# Tecniche di Specifica e di Verifica

Modeling with Transition Systems

# An example

***The Dining Philosophers***

- Possible problems:
  - ***Deadlock***: system state where no action can be taken (no meaningful transition possible)
  - ***Livelock***: When a system component is prevented to take any action, or a particular one (individual starvation)
  - ***Starvation***: obvious.

# Fairness

***The Dining Philosophers***

- A possible solution to deadlock:
  - ***pick up right fork only if both are present***

  **Useful assumptions on the system**:

  - ***weak fairness***: any phil. trans. ***continuously*** enabled will ***eventually*** fire (e.g. eating philosophers will finish)
  - ***strong fairness***: any phil. trans. enabled ***infinitely often*** will ***eventually*** occur (e.g. if 2 fork available infinitely often, phil. will eventually eat).

# Livelock

***The Dining Philosophers***

- Possible solution:
  - ***pick up fork only if both are present***

  **Assumptions**:

  - ***strong fairness***: any phil. trans. enabled infinitely often, will eventually occur (if 2 fork available infinitely often, philosopher will eventually eat).

  ***strong fairness*** is not enough to prevent ***livelock***

  ***Why***? Think of the case with 4 philosophers!

  Sol.(?): Try ***preventing consecutive eating***.

  Still suffers from ***livelock*** with 5 phils! ***Why***?

# Outline

- The model – Transition systems
- Some features
  - Paths
  - Computations
  - Branching
- First order representation

# Transition systems

- A transition system (***Kripke structure***) is a structure

$$\mathbf{TS = (S, S_0, R)}$$

where:

- – **S** is a finite set of states.
- – $\mathbf{S_0 \subseteq S}$ is the set of initial states.
- – $\mathbf{R \subseteq S \times S}$ is a transition relation
  - **R** must be ***total***, that is
    - – $\mathbf{\forall s \in S\ \exists s' \in S\ .\ (s, s') \in R}$ or, equivalently,
    - – for every state **s** in **S**, there exists **s'** in **S** such that **(s, s')** is in **R**.

# Notions and Notations

- **TS = (S, $S_0$, R)**
- **(s, s') Î R  R(s, s')  s ® s'**
- A (finite) ***path*** from **s** is a sequence

  **$s_1$, $s_2$,…,$s_n$**

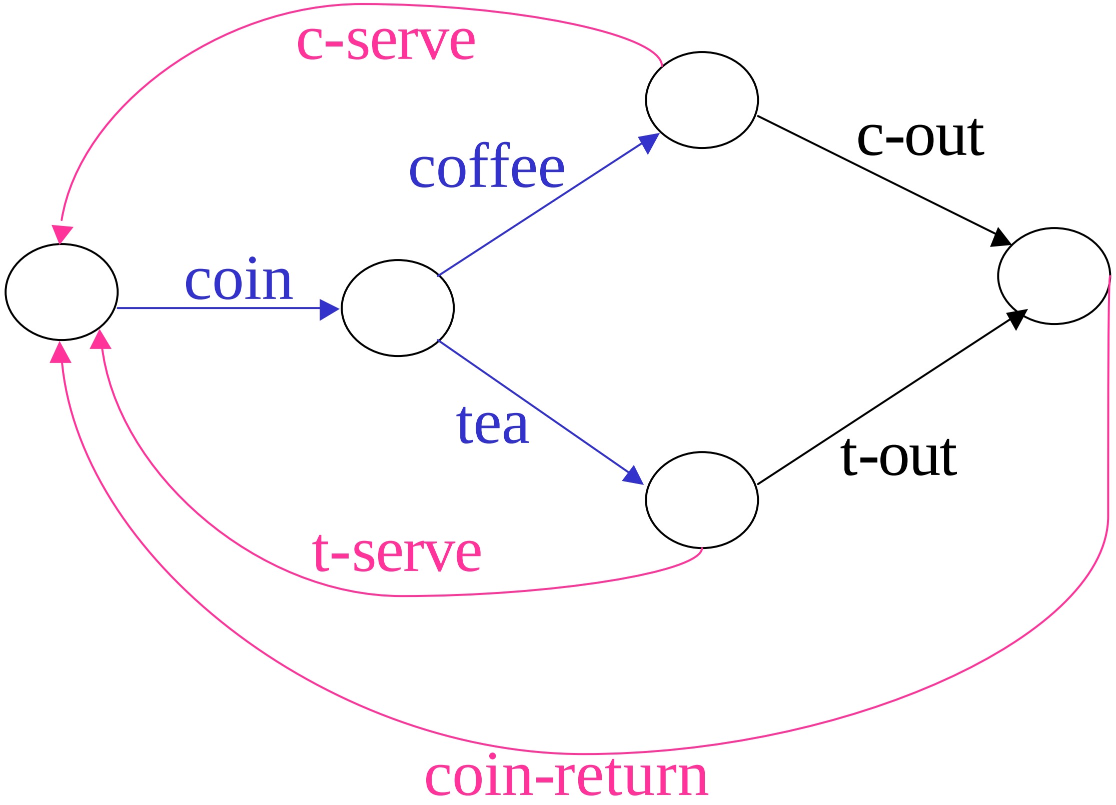
  such that

  - **s = $s_1$**
  - **$s_i$ ® $s_{i+1}$** for $0 < i < n$.
- It is from **s** to **s'** if **$s_n$ = s'**.
- An **infinite** path from **s** is an ***infinite sequence*** …..

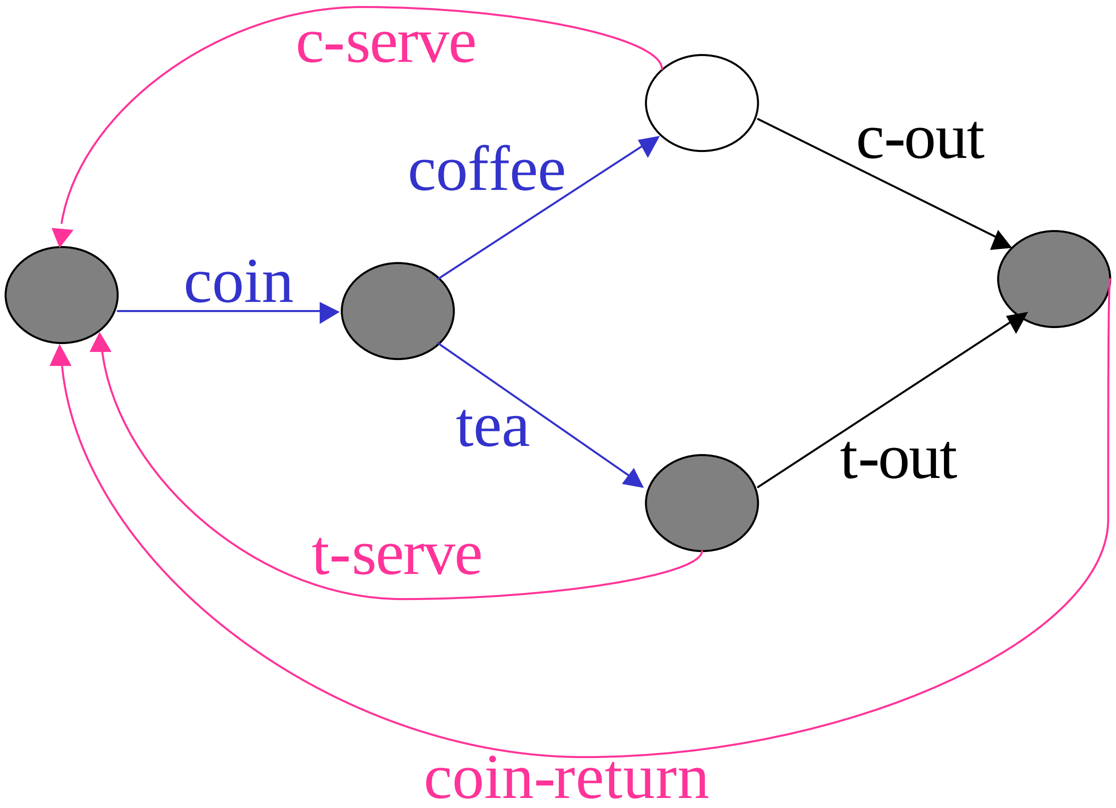
# Labeled transition systems

- Sometimes we may use a ***finite*** set of actions:
  - **Act = {a, b, ..}**
- The actions will be used to label the transitions.
- **TS = (S, $S_0$, Act, R)**
  - **$R \subseteq S \times Act \times S$**, labeled transitions.
  - **(s, a, s') $\in$ R  -  R(s, a, s')  -  $s \xrightarrow{a} s'$**

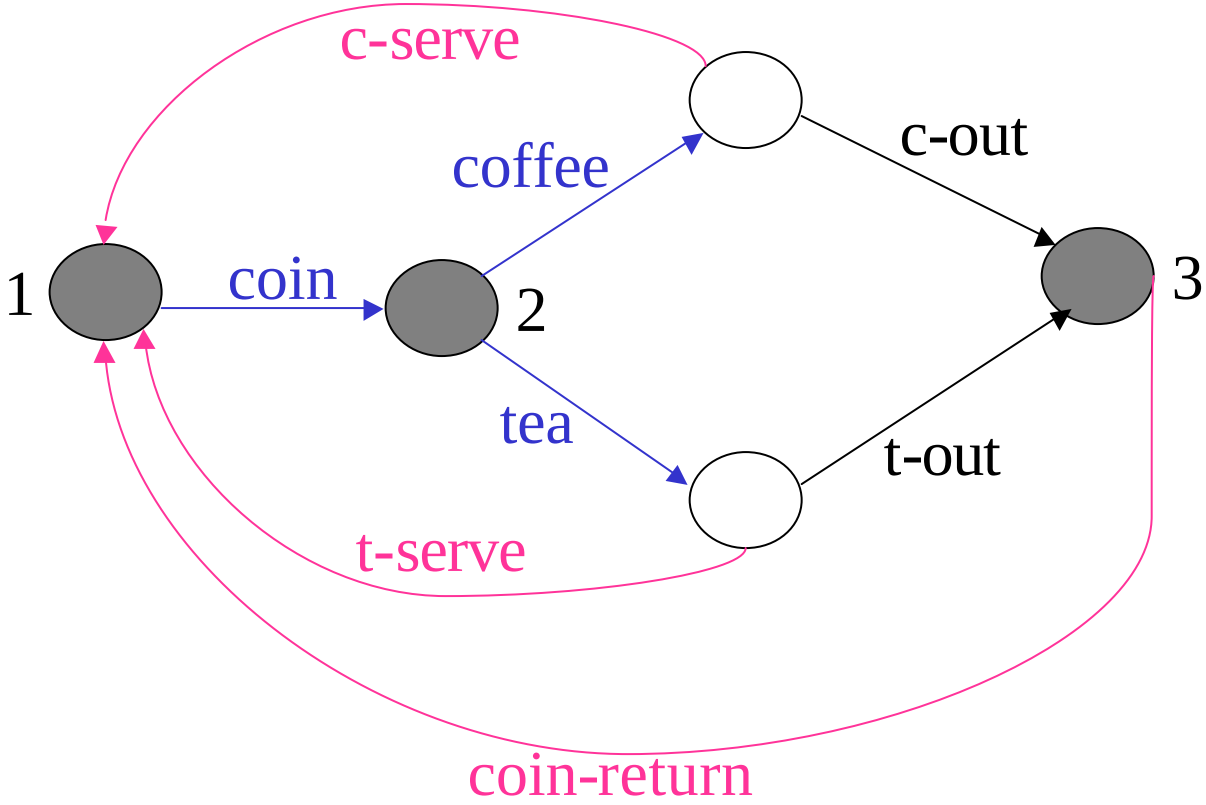
# A vending machine



# A path



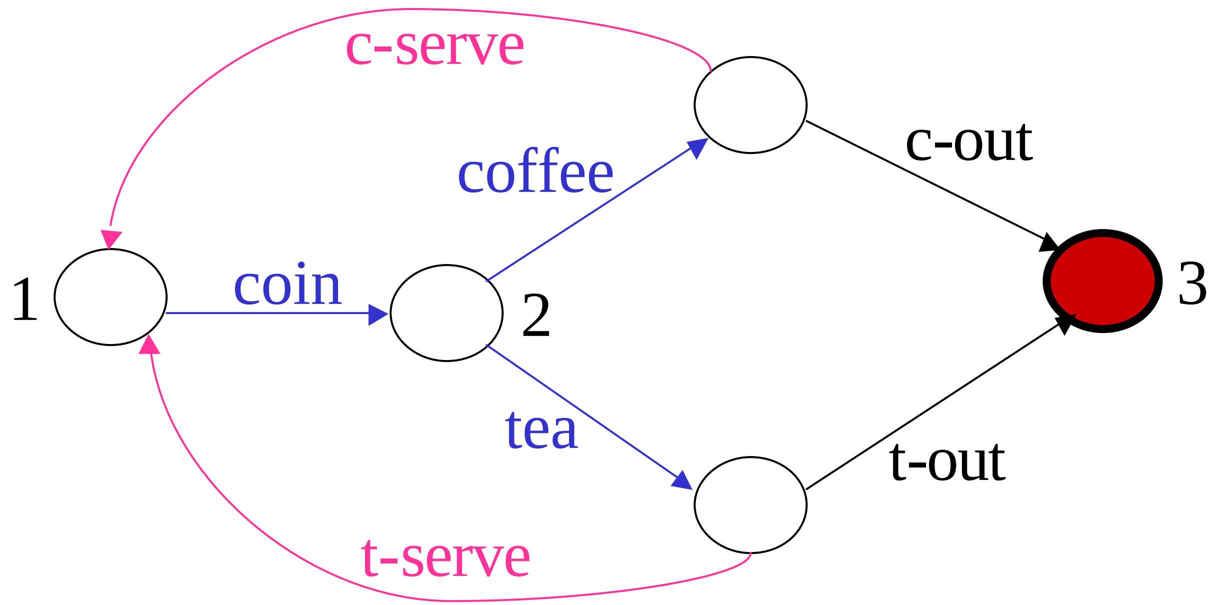
# A non-path




1 2 3 No!

3 1 2 yes!

# A non-total transition relation



# State space

- The ***state space*** of a system (e.g. program) is the set of ***all its possible states***.
- For example, if **V={a, b, c}** and the variables range over the naturals, then the ***state space*** includes:

<a=0,b=0,c=0>, <a=1,b=0,c=0>,

<a=1,b=1,c=0>, <a=932,b=5609,c=6658>

…

# Atomic transition

- Each ***atomic transition*** represents a small peace of code (or ***execution step***), such that ***no smaller*** peace of code (or ***step***) is observable.
- Is a:=a+1 atomic?
- In some systems it is, e.g., when a is a register and the transition is executed using an inc command.

# (Non)Atomicity (race conditions)

- Execute the following when **x=0** in two concurrent processes:

  **P1:a=a+1** **P2:a=a+1**

- Result: **a=2**.
- Is this always the case?

