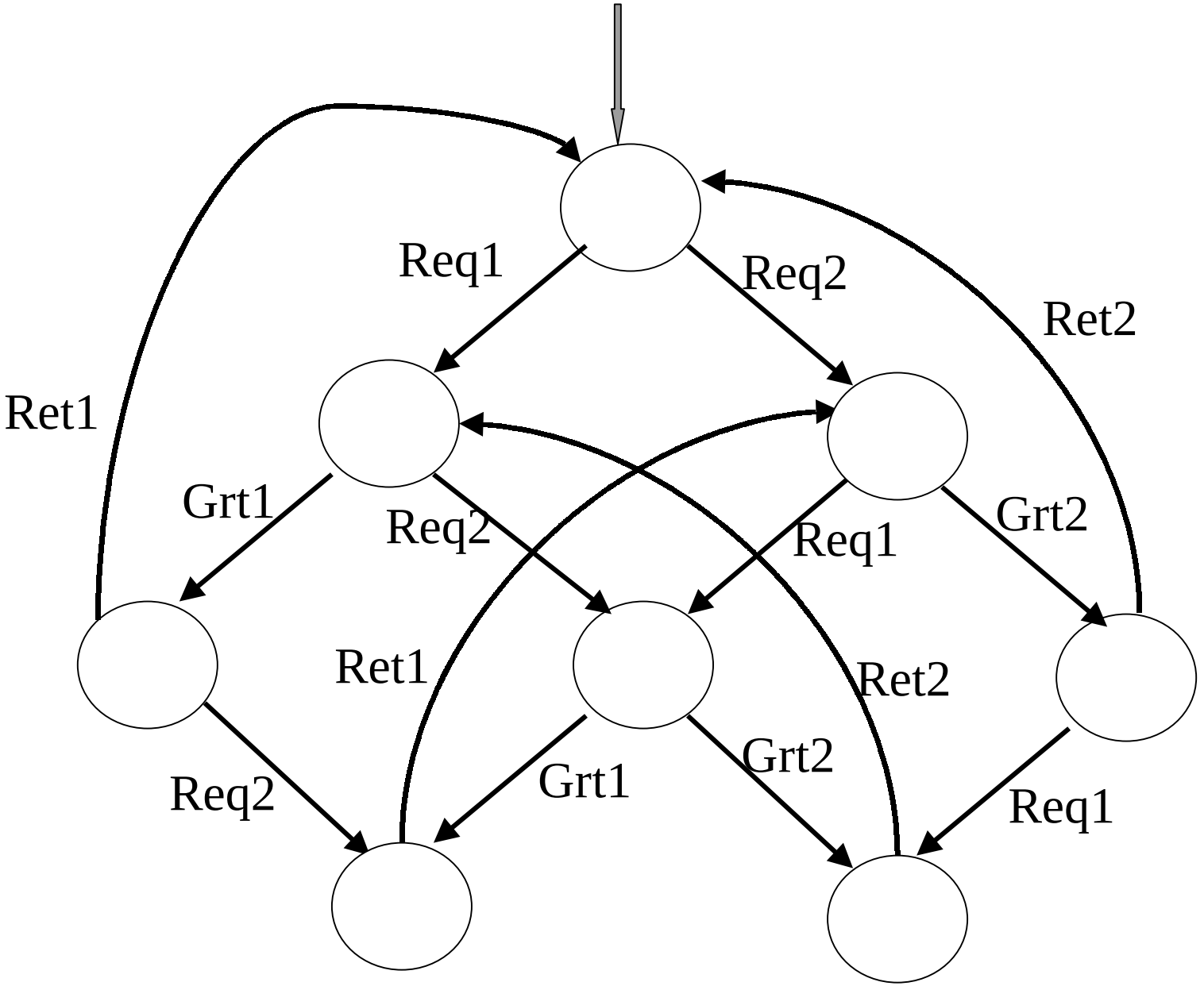
- Model a system to be verified as a Kripke structure:
  - Transition system **TS = (S, S_0, R)**
  - **AP** = A finite set of atomic propositions.
    - Basic assertions about the system
  - **L : S $\rightarrow$ 2^{AP}** = The set of subsets of **AP**.
  - **p $\in$ L(s) ---- p** is true at **s**.
  - **p $\notin$ L(s) ---- p** is not true at **s**.
- **K = (S, S_0, R, AP, L) ----** Kripke structure

The Application

- ***The computations*** of the Kripke structure **K** will be the models for **LTL** formulas.
- ***The property*** to be verified is captured as an LTL formula **φ**.
- The modeled system has the property **φ** ***iff every computation of*** **K** ***is a model of*** **φ**.
- Verify (***model check***) :
  - **K** $\models$ **φ**

An Example




Req1
Req2
Ret2
Ret1
Grt1
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Req1
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Req2
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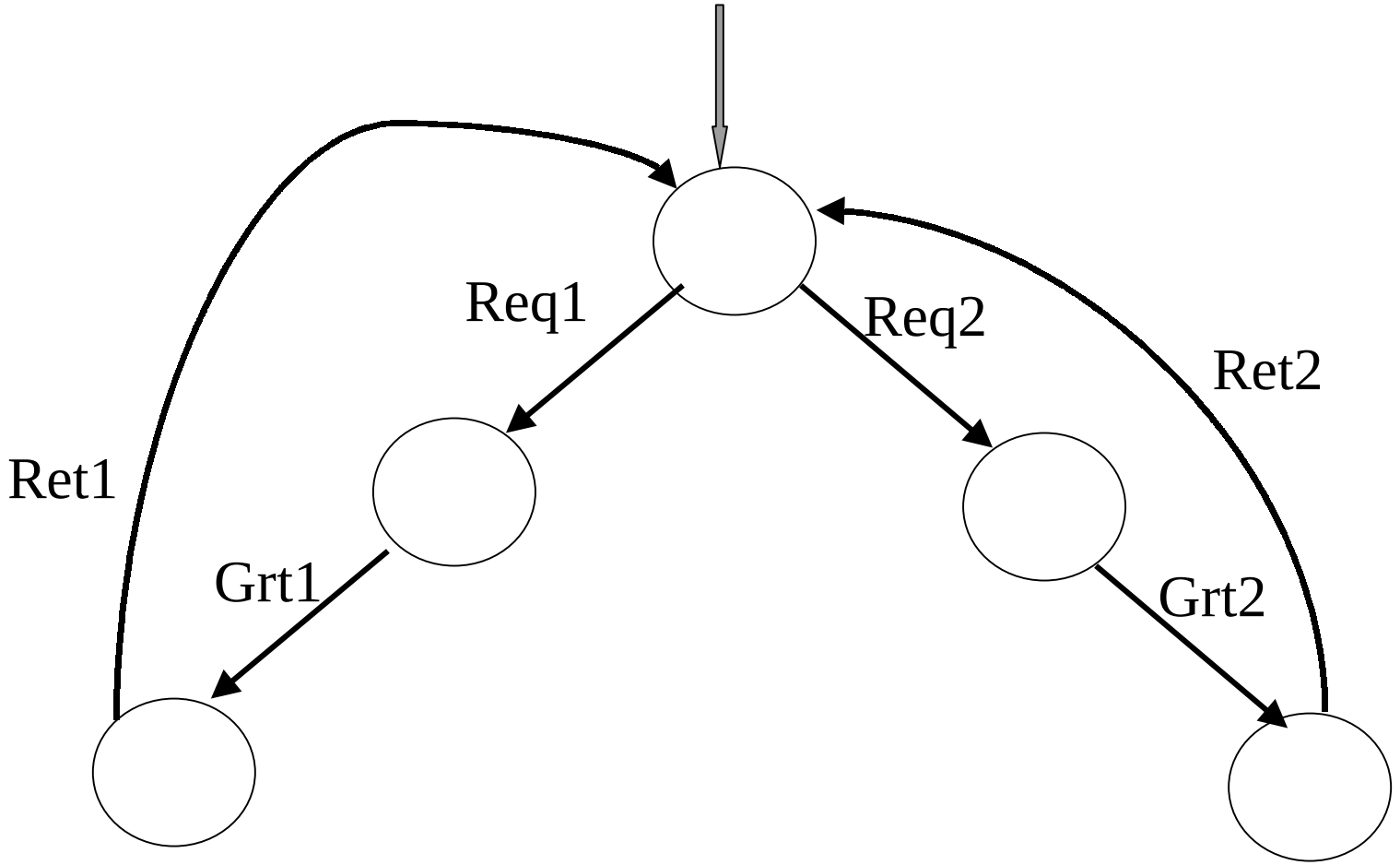
Grt2
Req1


