Tecniche di Specifica e di Verifica

Modeling with Transition Systems

An example

The Dining Philosophers

- Possible problems:
  - ***Deadlock***: system state where no action cen be taken (no transition possible)
  - ***Livelock***: When system component is prevented to take any action, or a particular one (individual starvation)
  - ***Starvation***: obvious.

Fairness

The Dining Philosophers

- Possible solutionto deadlock:
  - ***pick up right fork only if both are present***

  Assumptions:
  - ***weak fairness***: any trans. continuously enabled, will eventually fire (eating philosophers will finish)
  - ***strong fairness***: any trans. enabled infinitely often, will eventually occur (if 2 fork available infinitely often, phil. will eat).

Livelock

The Dining Philosophers

- Possible solution:
  - ***pick up fork only if both are present***

  Assumptions:

  - ***strong fairness***: any trans. enabled infinitely often, will eventually occur (if 2 fork available infinitely often, phil. will eat).

  strong fairness is not enough to prevent ***livelock***

  Why? Think of the case with 4 phil.!

  Sol.(?): Try ***preventing consecutive eating***.

  Still suffers from ***livelock*** with 5 phil.! ***Why***?

Outline

- The model – Transition systems
- Some features
  - Paths
  - Computations
  - Branching
- First order representation

Transition systems

- A transition system (***Kripke structure***) is a structure

$$\mathbf{TS = (S, S_0, R)}$$

where:

- $\mathbf{S}$ is a finite set of states.
- $\mathbf{S_0} \subseteq \mathbf{S}$ is the set of initial states.
- $\mathbf{R} \subseteq \mathbf{S} \times \mathbf{S}$ is a transition relation
  - $\mathbf{R}$ must be total, that is
    - $\forall \mathbf{s} \in \mathbf{S}\ \exists \mathbf{s'} \in \mathbf{S}\ .\ \mathbf{(s, s')} \in \mathbf{R}$ or, equivalently,
    - For every state **s** in **S,** there exists **s'** in **S** such that **(s, s')** is in **R**.

Notions and Notations

- **TS = (S, S_0, R)**
- **(s, s') $\in$ R   R(s, s')   s $\rightarrow$ s'**
- A (finite) ***path*** from **s** is a sequence

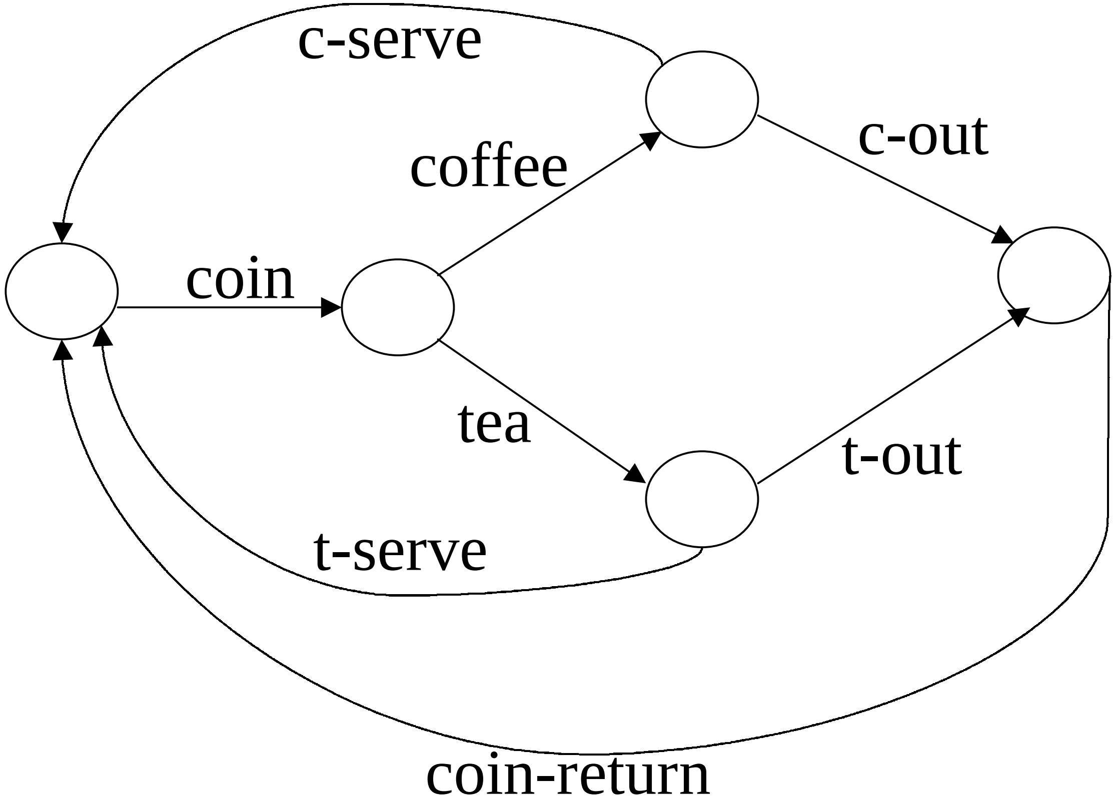
  $$\mathbf{s_1, s_2, \ldots, s_n}$$

  such that
  - **s = s_1**
  - **$s_i \rightarrow s_{i+1}$** for 0 < i < n.
- It is from **s** to **s'** if **s_n = s'**.
- An infinite path from **s** is a sequence …..

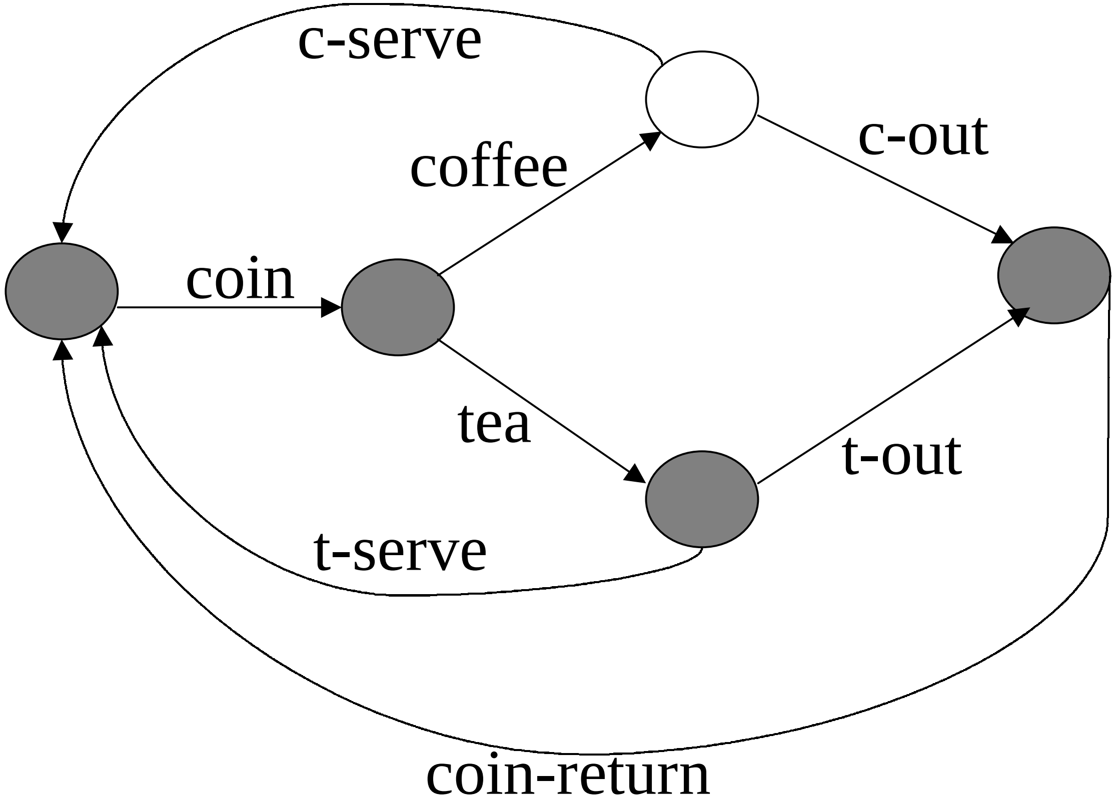
Labeled transition systems

- Sometimes we may use a finite set of actions:
  - **Act = {a, b, ..}**
- The actions will be used to label the transitions.
- **TS = (S, S_0, Act, R)**
  - $\mathbf{R} \subseteq \mathbf{S} \times \mathbf{Act} \times \mathbf{S}$, labeled transitions.
- **(s, a, s') $\in$ R - R(s, a, s') - $s \xrightarrow{a} s'$**

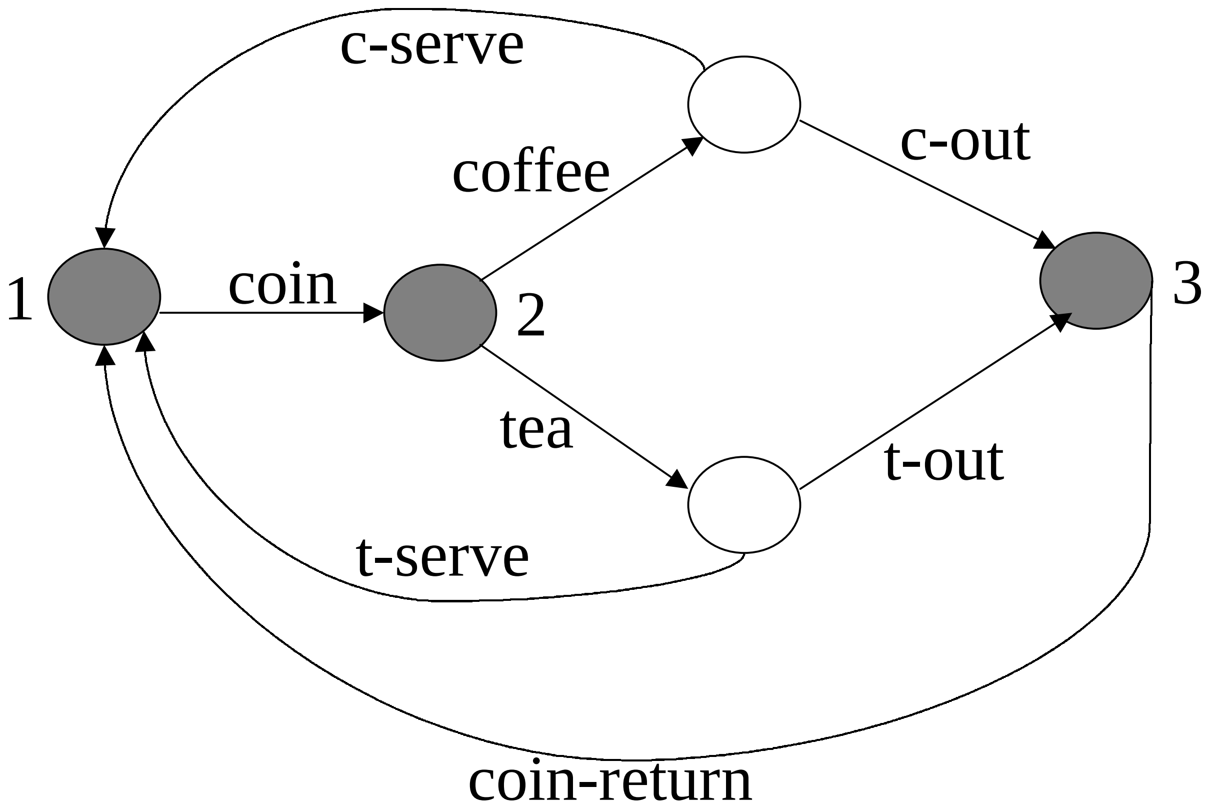
A vending machine



A path



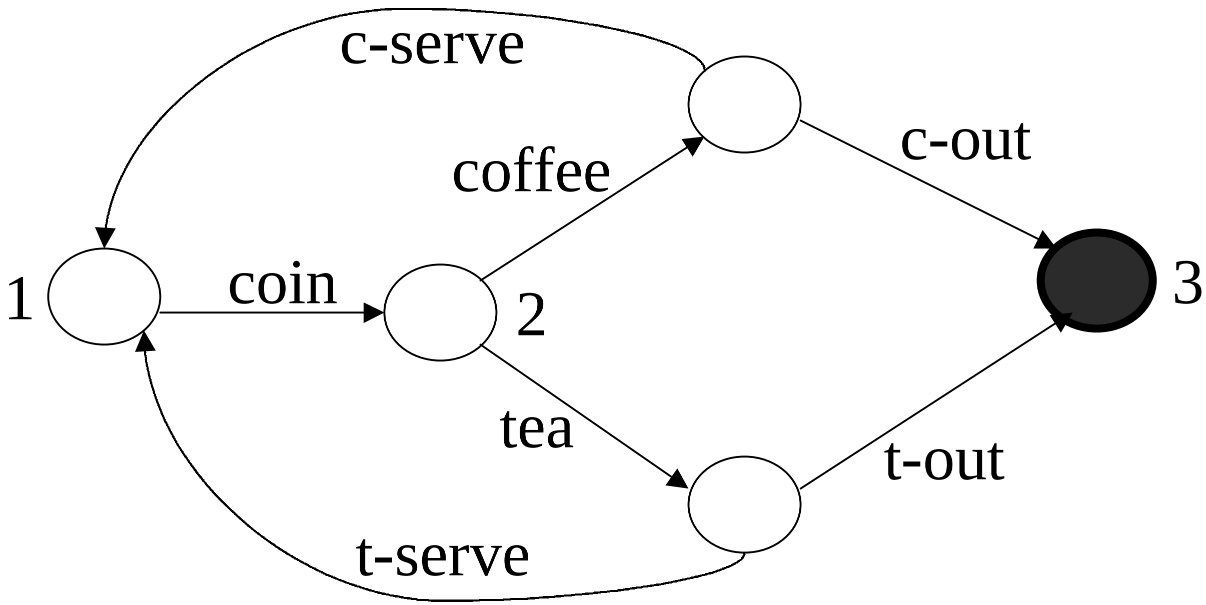
A non-path



1 2 3 No!

3 1 2 yes!

A non-total transition relation



State space

- The ***state space*** of a system (e.g. program) is the set of all possible states for it.
- For example, if **V={a, b, c}** and the variables are over the naturals, then the state space includes:

<a=0,b=0,c=0>,<a=1,b=0,c=0>,<a=1,b=1,c=0>,<a=932,b=5609,c=6658>…

Atomic transition

- Each ***atomic transition*** represents a small peace of code (or execution step), such that no smaller peace of code (or step) is observable.
- Is a:=a+1 atomic?
- In some systems, e.g., when a is a register and the transition is executed using an inc command.

(Non)Atomicity

- Execute the following when **x=0** in two concurrent processes:

  P1:a=a+1

  P2:a=a+1

- Result: **a=2**.
- Is this always the case?

