Tecniche di Specifica e di Verifica

Linear Time Temporal Logic

Temporal Logics: The context

- ***Kripke Structures*** model systems.
- ***Temporal logics*** model dynamic behavioral properties of systems.
  - **Linear Time**
  - Branching Time
- ***Model checking*** can be used to determine if a system has the desired behavioral property.

Linear time temporal logics.

- ***LTL (Linear Time Temporal Logic)***
  - **Syntax**
  - **Semantics**
  - The Model Checking Problem.
  - Its solution.

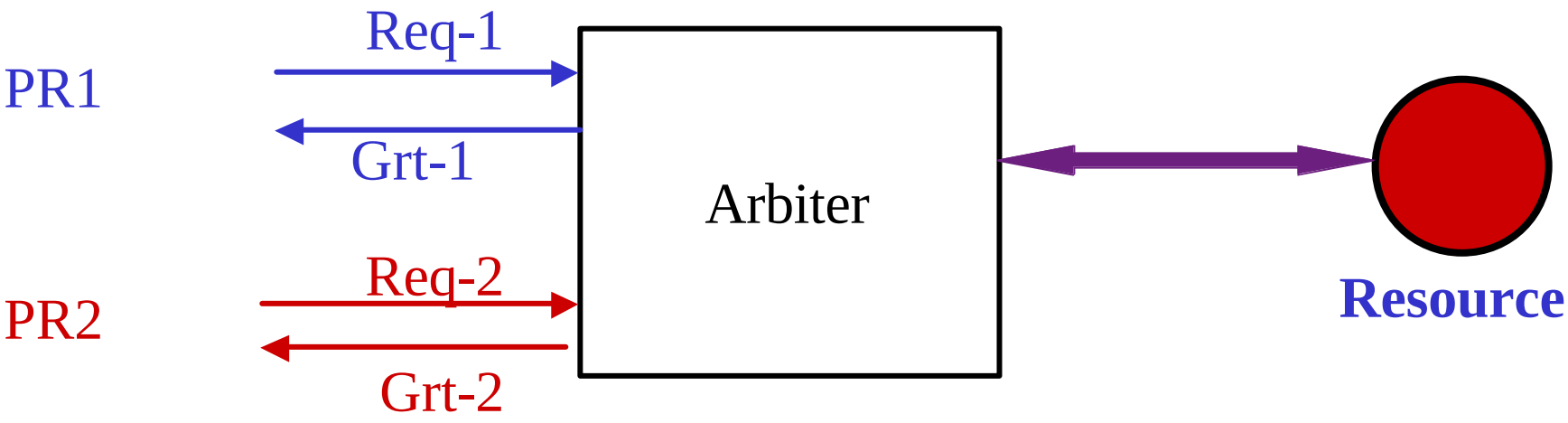
The Application

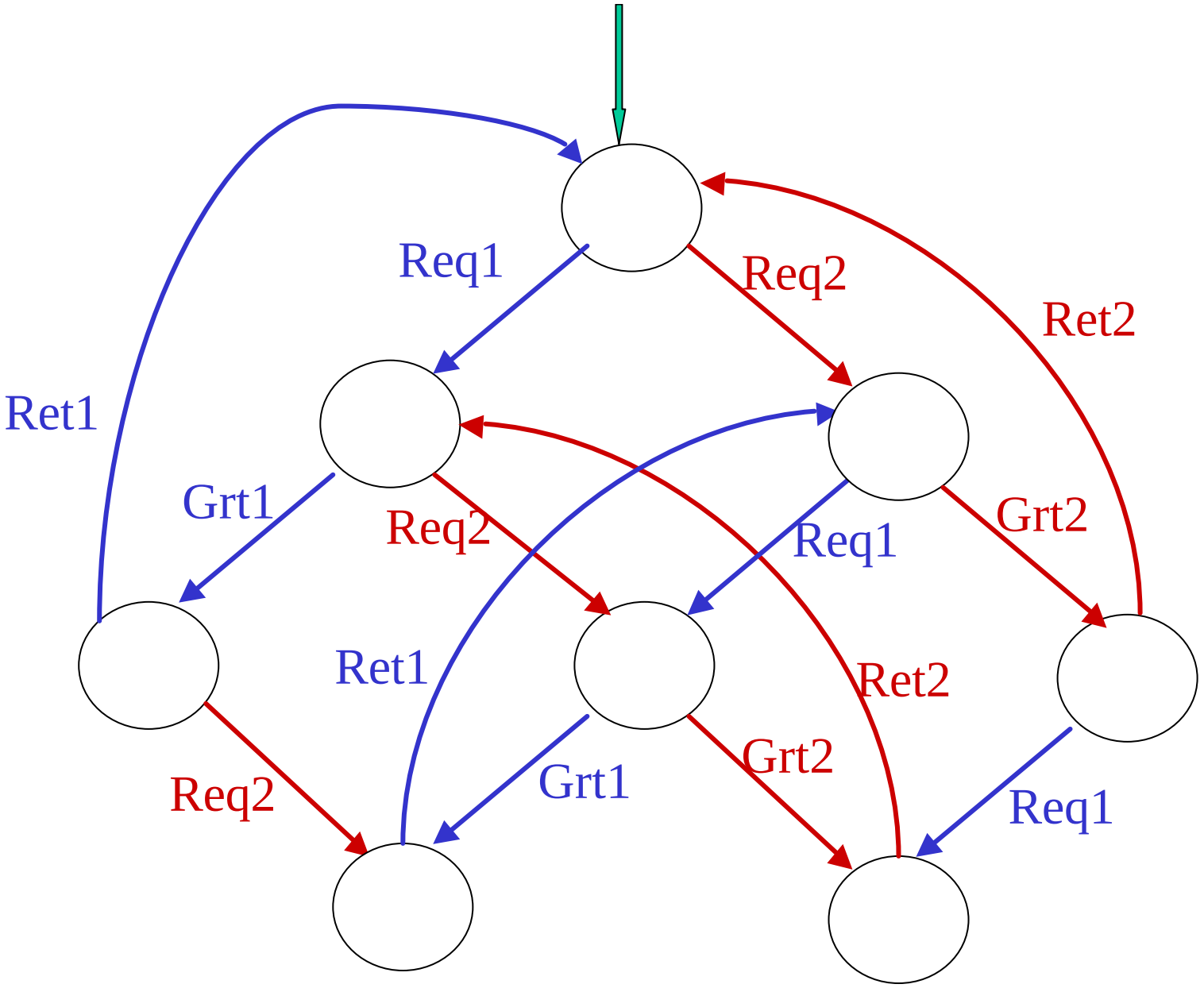
- Model a system to be verified as a Kripke structure:
  - Transition system **TS = (S, S_0, R)**
  - **AP** = A finite set of atomic propositions.
    - Basic assertions about the system
  - **L : S $\longrightarrow$ 2^{AP}** = The set of subsets of **AP**.
  - **p Î L(s) ---- p** is true at **s**.
  - **p Ï L(s) ---- p** is not true at **s**.
- **K = (S, S_0, R, AP, L)** ---- Kripke structure

The Application

- ***The computations*** of the Kripke structure **K** will be the ***models*** for **LTL** formulas.
- ***The property*** to be verified is captured as an LTL formula φ.
- The modeled system **K** has the property φ ***iff every computation of*** **K** ***is a model of*** φ.
- We need to verify (***model check***) whether:
  - $\mathbf{K} \models \varphi$