Req1
Req2
Ret2
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Grt1
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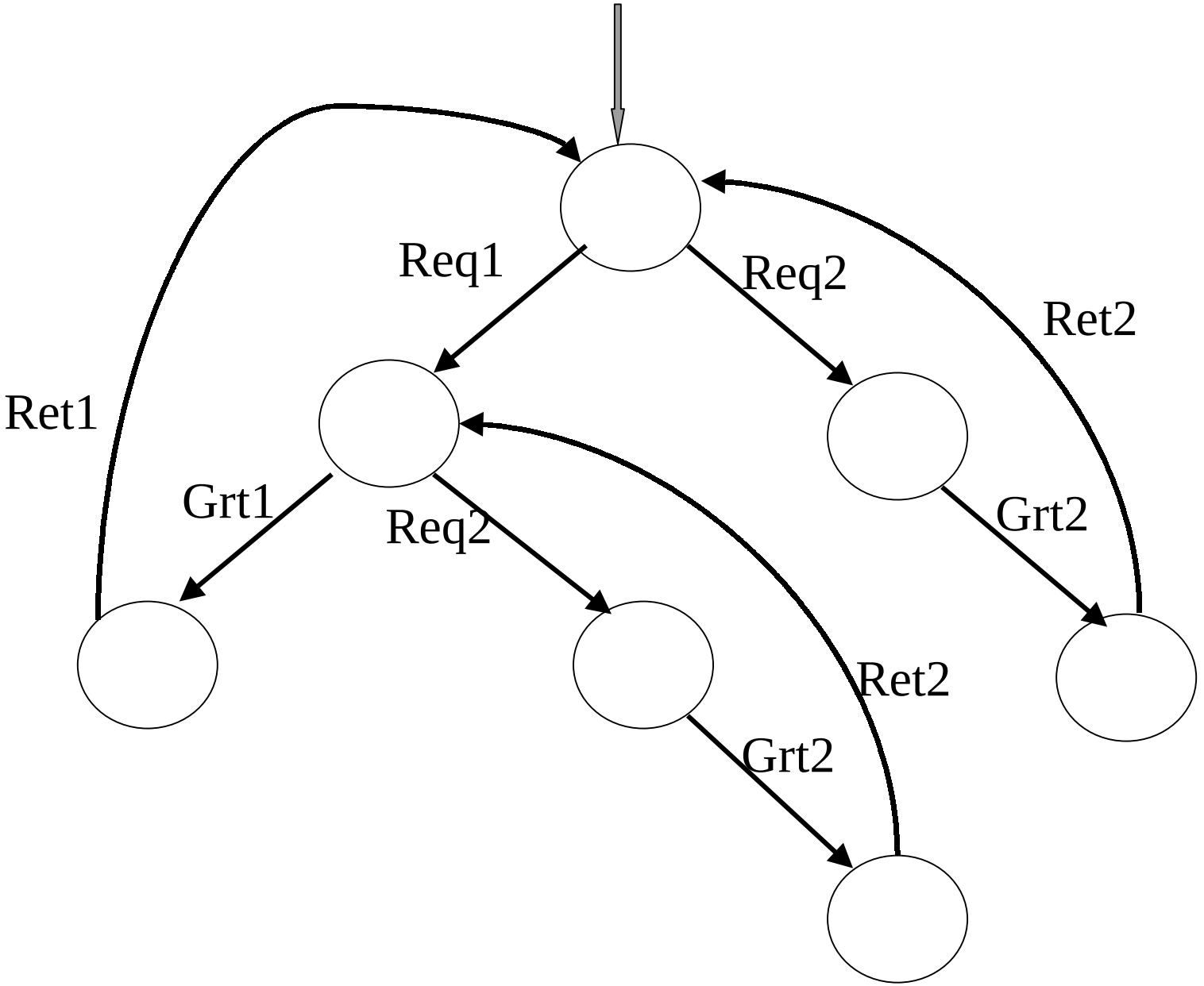
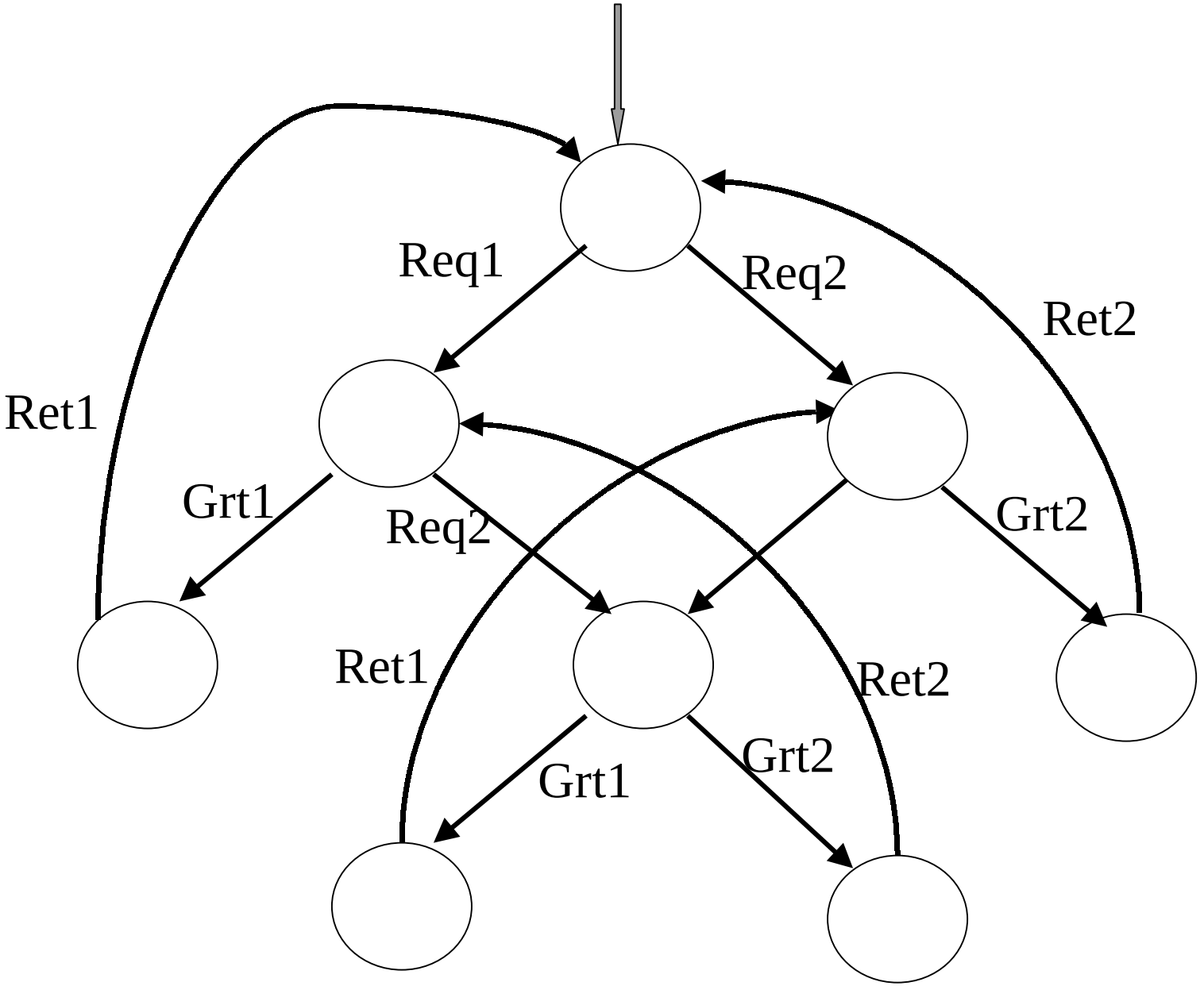