- Consider the actual translation:

P1:load R1,a
inc R1
store R1,a

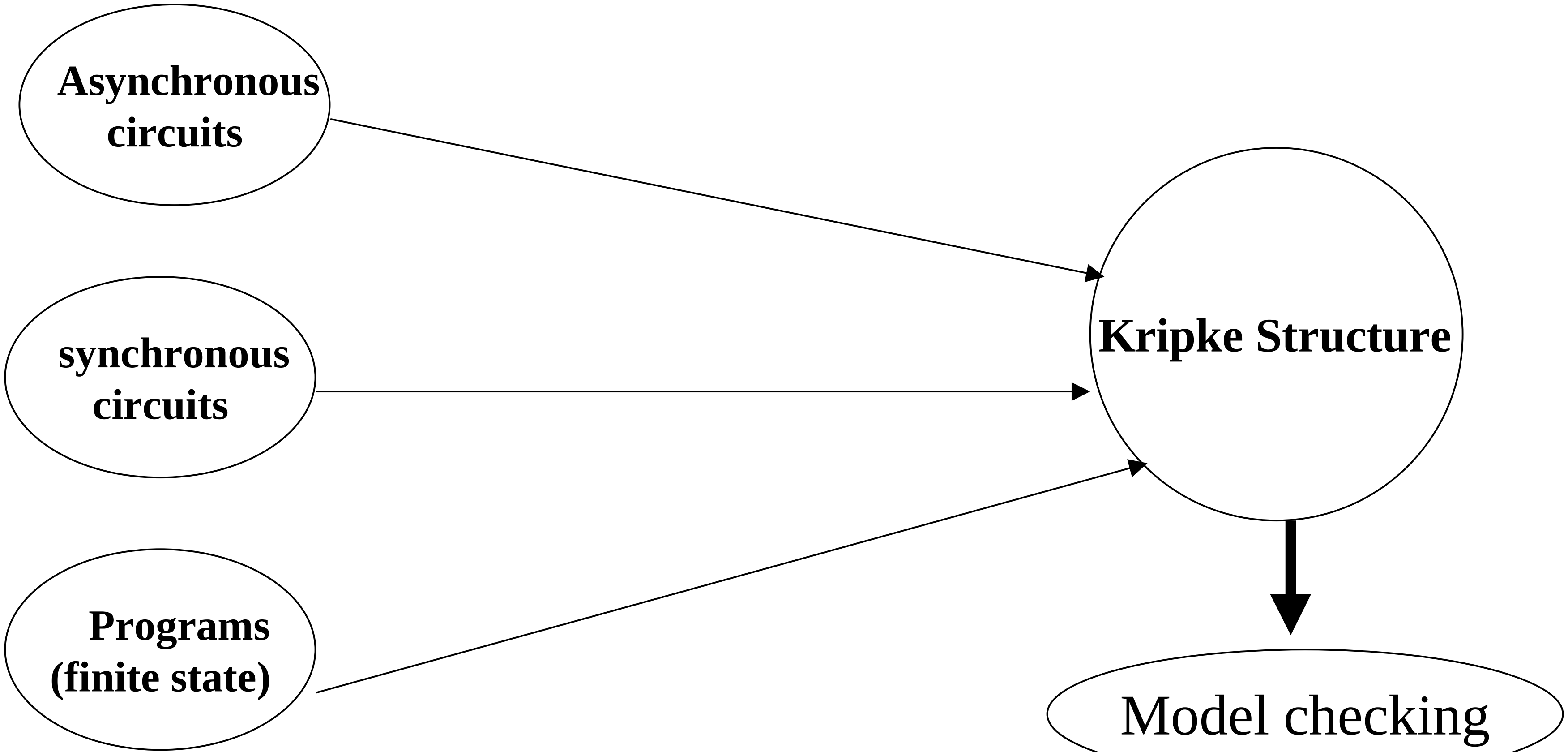
P2:load R2,a
inc R2
store R2,a

- **a may be also 1**

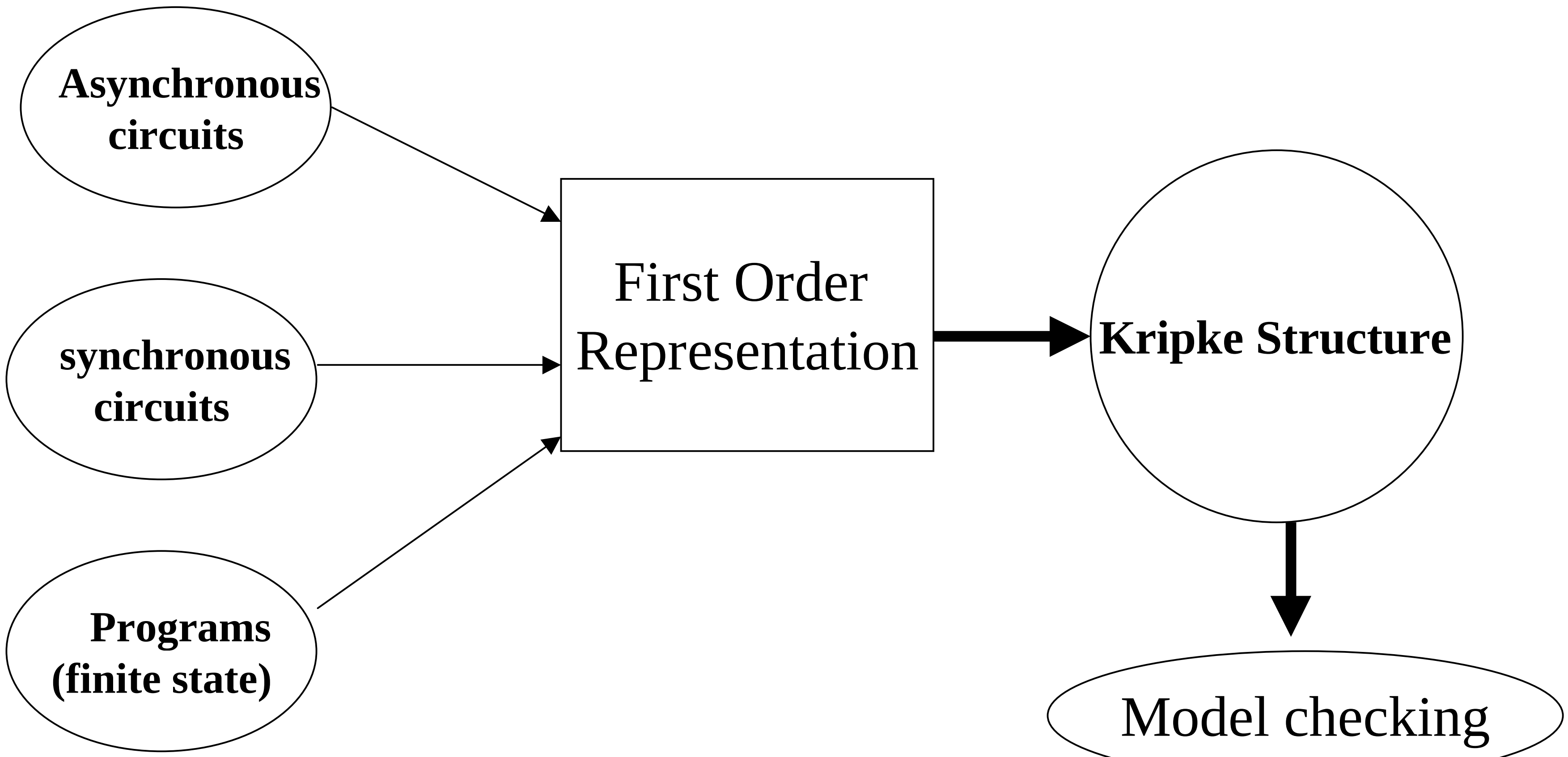
The common framework

- Many systems need to be modeled.
  - Digital circuits
    - **Synchronous**
    - **Asynchronous**
  - Programs
- Strategy : Capture the main features using a logical framework (nothing to do with temporal logics!) : ***First order representation***

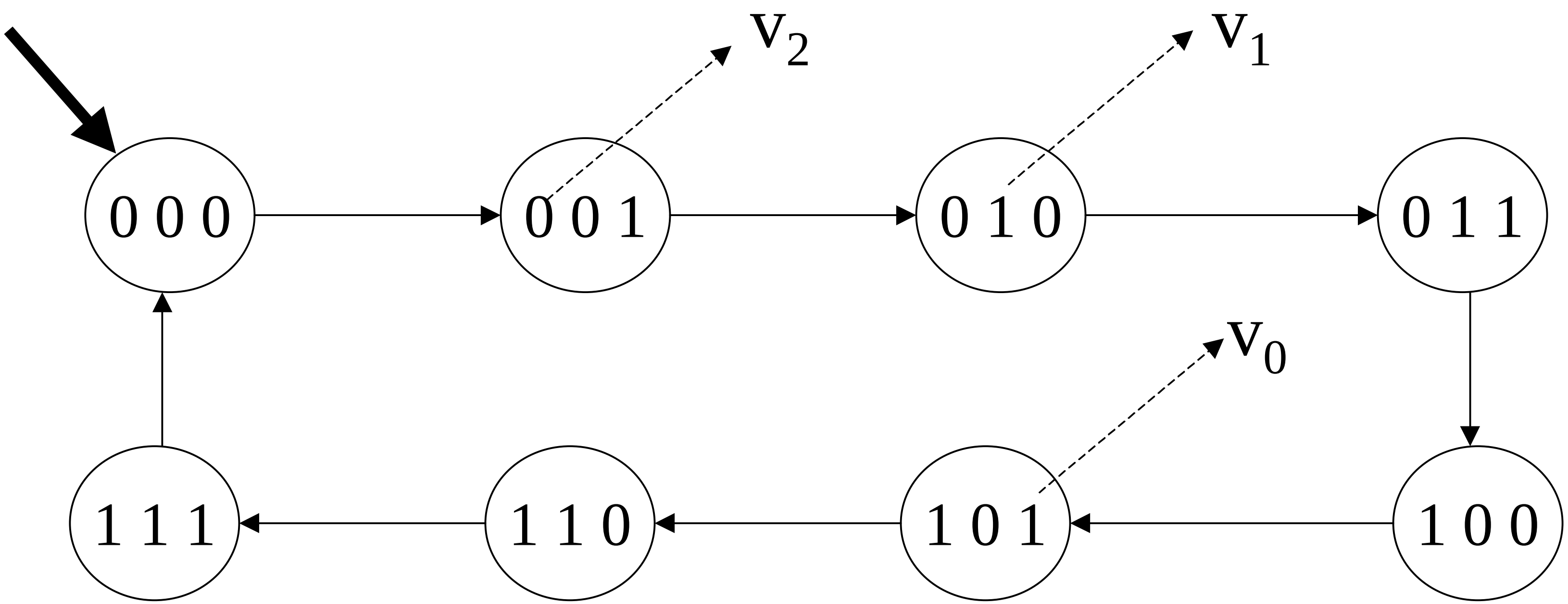
The inefficient way



The efficient way



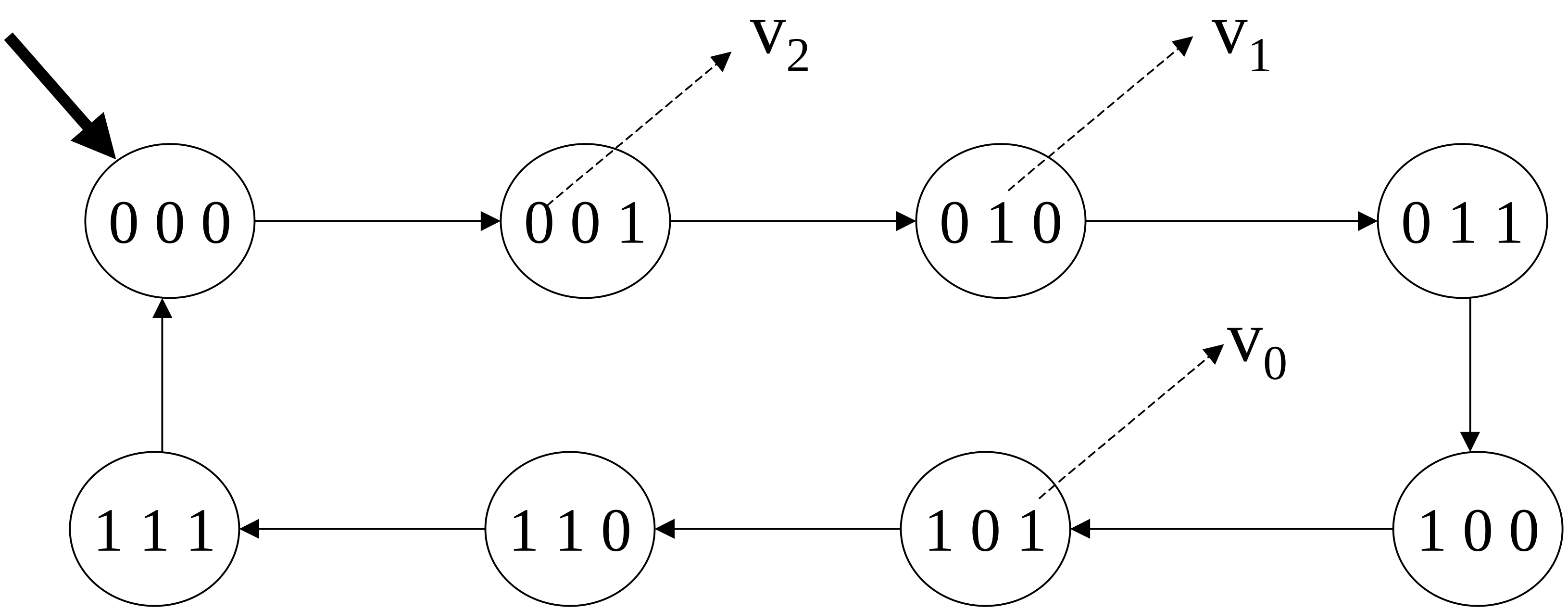
A mod-8 counter



The mod-8 counter

- System variables : $v_2\ v_1\ v_0$
- Domain of $v_2 = \{0, 1\}$
- Same for v_1 and v_0
- Special case : These variables are boolean
- A state is a function which assigns to each variable a value in its domain.
- $s(v_0) = 0 \quad s(v_1) = 1 \quad s(v_2) = 1$
- It is the state (1 1 0) !

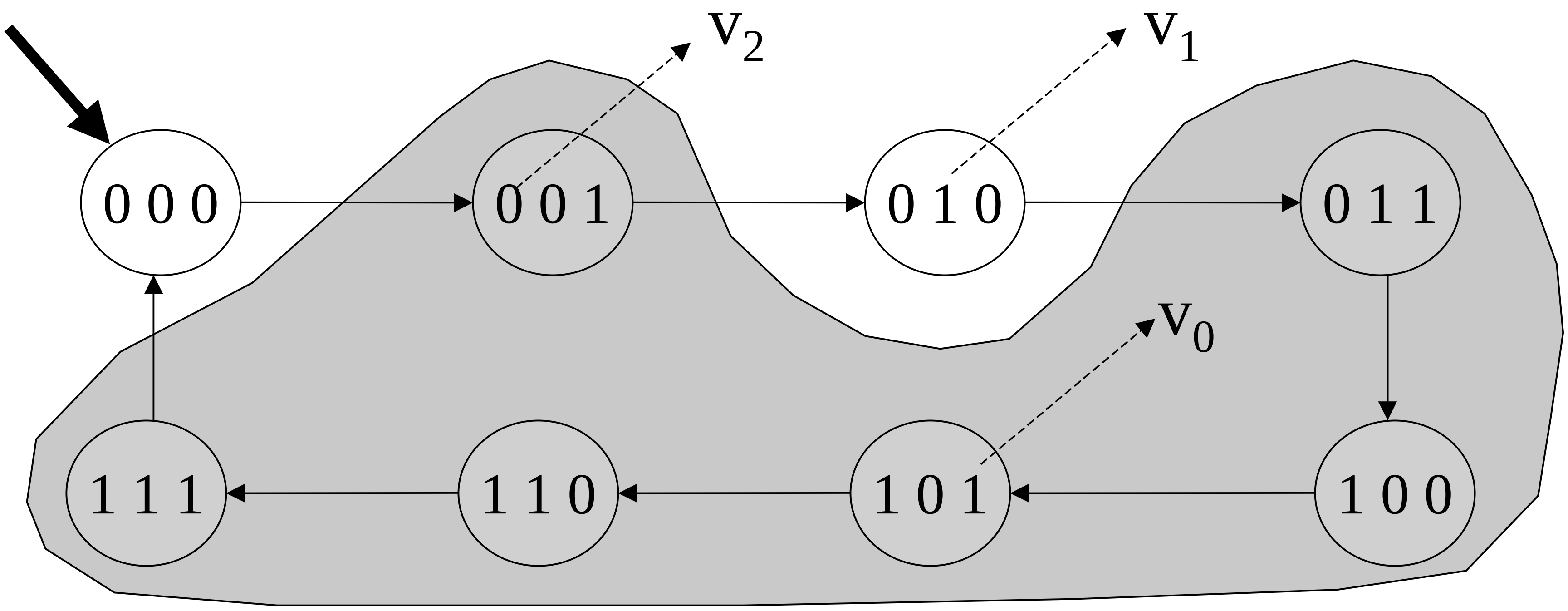
State Predicates



A set of states can be picked out by a formula;