An Example




Req1
Req2
Ret2
Ret1
Grt1
Req2
Req1
Grt2
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Req2
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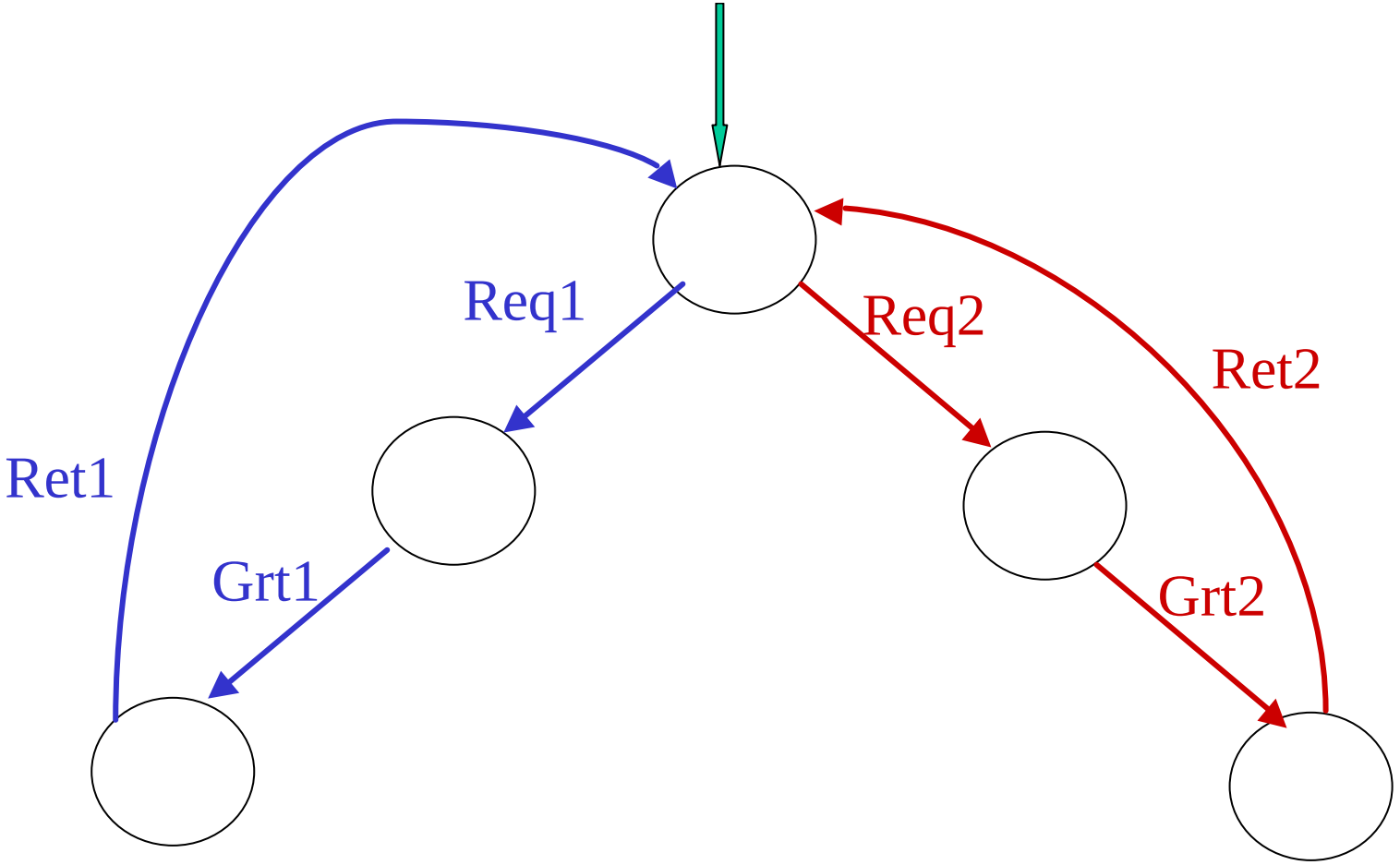
Grt2
Req1