Req1
Req2
Ret2
Ret1
Grt1
Req2
Grt2
Ret2
Grt2


Req1
Req2
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Grt1
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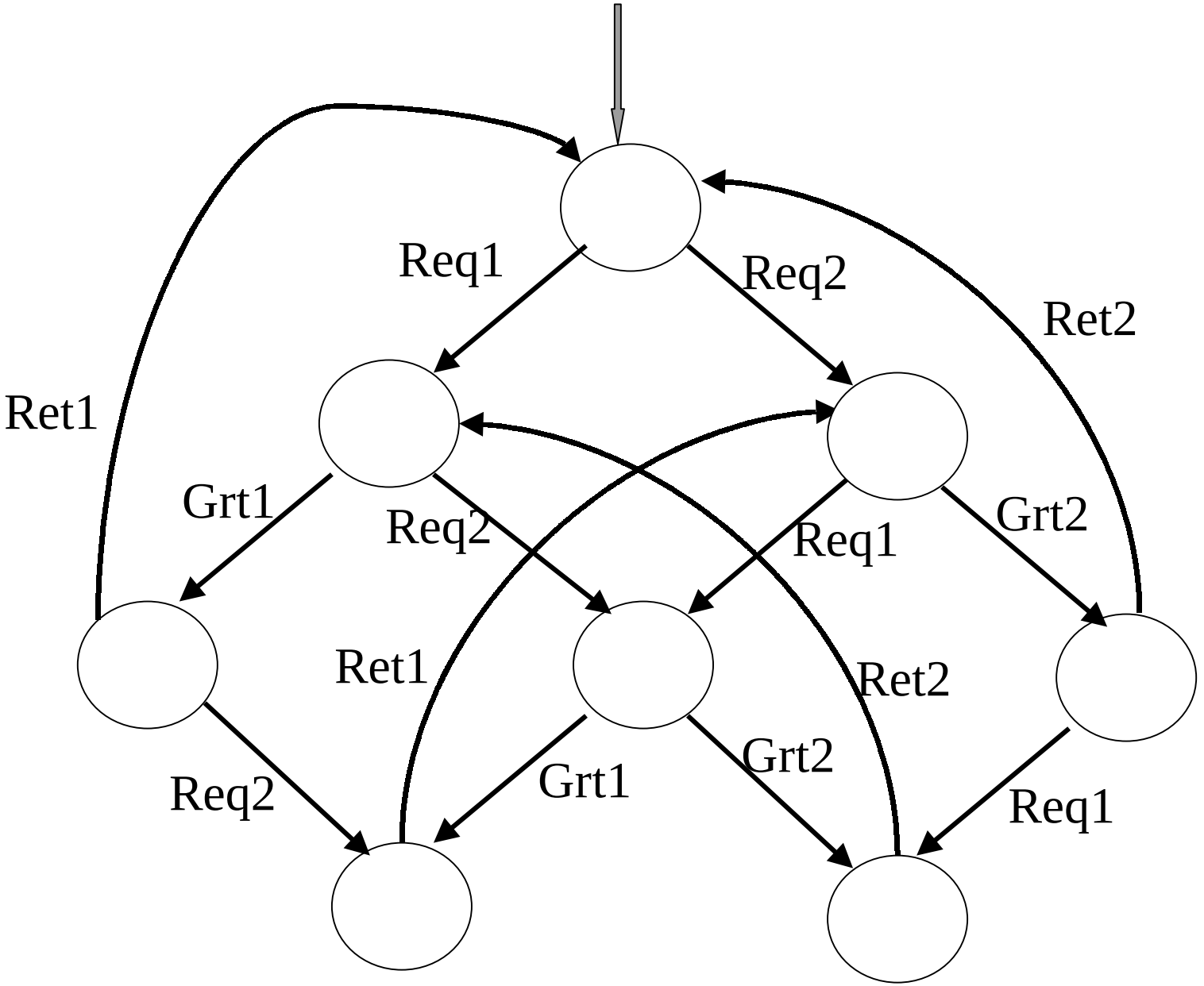
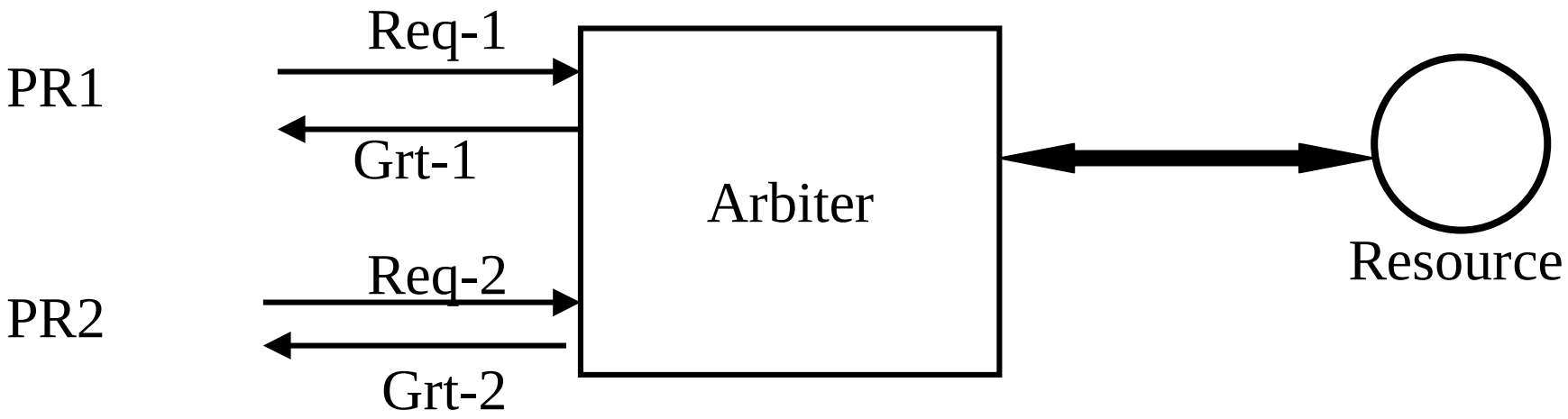
Grt2
Grt1