$\mathbf{X = v_2 \vee v_0}$ is the set {...}

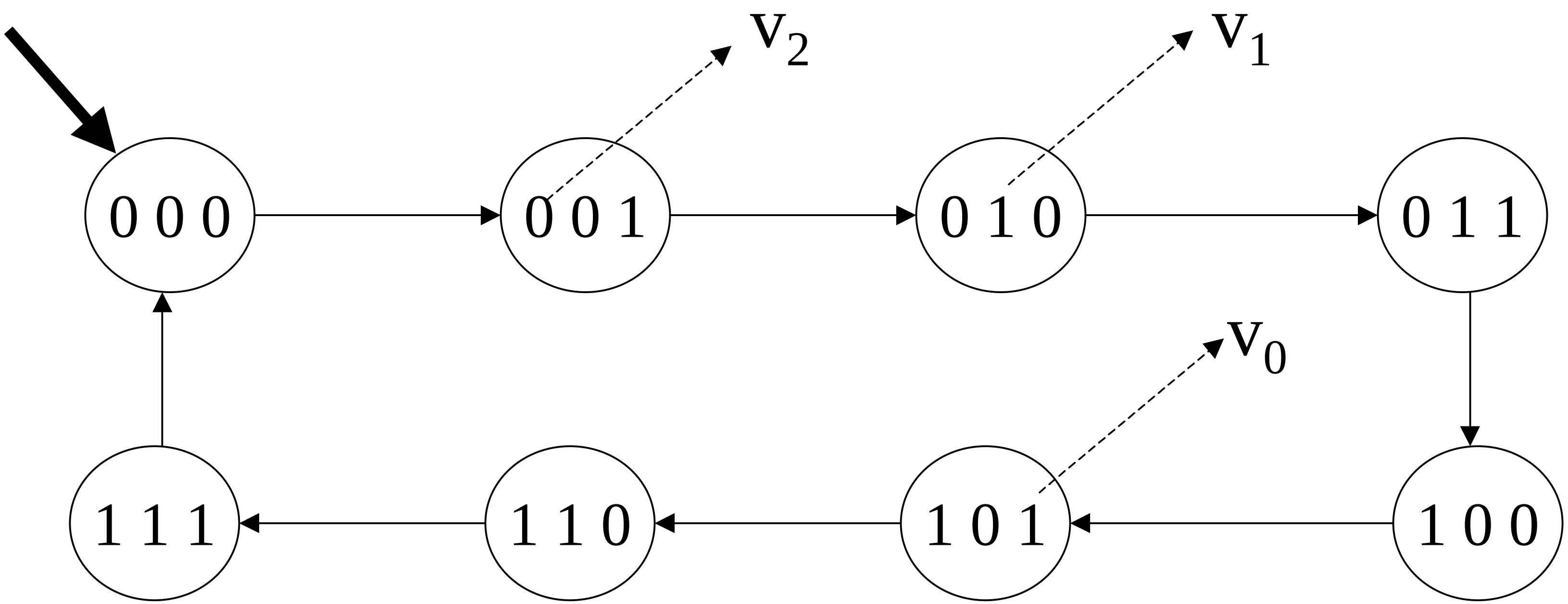
State Predicates



A set of states can be picked out by a formula;

$\mathbf{X = v_2 \vee v_0}$ is the set {100, 101, 110, 111, 001, 011}

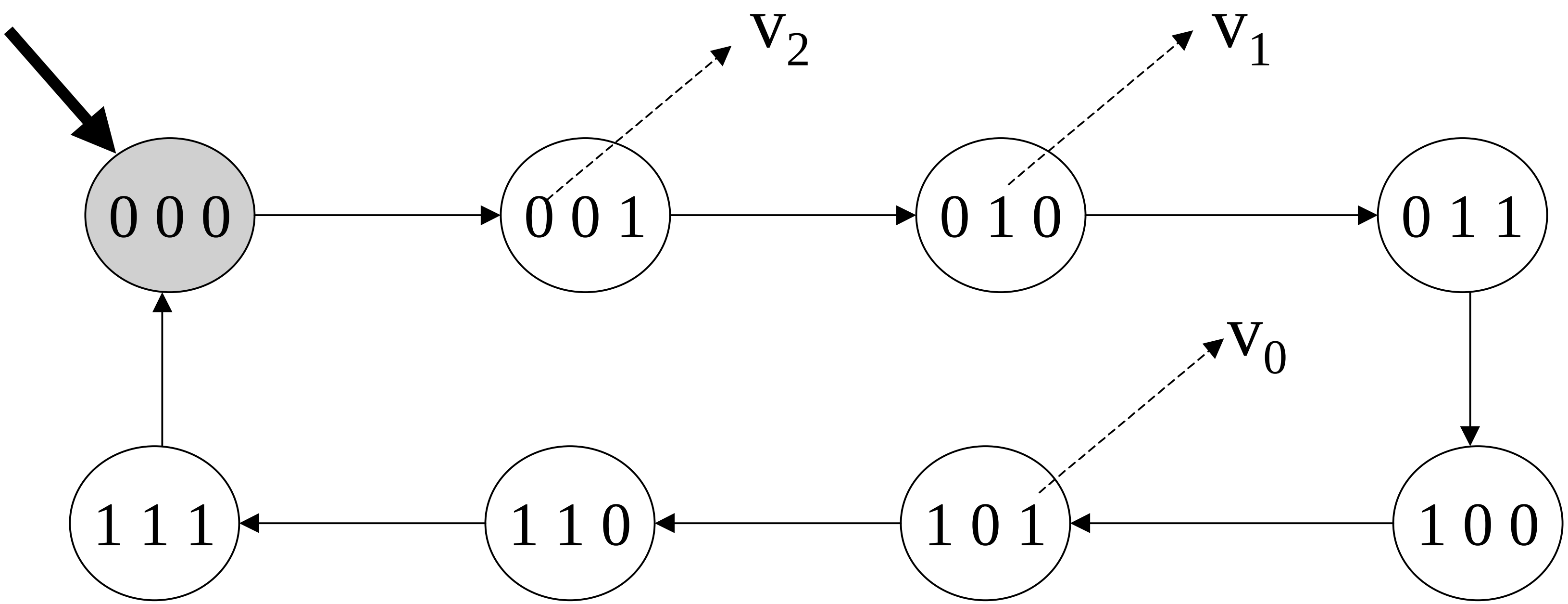
Initial States Predicate



A set of states can be picked out by a formula;

$\mathbf{X' = \neg v_2 \land \neg v_1 \land \neg v_0}$

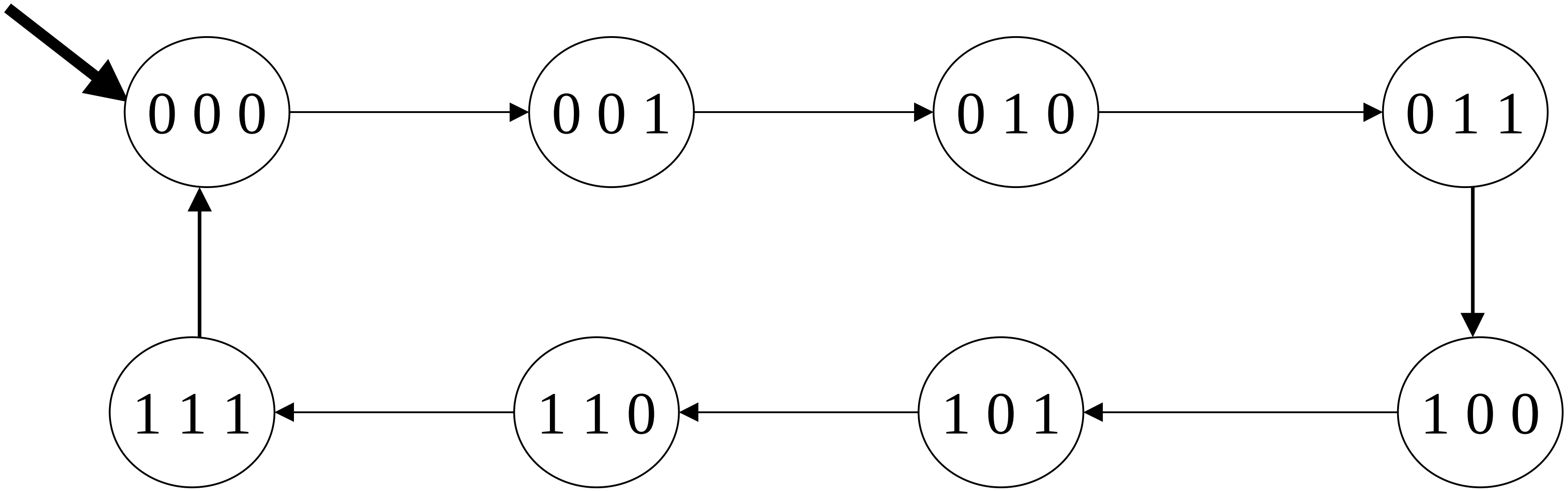
Initial States Predicate



A set of states can be picked out by a formula;

$\mathbf{X'} = \neg \mathbf{v_2} \wedge \neg \mathbf{v_1} \wedge \neg \mathbf{v_0} \quad \mathbf{X'} = \{ \mathbf{S_0} \} = \{ 000 \}$

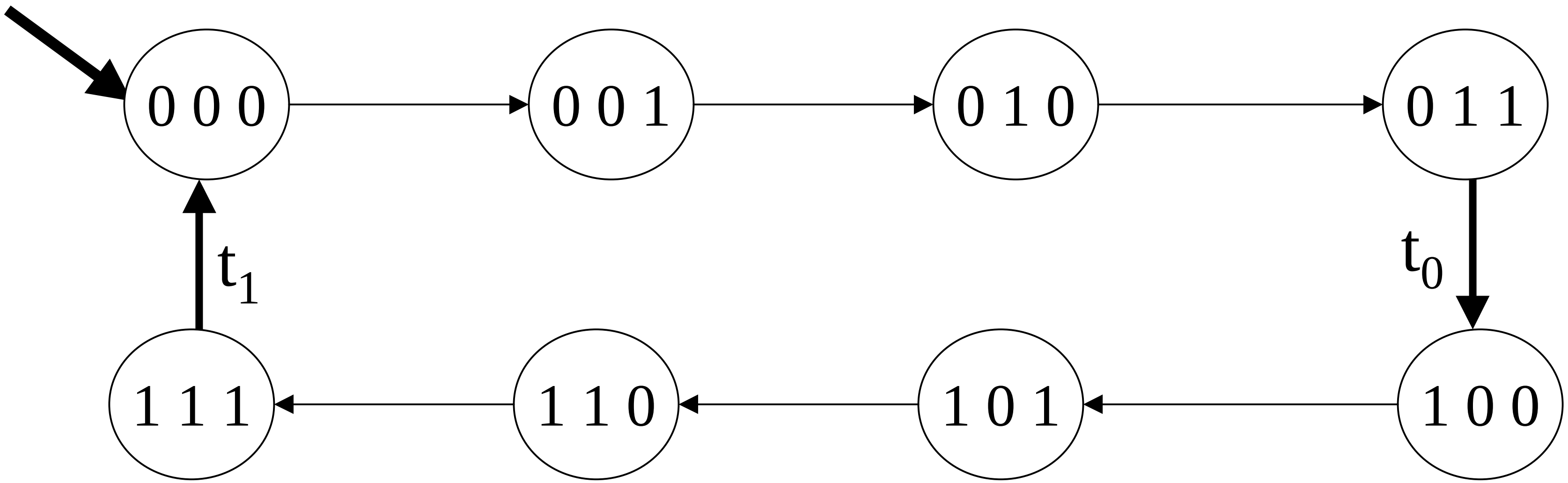
Transition relation predicate



A set of transitions can also be picked out by a formula.

$\mathbf{R_2} = \mathbf{v_2' = (v_0 * v_1) \oplus v_2}$ $\mathbf{v_2}$ – old value $\mathbf{v_2'}$ – new value

Transition relation predicate

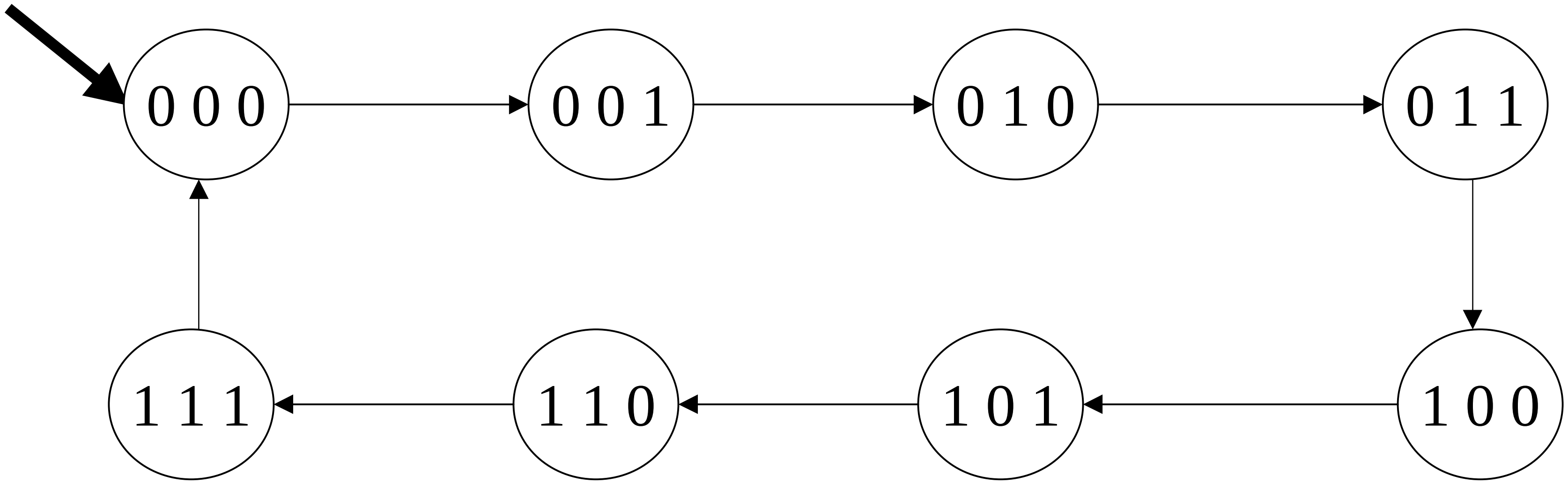


A set of transitions can also be picked out by a formula.

$\mathbf{R_2} = \mathbf{v_2' = (v_0 \wedge v_1) \oplus v_2}$ $\mathbf{v_2}$ – old value $\mathbf{v_2'}$ – new value

$\{t_0, t_1\} \subseteq \mathbf{R_2}$

Transition relation predicate

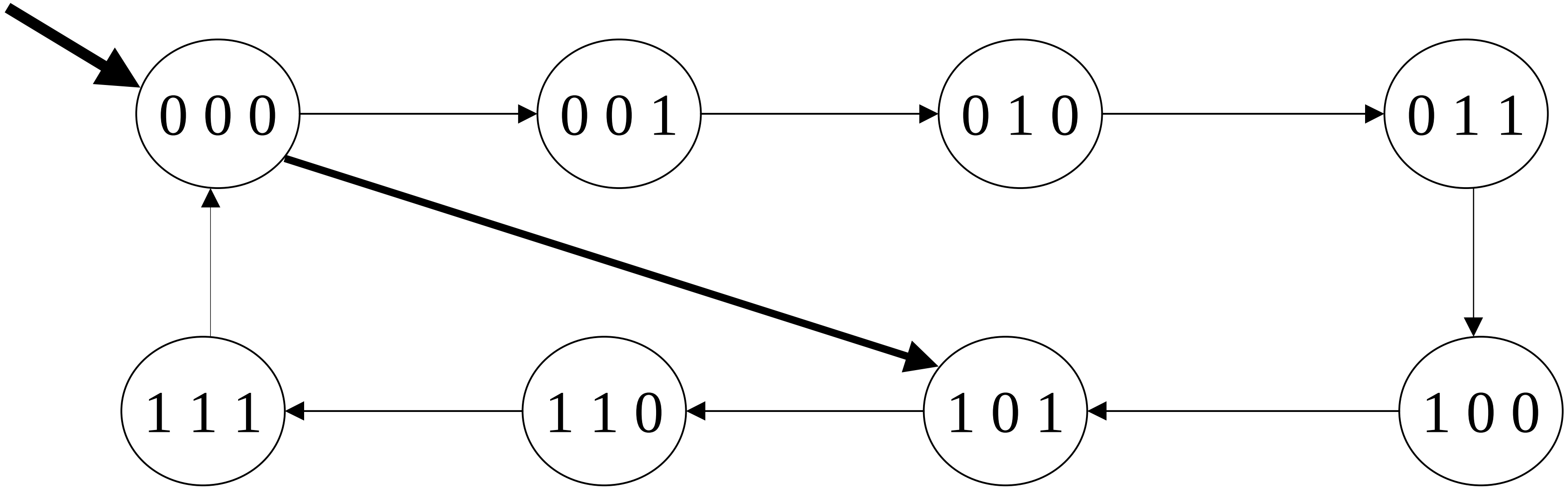


T = Formula(v_2, v_1, v_0, v_2', v_1', v_0'}

Not all formulas will define subsets of transitions.

You must pick the right formula .