- Consider the actual translation:

**P1:load R1,a**
**inc R1**
**store R1,a**

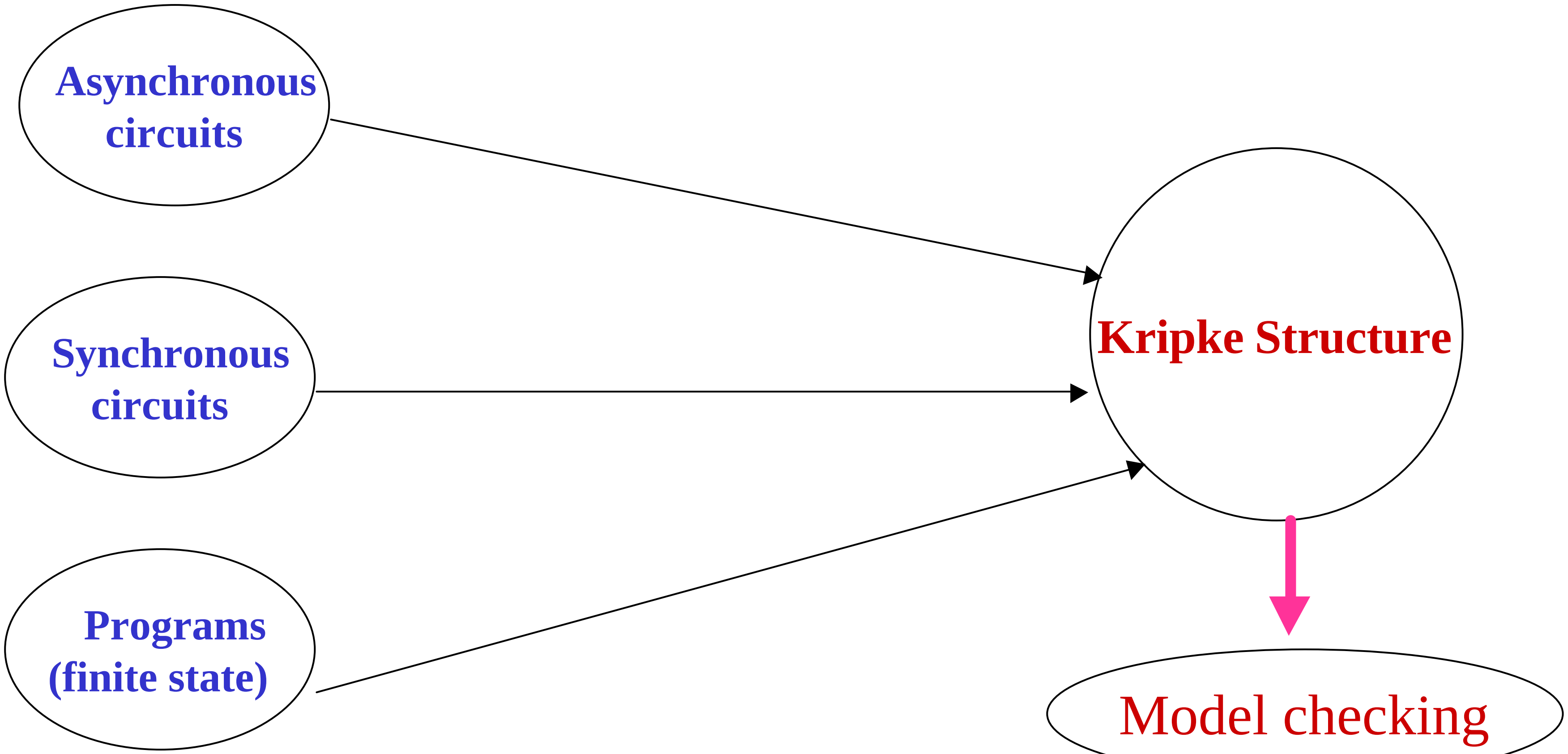
**P2:load R2,a**
**inc R2**
**store R2,a**

- **a may also be 1**

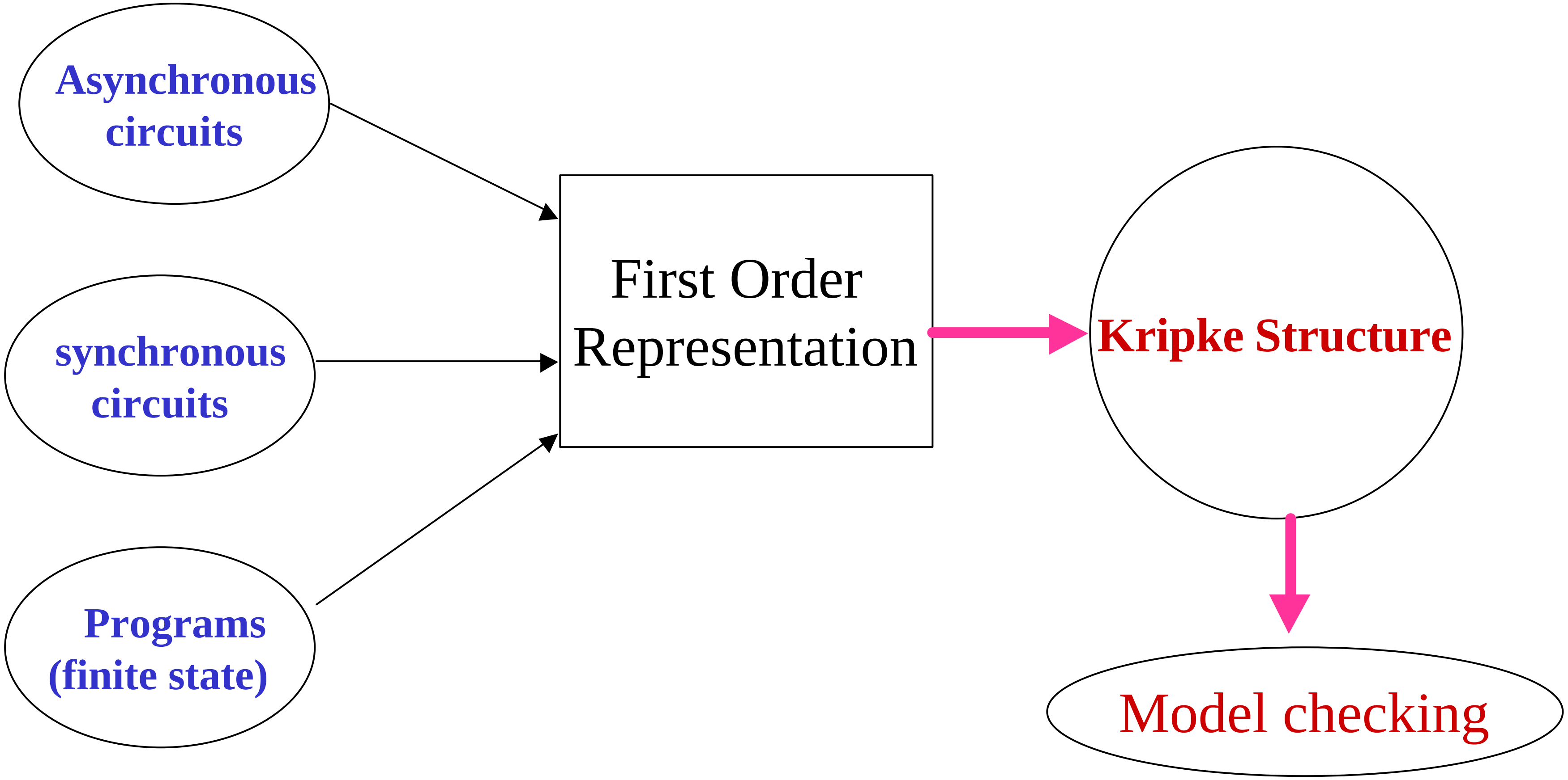
# The common framework

- Many systems need to be modeled.
  - Digital circuits
    - **Synchronous**
    - **Asynchronous**
  - Programs
- Strategy : Capture the main features using a logical framework (nothing to do with temporal logics!) : ***First order representation***

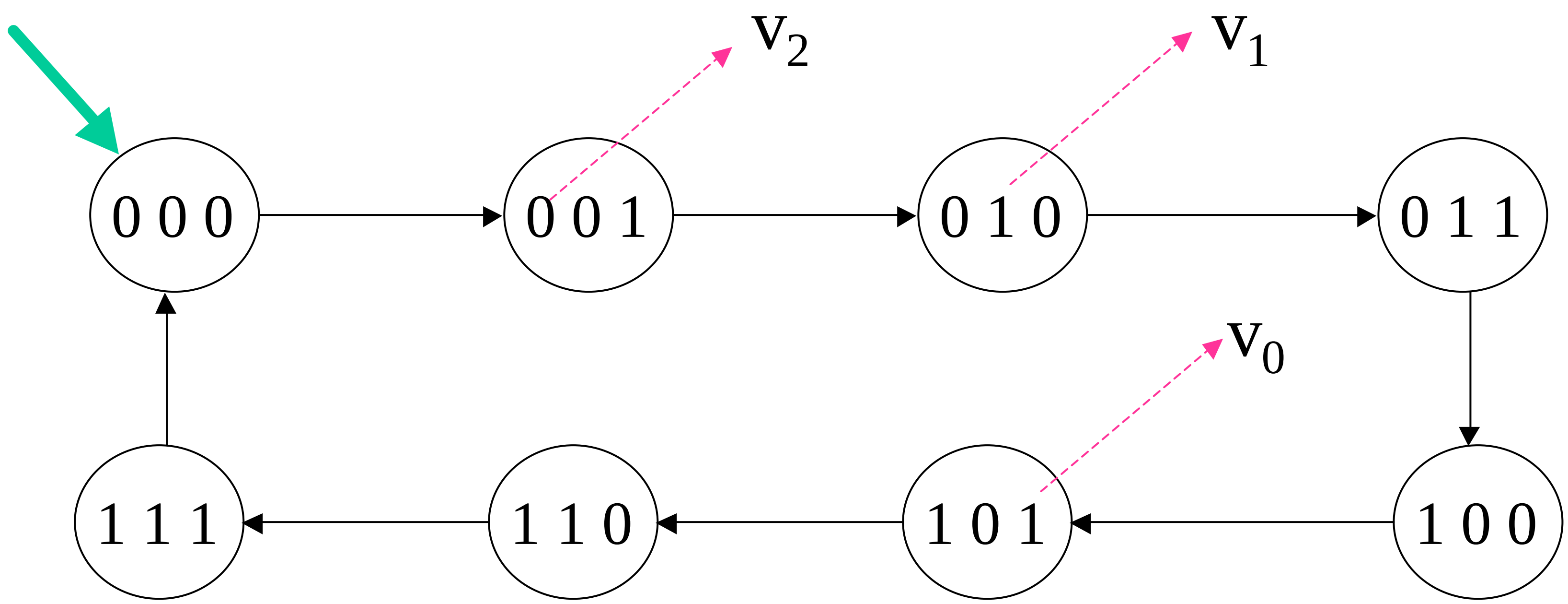
# The inefficient way



# The efficient way



# A mod-8 counter

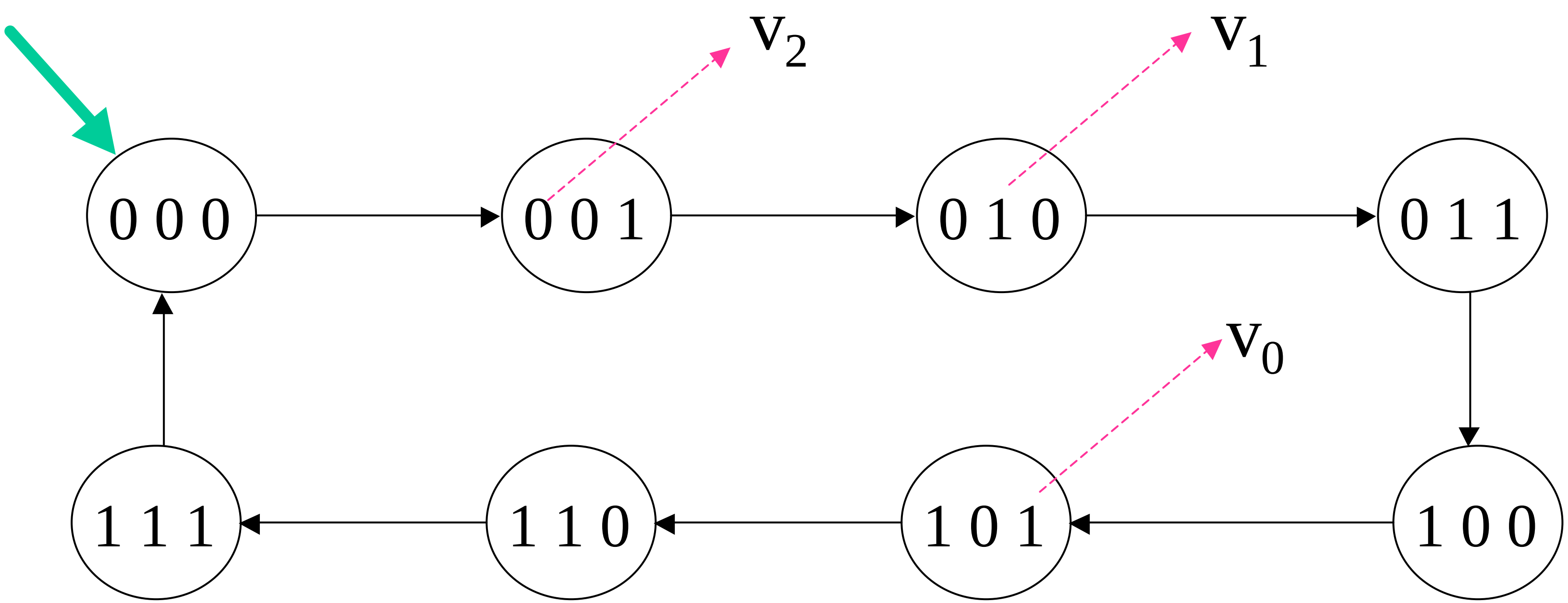


# The mod-8 counter

- System variables : $V = \{v_2\ v_1\ v_0\}$
- Domain of $v_2$ is $\{0, 1\}$

  Same domain for $v_1$ and $v_0$ as well.
- Special case : These variables are boolean
- Each state s can also be seen as a function assigning to each variable a value in its domain.
  - **s : V ® B**
  - **$s(v_0) = 0$  $s(v_1) = 1$  $s(v_2) = 1$**
  - This specifies the state **s** = (1 1 0) !