Req1
Req2
Ret2
Ret1
Grt1
Req2
Req1
Grt2
Ret1
Ret2
Req2
Grt1
Grt2
Req1

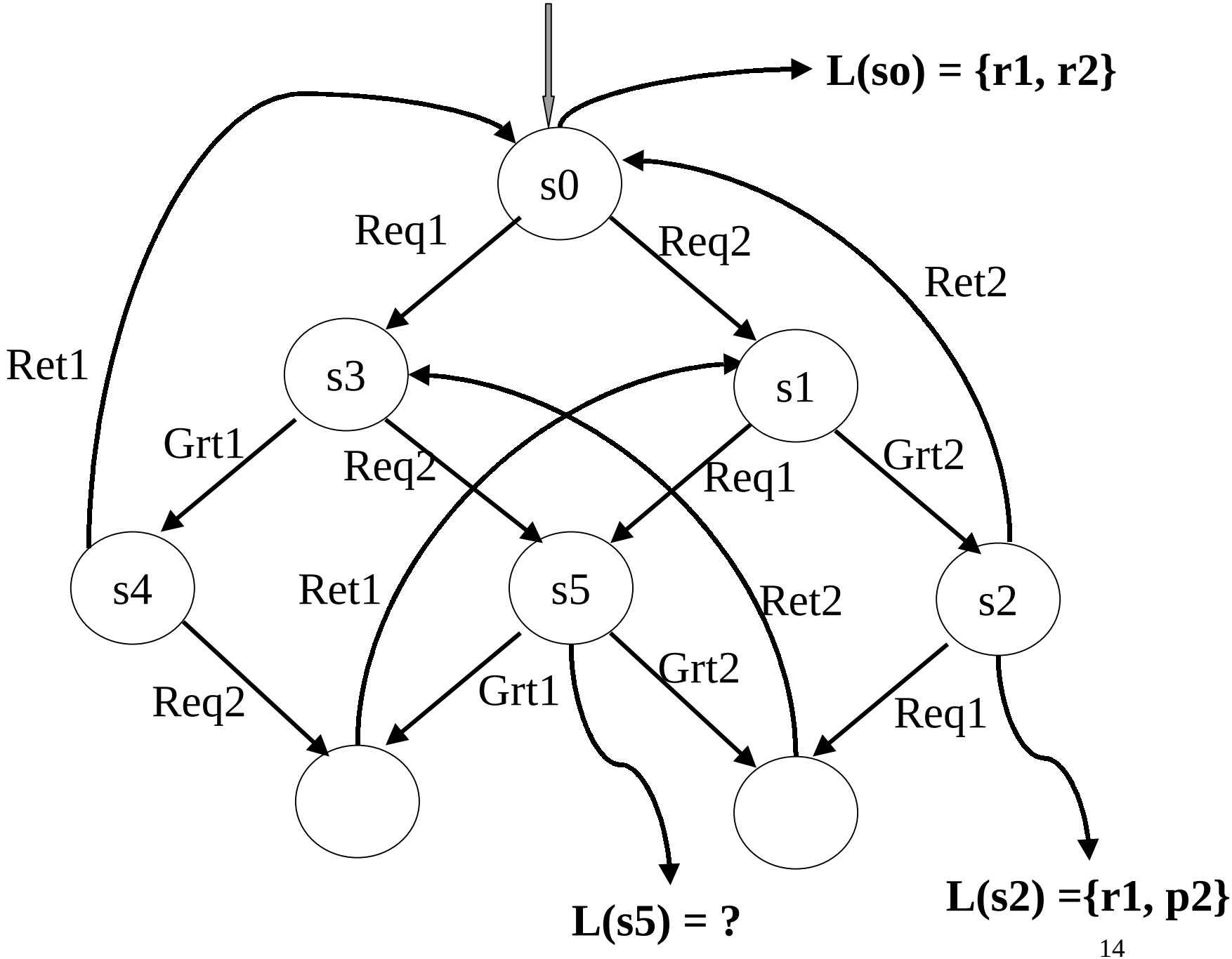
A set of Atomic Propositions

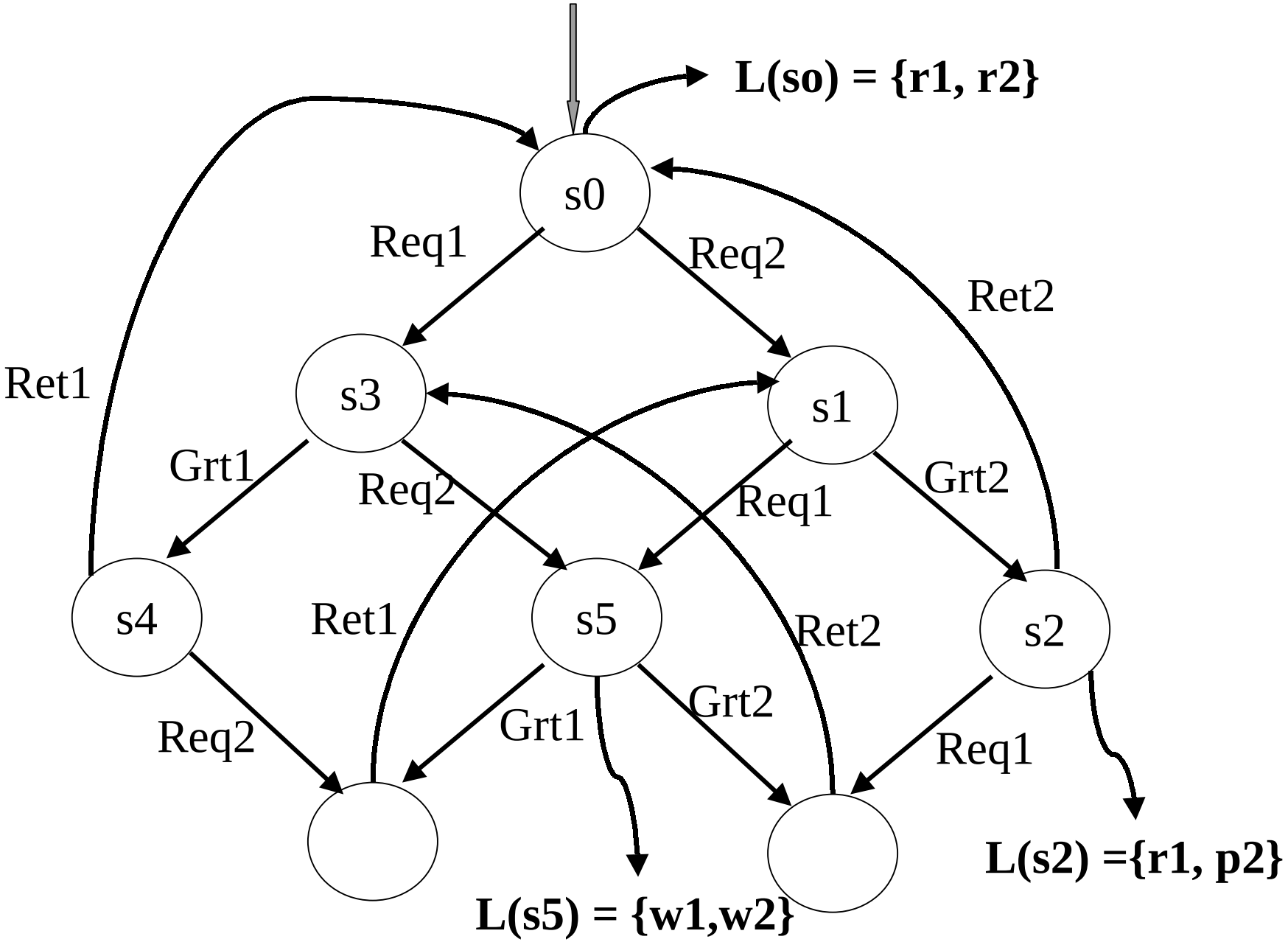


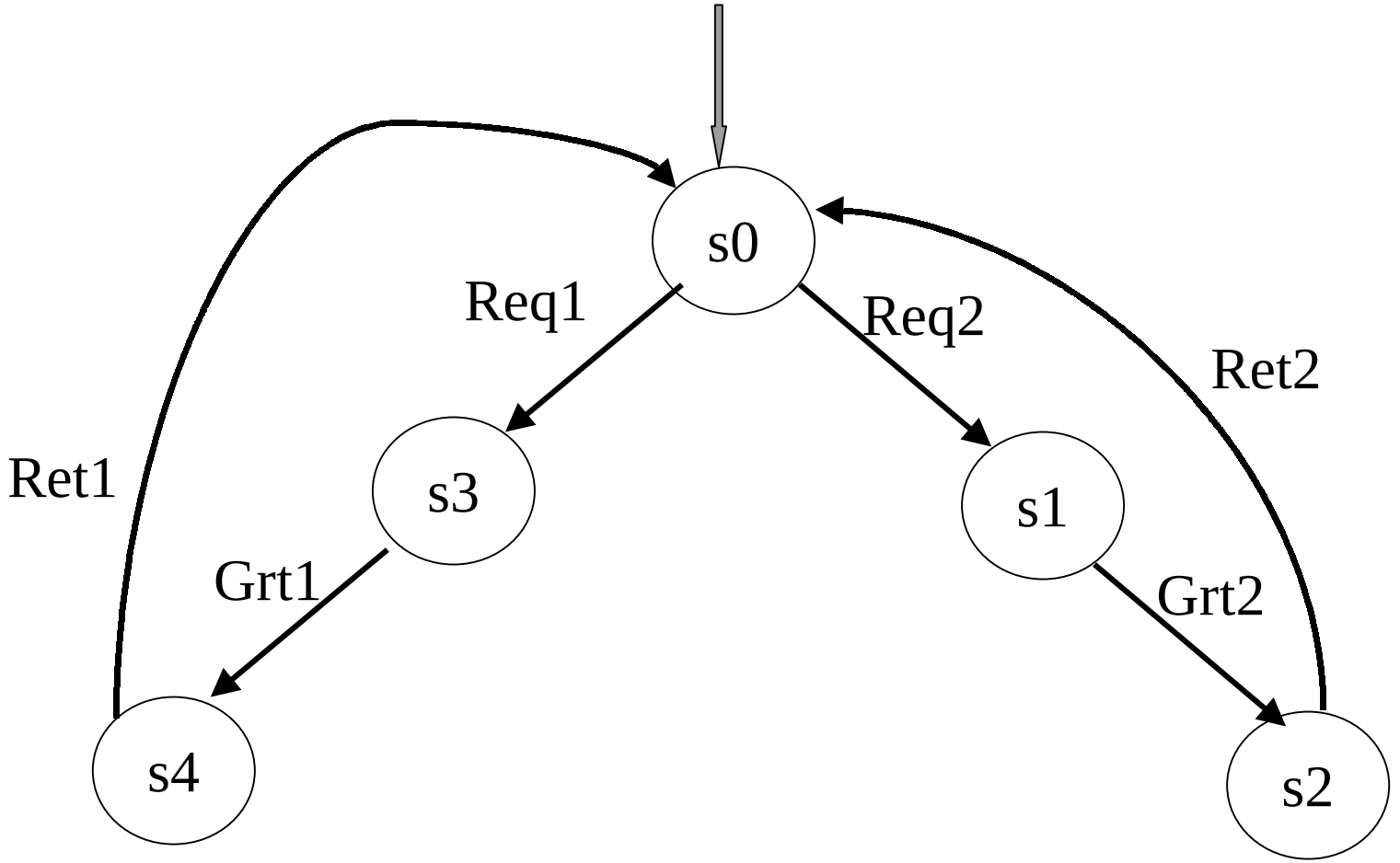
i1 – Process 1 is ***idle***
w1– Process 1 is ***waiting***
u1 – Process 1 is ***using*** the resource.
AP = { i1, w1, u1, i2, w2, u2}

The context

- Model a system to be verified as a Kripke structure:
  - Transition system **TS = (S, S_0, R)**
  - AP = A finite set of atomic propositions.
    - Basic assertions about the system
  - **L : S** $\longrightarrow$ $\mathbf{2^{AP}}$ = The set of subsets of AP.
  - **p** $\in$ **L(s) ---- p** is true at **s**
  - **p** $\notin$ **L(s) ---- p** is not true at **s**.
- **K = (S, S_0, R, AP, L) ----** Kripke structure


L(so) = {r1, r2}
s0
Req1