Transition relation predicate

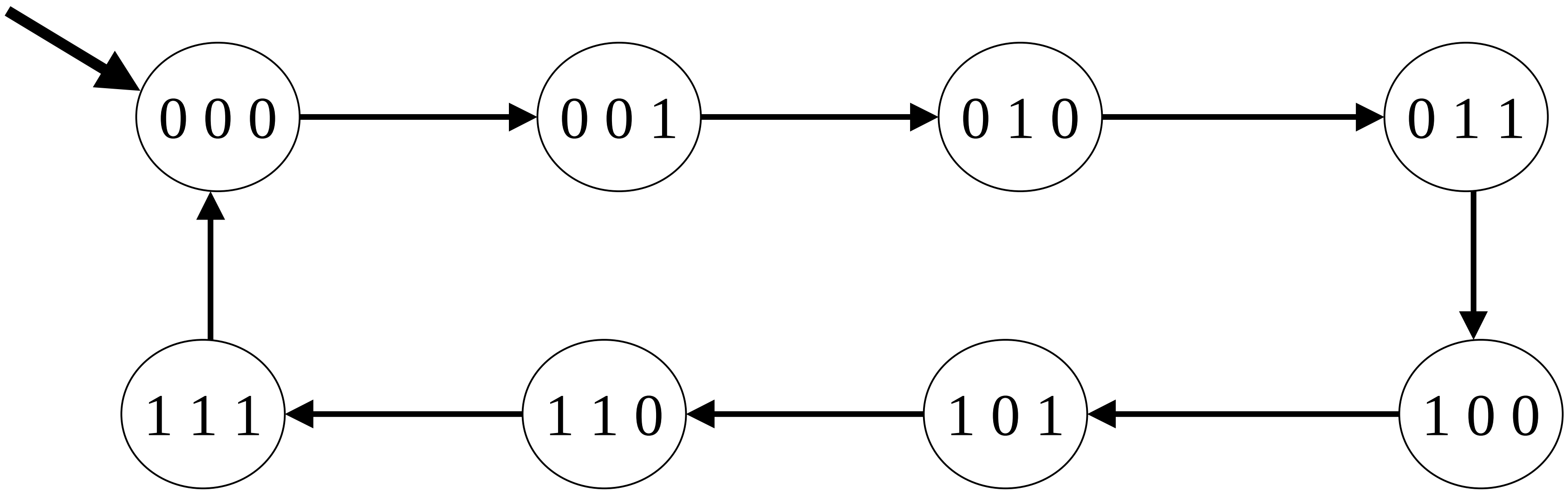


$\mathbf{R_0} = \mathbf{v_0'} \neq \mathbf{v_0}$ $\mathbf{v_0}$ – old value $\mathbf{v_0'}$ – new value

$\mathbf{R_0} = \{(000) \longrightarrow (101)$,……..$\}$

But this is not a transition!

Transition relation predicate



$\mathbf{R_0} = \mathbf{v_0' \neq v_0}$ $\mathbf{v_i}$ – old value $\mathbf{v_i'}$ – new value

$\mathbf{R_1} = \mathbf{v_1' = (v_0 \oplus v_1)}$

$\mathbf{R_2} = \mathbf{v_1' = (v_0 \wedge v_1) \oplus v_2}$

$\mathbf{R} = \mathbf{R_0 \wedge R_1 \wedge R_2}$

Summary of Predicates

- System variables v_0, v_1, v_2, ….v_n.
- Each $\mathbf{v_i}$ has a domain of values
  - Boolean , {a,b,c,..}, {5,8,0,7}…
  - Each domain is required to be finite.
- A state is a function **s** which assigns to each system variable a value in its domain.
- The set of states is ***finite.***

Summary

- Predicates can be used to pick out –succinctly- sets of states (useful for identifying initial states).
- **X** = Formula (v_0, v_1, v_2,...,v_n)
- But this works only when all domains are boolean.
- In general Formula will be a first order formula.

Summary

- A set of transitions can also be picked out using predicates.
- **T** = Formula $(v_0, v_1,..v_n, \ v_0', v_1'..,v_n'\}$
- **T** is the set of all transitions

  $(v_0, v_1, \ldots, v_n) \longrightarrow (v_0', v_1', \ldots, v_n')$

  such that Formula (above!) is satisfied.
- Not all (state or transition) formulas will be legitimate.

Why use formulas?

- Once and for all, say how to go from the “logical” description to Kripke structures.
- Once we have a Kripke structure, we are in business.
- We can use
  - temporal logics to specify properties
  - Model checking to verify these properties.

First Order Logic

- The general structure :
  - Syntax
    - Formulas
  - Semantics
    - When is a formula true ?
    - Models
      - Interpretations
      - Valuations

Syntax

- Terms
  - Variables
  - Functions symbols, constant symbols
- Atomic formulas
  - Relation symbols, equality, terms
- Formulas
  - Atomic formulas
  - Propositional connectives
  - ***Existential and universal quantifiers***

Syntax

- (individual) variables --- **x, y, v_3, v'**,…
  - System variables in our context
- Function symbols : $\mathbf{f}^{(\mathbf{n})}$
  - **n** is the arity of **f**.
  - Add $^{(2)}$
  - Next $^{(1)}$
- Function symbols will capture the functions used in the programs, circuits, …

Constant symbols

- Apart from variables, it will also be convenient to have constant symbols.
  - ***zero*** , ***five***, …
- Variables can be assigned different values but a constant symbol is assigned a fixed value.

Terms

- Terms are used to point at values.
- A variable is a term.
  - **x , v , v''**
- A constant symbol is a term.
- Suppose *f* is a function symbol of arity n and $t_1, t_2, \ldots, t_n$ are terms then $f(t_1, t_2, \ldots, t_n)$ is also a term.

Terms

- Let Plus be a function symbol of arity 2.
- $\mathbf{v_1}$ **,** $\mathbf{v_2}$, **Plus(**$\mathbf{v_2}$**, Plus(**$\mathbf{v_1}$**,** $\mathbf{v_1}$**))** are terms.
  - the semantics of the last term is intuitively $\mathbf{v_2 + 2v_1}$
- Let weird_op be a function symbol of arity 3
- Then

  Plus(weird_op(**v**, Plus($\mathbf{v_1}$, $\mathbf{v_2}$), ***five***), Plus(**v**, **v"**))

  is a term.

Predicates

- Relation (predicate) symbols :
  - ***P*** which also has an arity
  - ***Greater-Than*** has arity 2
  - ***Prime*** has arity 1
  - ***Middle*** has arity 3 **-- *Middle*(t$_1$, x, t$_2$)**
    - intuitively, **x** lies between **t$_1$** and **t$_2$**
- ***Equal*** has arity 2
  - will be denoted as **=**
  - It is a "**constant**" relation symbol.

Atomic formulas.

- If $\mathbf{t_1}$ and $\mathbf{t_2}$ are terms then $\mathbf{=(t_1, t_2)}$ is an atomic formula.
  - also written $\mathbf{t_1 = t_2}$
- Suppose $\boldsymbol{P}$ has arity $\mathbf{n}$ and $\mathbf{t_1, t_2, \ldots, t_n}$ are terms.
- Then $\boldsymbol{P}\mathbf{(t_1, t_2, \ldots, t_n)}$ is an atomic formula.

Atomic formulas

- ***Greater-Than***(***five***, ***zero***)
- ***Greater-Than***(***two***, ***four***)
- ***Prime*****(Plus(v_1, v”))**
- **Plus(v,Zero) = weird_op(v,v,four)**
- **v = *Greater_Than*(v_1,v_2)** is ***not*** an atomic formula !

Terms and Predicates

- A term is meant to denote a value.
  - Makes no sense to talk about a term being true or false.
- An atomic formula may be true or false (depends on the interpretation).
  - Does not make sense to associate a value with an atomic formula.

Formulas

- Every atomic formula is a formula.
- If φ is a formula then $\neg\varphi$ is a formula.
- If φ and φ' are formulas then $\varphi \vee \varphi'$ is a formula.
- $\varphi \wedge \varphi'$ abbreviates: $\neg(\neg\varphi \vee \neg\varphi')$
- $\varphi \supset \varphi'$ abbreviates : $\neg\varphi \vee \varphi'$
- $\varphi \equiv \varphi'$ abbreviates : $(\varphi \supset \varphi') \wedge (\varphi' \supset \varphi)$

Formulas

- If φ is a formula and $\mathbf{x}$ is a variable then $\exists \mathbf{x}.\, \varphi$ is a formula.
- $\forall \mathbf{x}.\, \varphi$ abbreviates : $\neg \exists \mathbf{x}.\, \neg \varphi$
- These are ***existential*** and ***universal*** quantifiers.
- The power of first order logic comes from these operators!

Semantics

- **Models** :
  - ***Domain of interpretation***
  - ***Interpretation***
    - For the function, constant and relation symbols.
      - ***Fixed for all formulas***.
    - For the individual variables, on a “per formula” basis.
      - ***Valuations***.

Semantics

- ***Domain***
  - Each variable will have its domain of values.
  - We pretend all these domains are the same.
  - Or rather, a big enough “universe” that will contain all these domains.
- Fix **D** the universe of values.

Semantics

Interpretation function I

- Assign a concrete function to each function symbol (of the same arity!)
- Assign a concrete member of **D** to each constant symbol.
- Assign a concrete relation to each relation symbol (of the same arity!).

Semantics

- **D ---** The set of integers.

- ***Plus*** $\xrightarrow{I}$ +
- ***Greater_Than*** $\xrightarrow{I}$ >
- ***Zero*** $\xrightarrow{I}$ **0**
- ***weird_op*** $\xrightarrow{I}$ $\boldsymbol{f}$ where for each i, j, k

  $f(i, j, k) = 2i + 3j - 17k$

Semantics

- Assume we have fixed an ***interpretation*** for all function symbols, constant symbols and relational symbols.
- Let φ be a formula. Fix a ***valuation*** **V** which assigns a member of **D** to each ***variable***.
- **V** : Variables $\longrightarrow$ **D**

Semantics

- Let φ be a formula. Fix a valuation **V** which assigns a member of **D** to each variable.
- **V** : Variables $\longrightarrow$ **D**
- This extends to a valuation **V_T** for **all terms**!
  - **V_T**(***v***) = **V**(***v***)   if ***v*** is a variable.
  - **V_T**(***c***) = **d**   if ***c*** is a constant symbol and the interpretation we have fixed assigns the value d to ***c***.

Semantics

- Let φ be a formula. Fix a valuation **V** which assigns a member of **D** to each variable.
- **V** : Variables $\longrightarrow$ **D**
- This extends to a valuation **V_T** for **all terms**!
  - Suppose $\boldsymbol{f}$ is of arity n and $\boldsymbol{t_1, t_2, \ldots, t_n}$ are terms with $\mathbf{V\_T}(\boldsymbol{t_1}) = d_1$, …. $\mathbf{V\_T}(\boldsymbol{t_n}) = d_n$.
  - Suppose $\boldsymbol{f}$ has been assigned the function F by our interpretation. Then
  - $\mathbf{V\_T}(\boldsymbol{f(t_1, t_2, \ldots, t_n)}) = F(d_1, d_2, \ldots, d_n)$.

Semantics

- Let φ be a formula. Fix a valuation **V** which assigns a member of **D** to each variable.
- So we now have **V_T** that assigns a member of **D** each term.
- φ is satisfied under **V** (and the interpretation we have fixed for all formulas) if :
- suppose $\boldsymbol{P(t_1, t_2,.., t_n)}$ is an atomic formula

  and **V_T**($\boldsymbol{t_1}$) = d_1, ….**V_T**($\boldsymbol{t_n}$) = d_n

  and PCON is the relation assigned to ***P*** by our interpretation.

Semantics

- Suppose ***P(t1, t2,.., tn)*** is an atomic formula and $\mathbf{V\_T}(\boldsymbol{t_1}) = d_1$, ….$\mathbf{V\_T}(\boldsymbol{t_n}) = d_n$ and PCON is the relation assigned to ***P*** by our interpretation.
- Then $\boldsymbol{P(t_1, t_2,.., t_n)}$ is satisfied under **V** iff PCON$(d_1, d_2,\ldots,d_n)$ holds in **D**.
  $(d_1, d_2, \ldots.,d_n) \in$ PCON $\subseteq \mathbf{D} \times \mathbf{D} \times \ldots \times \mathbf{D}$

Semantics

- Suppose φ is of the form $\neg\varphi'$.

  then φ is satisfied under **V** iff φ' is **not** satisfied under **V**.
- Suppose φ is of the form $\varphi_1 \vee \varphi_2$

  then φ is satisfied under **V** iff φ_1 is satisfied under **V** or φ_2 is satisfied under **V**.

Semantics

- The only case left is when φ is of the form $\exists \mathbf{x}.\varphi'$.
- φ is satisfied under **V** iff there is a valuation **V'** such that φ' is satisfied under **V'**, and **V'** is required to meet the condition :
  - **V'** is exactly **V** for all variables except **x**.
  - for **x** , **V'** can assign any value in **D** of its choosing.

Semantics II

- Models :
  - **Domain** of interpretation
  - **Interpretation**
    - For the function, constant and relation symbols.
      - Fixed for all formulas.
    - For the individual variables, on a per formula basis.
      - **Valuations**.

Semantics II

- Assign a concrete function to each function symbol (of the same arity!)
- Assign a concrete member of **D** to each constant symbol.
- Assign a concrete relation to each relation symbol (of the same arity!).

Semantics II

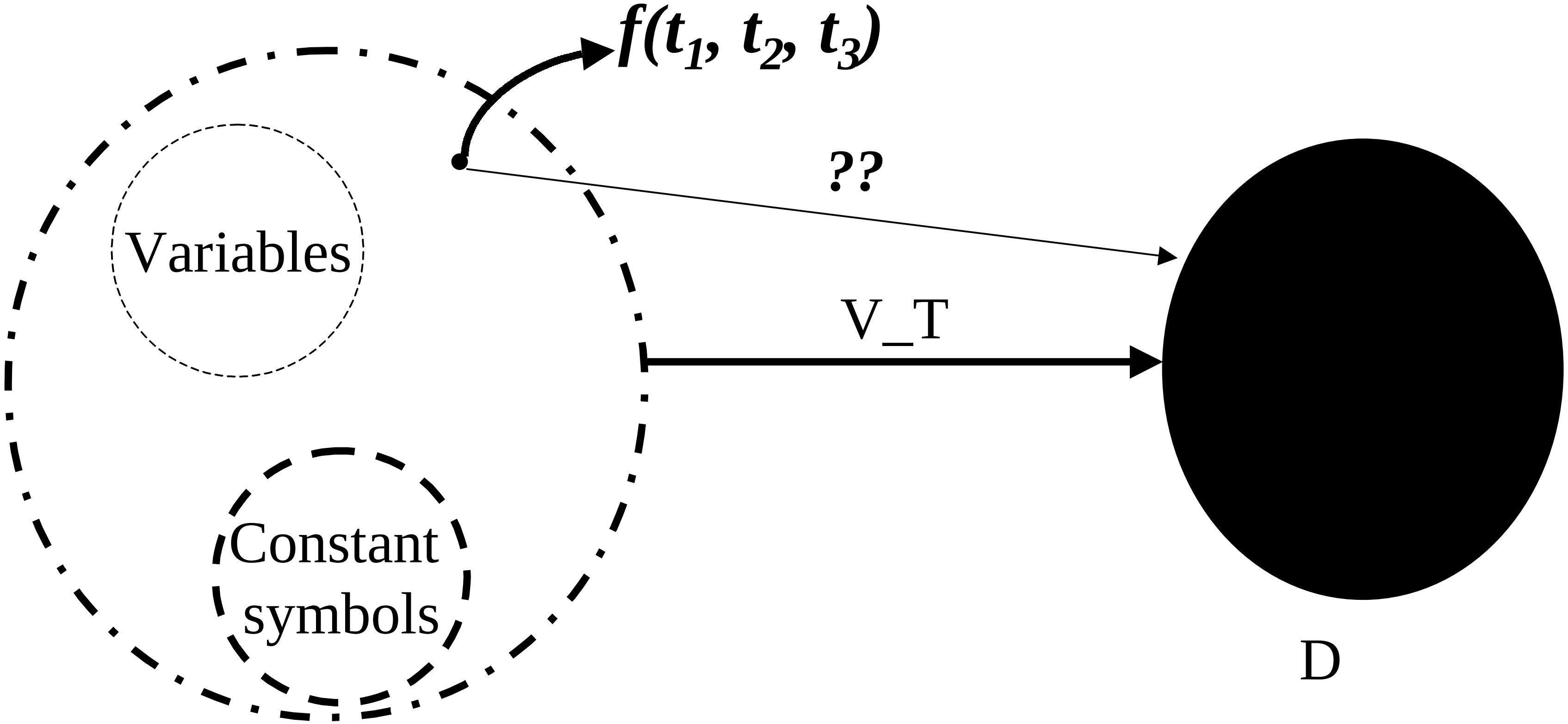
- Assume we have fixed an interpretation for all function symbols, constant symbols and relational symbols.
- Let φ be a formula. Fix a **valuation V** which assigns a member of **D** to each variable.
- $\mathbf{V} : \text{Variables} \longrightarrow \mathbf{D}$