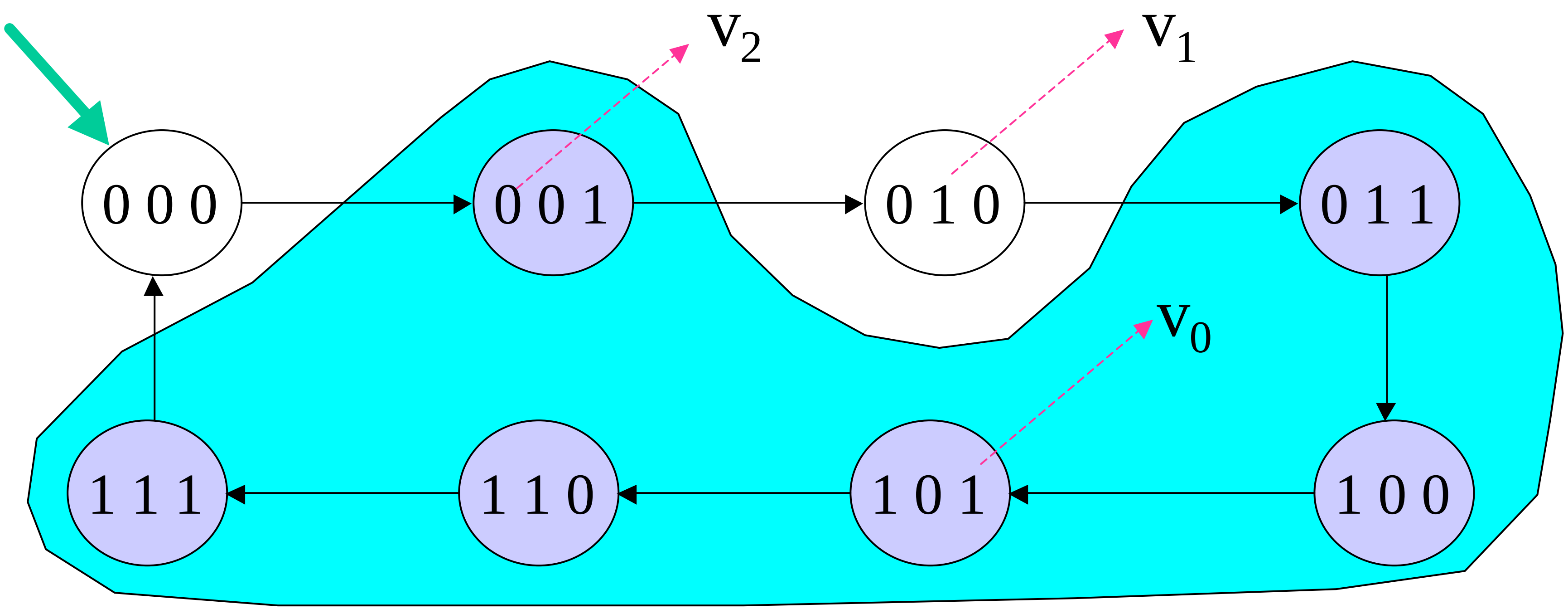
# State Predicates




**A set of states can be picked out by a propositional formula**:

**$X = v_2$ Ú $v_0$** is the set {…}

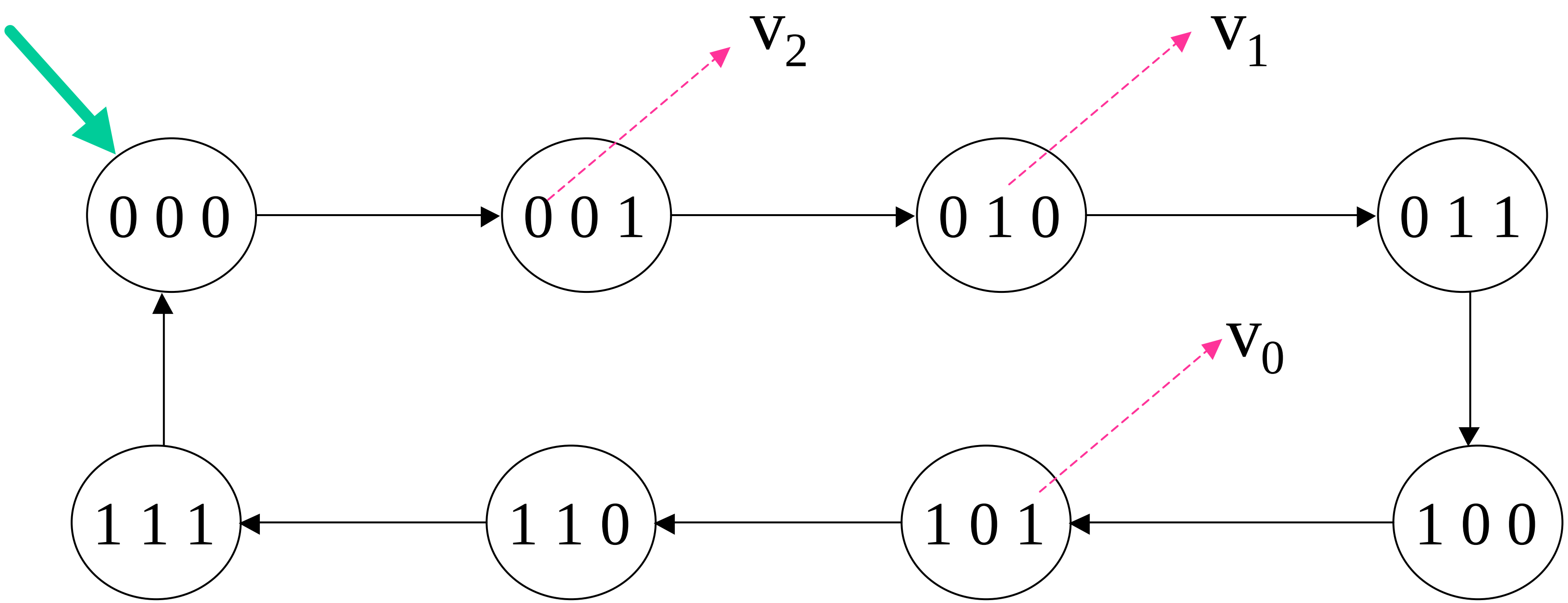
# State Predicates




**A set of states can be picked out by a propositional formula**:

$\mathbf{X} = \mathbf{v_2} \vee \mathbf{v_0}$  is the set {100, 101, 110, 111, 001, 011}

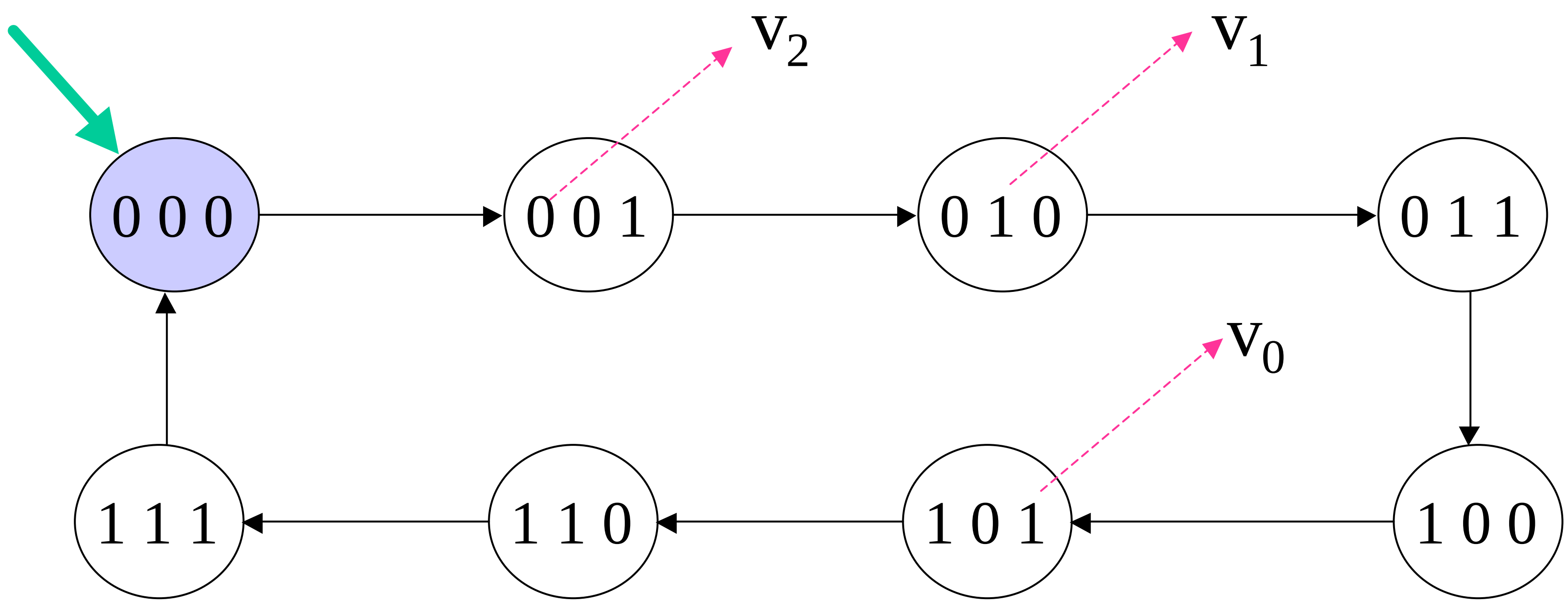
# Initial States Predicate




A set of states can be picked out by a formula;

$\mathbf{X'} = \neg \mathbf{v_2} \wedge \neg \mathbf{v_1} \wedge \neg \mathbf{v_0}$

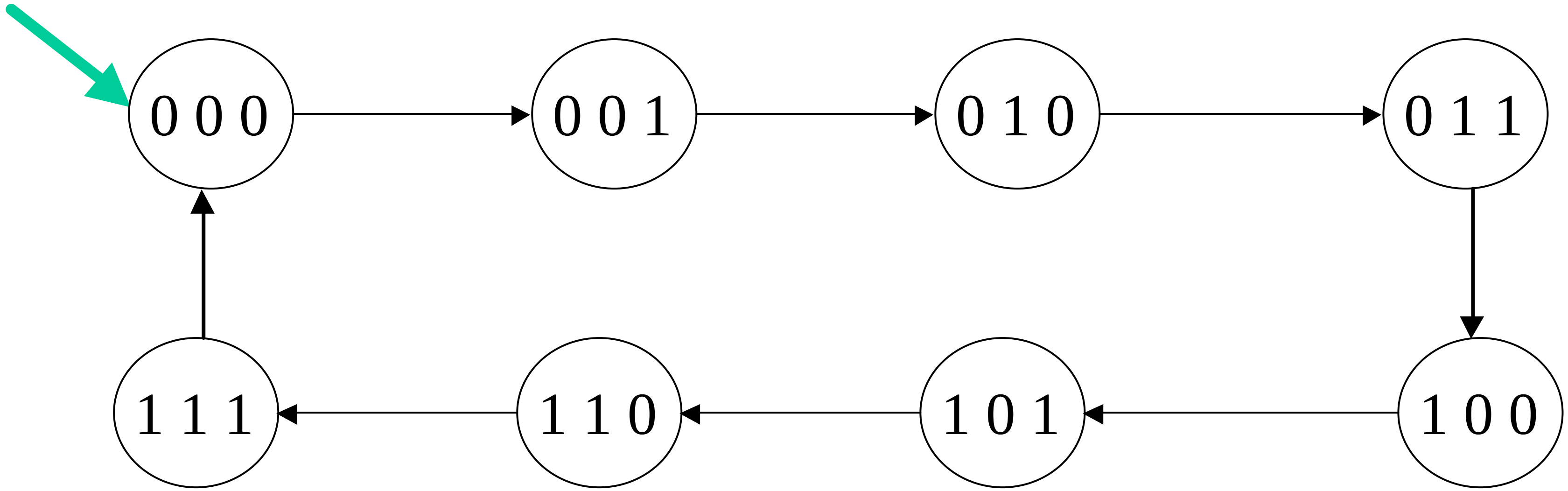
# Initial States Predicate




A set of states can be picked out by a formula;

$\mathbf{X_1} = \neg \mathbf{v_2} \wedge \neg \mathbf{v_1} \wedge \neg \mathbf{v_0}$ therefore $\mathbf{X_1} = \{ \mathbf{S_0} \} = \{ 000 \}$

# Transition relation predicate

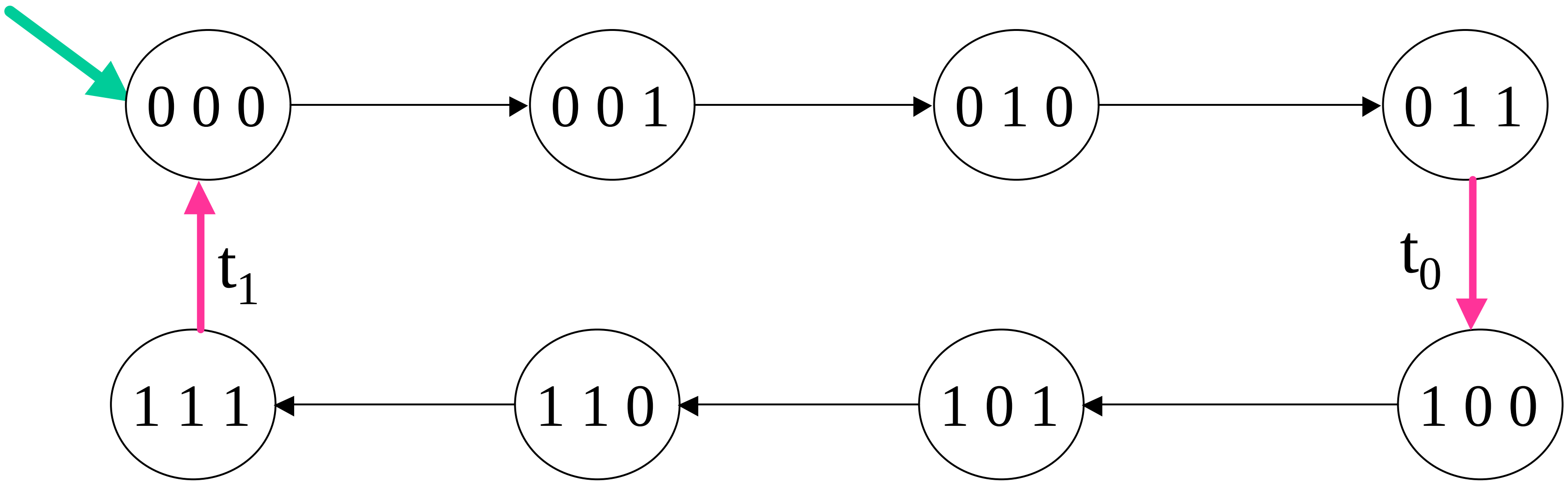



**A set of *transitions* can also be picked out by a formula.**

$\mathbf{R_2} = v_2' \Leftrightarrow (v_0$ Ù $v_1)$ Å $v_2$

$v_2$ – current value   $v_2'$ – next value

# Transition relation predicate

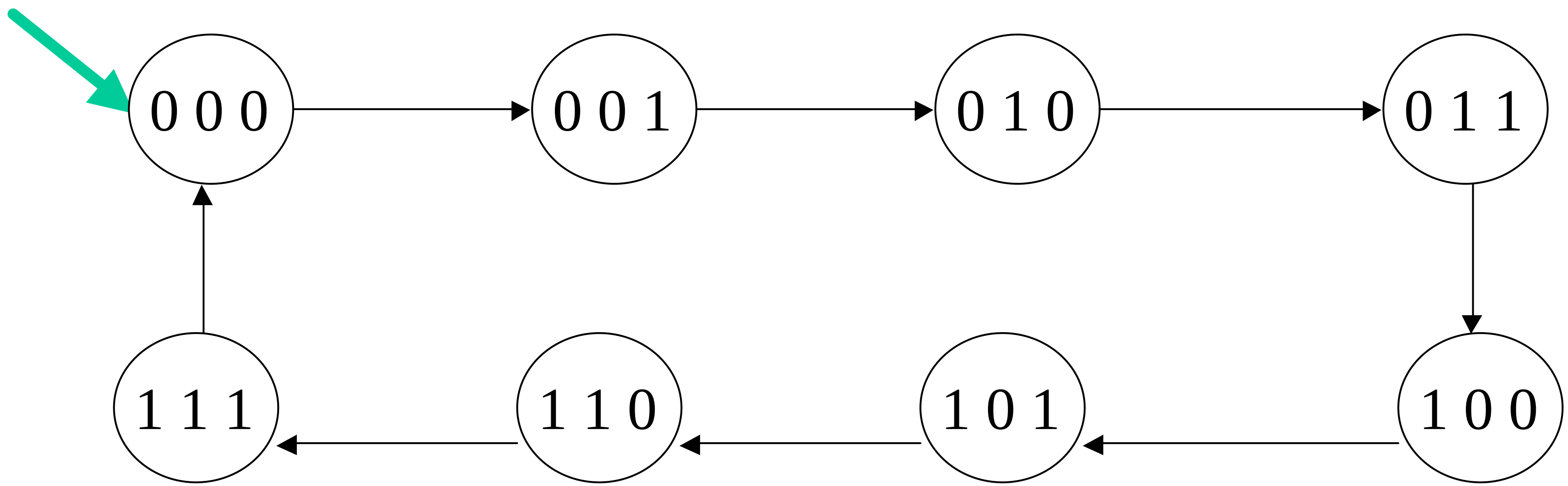



A set of transitions can also be picked out by a formula.

$\mathbf{R_2}$= $\mathbf{v_2'}$ ⇔ ($\mathbf{v_0}$ Ù $\mathbf{v_1}$) Å $\mathbf{v_2}$      $\mathbf{v_2}$ – current value   $\mathbf{v_2'}$ – next value

$\{t_0, t_1\} \subseteq \mathbf{R_2}$

# Transition relation predicate

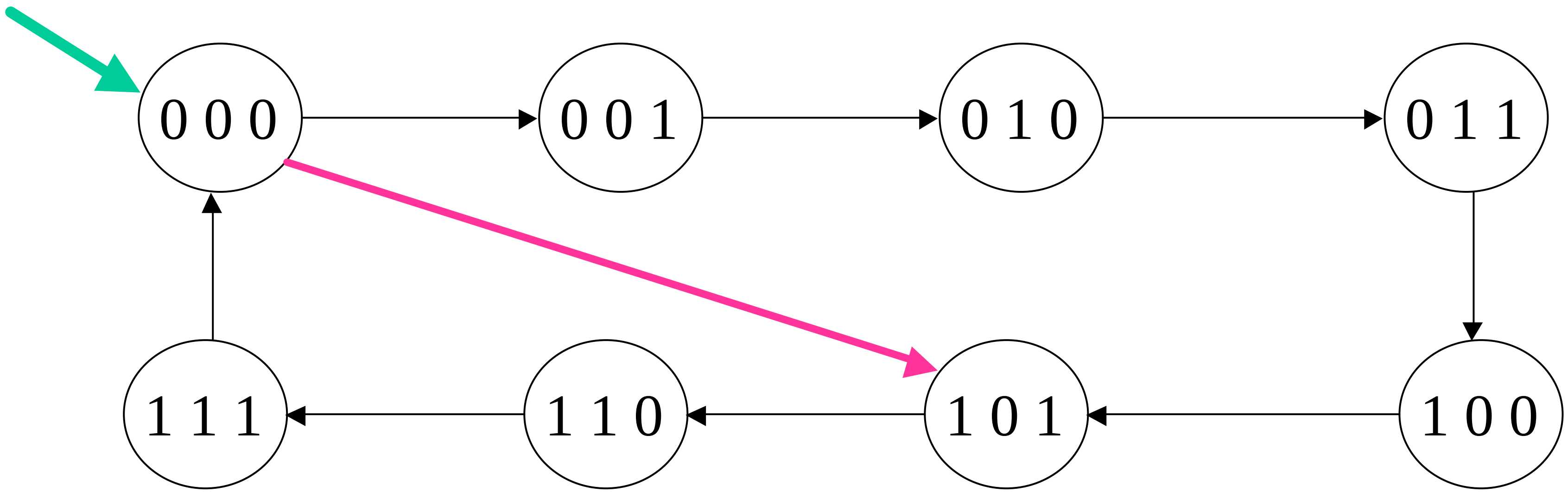



**R** = **Formula**($\mathbf{v_2}$, $\mathbf{v_1}$, $\mathbf{v_0}$, $\mathbf{v_2'}$, $\mathbf{v_1'}$, $\mathbf{v_0'}$)

Not all formulae will define subsets of transitions.