Req2
Ret2
Ret1
s3
s1
Grt1
Req2
Req1
Grt2
s4
Ret1
s5
Ret2
s2
Req2
Grt1
Grt2
Req1
L(s5) = ?
L(s2) ={r1, p2}


L(so) = {r1, r2}
s0
Req1
Req2
Ret2
Ret1
s3
s1
Grt1
Req2
Req1
Grt2
s4
Ret1
s5
Ret2
s2
Req2
Grt1
Grt2
Req1
L(s2) ={r1, p2}
L(s5) = {w1,w2}



s0 s3 s4 s0 s1 s2 s0 s3 …

{r1, r2} {w1, r2} {u1,r2} {r1, r2} {r1,w2} {r1,p2} {r1,r2} {w1,r2}...

Assertions about a computation

s0	s3	s4	s0	s1	s2	s0	s3 …
{r1, r2}	**{w1, r2}**	**{p1,r2}**	**{r1, r2}**	**{r1,w2}**	**{r1,p2}**	**{r1,r2}**	**{w1,r2}..**

- If at some stage Process 1 is **waiting** then at some later stage it is **printing** (i.e. using the resource).
- **At no stage** are both processes using the resource.
- If a process is waiting then it does so **until** it starts to use the resource.
- **There is a stage** at which both processes are waiting.