Lift V to All Terms

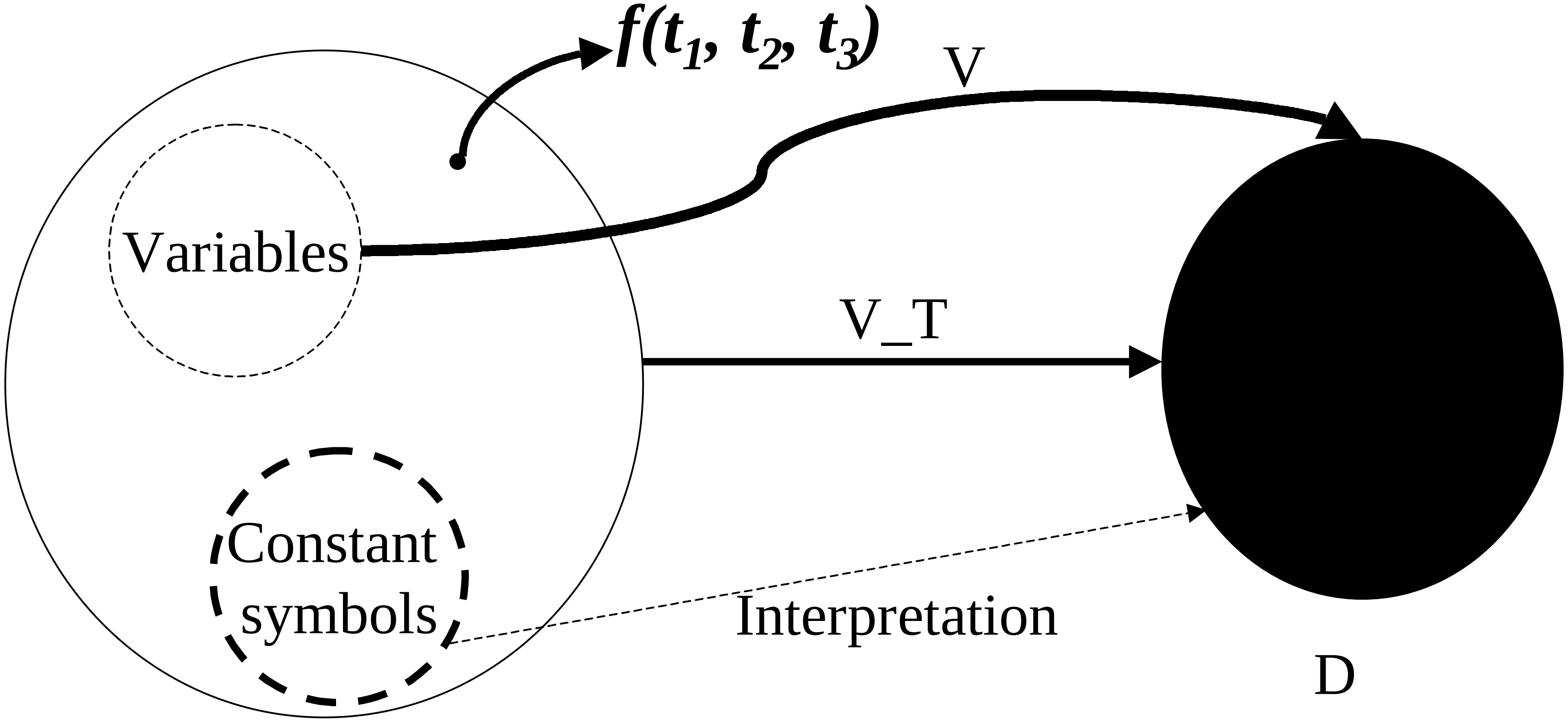
- We have :
  - An interpretation for the function symbols and constant symbols.
  - **V** : Variables $\longrightarrow$ **D**
- Using this, we can construct (uniquely!)

V_T : Terms $\longrightarrow$ **D**

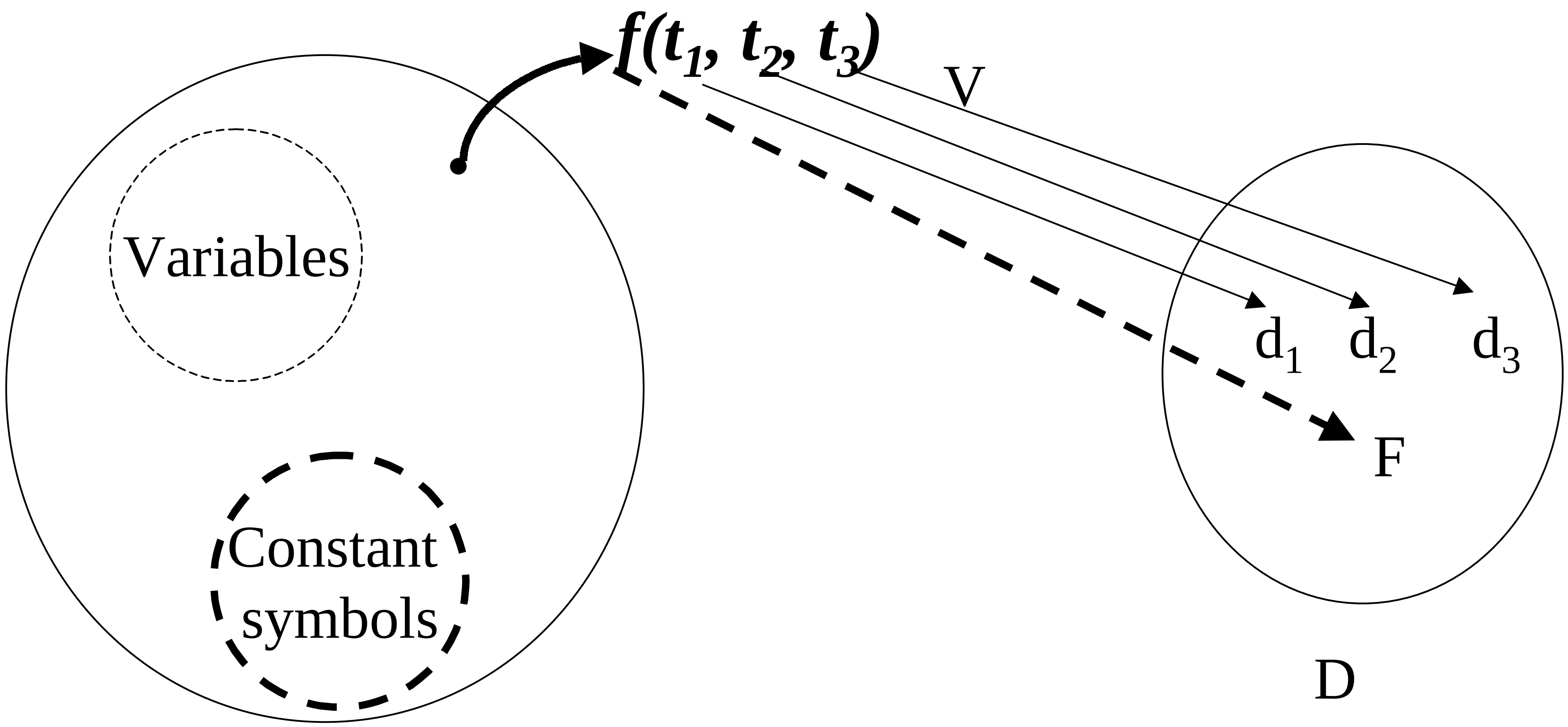
Constructing V_T



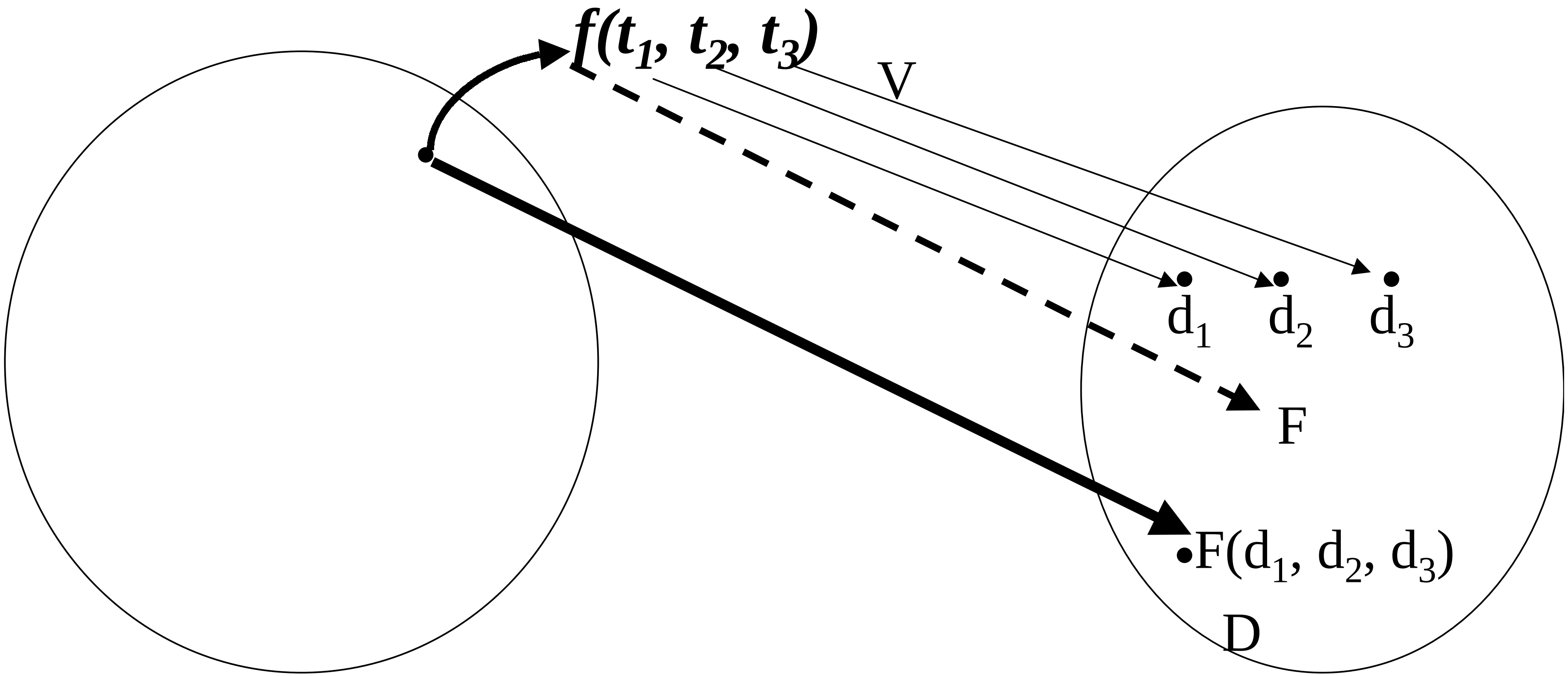
Constructing V_T



Constructing V_T



Constructing V_T



Semantics II

- Let φ be a formula. Fix a valuation **V** which assigns a member of **D** to each variable.
- So we now have **V_T** that assigns a member of **D** each term.
- φ is satisfied under **V** (and the interpretation we have fixed for all formulas) if :

Semantics II

- Suppose $\boldsymbol{P(t_1, t_2,.., t_n)}$ is an atomic formula and $\mathbf{V\_T}(\boldsymbol{t_1}) = d_1, \ldots .\mathbf{V\_T}(\boldsymbol{t_n}) = d_n$ and PCON is the relation assigned to ***P*** by our interpretation.
- Then $\boldsymbol{P(t_1, t_2,.., t_n)}$ is satisfied under **V** iff PCON$(d_1, d_2, \ldots, d_n)$ holds in **D**.
  $(d_1, d_2, \ldots., d_n) \in \text{PCON} \subseteq \mathbf{D} \times \mathbf{D} \times \ldots \times \mathbf{D}$

Semantics II

- Suppose φ is of the form $\neg\varphi'$.
- Then φ is satisfied under **V** iff φ' is not satisfied under **V**.
- Suppose φ is of the form $\varphi_1 \vee \varphi_2$
- Then φ is satisfied under **V** iff φ_1 is satisfied under **V** or φ_2 is satisfied under **V**.

Semantics II

- ***Greater-Than***(**Plus**(***v***, ***3***), **Multi**(***x***, ***2***))

  t_1 t_2

- V(v) = 2 V(x) = 1
- $V\_T(t_1) = 5$ $V\_T(t_2) = 2$
- $(5, 2) \in > \subseteq$ Integers $\times$ Integers
- V'(v) = 1 V'(x) = 6
- Under V', the atomic formula is not true.

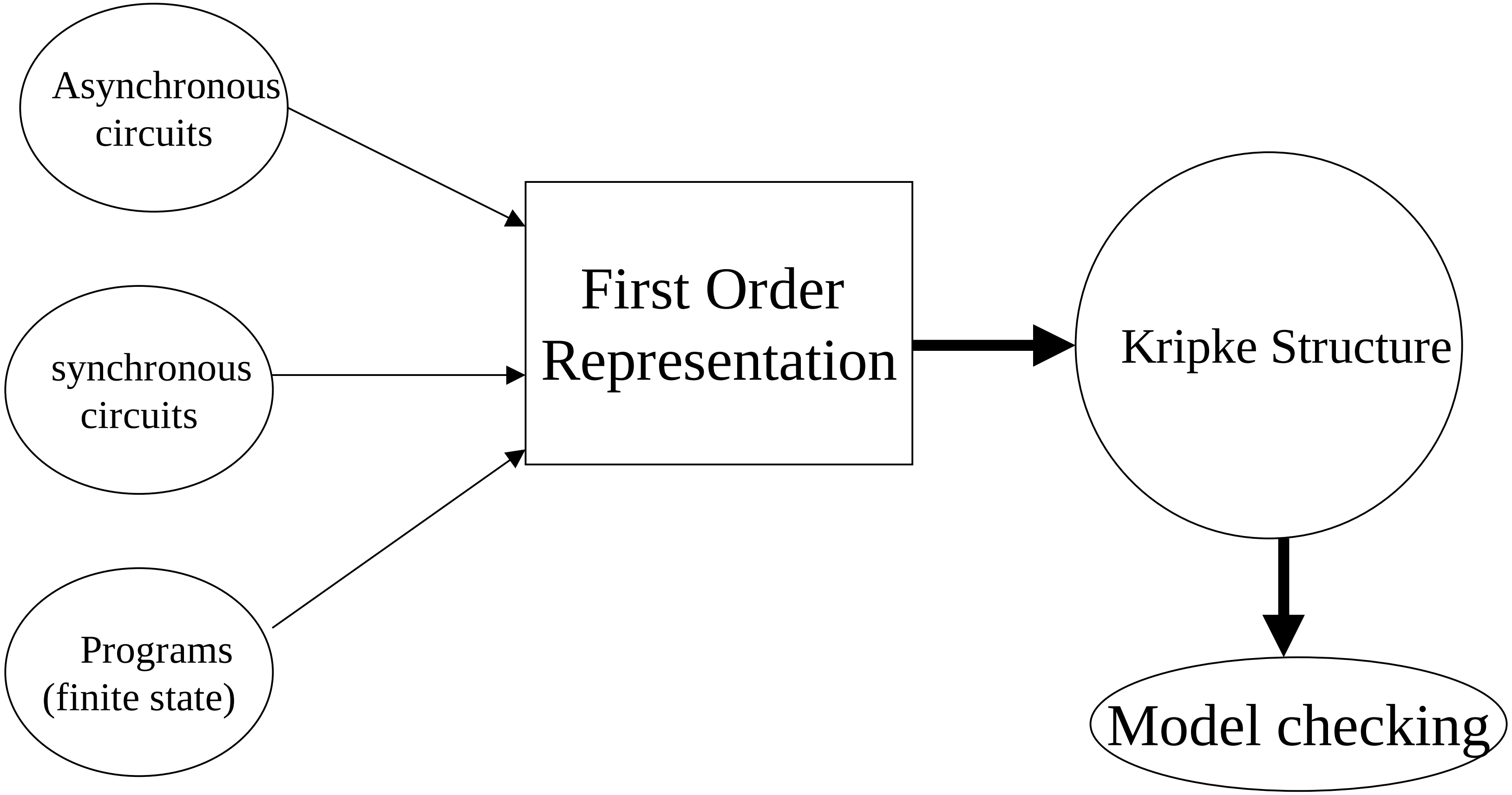
Semantics II

- The only case left is when φ is of the form $\exists \mathbf{x}.\,\varphi'$
- φ is satisfied under **V** iff there is a valuation **V'** such that φ' is satisfied under **V'** and **V'** is required to meet the condition :
  - **V'** is exactly **V** for all variables except **x**.
  - For **x** , **V'** can assign any value of its choosing.