**You** must pick the right formula .

# Transition relation predicate

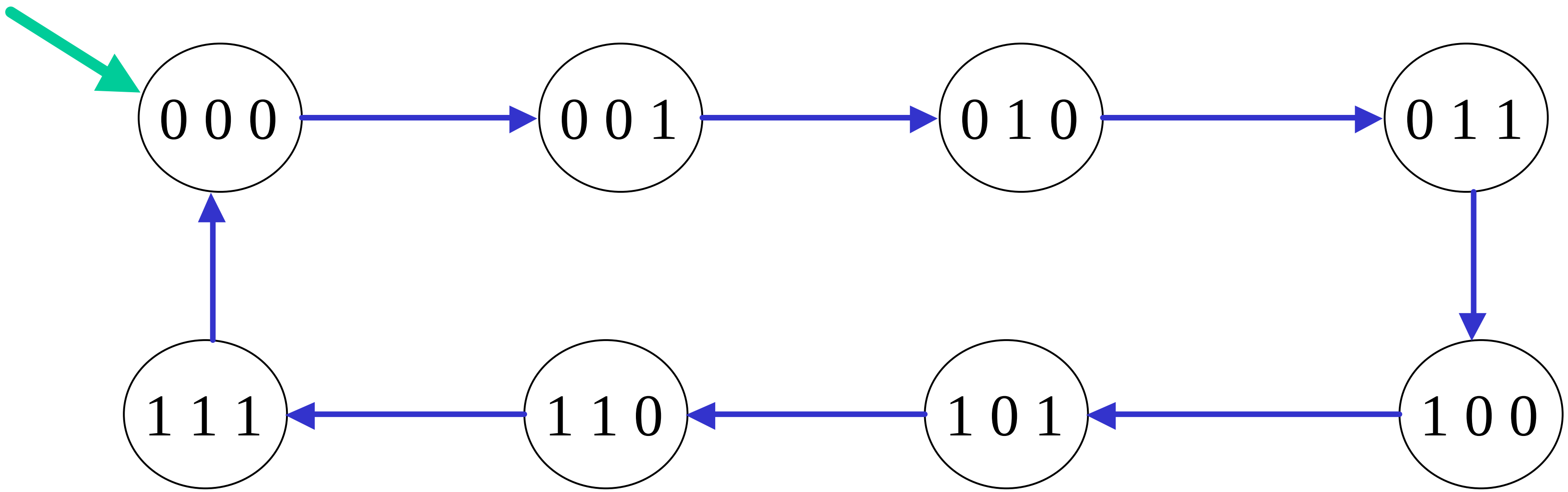



$\mathbf{R_0} = v_0' \;{}^{1}\; v_0$ $\qquad$ $v_0$ – current value $\quad v_0'$ – next value

$\mathbf{R_0} = \{(000) \longrightarrow (101)$ ,……..}

But this is not a transition!

# Transition relation predicate




$\mathbf{R_0}$ = $v_0$’ ¹ $\mathbf{v_0}$     $\mathbf{v_i}$ – current value   $\mathbf{v_i}$’ – next value

$\mathbf{R_1}$ = $v_1$’ **=** **($v_0$** Å **$v_1$)**

$\mathbf{R_2}$ = $v_2$’ **=** **($v_0$** Ù **$v_1$)** Å **$v_2$**

**R** = **$R_0$** Ù **$R_1$** Ù **$R_2$**

# Summary of Predicates

- System variables $\mathbf{v_0}$, $\mathbf{v_1}$, $\mathbf{v_2}$, …..,$\mathbf{v_n}$.
- Each $\mathbf{v_i}$ has a domain of values
  - Boolean , {a,b,c,..}, {5,8,0,7}…
  - We require that each domain be ***finite***.
- A state is a function **s** which assigns to each system variable a value in its domain.
- The set of states is ***finite*****.**

# Summary

- Predicates can be used to pick out –succinctly- sets of states (useful for identifying initial states).
- **X** = **Formula**($\mathbf{v_0}$, $\mathbf{v_1}$, $\mathbf{v_2}$,…,$\mathbf{v_n}$)
- But this works well only when all domains are boolean.
- In general, we can use ***first order formulae***.

# Summary

- A set of transitions can also be picked out using predicates.
- **T** = **Formula**($v_0, v_1, \ldots, v_n, v_0', v_1', \ldots, v_n'$)
- **T** is the set of all transitions

  $$(v_0, v_1, \ldots, v_n) \longrightarrow (v_0', v_1', \ldots, v_n')$$

  such that Formula (above!) is satisfied.
- Not all (state or transition) formulas will be legitimate.

# Why use formulae?

- ***Formulae*** allow us to compactly describe a system and its dynamics
- It's easy to go from a "***logical***" description to ***Kripke structures***.
- Once we have a ***Kripke structure***, we are in business.
- We can use
  - ***Temporal Logics*** to specify properties
  - ***Model checking*** to verify these properties.

# First Order Logic

- The general structure :
  - **Syntax**
    - Formulae
  - **Semantics**
    - When is a formula  true?
    - Models
      - Interpretations
      - Valuations

# Syntax

- **Terms**
  - Variables
  - Functions symbols, constant symbols
- **Atomic formulas**
  - Relation symbols, equality, terms
- **Formulas**
  - Atomic formulas
  - Propositional connectives
  - ***Existential and universal quantifiers***

# Syntax

- (individual) variables --- **x, y, $v_3$, v’**,…
  - System variables in our context
- Function symbols : $\mathbf{f}^{(\mathbf{n})}$
  - **n** is the arity of **f**.
  - Add $^{(2)}$
  - Next $^{(1)}$
- Function symbols will capture the functions used in the programs, circuits, …

# Constant symbols

- Apart from variables, it will also be convenient to have constant symbols.
  - ***zero*** , ***five***, ….
- Variables can be assigned different values but a constant symbol is assigned a fixed value.

# Terms

- Terms are used to point at values.
- Any variable **v** is a term.
  - **x , v , v''**
- Any constant symbol ***c*** is a term.
- Suppose ***f*** is a function symbol of arity ***n*** and $t_1, t_2, \ldots, t_n$ are terms, then $f(t_1, t_2, \ldots, t_n)$ is a also term.

# Terms

- Let Plus be a function symbol of arity 2.
- $\mathbf{v_1}$ , $\mathbf{v_2}$, **Plus**($\mathbf{v_2}$, **Plus**($\mathbf{v_1}$, $\mathbf{v_1}$)) are terms.
  - the semantics of the last term is intuitively
    $\mathbf{v_2 + 2v_1}$
- Let weird_op be a function symbol of arity 3
- Then

  Plus(weird_op(**v**, Plus($\mathbf{v_1}$, $\mathbf{v_2}$), ***five***), Plus(**v**, **v''**))

  is a term.

# Predicates

- Relation (predicate) symbols :
  - ***P*** which also has an arity
  - ***Greater-Than*** has arity 2
  - ***Prime*** has arity 1
  - ***Middle*** has arity 3 -- ***Middle*(t$_1$, x, t$_2$)**
    - intuitively, **x** lies between **t$_1$** and **t$_2$**
- ***Equal*** has arity 2
  - will be denoted as **=**
  - It is a "**constant**" relation symbol.

# Atomic formulas.

- If $\mathbf{t_1}$ and $\mathbf{t_2}$ are terms then $\mathbf{=(t_1, t_2)}$ is an atomic formula.
  - also written $\mathbf{t_1 = t_2}$
- Suppose $\boldsymbol{P}$ has arity **n** and $\mathbf{t_1, t_2, \ldots, t_n}$ are terms.
- Then $\boldsymbol{P}\mathbf{(t_1, t_2, \ldots, t_n)}$ is an atomic formula.

# Atomic formulas

- ***Greater-Than****(five, zero)*
- ***Greater-Than****(two, four)*
- ***Prime*****(Plus($v_1$, v”))**
- **Plus(v,Zero) = weird_op(v,v,four)**
- **v = *Greater_Than*($v_1$,$v_2$)** is ***not*** an atomic formula !

# Terms and Predicates

- A ***term*** is meant to denote a domain value.
  - **It makes no sense to talk about a term being true or false**.
- An ***atomic formula*** may be ***true*** or ***false*** (depends on the interpretation).
  - **It does not make sense to associate a domain value with an atomic formula**.

# Formulas

- Every atomic formula is a formula.
- If j is a formula then Øj is a formula.
- If j and j' are formulas then j Ú j' is a formula.
- j Ù j' abbreviates: Ø(Øj Ú Øj')
- j É j' abbreviates : Øj Ú j'
- j º j' abbreviates : (j É j') Ù (j' É j)

# Formulas

- If $j$ is a formula and $\mathbf{x}$ is a variable then $\$\mathbf{x}.\ j$ is a formula.
- $"\ \mathbf{x}.\ j$ abbreviates : $\O\$\mathbf{x}.\O\ j$
- These are ***existential*** and ***universal*** quantifiers.
- The power of first order logic comes from these operators!

# Semantics

- **Models** :
  - *Domain of interpretation*
  - *Interpretation*
    - For the function, constant and relation symbols.
      - ***Fixed for all formulas***.
    - For the individual variables, on a “per formula” basis.
      - ***Valuations***.

# Semantics

- ***Domain***
  - Each variable will have its domain of values.
  - We pretend all these domains are the same.
  - Or rather, a big enough “universe” that will contain all these domains.
- Fix **D** the universe of values.

# Semantics

***Interpretation function I***

- Assign a concrete function to each function symbol (of the same arity!)
- Assign a concrete member of **D** to each constant symbol.
- Assign a concrete relation to each relation symbol (of the same arity!).

# Semantics

- Assume we have fixed an interpretation for all function symbols, constant symbols and relational symbols.
- Let j be a formula. Fix a ***valuation*** (or ***assignment***) **V** which assigns a member of **D** to each variable.
- **V** : **Var** $\longrightarrow$ **D**

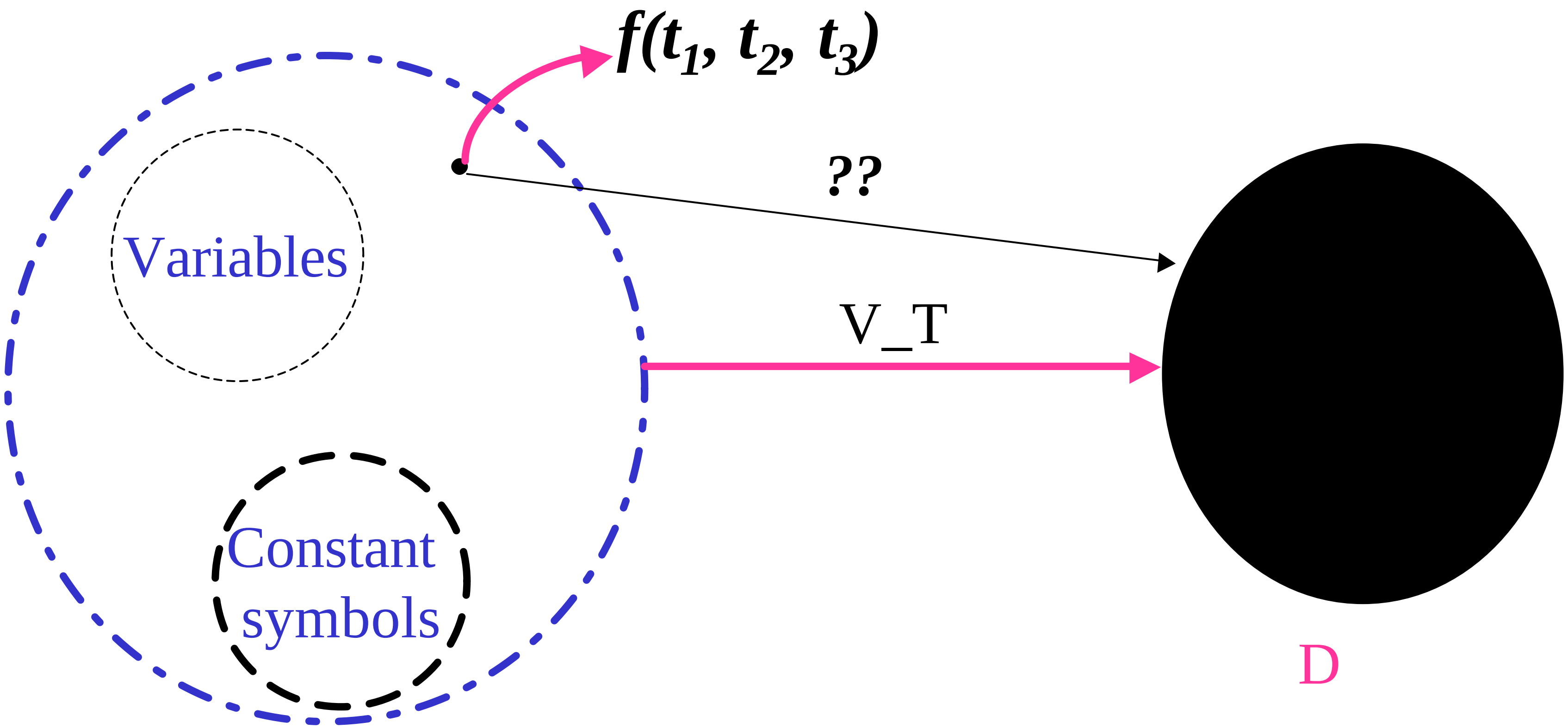
# Lift V to All Terms

- We have :
  - An ***interpretation*** for the function symbols and constant symbols.
  - An ***assignment*** **V** : **Var** $\longrightarrow$ **D**
- Using these, we can construct (uniquely!)

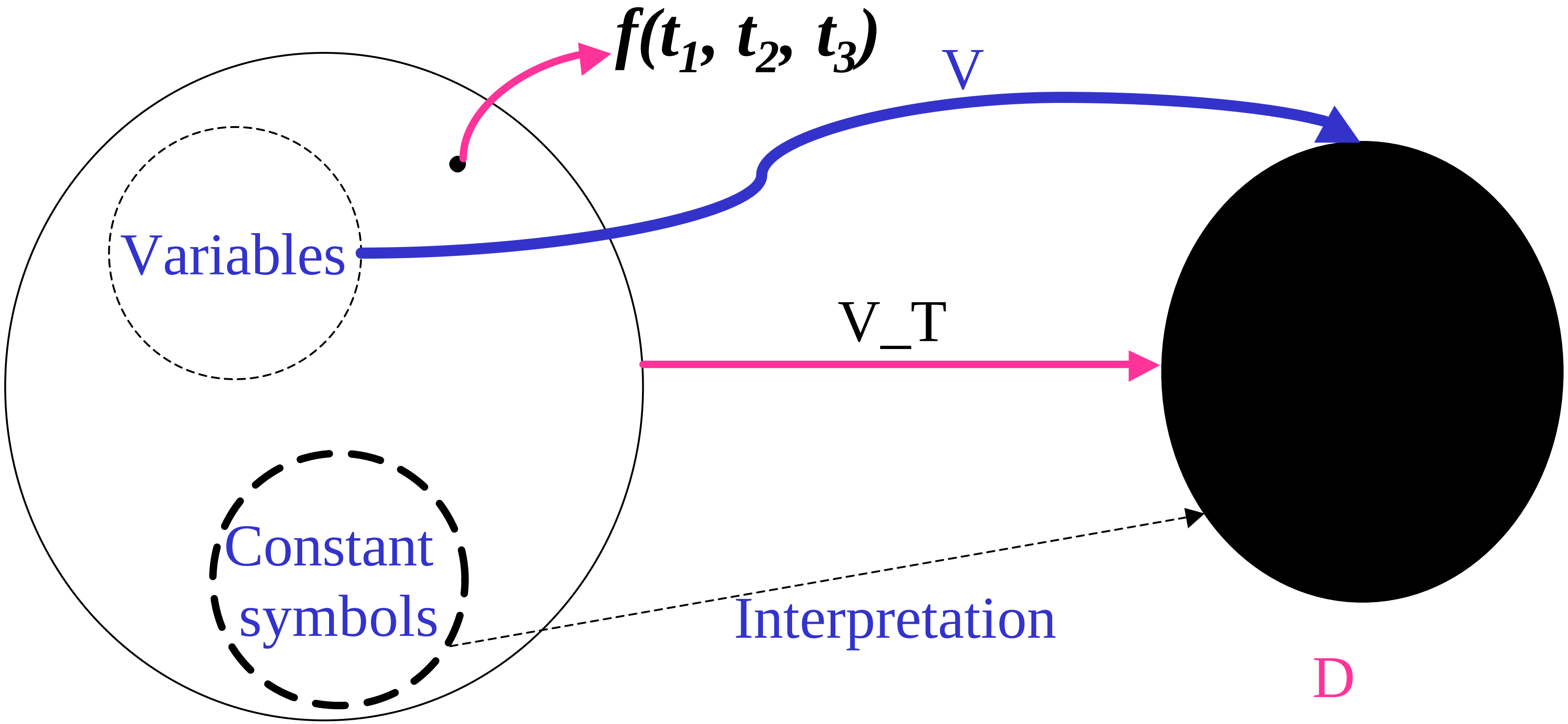
  **V_T** : Terms $\longrightarrow$ **D**

  the interpretation of terms!