The Application

- **K = (S, S_0, R, AP, L)**
- Every computation (sequence of states) can be viewed as a sequence of subsets of **AP**.
- **s_0 s_1 s_2 …. ---- L(s_0) L(s_1) L(s_2)** ….
- These **AP-computations** will be the models for the formulas of LTL.

- Verification :
  - Every **AP**-computation of **K** is a model of φ

Linear Time Temporal Logic (LTL)

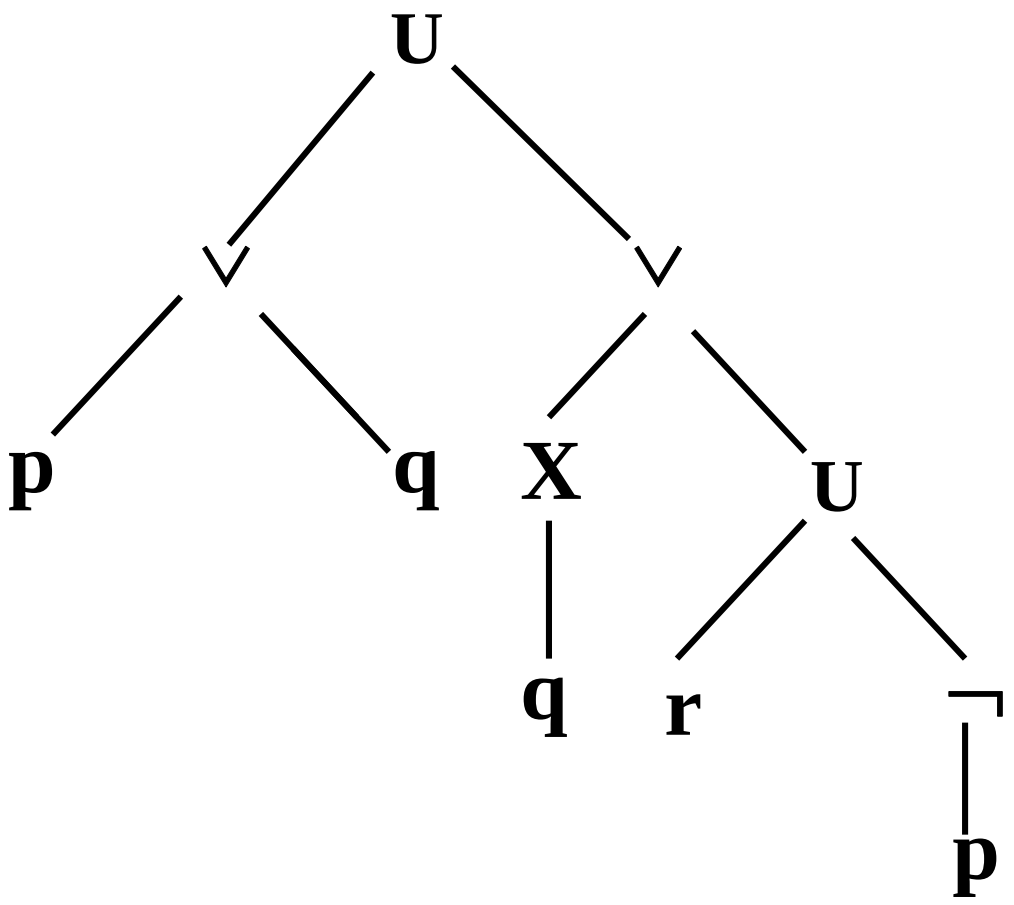
- Syntax :
  - **AP = {p_0, p_1, …p_n}**, a finite set of atomic propositions.
- Formulas :
  - Every ***p*** in AP is a formula.
  - If φ is a formula then $\neg\varphi$ is a formula.
  - If φ_1 and φ_2 are formulas then $(\varphi_1 \vee \varphi_2)$ is a formula.
  - If φ is a formula then $\mathbf{X}\varphi$ is a formula (Next).
  - If φ_1 and φ_2 are formulas then $(\varphi_1\ \mathbf{U}\ \varphi_2)$ is a formula (Until).

Formulas

- **LTL** ::= $p \mid \neg\varphi \mid \varphi_1 \vee \varphi_2 \mid \mathbf{X}\varphi \mid \varphi_1 \mathbf{U} \varphi_2$

- $p \;;\; p \vee q \;;\; (\neg p \vee q) \vee \neg(r \vee q)$
- $\mathbf{X} q \;;\; \mathbf{X}(p \vee q) \;;\; \mathbf{X}((\neg p \vee q) \vee \mathbf{X}\neg(r \vee q))$
- $(p \vee q)\,\mathbf{U}\,(\mathbf{X} q \vee (r\,\mathbf{U} \neg p))$

$$(p \vee q)\ U\ (X\, q \vee (r\ U \neg p))$$



Semantics

- **AP** = A finite set of atomic propositions.
- $\Sigma = 2^{AP}$ = The set of subsets of **AP**
- **AP = { p, q, r }**
- **Σ = {ϕ, { p }, { q }, ….,{ p, q, r } }**

- Σ^{ω} = The set of infinite sequences over Σ.

Semantics

- **AP = {p, q, r} $\Sigma = 2^{AP}$**
- **Σ = {ϕ, { p }, { q }, ….,{ p, q, r } }**
- **{ p,r } { q } $\varnothing$ { p, q, r } { r }**…….

  0 1 2 3 4 ……..
- At the **0^{th}** stage **p** and **r** are true but not **q**; at the **2^{nd}** stage no member of **AP** is true….

Semantics

- Σ^{ω} = The set of infinite sequences over Σ.
- $\sigma \in \Sigma^{\omega}$ **---** A model
- **σ(i) ---- i-**th position of σ
- **{p} {q,r} ∅ {r, q} {p, q, r}……..**
  | | | | |
- **0 1 2 3 4 ……**
- **σ(0) = {p} σ(2) = ∅ σ(3) = ?**

Semantics

- **AP** $\Sigma = 2^{AP}$
- Σ^{ω} = The set of infinite sequences over Σ.
- $\sigma \in \Sigma^{\omega}$ --- A model
- $\sigma(i)$ ---- **i**-th position of σ
- φ, a formula.
- $\sigma(i) \models \varphi$
  - $\sigma(i)$ ***satisfies*** φ
  - φ is true in the **i**-th position of σ

Semantics

- **LTL ::= *p*** $\mid \neg \varphi \mid \varphi_1 \vee \varphi_2 \mid$ **X** $\varphi \mid \varphi_1$ **U** φ_2
- $\sigma = \Gamma_0\ \Gamma_1\ \Gamma_2 \ldots \Gamma_i\ \Gamma_{i+1} \ldots\ldots$
- Each Γ_j is a subset of **AP**.
- $\sigma(i) \models p$ ***iff*** $p \in \Gamma_i$

Semantics

- **LTL ::= *p* $\mid \neg \varphi \mid \varphi_1 \vee \varphi_2 \mid$ X $\varphi \mid \varphi_1$ U φ_2**
- **AP = {p, q, r}**
- **σ = {p, q} {r} ϕ {q, r} {p, q, r} …..**

0 1 2 3 4

- **σ(0) *satisfies* q**
- **σ(1) *satisfies* r**
- **σ(2)** does ***not*** *satisfy* **q !**