Semantics II

- Whether $\exists \mathbf{x}.\, \varphi$ is true or not under **V**
  - does not depend on what **V** does on **x** !
- $\exists \mathbf{x}.\, \mathbf{2x = y}$ is true under $\mathbf{V(y) = 4}$
  $\mathbf{V(x) = 1}$!
- Because, we can find **V'** with $\mathbf{V'(y) = 4}$ but
  $\mathbf{V'(x) = 2}$.
- One says **x** is bound in the formula and **y** is free.

The efficient way



First Order Representation to Transition Systems

- $\{\mathbf{v_1, v_2, .., v_n}\}$--- System variables.
- $\mathbf{D_1, D_2, .., D_n}$ --- The corresponding domains.
- $\mathbf{D = \bigcup D_i}$
- $\mathbf{s : \{v_1, v_2, \ldots, v_n\}} \longrightarrow \mathbf{D}$ such that

  $\mathbf{s(v_1) \in D_1}$ **…..**
- **S ---** The set of states.

Initial States

- $S_0(v_1, v_2, .., v_n)$ is a FO formula describing the set of initial states.
- Atomic formula
  - $v = d$ where ***v*** is is a system variable and ***d*** is a constant symbol interpreted as a member of the domain of ***v***.

Example:

- "$\mathbf{S_0}$ is the set of all states where the **pc = 0** and input is a power of 2"
- $\exists n. (input = EXP(n)) \wedge (pc = 0)$

Transition relation

- ***R*** $\boldsymbol{(v_1, v_2, ..v_n, v_1', v_2',..,v_n')}$ is a FO formula involving the variables $v_1, v_2,..v_n$ (the system variables) and the new variables $(v_1', v_2',.., v_n')$.
- $\mathbf{(d_1, d_2,..,d_n)} \longrightarrow \mathbf{(d_1', d_2',..,d_n')}$ iff

  R $\boldsymbol{(v_1, v_2, ..v_n, v_1', v_2',..,v_n')}$ is true under the valuation $\mathbf{v_1 = d_1,\ldots, v_n = d_n, v_1' = d_1',..v_n' = d_n'}$.

Transition Relation

- V = {x, y, z}
- Program : {x, y, z, pc}

  l_0 : begin

  l_1 : $statement_1$

  l_2 : $statement_2$

  …

  l_5 : if even(x) then x = x/2 else x = x –1

  l_6 : …

Transition Relation

- **V = {x, y, z}**
- Program : **{x, y, z, pc}**

  l_5 : if even(x) then x = x/2 else x = x –1

  l_6 : …

- **φ (x, y, z, pc, x’, y’, z’, pc’)**
- **$pc = l_5 \wedge \ pc' = l_6 \ \wedge (\exists n. (x = 2n) \supset x' = x/2) \wedge$**
  $(\neg \exists n. (x = 2n) \supset x' = x\text{-}1) \wedge$ same(y, z)

which is equivalent to

- **$pc = l_5 \wedge \ pc' = l_6 \ \wedge$**
  $((\exists n.(x=2n) \wedge x'=x/2) \vee (\neg\exists n.(x=2n) \wedge x'=x\text{-}1))) \wedge$
  same(y, z)
- **same(y, z) --- $y' = y \ \wedge z' = z$**

Transition Relation

- In a similar fashion , we can construct transition relation formulas for :
  - Assignment statement
  - While statements
  - etc.etc.
  - See the text book!

Kripke Structures

- **AP** is a finite set of atomic propositions.
  - "**value of x is 5**"
  - "**x = 5**"
- **M = (S, S_0, R, L)**, a Kripke Structure.
  - **(S, S_0, R)** is a transition system.
  - **L : S $\longrightarrow$ 2^{AP}**
  - **2^{AP}** **----** The set of subsets of AP

Kripke Structures

- The atomic propositions and **L** together convert a transitions system into a model.
- We can start interpreting ***formulas*** over the ***Kripke structure***.
- The atomic propositions make basic (easy) assertions about system states.

Automata and Kripke Structures

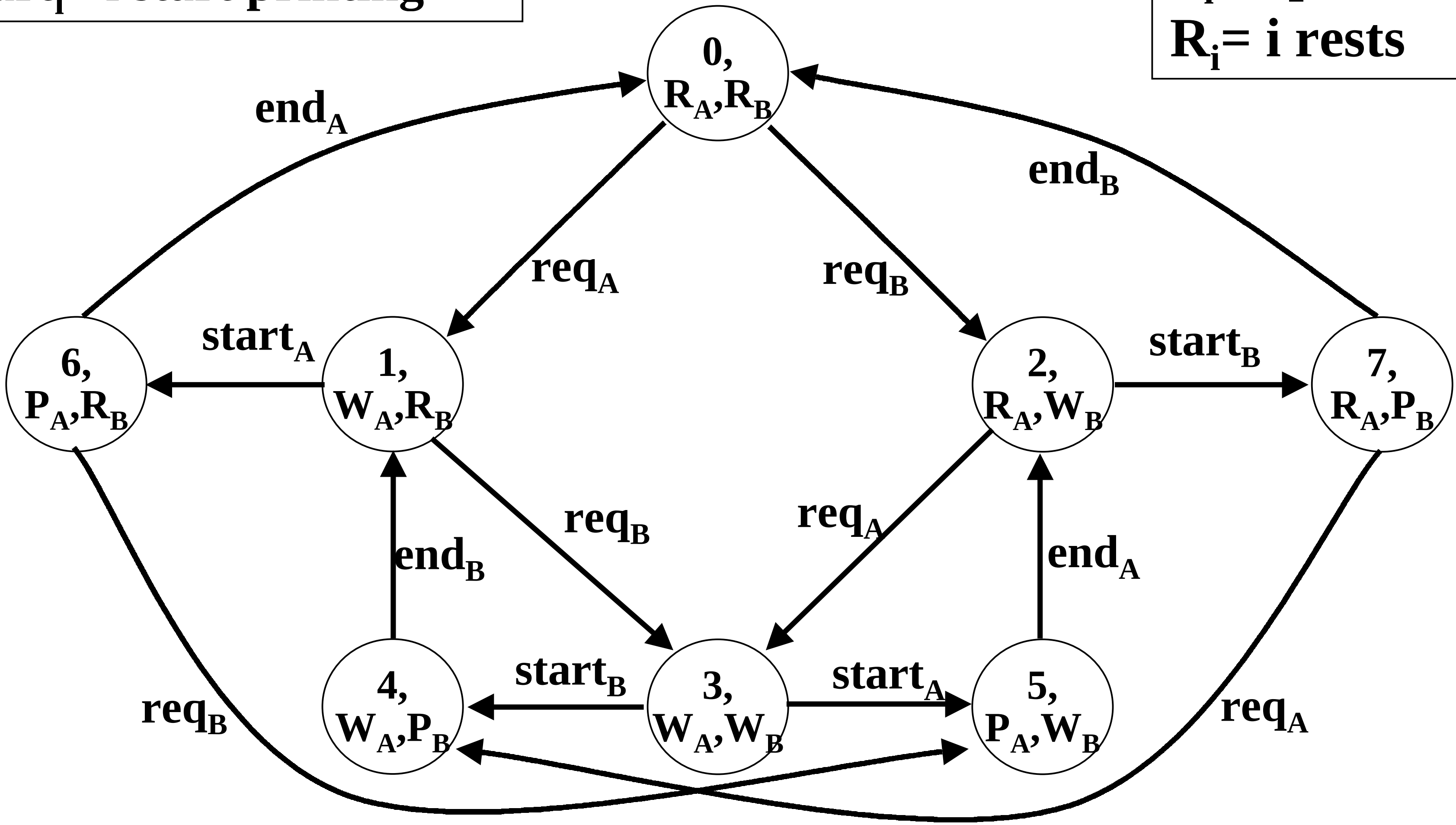
- **AP - set of elementary property**
- **$\langle S,A,R,s_0,L \rangle$**
- **S - set of states**
- **A - set of transition labels**
- **$R \subseteq S \times A \times S$ - (labeled) transition relation**
- **L - interpretation mapping $L: S \longrightarrow 2^{AP}$**

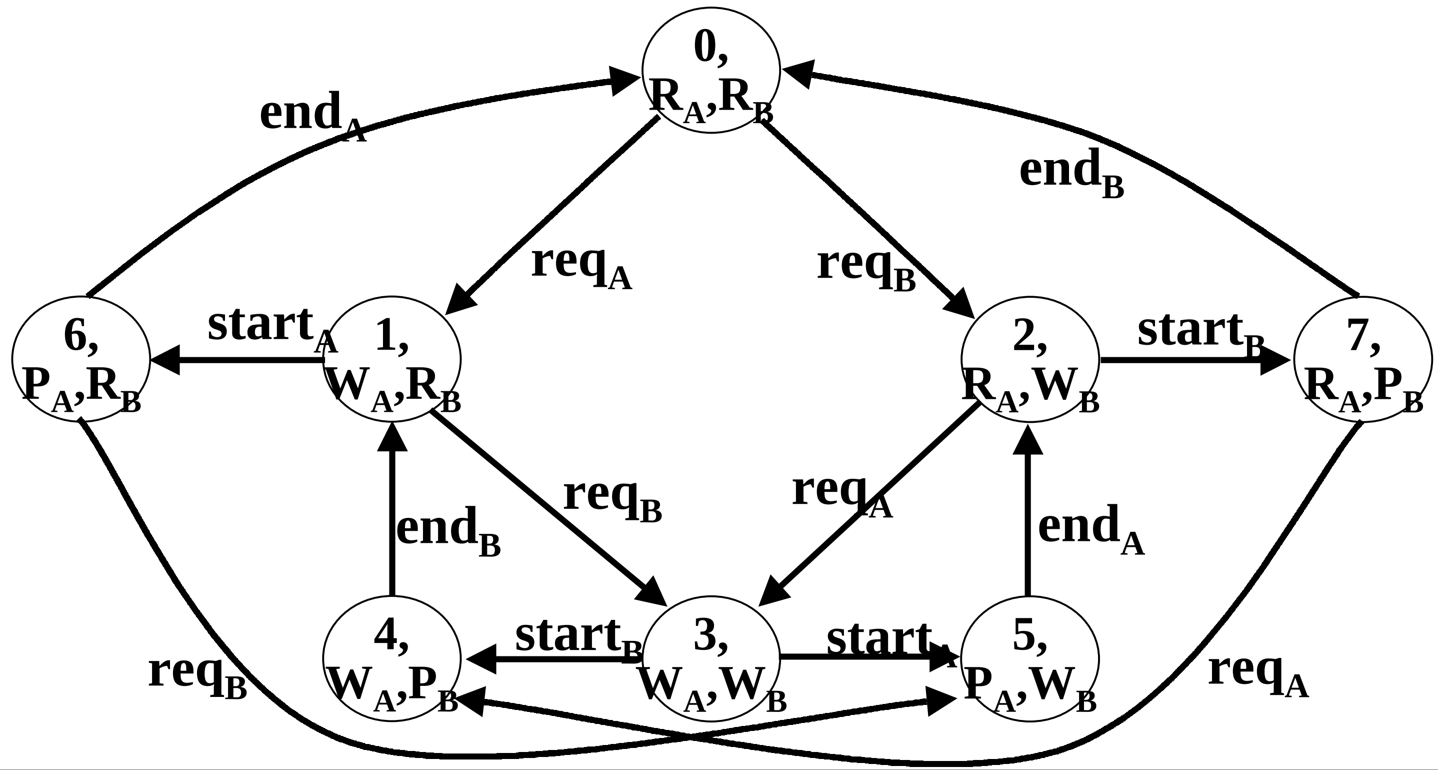
Example: a print manager

end_i= i ends printing
req_i= i requests printing
$start_i$= i start printing

AP

W_i= i waits
P_i= i prints
R_i= i rests





- **S = {0,1,2,3,4,5,6,7}**
- **A = {end_A,end_B, req_A, req_B, $start_A$, $start_B$}**
- **R = {(0,req_A,1), (0,req_B,2), (1,req_B,3), (1,$start_A$,6), (2,req_A,3), (2,$start_B$,7), (3,$start_A$,5), (3,$start_B$,4), (4,end_B,1), (5,end_A,2), (6,end_A,0), (6,req_B,5), (7,end_B,0), (7,req_A,4),}**
- **L = {0$\rightarrow$ {R_A,R_B}, 1$\rightarrow$ {W_A,R_B}, 2$\rightarrow$ {R_A,W_B}, 3$\rightarrow$ {W_A,W_B}, 4$\rightarrow$ {W_A,P_B}, 5$\rightarrow$ {P_AW_B}, 6$\rightarrow$ {P_A,R_B}, 7$\rightarrow$ {R_AP_B} }**

Properties of the printing systems

- Every state in which $\mathbf{P_A}$ holds, is preceded by a state in which $\mathbf{W_A}$ holds
- In any state in which $\mathbf{W_A}$ holds is followed (possibly not immediately) by a state in which $\mathbf{P_A}$ holds.
- The first can easily be checked to be true
- The second is false (e.g. 0134134134…) - in other words the system is ***not fair***.

Synchronization

- Usually complex systems are composed of a number of smaller ***subsystems*** (***modules***)
- It is natural to model the whole system starting from the models of the subsystems.
- And then define how they cooperate.
- There are many ways to define cooperation (***synchronization***).

Synchronization: no interaction

The system model is just the ***cartesian product*** of the simple modules.

Let $\boldsymbol{TS_1},\ldots,\boldsymbol{TS_n}$ be $\boldsymbol{n}$ automata (or **TS**), where $\boldsymbol{TS_i=<S_i,A_i,R_i,s_{i0}>}$

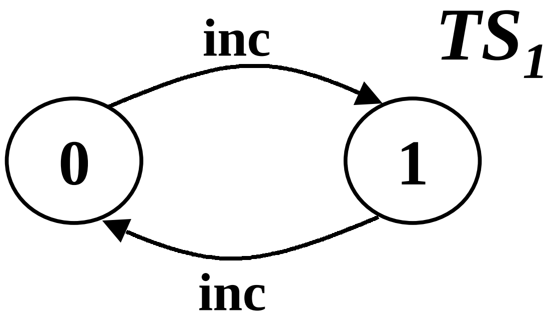
The system is then defined as $\boldsymbol{TS=<S,A,R,s_0>}$ where

$$\boldsymbol{S = S_1 \times S_2 \times \ldots \times S_n}$$

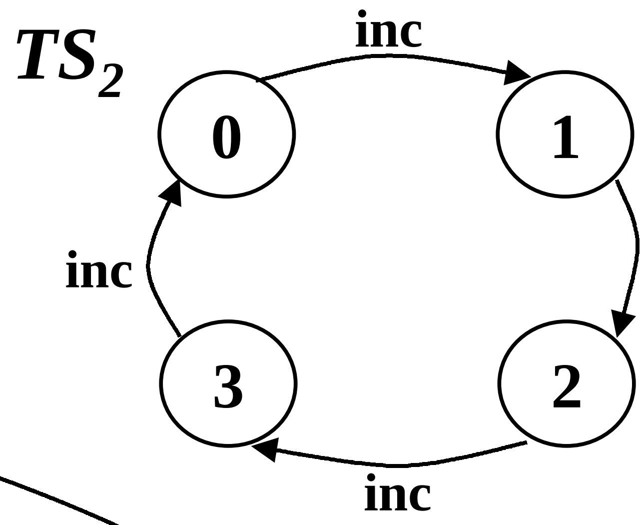
$$\boldsymbol{A = A_1 \cup \{-\} \times A_2 \cup \{-\} \times \ldots \times A_n \cup \{-\}}$$

$$\boldsymbol{R = \{((s_1,s_2,\ldots,s_n),(a_1,a_2,\ldots,a_n),(s'_1,s'_2,\ldots,s'_n)) \mid \text{forall } i, \; a_1 \neq - \text{ and } (s_i,a_i,s'_i) \in R_i, \text{ or } a_1 = - \text{ and } s'_i = s_i\}}$$