# Constructing V_T



# Constructing V_T



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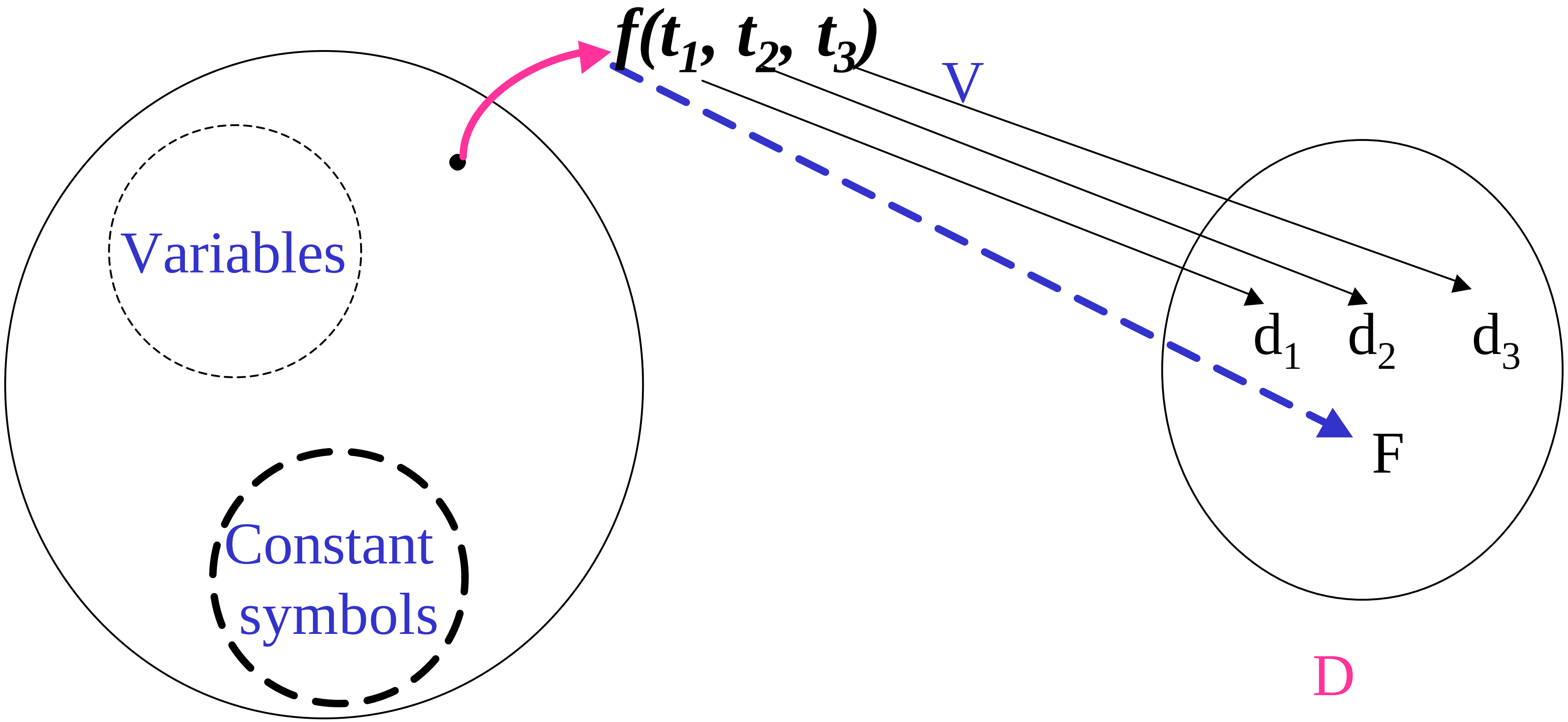
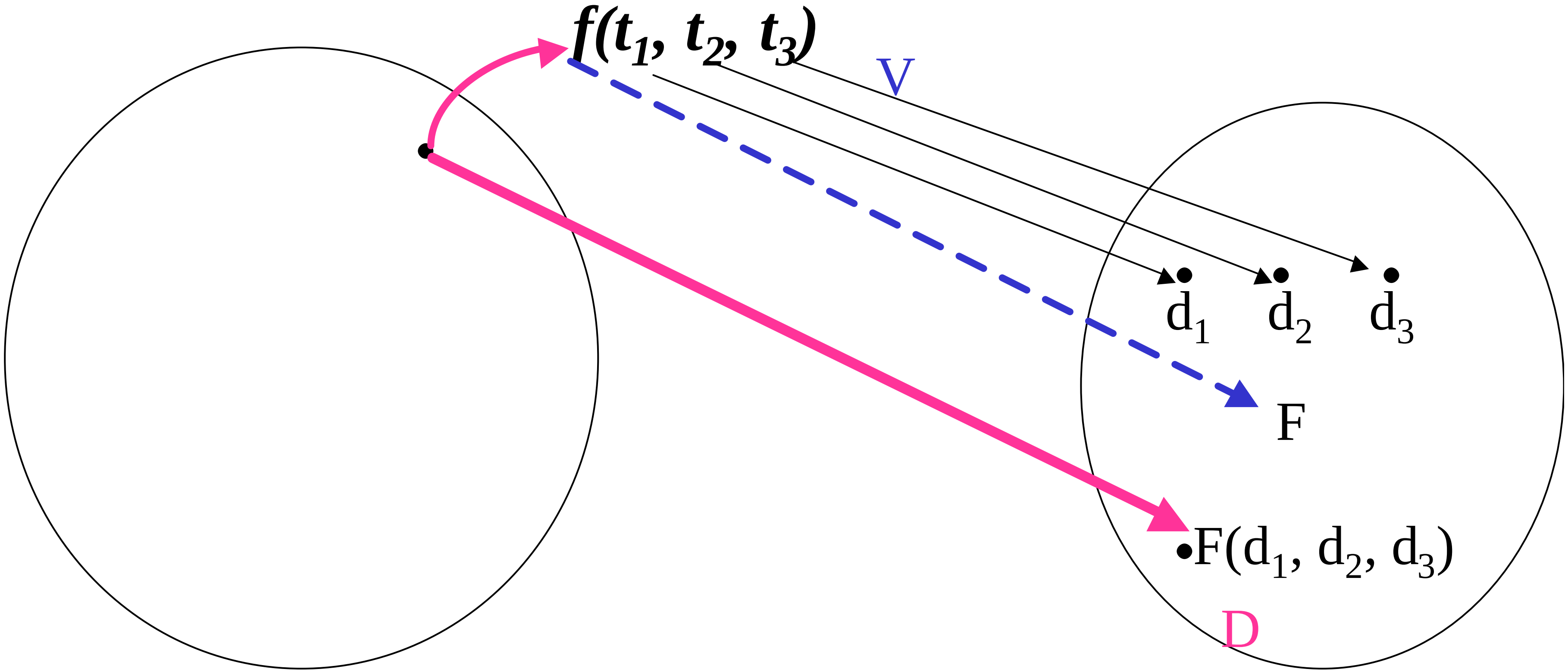


# Constructing V_T



# Semantics

- Let j be a formula. Fix a valuation **V** which assigns a member of **D** to each variable.
- So we now have **V_T** that assigns a member of **D** to each term.
- j is satisfied under **V** (and the interpretation we have fixed, for all formulae) if :

# Semantics

- Suppose $P(t_1, t_2,.., t_n)$ is an atomic formula
  and $\mathbf{V\_T}(t_1) = d_1, \ldots.\mathbf{V\_T}(t_n) = d_n$
  and PCON is the relation assigned to
  symbol ***P*** by our interpretation **I**.
- Then $P(t_1, t_2,.., t_n)$ is satisfied under **V** iff
  $\mathrm{PCON}(d_1, d_2,\ldots,d_n)$ holds in **D**, that is:
  $\boxed{(d_1, d_2, \ldots.,d_n) \in \mathrm{PCON}}$ **Í D ´ D ´…´ D**

# Semantics

- Suppose j is of the form Ø j'.

  Then j is satisfied under **V** iff j' is **not** satisfied under **V**.
- Suppose j is of the form $j_1$Ú $j_2$

  Then j is satisfied under **V** iff $j_1$ is satisfied under **V** **or** $j_2$ is satisfied under **V**.

# Semantics

- ***Greater-Than*(Plus(*v*, *3*), Multi(*x*, *2*))**

  $t_1$ $t_2$

- V(v) = 2 V(x) = 1

  $V\_T(t_1) = 5$ $V\_T(t_2) = 2$

  $(5, 2) \in > \subseteq \text{Integers} \times \text{Integers}$

- V'(v) = 1 V'(x) = 6 and $V'\_T(t_1) = 3$ $V'\_T(t_2) = 12$

  $(3, 12) \notin > \subseteq \text{Integers} \times \text{Integers}$

- Under **V** the atomic formula is true, but under **V'** the atomic formula is not.

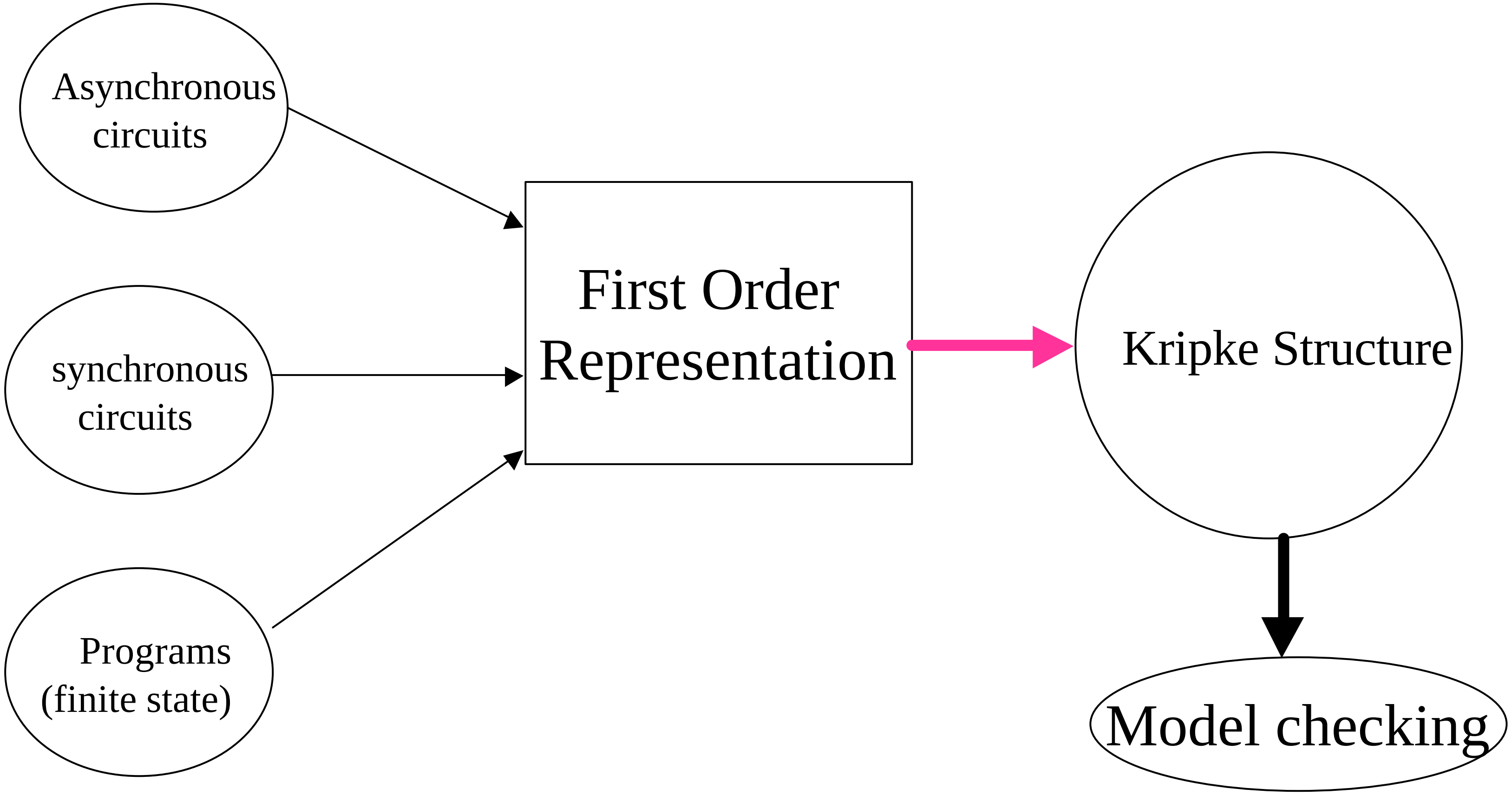
# Semantics

- The only case left is when j is of the form
  **$x.** j **’**
- j is satisfied under **V** iff there is a valuation **V’** such that j **’** is satisfied under **V’** and **V’** is required to meet the condition:
  - **V’** is exactly **V** for all variables except **x**.
  - To **x** , **V’** can assign ***any value*** of its choosing.

# Semantics

- Whether **$x.** j is true or not under **V**
  - does not depend on what **V** does on **x** !
- $**x. 2x = y** is true under **V(y) = 4**, **V(x) = 1**!
- Because, we can find **V'**, with **V'(y) = 4** but **V'(x) = 2**.
- One says **x** is ***bound*** in the formula and **y** is ***free***.

# The efficient way



# First Order Representation to Transition Systems

- $\{\mathbf{v_1, v_2, \ldots, v_n}\}$--- System variables.
- $\mathbf{D_1, D_2, \ldots, D_n}$ --- The corresponding domains.
- $\mathbf{D} = \bigcup \mathbf{D_i}$
- $\mathbf{s} : \{\mathbf{v_1, v_2, \ldots, v_n}\} \longrightarrow \mathbf{D}$ such that

  $\mathbf{s(v_1)} \in \mathbf{D_1} \ldots..$
- **S** --- The set of states.

# Initial States

- $S_0(v_1, v_2, \ldots, v_n)$ is a FO formula describing the set of initial states.
- Atomic formula
  - $v = d$ where $v$ is is a system variable and $d$ is a constant symbol interpreted as a member of the domain of $v$.

***Example:***

- "$S_0$ is the set of all states where the **pc = 0** and ***input*** is a power of 2"
- $\$n.\ (input = EXP(n))$ Ù $(pc = 0)$

# Transition relation

- $R(v_1,v_2,\ldots v_n,v_1',v_2',\ldots,v_n')$ is a FO formula involving the ***current variables*** $v_1,v_2,\ldots,v_n$ (the system variables) and the ***next variables*** ($v_1'$, $v_2'$,...,$v_n'$).

- $(d_1, d_2,\ldots,d_n) \longrightarrow (d_1', d_2',\ldots,d_n')$ iff $R(v_1,v_2,\ldots v_n,v_1',v_2',\ldots,v_n')$ is true under the valuation $v_1 = d_1,\ldots,v_n = d_n$, $v_1' = d_1',\ldots v_n' = d_n'$.

# Transition Relation

- V = {x, y, z}
- Program : {x, y, z, pc}

  $l_0$ : begin

  $l_1$ : statement$_1$

  $l_2$ : statement$_2$

  ….

  $l_5$ : if even(x) then x = x/2  else x = x –1

  $l_6$ : ….