Semantics

- **LTL ::= *p* $\mid \neg \varphi \mid \varphi_1 \vee \varphi_2 \mid$ X $\varphi \mid \varphi_1$ U φ_2**
- $\sigma = \quad \Gamma_0 \ \Gamma_1 \ \Gamma_2 \ \ldots\ldots \ \Gamma_i \quad \Gamma_{i+1} \ldots\ldots\ldots \ldots$
- Each Γ_j is a subset of **AP**.
- $\sigma(i) \models \neg \varphi$ ***iff*** $\sigma(i) \not\models \varphi$

Semantics

- **LTL ::= *p*** $\mid \neg \varphi \mid \varphi_1 \vee \varphi_2 \mid$ **X** $\varphi \mid \varphi_1$ **U** φ_2
- $\sigma = \quad \Gamma_0 \ \Gamma_1 \ \Gamma_2 \ \ldots\ldots \ \Gamma_i \quad \Gamma_{i+1} \ \ldots\ldots\ldots \ldots \ldots$
- Each Γ_j is a subset of **AP**.
- $\sigma(i) \models \varphi_1 \vee \varphi_2$ ***iff*** $\sigma(i) \models \varphi_1$ ***OR*** $\sigma(i) \models \varphi_2$

Semantics

- **LTL ::= *p* $\mid \neg \varphi \mid \varphi_1 \vee \varphi_2 \mid$ X $\varphi \mid \varphi_1$ U φ_2**
- **AP = {p, q, r}**
- **σ = {p, q} {r} ϕ {q, r} {p, q, r} …..**

  0 1 2 3 4
- **σ(0) *satisfies* $\neg$r ; σ(0)** does ***not satisfy*** **r**
- **σ(1) *satisfies* p $\vee$ r ; σ(1) *satisfies* r**
- **σ(2) *satisfies* $\neg$(p $\vee$ r) ?**

Semantics

- **LTL ::= $p \mid \neg \varphi \mid \varphi_1 \vee \varphi_2 \mid \mathbf{X}\, \varphi \mid \varphi_1\, \mathbf{U}\, \varphi_2$**
- **AP = {p, q, r}**
- **σ = {p, q} {r} ϕ {q, r} {p, q, r} …..**

  0 1 2 3 4
- **σ(2) *satisfies* $\neg$(p $\vee$ r)** ? ***Yes!***
- **σ(2)** does ***not satisfy*** **p $\vee$ r**

Semantics

- **LTL ::= *p* $\mid \neg \varphi \mid \varphi_1 \vee \varphi_2 \mid$ X $\varphi \mid \varphi_1$ U φ_2**
- $\sigma = \Gamma_0 \quad \Gamma_1 \quad \Gamma_2 \ldots\ldots \Gamma_i \quad \Gamma_{i+1} \ldots\ldots\ldots$ ..

  Γ_i : Xφ ; Γ_{i+1} : φ

- $\sigma(i) \models$ X φ *iff* $\sigma(i+1) \models \varphi$

Semantics

- **LTL ::= *p* $\mid \neg \varphi \mid \varphi_1 \vee \varphi_2 \mid$ X $\varphi \mid \varphi_1$ U φ_2**
- **AP = {p, q, r}**
- **σ = {p,q} {r} $\varnothing$ {q,r} {p,q,r} …..**

  0 1 2 3 4
- **σ(2) *satisfies* X r ; σ(3) *satisfies* r**
- **σ(0) *satisfies* X(p $\vee$ r) ; σ(1) *satisfies* r**
- **σ(1)** does ***not satisfy*** **X(p $\vee$ r)**
  - **σ(2)** does ***not satisfy*** **p $\vee$ r**

Semantics

- **LTL ::= *p* $\mid \neg \varphi \mid \varphi_1 \vee \varphi_2 \mid$ X $\varphi \mid \varphi_1$ U φ_2**
- **AP = {p, q, r}**
- **σ = {p,q} {r} $\varnothing$ {q,r} {p,q,r} …..**

  0 1 2 3 4
- **σ(1) *satisfies* X(X $\neg$ p) *iff***
  - **σ(2) *satisfies* X $\neg$ p *iff***
  - **σ(3) *satisfies* $\neg$ p *iff***
  - **σ(3)** does ***not satisfy*** **p**

Semantics

- **LTL ::= *p* $\mid \neg \varphi \mid \varphi_1 \vee \varphi_2 \mid$ X $\varphi \mid \varphi_1$ U φ_2**
- $\sigma = \Gamma_0 \quad \Gamma_1 \ldots \Gamma_i \quad \Gamma_{i+1} \ldots \Gamma_{k-1} \quad \Gamma_k \ldots$

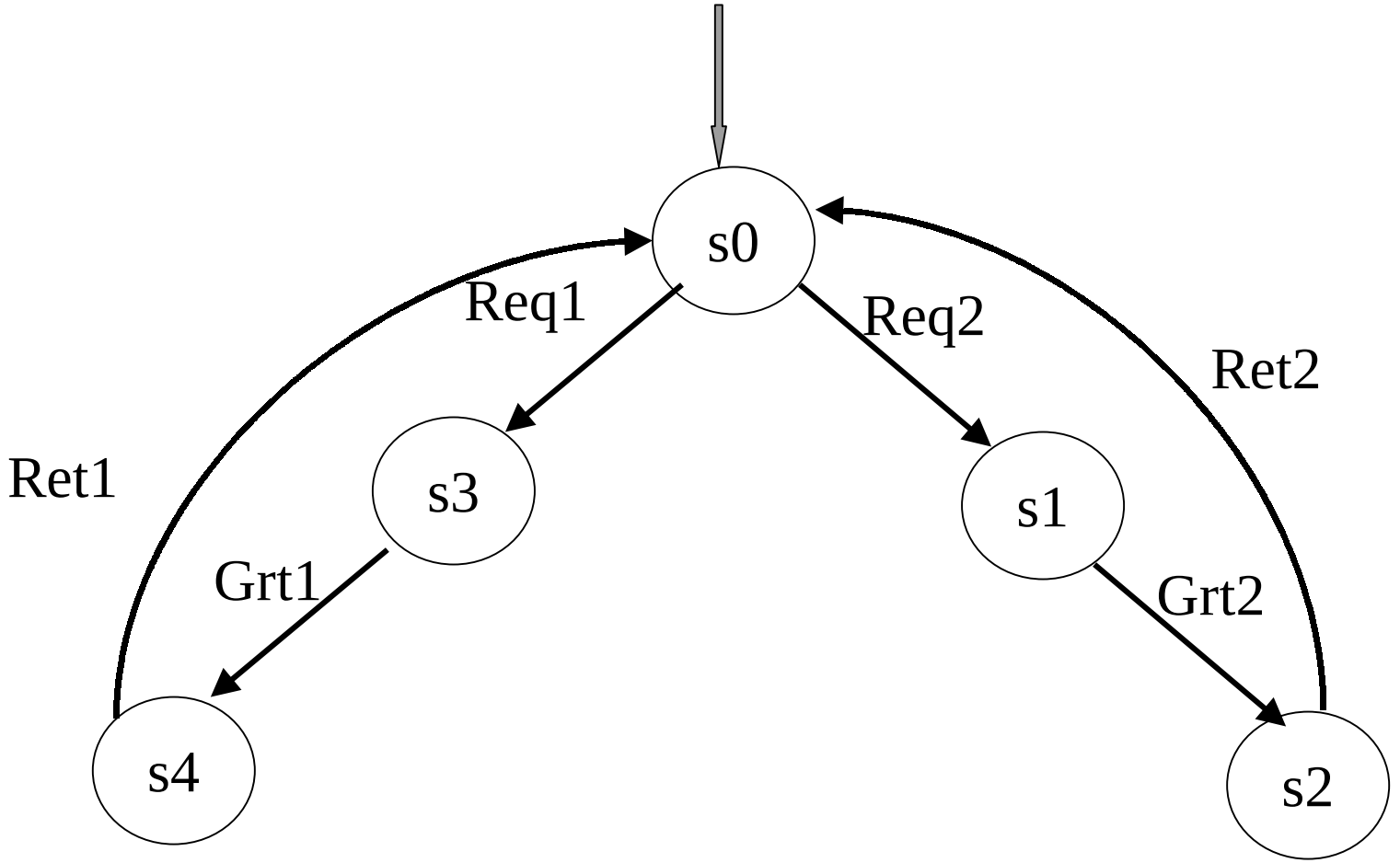
  $\varphi_1 \ldots \quad \varphi_1 \quad \ldots \varphi_1 \quad \varphi_2$