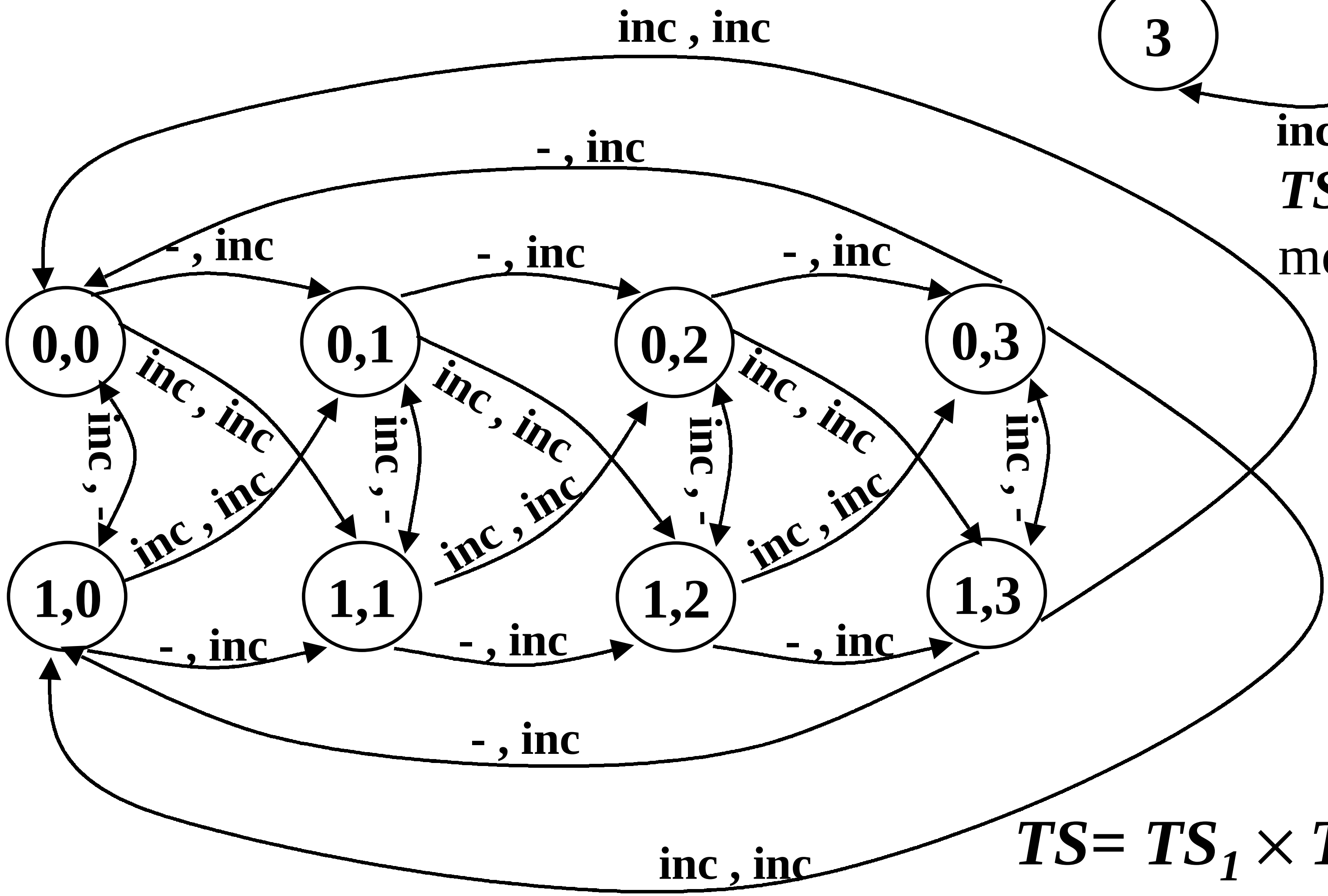
$$\boldsymbol{s_0 = (s_{10},s_{20},\ldots,s_{n0})}$$



TS_1 contatore modulo 2



TS_2: contatore modulo 4



$TS = TS_1 \times TS_2$

Synchronization: interaction

To allow for interaction, or synchronization on specific actions we can introduce a Synchronization Set (to inhibit undesired transitions) :

- ***Synchronization set is just a subset of the composite actions:***

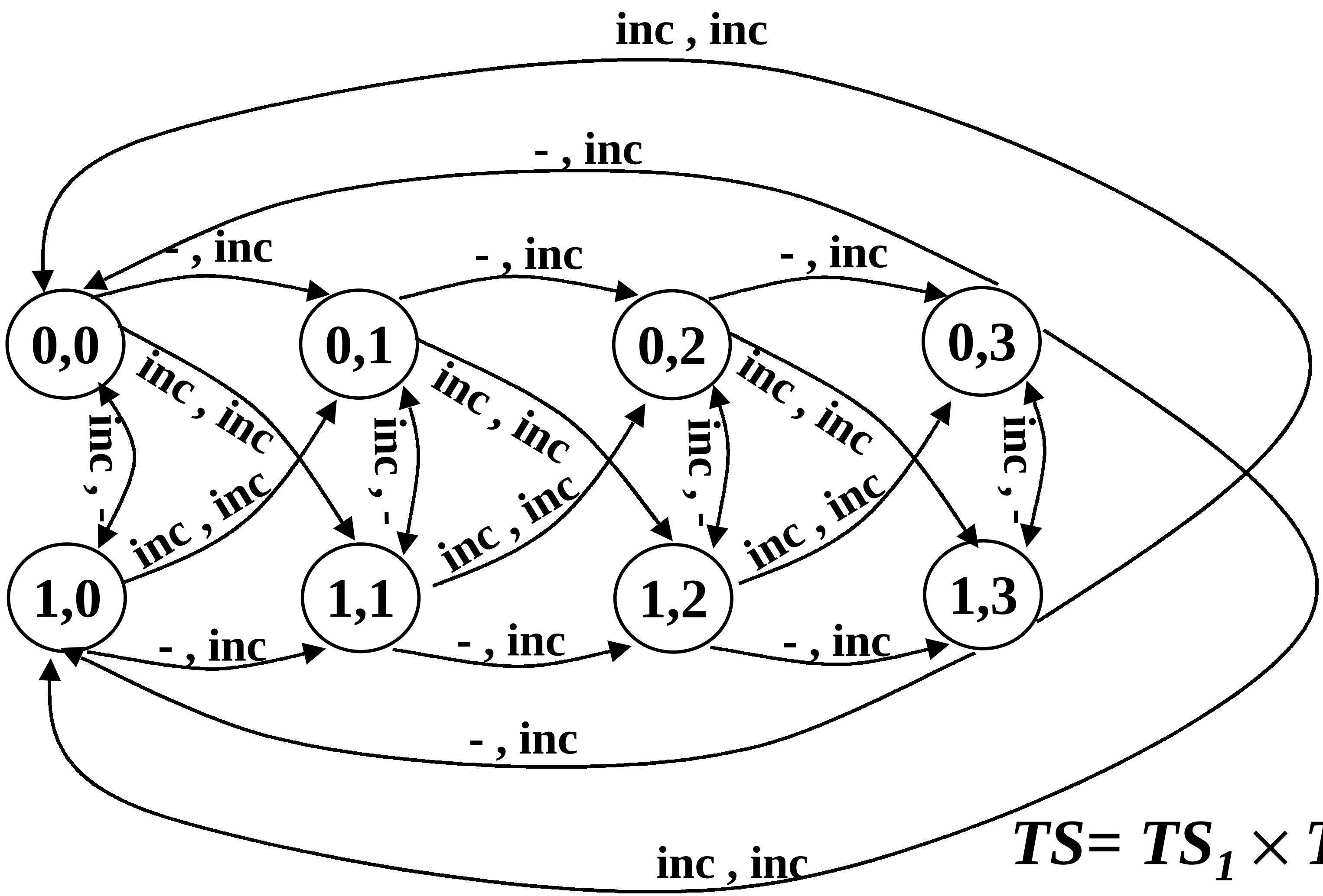
$$Sync \subseteq A_1 \cup \{-\} \times A_2 \cup \{-\} \times \ldots \times A_n \cup \{-\}$$

- ***Then we will have to define the possible transitions as:***

$$R=\{((s_1,s_2,\ldots,s_n),(a_1,a_2,\ldots,a_n),(s'_1,s'_2,\ldots,s'_n)) \mid$$
$$(a_1,a_2,\ldots,a_n) \in Sync \text{ and for all } i,$$
$$a_1 \neq - \text{ and } (s_i,a_i,s'_i) \in R_i, \text{ or } a_1 = - \text{ and } s'_i = s_i\}$$

Free synchronization (Asynchronous systems):

Sync = {*inc*,-}$\times$ {*inc*,-}



TS= $TS_1 \times TS_2$

Free synchronization

Asynchronous systems:

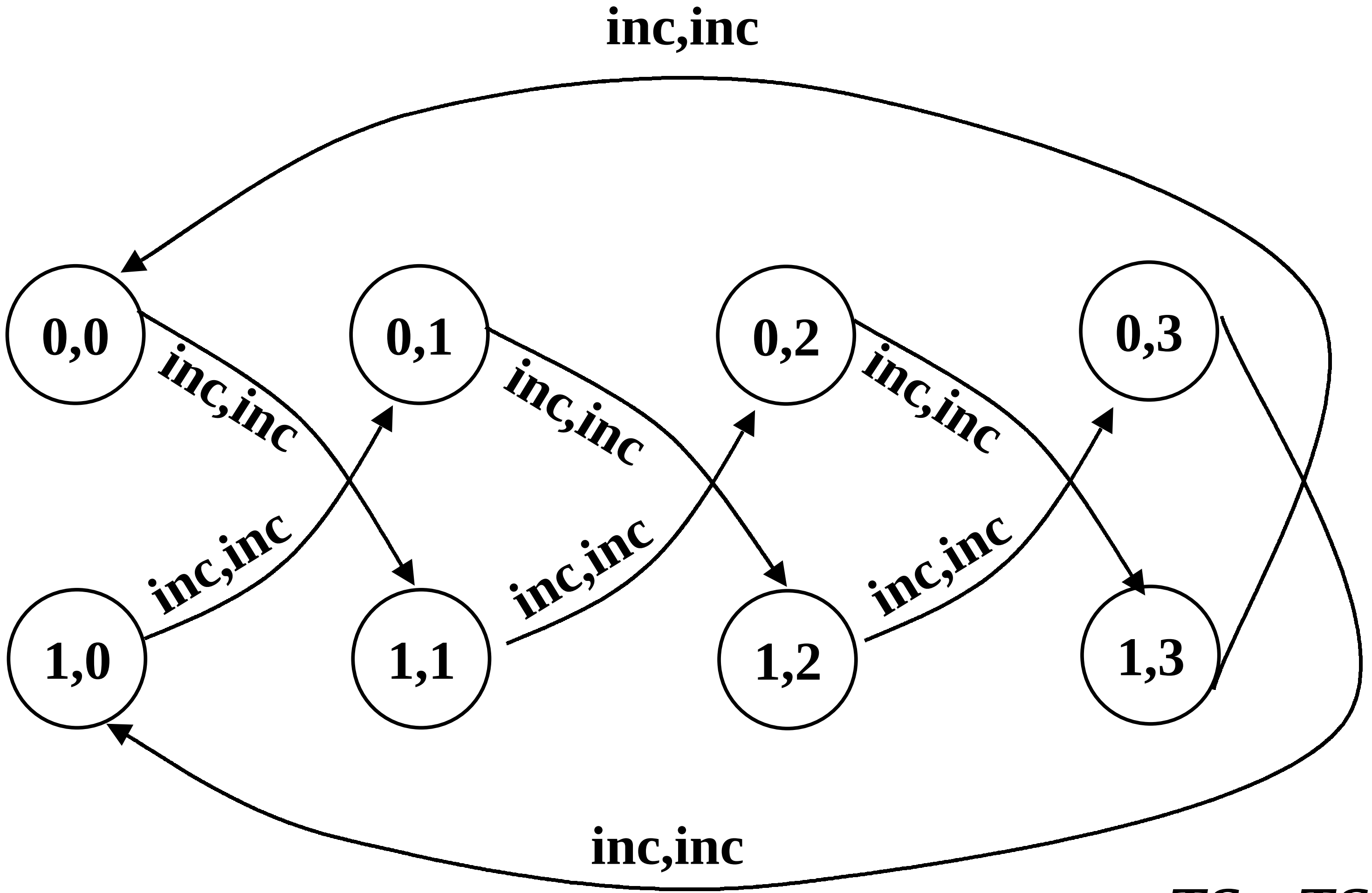
$Sync = \{inc,-\} \times \{inc,-\}$

- $R(V,V') = \bigwedge_{i \in I}(R_i(v_i,v_i') \vee \text{same}(v_i))\ \boxed{\wedge \neg \bigwedge \text{same}(v_i)}$

if one wants to *discard* the situation where *no component acts*

Synchronization on all actions (Synchronous systems):

***Sync* = {(*inc*,*inc*)}**



$TS = TS_1 \times TS_2$

Synchronous systems

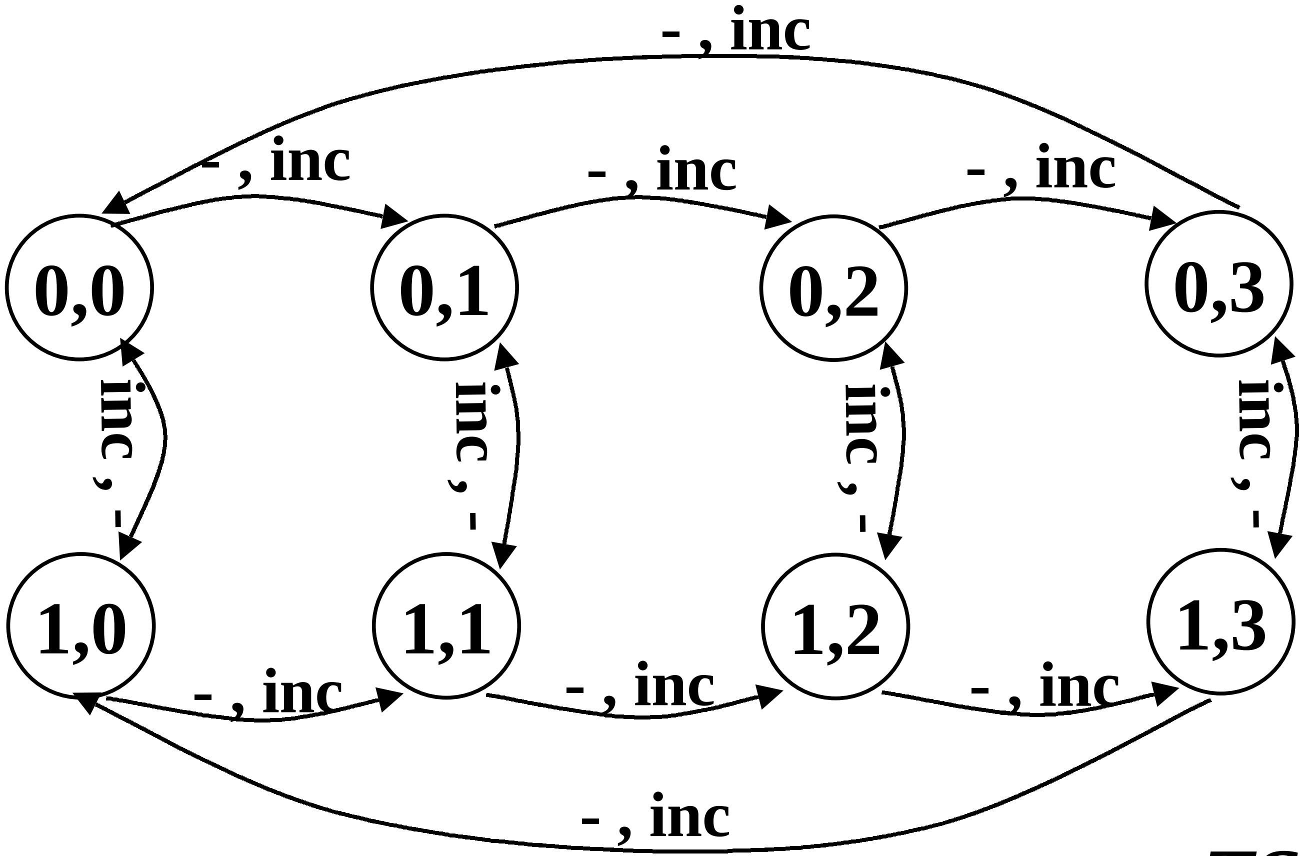
Synchronous systems:

$\boldsymbol{Sync = \{(inc, inc)\}}$

- $\boldsymbol{R(V,V') = \bigwedge_{i \in I} R_i(v_i, v_i')}$

Asynchronous systems with interleaving (only one component acts at any time):