# Transition Relation

- **V = {x, y, z}**
- Program : **{x, y, z, pc}**

  **$l_5$ : if even(x) then x = x/2  else x = x –1**

  **$l_6$ : ….**

- j **(x, y, z, pc, x’, y’, z’, pc’)**
- **pc = $l_5$** Ù **pc’ = $l_6$** Ù **(**$**n.(x = 2n)** É **x’ = x/2)** Ù
  **(**Ø$**n.(x = 2n)** É **x’ = x-1)** Ù **same(y, z)**

**Notice that the formula above is equivalent to:**

- **pc = $l_5$** Ù **pc’ = $l_6$** Ù
  **((**$**n.(x=2n)** Ù **x’=x/2)** Ú **(**Ø$**n.(x=2n)** Ù **x’=x-1))** Ù
  **same(y, z)**
- **where same(y, z) stands for y’ = y** Ù **z’ = z**

# Transition Relation

- In a similar fashion , we can specify the transition relation formulae for :
  - Assignment statement
  - While statements
  - etc.etc.
  - See the text book!

# Kripke Structures

- **AP** is a finite set of atomic propositions.
  - “**value of x is 5**”
  - “**x = 5**”
- **M = (S, $S_0$, R, L)**, a Kripke Structure.
  - **(S, $S_0$, R)** is a transition system.
  - **L : S** $\longrightarrow$ **$2^{AP}$**
  - **$2^{AP}$** ---- The set of subsets of AP

    (**L(s) Î $2^{AP}$** identifies a **state**

    **$2^{AP}$** identifies the **state space**)

# Kripke Structures

- The atomic propositions and **L** together convert a transitions system into a model.
- We can start interpreting ***formulas*** over the ***Kripke structure***.
- The atomic propositions make basic (easy) assertions about system states.

# Automata and Kripke Structures

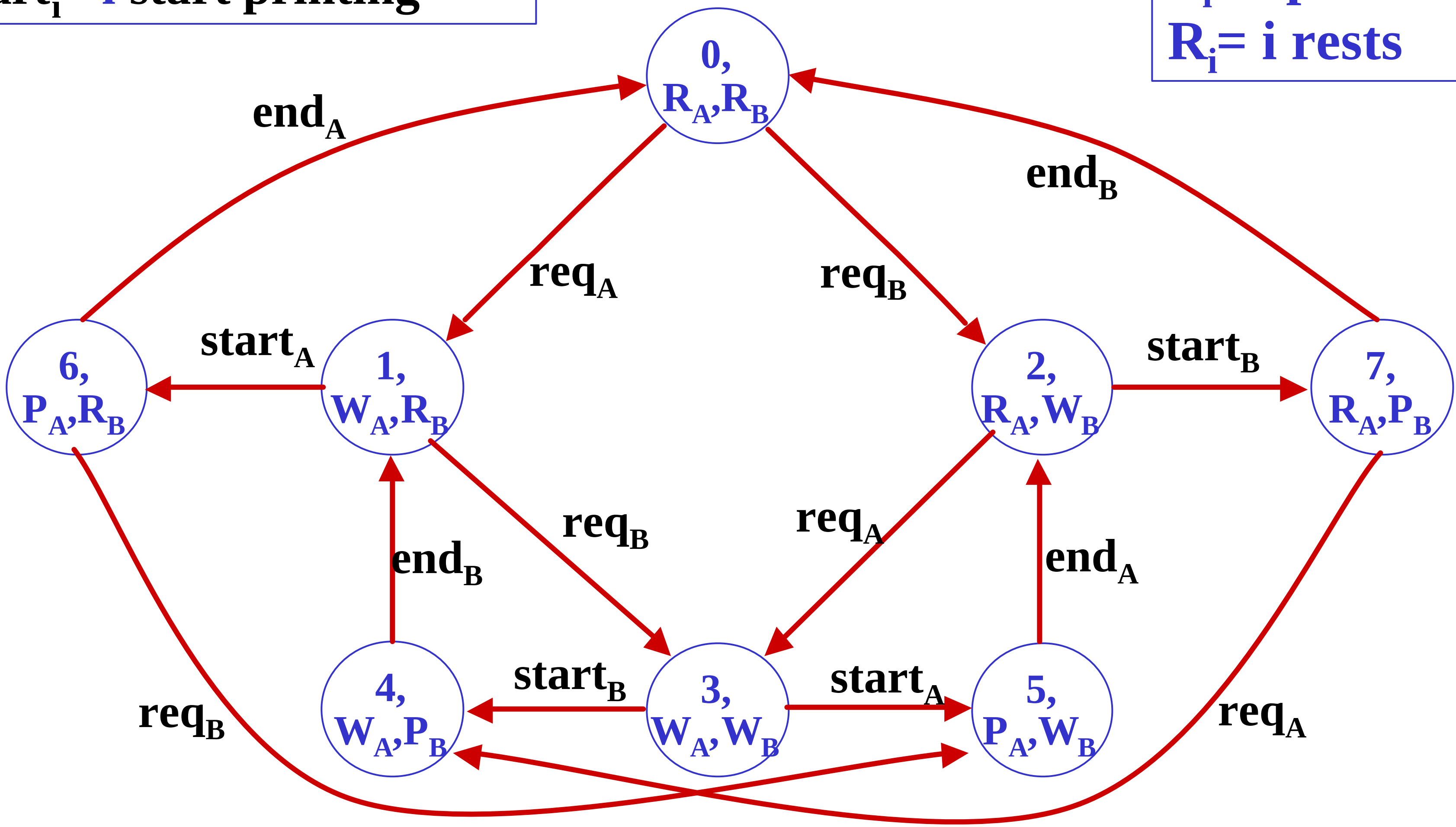
- **AP - set of elementary property**
- $\mathbf{<S,A,R,s_0,L>}$
- **S - set of states**
- **A - set of transition labels**
- **R Í S´A´S - (labeled) transition relation**
- **L - interpretation mapping** $\mathbf{L:S \longrightarrow 2^{AP}}$
- **In *FO representation* we would need two sets of variables: V and Act (for actions or input).**

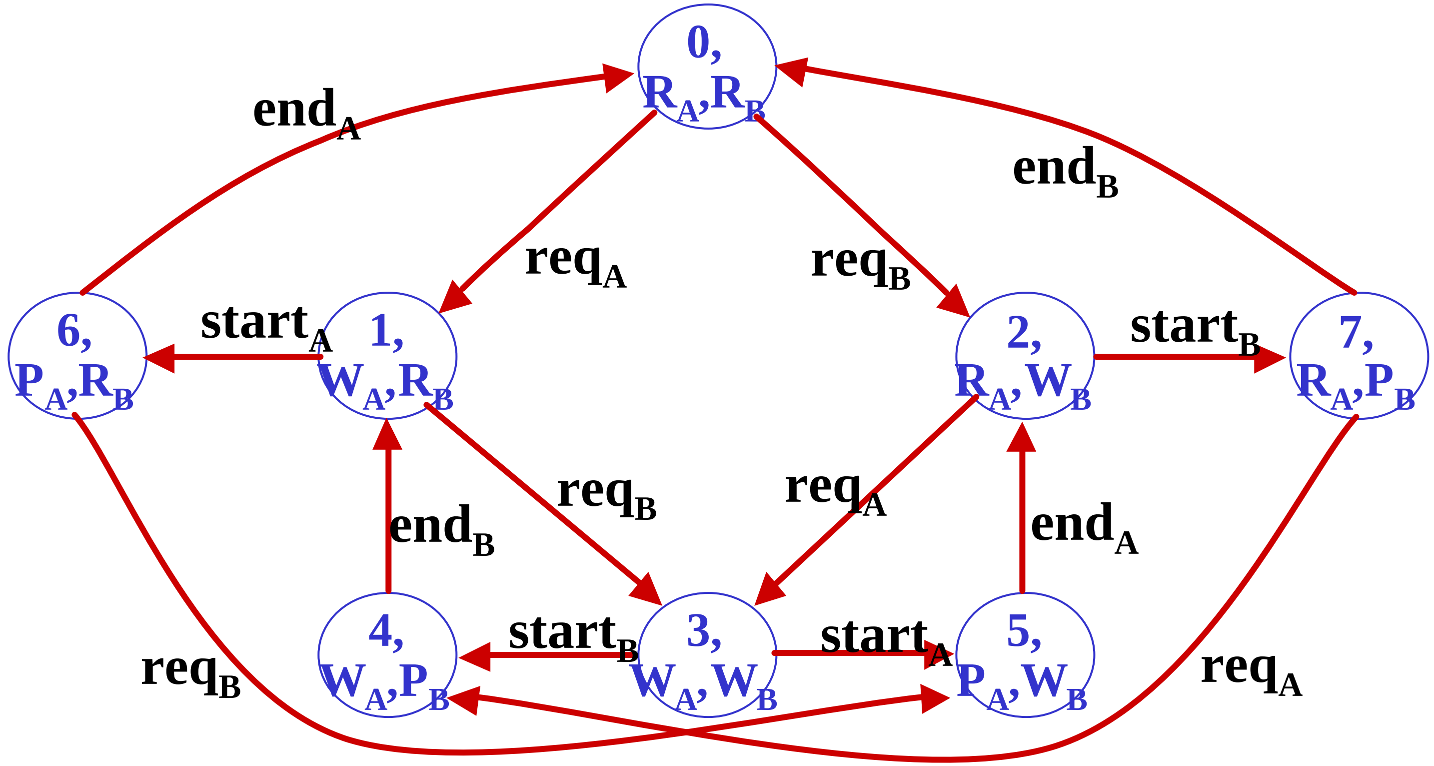
# Example: a print manager

**$end_i$ = i ends printing**
**$req_i$ = i requests printing**
**$start_i$ = i start printing**

**AP**

**$W_i$ = i waits**
**$P_i$ = i prints**
**$R_i$ = i rests**






- $S = \{0,1,2,3,4,5,6,7\}$
- $A = \{end_A, end_B, req_A, req_B, start_A, start_B\}$
- $R = \{(0,req_A,1), (0,req_B,2), (1,req_B,3), (1,start_A,6), (2,req_A,3), (2,start_B,7), (3,start_A,5), (3,start_B,4), (4,end_B,1), (5,end_A,2), (6,end_A,0), (6,req_B,5), (7,end_B,0), (7,req_A,4),\}$
- L = {0® $\{R_A,R_B\}$, 1® $\{W_A,R_B\}$, 2® $\{R_A,W_B\}$, 3® $\{W_A,W_B\}$, 4® $\{W_A,P_B\}$, 5® $\{P_A W_B\}$, 6® $\{P_A,R_B\}$, 7® $\{R_A P_B\}$ }

# Properties of the printing systems

1. Every state in which $\mathbf{P_A}$ holds, is preceded by a state in which $\mathbf{W_A}$ holds
2. Any state in which $\mathbf{W_A}$ holds is followed (possibly not immediately) by a state in which $\mathbf{P_A}$ holds.

- The first can easily be checked to be true
- The second is ***false*** (e.g. 0134134134…) - in other words the system is ***not fair***.

# Synchronization

- Usually complex systems are composed of a number of smaller ***subsystems*** (***modules***)
- It is natural to model the whole system starting from the models of the subsystems.
- And then define how they cooperate.
- There are many ways to define cooperation (***synchronization***).

# Synchronization: no interaction

The system model is just the ***cartesian product*** of the simpler modules.

Let $TS_1,\ldots,TS_n$ be $n$ automata (or **TSs**), where

$$TS_i = \langle S_i, A_i, R_i, s_{i0} \rangle$$

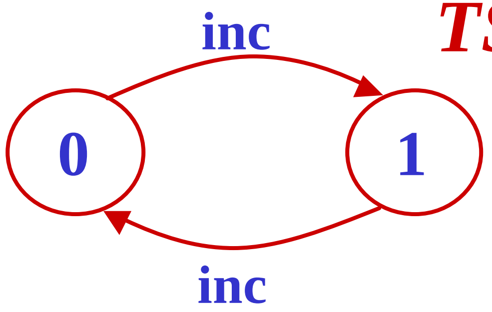
The system is then defined as $TS=\langle S,A,R,s_0\rangle$ where

$$S = S_1 \times S_2 \times \ldots \times S_n$$

$$A = A_1 \cup \{-\} \times A_2 \cup \{-\} \times \ldots \times A_n \cup \{-\}$$

$$R = \{(\langle s_1,\ldots,s_n\rangle, \langle a_1,\ldots,a_n\rangle, \langle s'_1,\ldots,s'_n\rangle) \mid \textit{forall } i,\ a_i \neq - \textit{ and } (s_i,a_i,s'_i) \in R_i, \textit{ or } a_i = - \textit{ and } s'_i = s_i\}$$

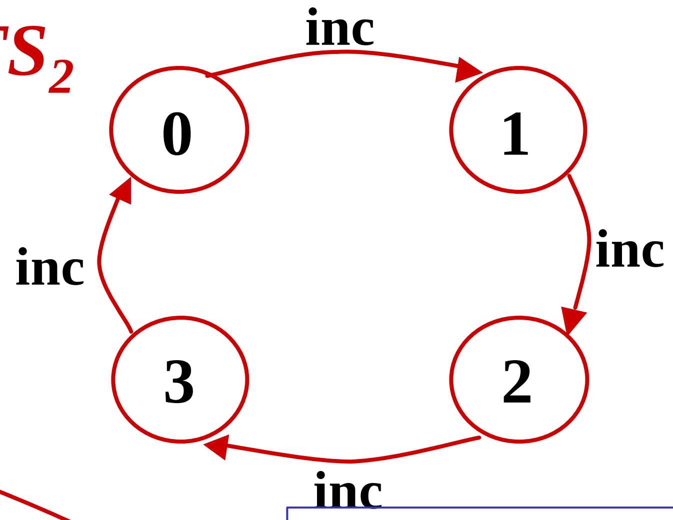
$$s_0 = \langle s_{10}, s_{20}, \ldots, s_{n0} \rangle$$



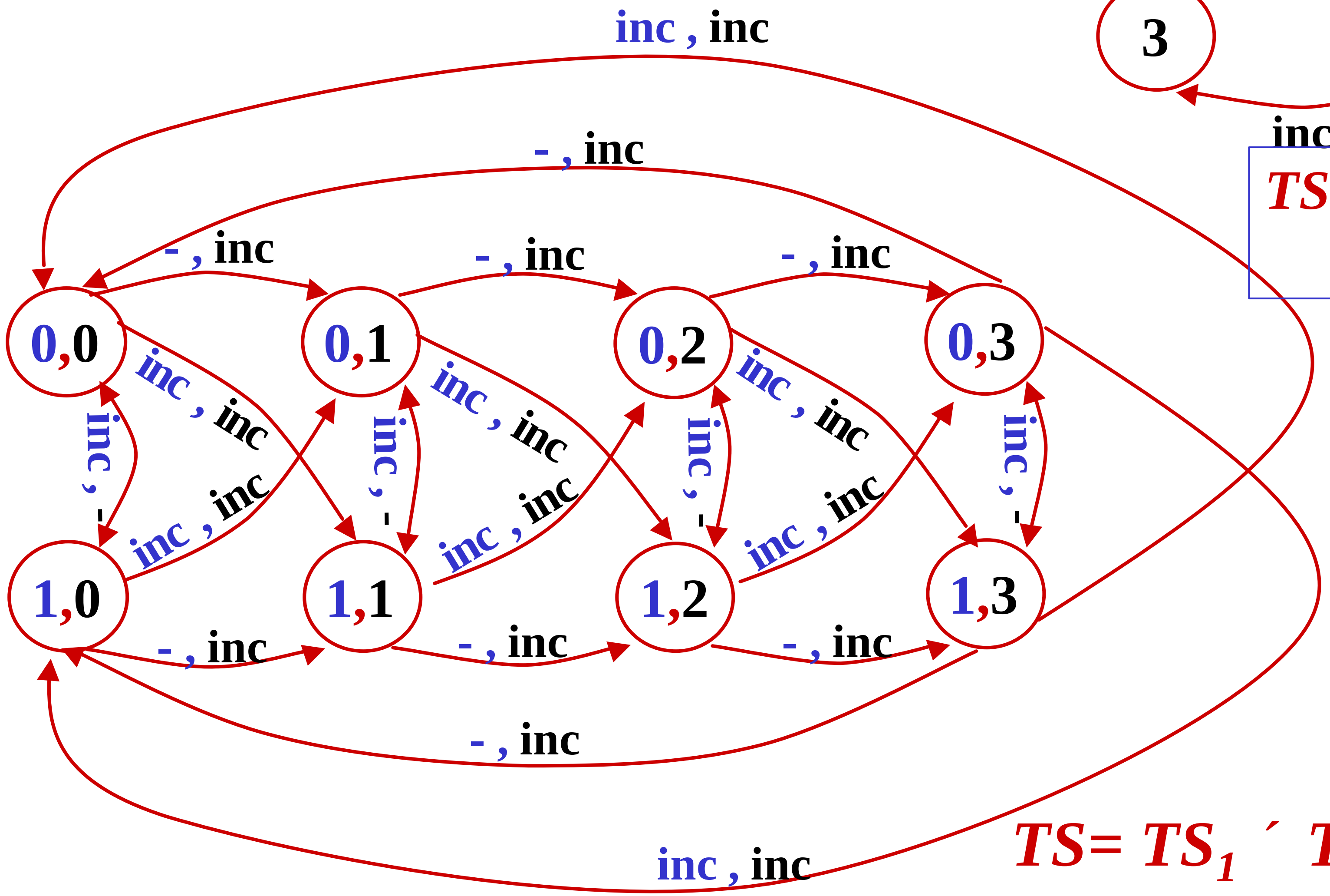

$TS_1$

$TS_1$ counter
modulo 2

$TS_2$




$TS_2$: counter
modulo 4




$TS = TS_1 \times TS_2$

# Synchronization: interaction

***To allow for interaction, or synchronization on specific actions we can introduce a Synchronization Set (to inhibit undesired transitions) :***