- $\sigma(i) \models \varphi_1$ U φ_2 ***iff*** there exists $k \geq i$
  - $\sigma(k) \models \varphi_2$
  - $\sigma(j) \models \varphi_1$ for every $i \leq j < k$

Semantics

- **LTL ::= *p*** $\mid \neg \varphi \mid \varphi_1 \vee \varphi_2 \mid$ **X** $\varphi \mid \varphi_1$ **U** φ_2
- **k** could be arbitrarily greater than **i**.
- **k = i** is allowed and there is **no i ≤ j < k**

- $\sigma(i) \models \varphi_1$ **U** φ_2 ***iff*** there exists **k ≥ i**
  - $\sigma(k) \models \varphi_2$
  - $\sigma(j) \models \varphi_1$ for every **i ≤ j < k**



s0 s3 s4 s0 s1 s2 s0 s3 …

{r1,r2} {w1,r2} {p1,r2} {r1,r2} {r1,w2} {r1,p2} {r1,r2} {w1,r2}...

An Example

AP = {r1, w1, p1, r2, w2, p2}

{r1,r2}	**{w1,r2}**	**{p1,r2}**	**{r1,r2}**	**{r1,w2}**	**{r1,p2}**	**{r1,r2}**	**{w1,r2}...**
0	1	2	3	4	5	6	7

- **σ(1) *satisfies* (r2 U w2)** ;
  - **σ(4) *satisfies* w2** and
  - **σ(1), σ(2), σ(3) *satisfy* r2**.

An Example

AP = {r1, w1, p1, r2, w2, p2}

{r1,r2}	**{w1,r2}**	**{p1,r2}**	**{r1,r2}**	**{r1,w2}**	**{r1,p2}**	**{r1,r2}**	**{w1,r2}...**
0	1	2	3	4	5	6	7

- **σ(1)** does ***not satisfy*** **(r2 U p2)** ;
  - **σ(5)** ***satisfies*** **p2** and
  - **σ(1), σ(2), σ(3)** ***satisfy*** **r2**.
  - But **σ(4)** does ***not satisfy*** **r2 !**

An Example

AP = {r1, w1, p1, r2, w2, p2}

{r1,r2}	**{w1,r2}**	**{p1,r2}**	**{r1,r2}**	**{r1,w2}**	**{r1,p2}**	**{r1,r2}**	**{w1,r2}...**
0	1	2	3	4	5	6	7

- **σ(1) *does satisfy* ((r2 ∨ w2) U p2)** ;
  - **σ(5) *satisfies* p2** and
  - **σ(1), σ(2), σ(3) *satisfy* r2, hence also (r2 ∨ w2)**.
  - **σ(4) *satisfies* w2, hence also (r2 ∨ w2) !**

Models

- **AP** $\Sigma^{\mathbf{AP}} = 2$
- Σ^{ω} = The set of infinite sequences over Σ.
- $\sigma \in \Sigma^{\omega}$
- φ an **LTL** formula.

- A path σ is a ***model*** of φ **(**$\sigma \models \varphi$ **)** ***iff***
  - $\sigma(0) \models \varphi$

Validity in LTL

- **AP** $\Sigma^{\mathbf{AP}} = 2$
- Σ^{ω} = The set of infinite sequences over Σ.
- $\sigma \in \Sigma^{\omega}$
- φ an **LTL** formula.

- φ is ***LTL-valid*** $(\models \varphi)$ ***iff for every*** $\sigma \in \Sigma^{\omega}$
  - $\sigma \models \varphi$

Derived Operators

- **LTL ::= $p \mid \neg \varphi \mid \varphi_1 \vee \varphi_2 \mid X \varphi \mid \varphi_1 \, U \, \varphi_2$**
- **$\varphi 1 \wedge \varphi 2$ --- $\neg (\neg \varphi_1 \vee \neg \varphi_2)$ (And)**
- **$\varphi 1 \supset \varphi 2$ --- $\neg \varphi_1 \vee \varphi_2$ (Implies)**
- **$\varphi 1 \equiv \varphi 2$ ---- $(\varphi_1 \supset \varphi_2) \wedge (\varphi_2 \supset \varphi_1)$ (Iff)**
- **AP = $\{p_1, p_2, \ldots, p_n\}$**
- **$\top$ ---- $p_1 \vee \neg p_1$ (true)**

- **Fact : In *every model* σ, *at every* i,**
  - **σ(i) *satisfies* $\top$**

Derived Operators

- **LTL ::= $p \mid \neg \varphi \mid \varphi_1 \vee \varphi_2 \mid \mathbf{X}\, \varphi \mid \varphi_1\, \mathbf{U}\, \varphi_2$**
- **$\mathbf{F}\varphi$ ---- $(\top\, \mathbf{U}\, \varphi)$** (future ; diamond: $\Diamond$)

- **Fact** :
  - **σ(i)** ***satisfies*** **$\mathbf{F}\varphi$** ***iff*** **there exists k $\geq$ i** such that **σ(k)** ***satisfies*** **φ**.

Derived Operators

- **LTL ::= $p \mid \neg \varphi \mid \varphi_1 \vee \varphi_2 \mid \mathbf{X}\, \varphi \mid \varphi_1\, \mathbf{U}\, \varphi_2$**
- **$\mathbf{F}\varphi$ ---- $(\top\, \mathbf{U}\, \varphi)$**
- **$\mathbf{G}\varphi$ ---- $\neg \mathbf{F} \neg \varphi$** (invariant; box: $\Box$)

- **Fact** :
  - **σ(i)** ***satisfies*** **$\mathbf{F}\varphi$** ***iff*** **for every k $\geq$ i ,**
    σ(k) ***satisfies*** **φ .**

Model Checking

- **K = (S, S_0, R, AP, L)** (the system)
- φ, an **LTL** formula. (the property)
- **K** ⊨ φ ***iff*** **every AP-computation** of **K** is a model of φ.
- Determining this is the ***model checking problem***.
- This can be automated!