***Sync* = {(-,*inc*),(*inc*,-)}**



$$TS = TS_1 \times TS_2$$

Asynchronous systems: Interleaving

Asynchronous systems:

$$\mathit{Sync} = \{\mathit{inc},\text{-}\} \times \{\mathit{inc},\text{-}\}$$

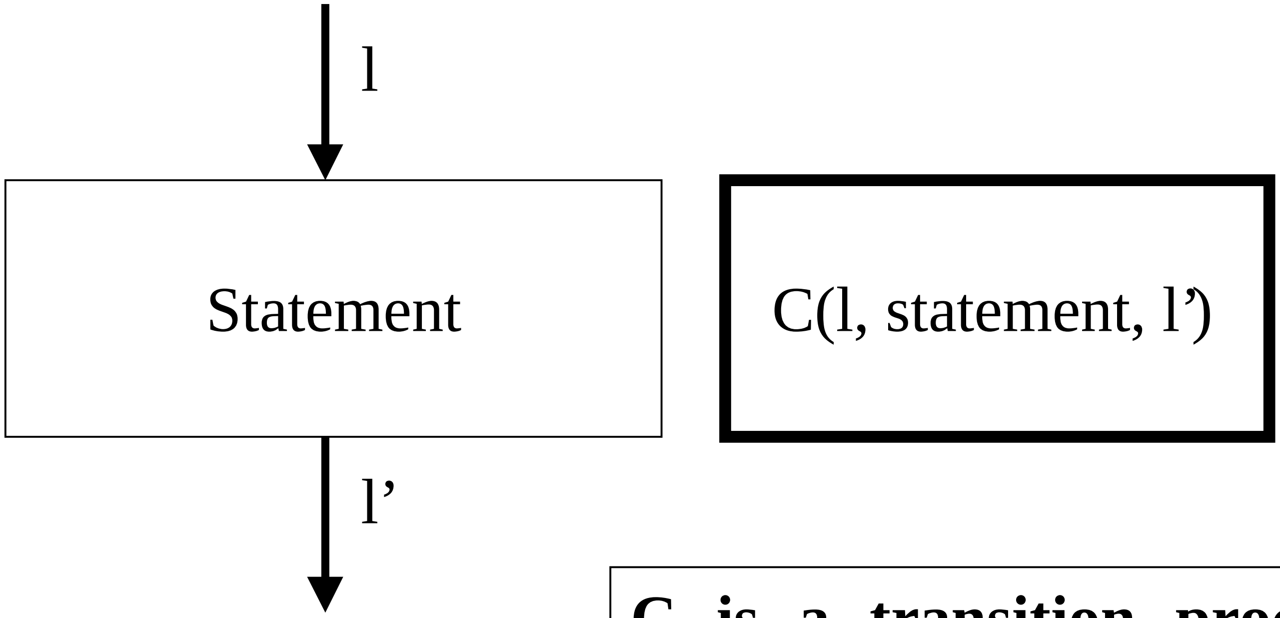
- $R(V,V') = \bigvee_{i \in I} \left(R_i(v_i, v_i') \wedge \bigwedge_{i \neq i} \mathrm{same}(v_j)\right)$

Concurrent programs

- Many systems to be verified can be viewed as concurrent programs
  - operating system routines
  - cache protocols
  - communication protocols
- **P = cobegin (P_1 || P_2 || …|| P_n) coend**
- **P_1, P_2,..P_n ---** Sequential Programs.
- ***Usually interleaving semantics is assumed***

Sequential Programs

General Structure



C is a transition procedure which takes a *label*, a *program statement* and a *label* and gives the *FOL* formula specifying the transition relation.

Assignments

l

v:= expression
($v := 2x - v + 3 \cdot y$)

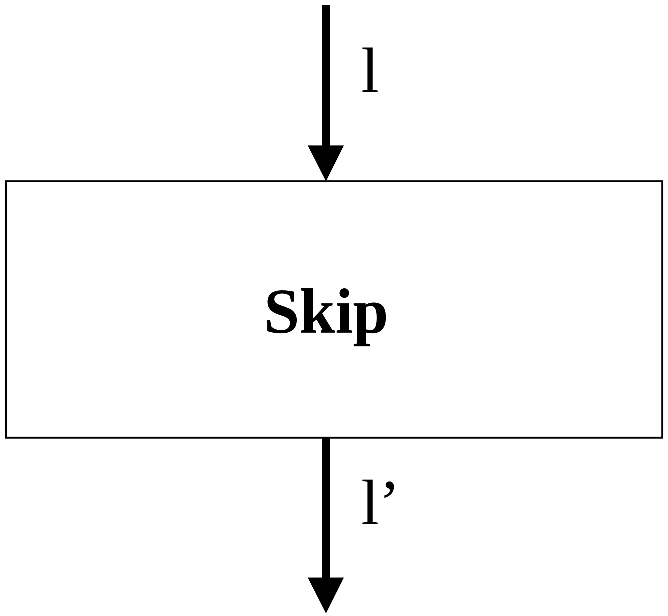
l'

C(l, assignment, l')

$$pc = l \wedge pc' = l' \wedge$$
$$v' = expression \wedge same\ (V - \{v\})$$

$$[Y = \{y_1, y_2, ..y_m\}$$
$$y_1' = y_1 \wedge y_2' = y_2 \wedge \ldots \wedge y_m]$$

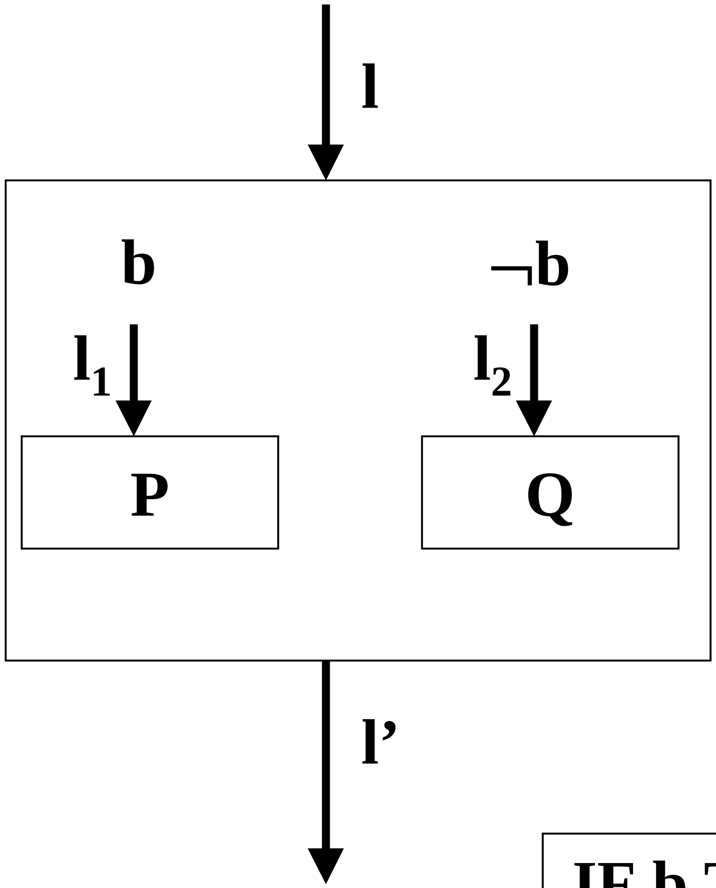
Skip



C(l, skip, l’)

$$pc = l \wedge pc' = l' \wedge \text{same}(V)$$

Conditional statement



$C(l, b, \text{IF-THEN-ELSE}, l_1, l_2, l')$

$(pc = l \wedge pc' = l_1 \wedge b \wedge same(V)) \vee$

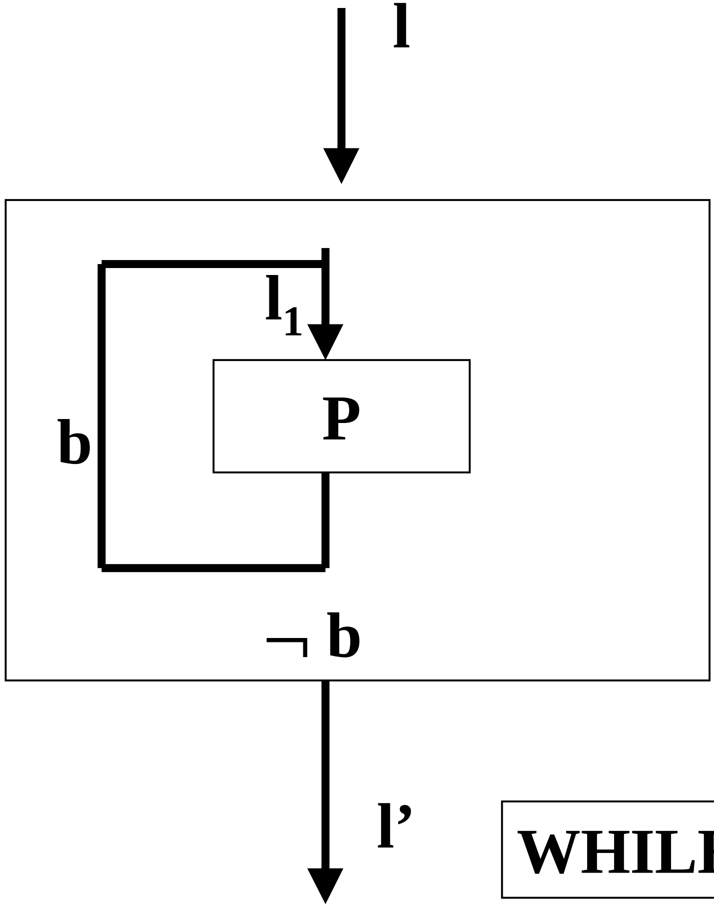
$(pc = l \wedge pc' = l_2 \wedge \neg b \wedge same(V)) \vee$

$C(l_1, P, l') \vee$

$C(l_2, Q, l')$

IF b THEN P ELSE Q FI

While statement



$$C(l, b, l_1, \text{ WHILE}, l')$$

$(pc = l \ \wedge \ pc' = l_1 \ \wedge b \wedge same(V)) \vee$

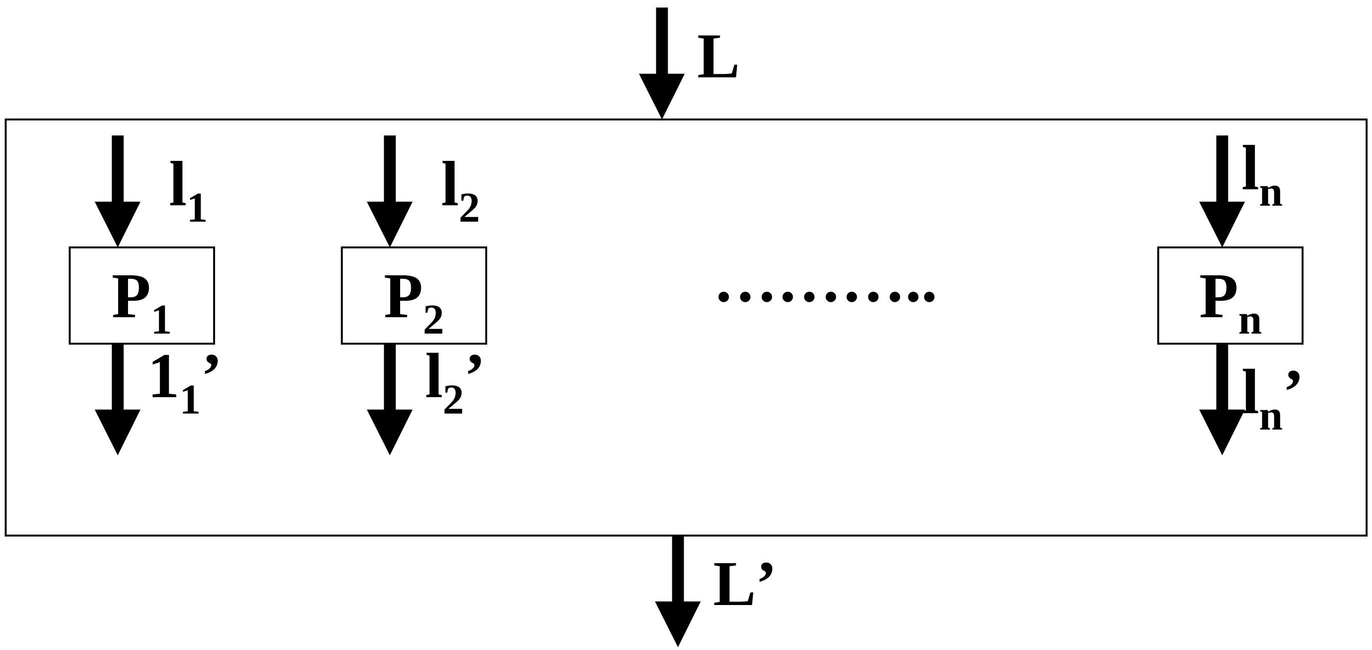
$(pc = l \ \wedge \ pc' = l' \wedge \neg\, b \wedge same(V)) \vee$

$C(l_1, P, l)$

WHILE b DO P END_WHILE

Concurrent programs

- **P = cobegin (P_1 || P_2 || …|| P_n) coend**
- **P_1, P_2,..P_n ---** Sequential Programs.



Concurrent programs

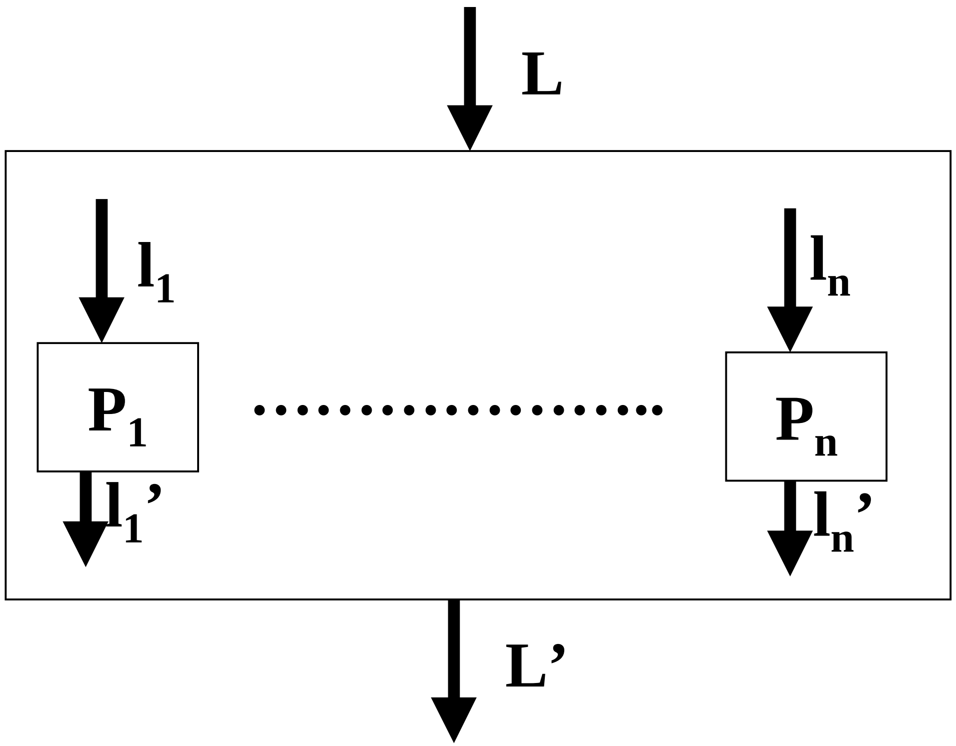
- $\mathbf{P = cobegin\ (P_1 \parallel P_2 \parallel \ldots \parallel P_n)\ coend}$
- $\mathbf{P_1, P_2,..P_n}$ --- ***Sequential Programs***.
- $\mathbf{C(l_1, P_1, l_1')}$ --- The transitions of $\mathbf{P_1}$ (defined inductively!).
- $\mathbf{V_i}$ ---- The set of variables of $\mathbf{P_i}$.
- Programs may ***share*** variables !
- $\mathbf{pc_i}$ – The program counter of $\mathbf{P_i}$.

Concurrent programs

- **pc ----** the program counter of the concurrent program; it could be part of a larger program!
- $\perp$ denotes the program counter value is ***undefined*** .
- $\boldsymbol{S_0(V, PC) = \mathbf{pre(V)} \wedge \mathbf{pc} = \mathbf{L} \wedge}$

  $$\mathbf{pc_1} = \perp \wedge \ldots\ldots \wedge \mathbf{pc_n} = \perp$$

The Transition Predicate



$$
\begin{gathered}
(pc = L \wedge pc_1' = l_1 \wedge \ldots. \\
\wedge pc_n' = l_n \wedge pc' = \bot) \vee \\
(pc = \bot \wedge pc_1 = l_1' \wedge \ldots \\
pc_n = l_n' \wedge pc' = L' \wedge \\
pc_1' = \bot \wedge \ldots pc_n' = \bot) \vee \\
(C(l_1, P_1, l_1') \wedge Same\ (V - V_1) \\
\wedge Same(PC - \{pc_1\})) \vee \ldots \\
C(l_n, P_n, l_n') \wedge Same\ (V - V_n) \\
\wedge Same(PC - \{pc_n\}))
\end{gathered}
$$

Summary

- System variables
- Domain of values
- States
- Initial state predicate
- Transition predicate
- pc values (for programs)