- ***Synchronization set is just a subset of the composite actions:***

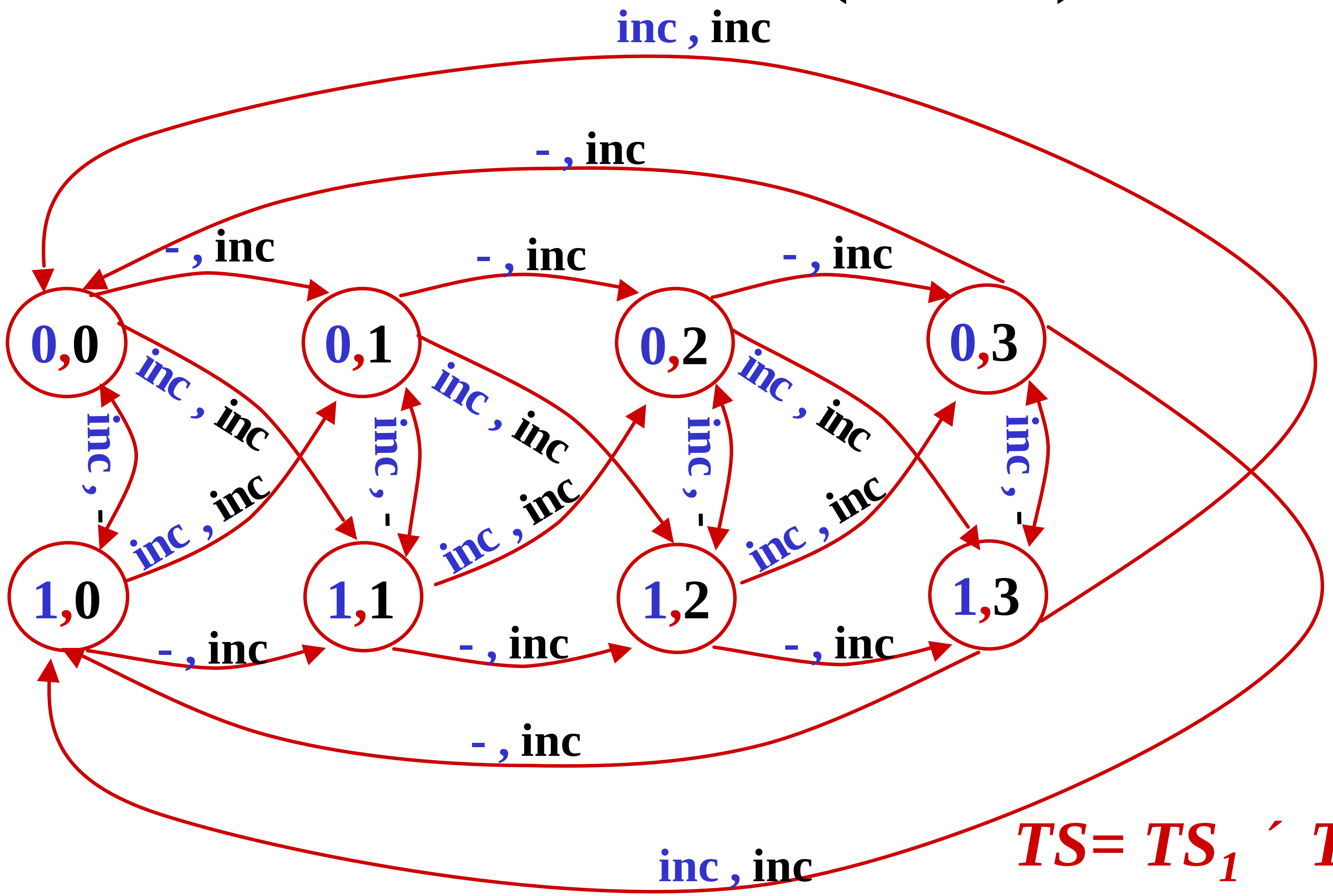
$$Sync \subseteq A_1 \cup \{-\} \times A_2 \cup \{-\} \times \ldots \times A_n \cup \{-\}$$

- ***Then we will have to define the possible transitions as:***

$$R=\{(\langle s_1,\ldots,s_n\rangle,\langle a_1,\ldots,a_n\rangle,\langle s'_1,\ldots,s'_n\rangle) \mid$$
$$(a_1,\ldots,a_n) \in Sync \text{ and forall } i,\ a_i \neq -$$
$$\text{and } (s_i,a_i,s'_i) \in R_i, \text{ or } a_i = - \text{ and } s'_i = s_i\}$$

# *Free synchronization (Asynchronous systems):*

*Sync* = {*inc*,-} ´ {-,*inc*} = {(-,-), (*inc*,-), (-,*inc*),
(*inc*,*inc*)}




*TS*= *TS*$_1$ ´ *TS*$_2$

# *Free synchronization*

*Asynchronous systems:*

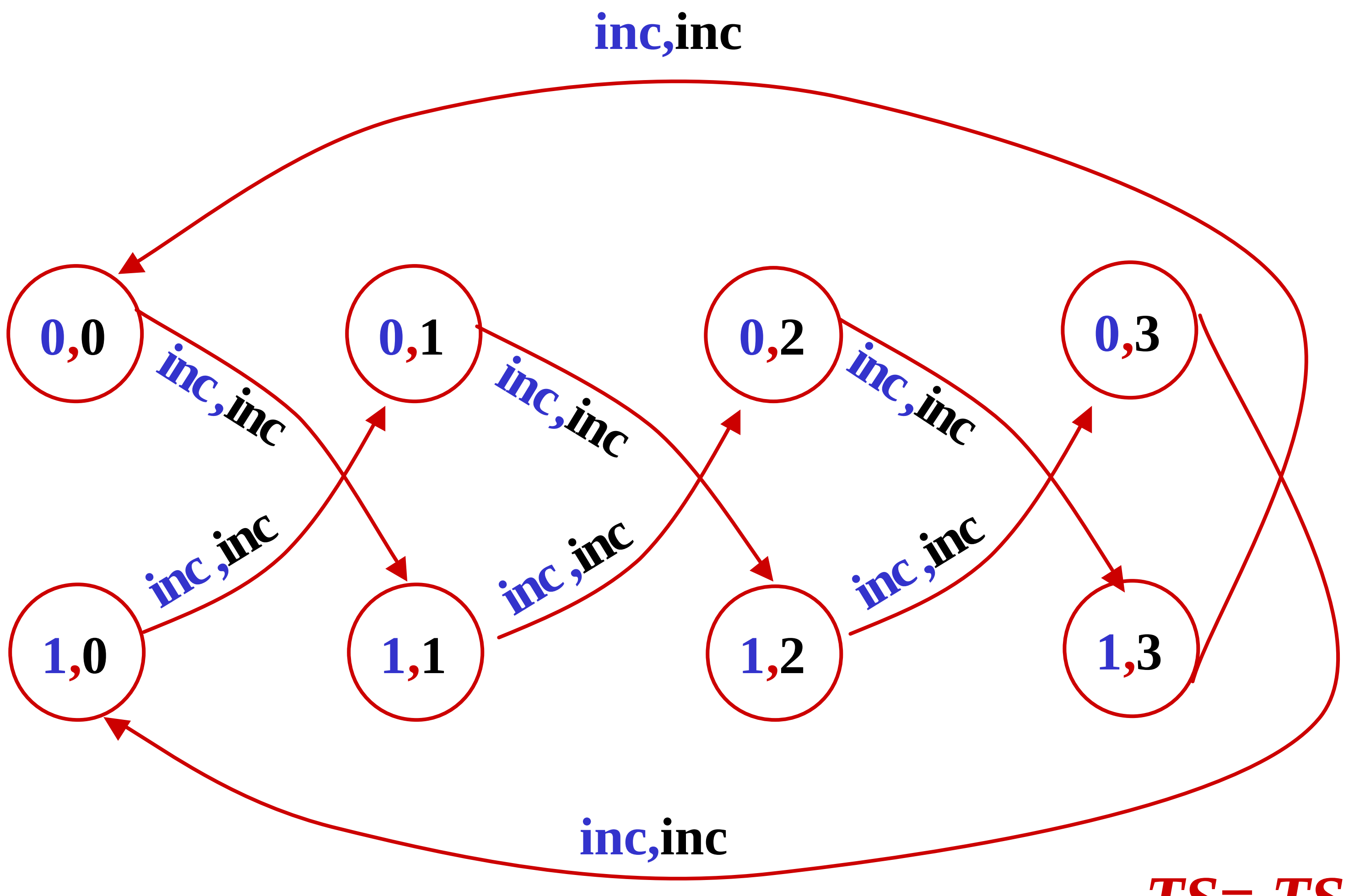
$$Sync = \{inc,-\} \times \{-,inc\} \quad \boxed{\setminus \ \{(-,-)\}}$$

$$R(V,V') = \bigwedge_{i \in I} \big(R_i(v_i,v_i') \vee \mathbf{same}(v_i)\big) \quad \boxed{\wedge \neg \bigwedge_{i \in I} \mathbf{same}(v_i)}$$

**if one wants to *discard* the situation where *no component acts***

***Synchronization on all actions (Synchronous systems):***

***Sync* = {(*inc*,*inc*)}**




***TS*= *TS*$_1$ ´ *TS*$_2$**

# Synchronous systems

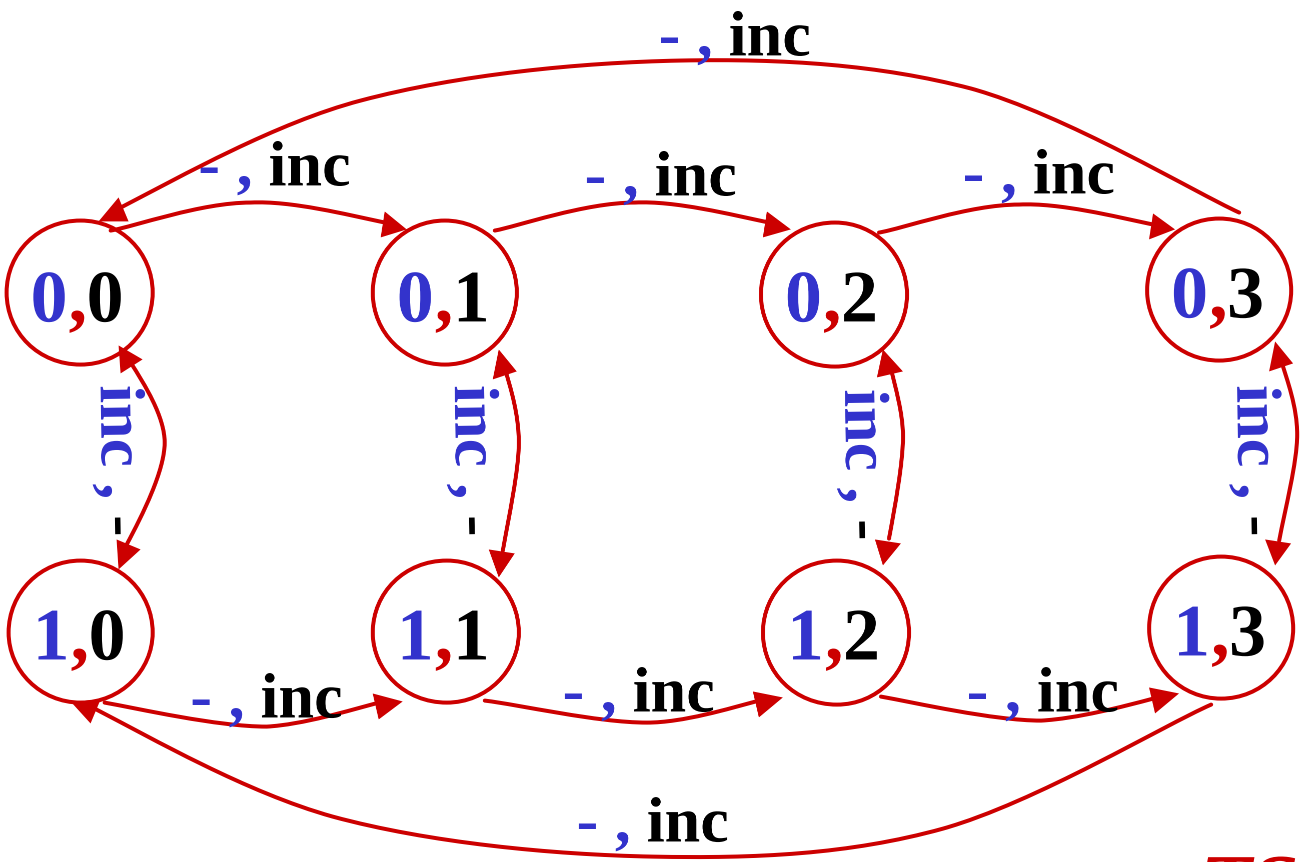
***Synchronous systems:***

***Sync* = {(*inc*,*inc*)}**

$$R(V,V') = \bigwedge_{i \in I} R_i(v_i,v_i')$$

## *Asynchronous systems with interleaving (only one component acts at any time):*

$\mathit{Sync} = \{(-,\mathit{inc}),(\mathit{inc},-)\}$




$TS = TS_1 \times TS_2$

# *Asynchronous systems: Interleaving*

***Asynchronous systems**: only one component acts at any time.*

***Sync** = {(-,inc),(inc,-)}*

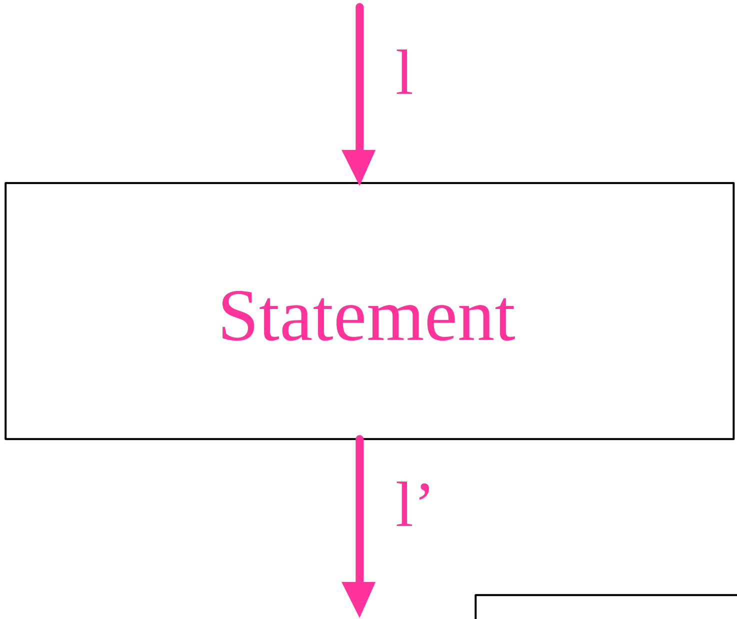
$$R(V,V') = \bigvee_{i \in I} \left( R_i(v_i,v_i') \wedge \bigwedge_{j \neq i} \mathrm{same}(v_j) \right)$$

# Concurrent programs

- Many systems to be verified can be viewed as concurrent programs
  - operating system routines
  - cache protocols
  - communication protocols
- **P = cobegin ($P_1$ || $P_2$ || …|| $P_n$) coend**
- **$P_1$, $P_2$,..$P_n$** --- Sequential Programs.
- ***Program variables*** set **$V$ = $V_1$**È…È**$V_n$** (set **$V_i$** for program **i**)
- ***Program counters*** set **PC** (one for each program)
- ***Usually interleaving semantics is assumed***

# Sequential Programs: the transition predicate C

General Structure




**C(l, statement, l’)**

**C is essentially a *translation function* taking a *label*, a *program statement* and a *label* and giving the *FOL* formula specifying the transition relation for the statement.**

# Assignments

**C(l, v:=expr., l’)**

l

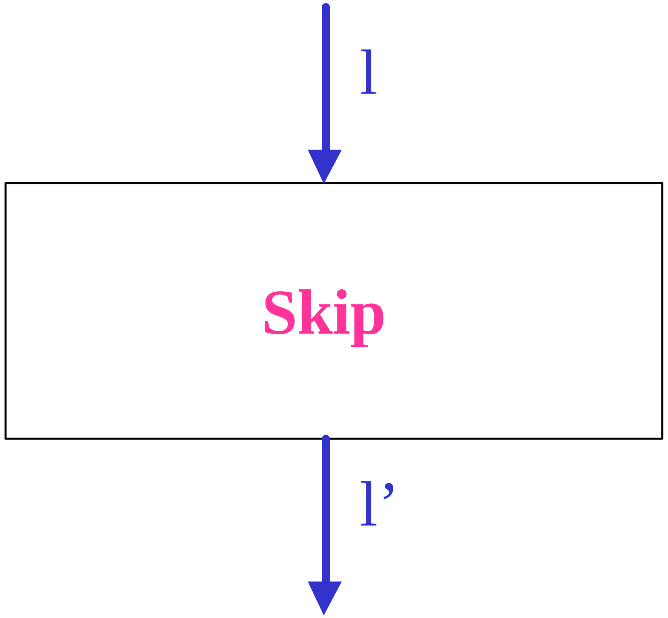
**v:= expr.**
**(v := 2x – v +3·y)**

l’

**pc = l Ù pc’ = l’ Ù v’ = expr. Ù**
**Ù same (V – {v}) Ù same (PC – {pc})**

**[for Y = {$y_1$, $y_2$, ..$y_m$},**
**same(Y) º $y_1$’ = $y_1$ Ù $y_2$’ = $y_2$ Ù… Ù $y_m$]**

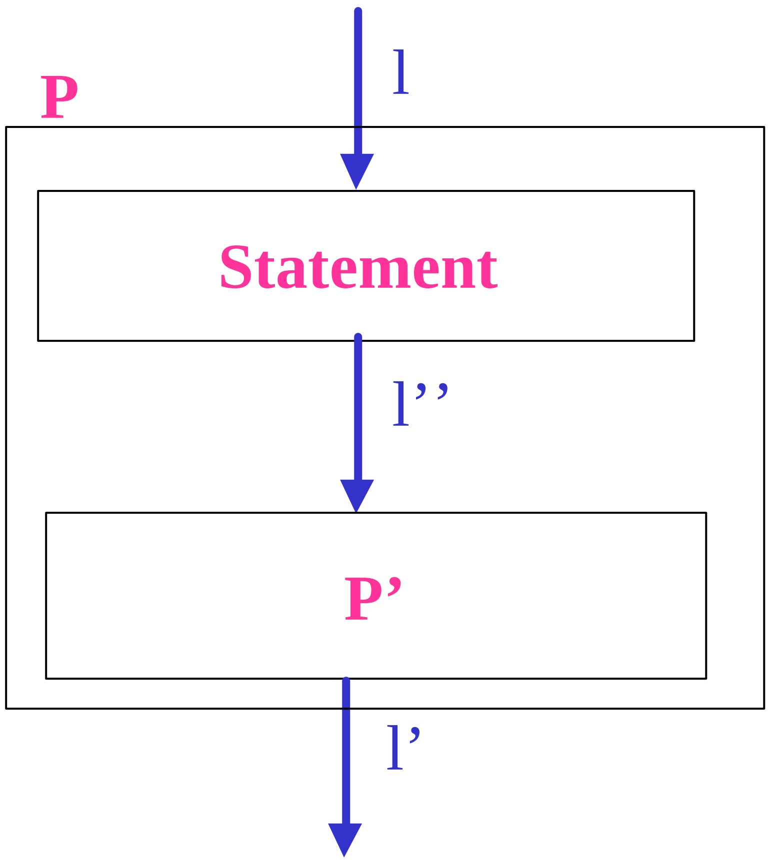
# Skip




**C(l, skip, l’)**

**pc = l Ù pc’ = l’ Ù same (V)**
**Ù same (PC – {pc})**

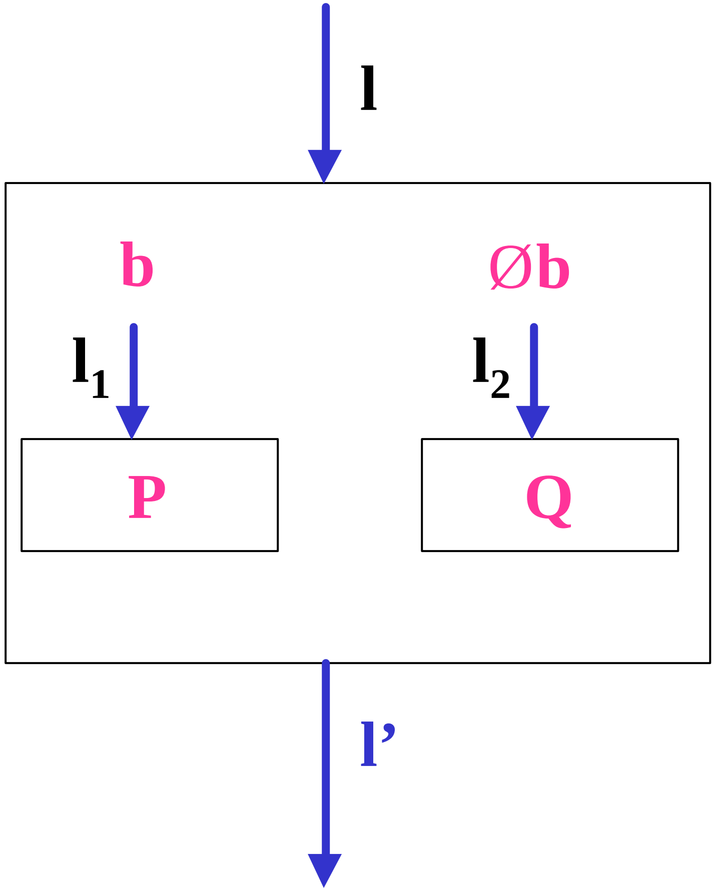
# Sequential composition




C(l, **P** , l')

C(l, **Statement** , l'') Ú
C(l'', **P'** , l')

# Conditional statement




**C(l, IF-THEN-ELSE(b,$l_1$,$l_2$), l')**

**(pc = l Ù pc' = $l_1$ Ù b Ù same(V) Ù same (PC – {pc}) Ú**

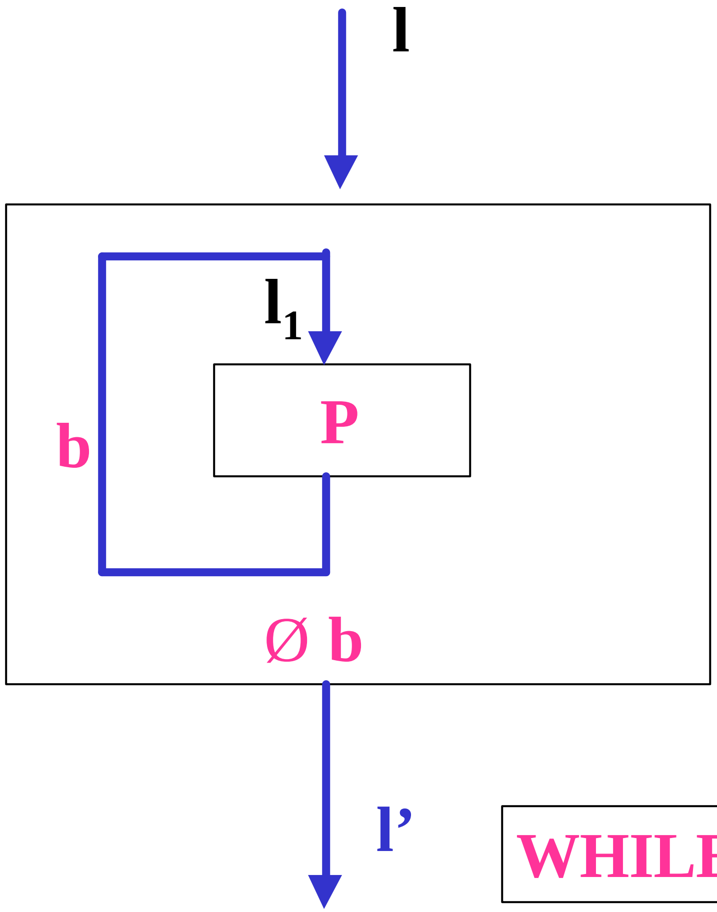
**(pc = l Ù pc' = $l_2$ Ù Ø b Ù same(V) Ù same (PC – {pc}) Ú**

**C($l_1$, P, l') Ú**

**C($l_2$, Q, l')**

**IF b THEN P ELSE Q FI**

# While statement




C(l,WHILE(b, $l_1$), l’)

(pc = l Ù pc’ = $l_1$ Ù b Ù same(V) Ù same (PC – {pc}) Ú

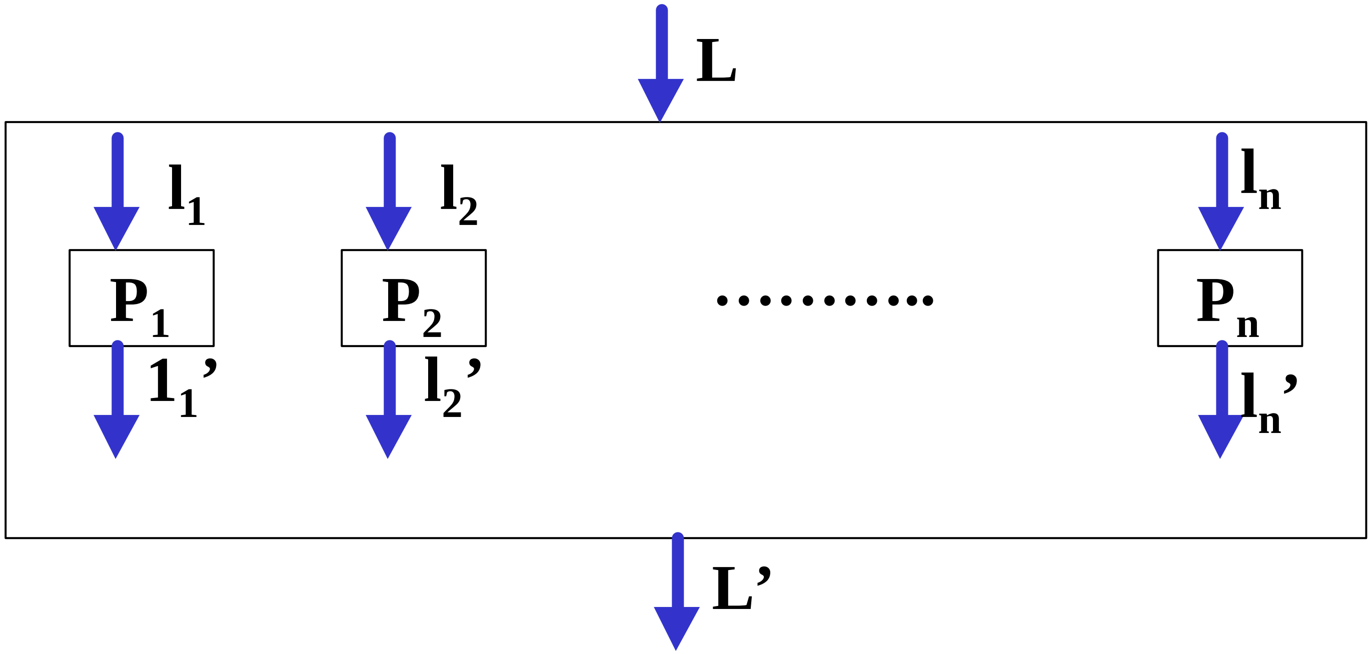
(pc = l Ù pc’ = l’ Ù Ø b Ù same(V) Ù same (PC – {pc}) Ú

C($l_1$, P, l)

WHILE b DO P END_WHILE

# Concurrent programs

- **P =** **cobegin** **($P_1$ || $P_2$ || …|| $P_n$)** **coend**
- **$P_1$, $P_2$,..$P_n$** --- Sequential Programs.



# Concurrent programs

- **P** = **cobegin** **($P_1$ || $P_2$ || …|| $P_n$)** **coend**
- **$P_1$, $P_2$,..$P_n$** --- ***Sequential Programs***.
- **$C(l_1, P_1, l_1')$** --- The transitions of program **$P_1$** (defined ***inductively*** on the structure of **$P_1$**!).
- **$V_i$** ---- The set of variables of program **$P_i$**.
- Programs may ***share*** variables!
- **$pc_i$** – The program counter of program **$P_i$**.

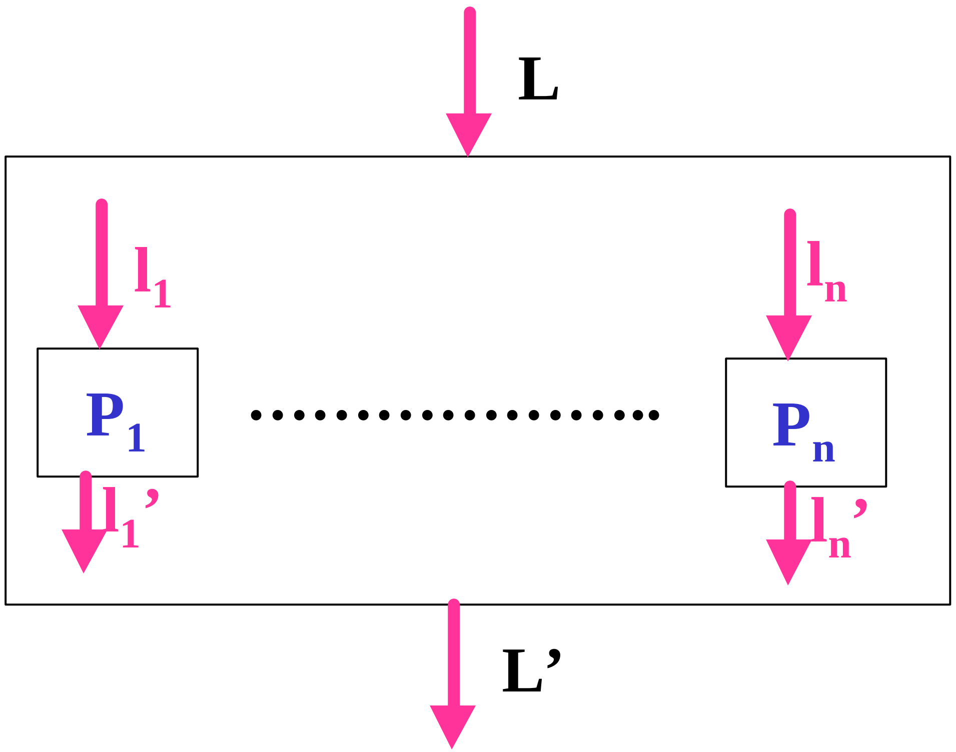
# Concurrent programs

- **pc** ---- the program counter of the ***concurrent program***; it could be part of a larger program!
- ^ denotes an ***undefined*** program counter value.
- ***$S_0$(V, PC)*** **= pre(V)** Ù **(pc=L)** Ù

  **($pc_1$=^)** Ù **……** Ù **($pc_n$=^)**

  Ù **same(V)**

# The Transition Predicate




**C(L, P, L')**

(pc = L Ù
Ù $pc_1$' = $l_1$ Ù…. Ù $pc_n$' = $l_n$ Ù
∧ pc' = ∧ Ù same(V)) Ú
(C($l_1$, $P_1$, $l_1$') Ù Same (V – $V_1$)
Ù Same(PC –{$pc_1$})) Ú…
C($l_n$, $P_n$, $l_n$') Ù Same (V – $V_n$)
Ù Same(PC –{$pc_n$})) Ú
(pc = ∧ Ù
$pc_1$ = $l_1$' Ù… Ù $pc_n$ = $l_n$' Ù
Ù pc' = L' Ù
$pc_1$' = ∧ Ù …$pc_n$' = ∧ Ù same(V))

# Summary

- System variables
- Domain of values
- States
- Initial state predicate
- Transition predicate
- pc values (for programs)
- Synchronization mechanisms