Req1
Req2
Ret2
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Grt1
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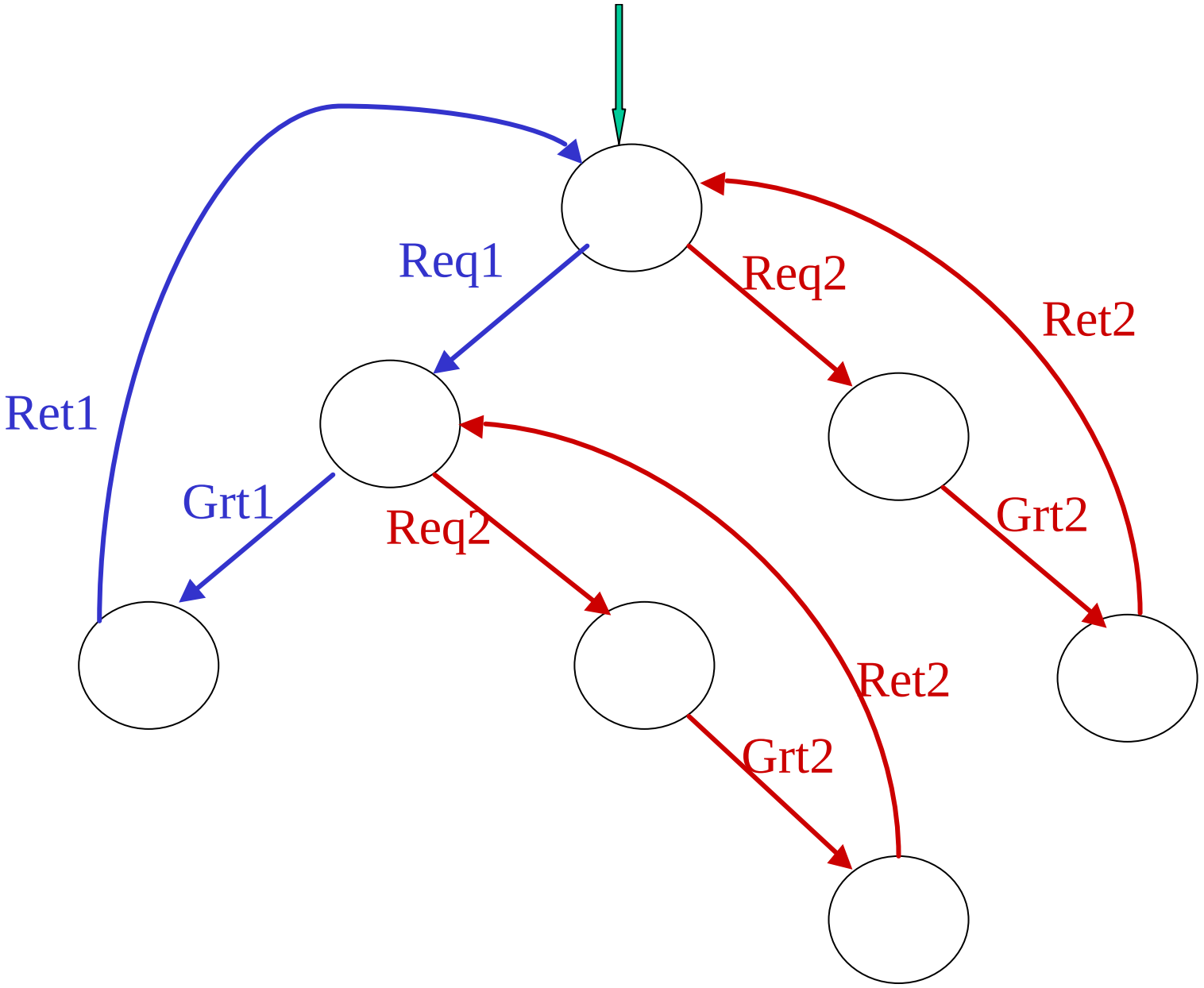
Req1
Req2
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Req2
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Ret2
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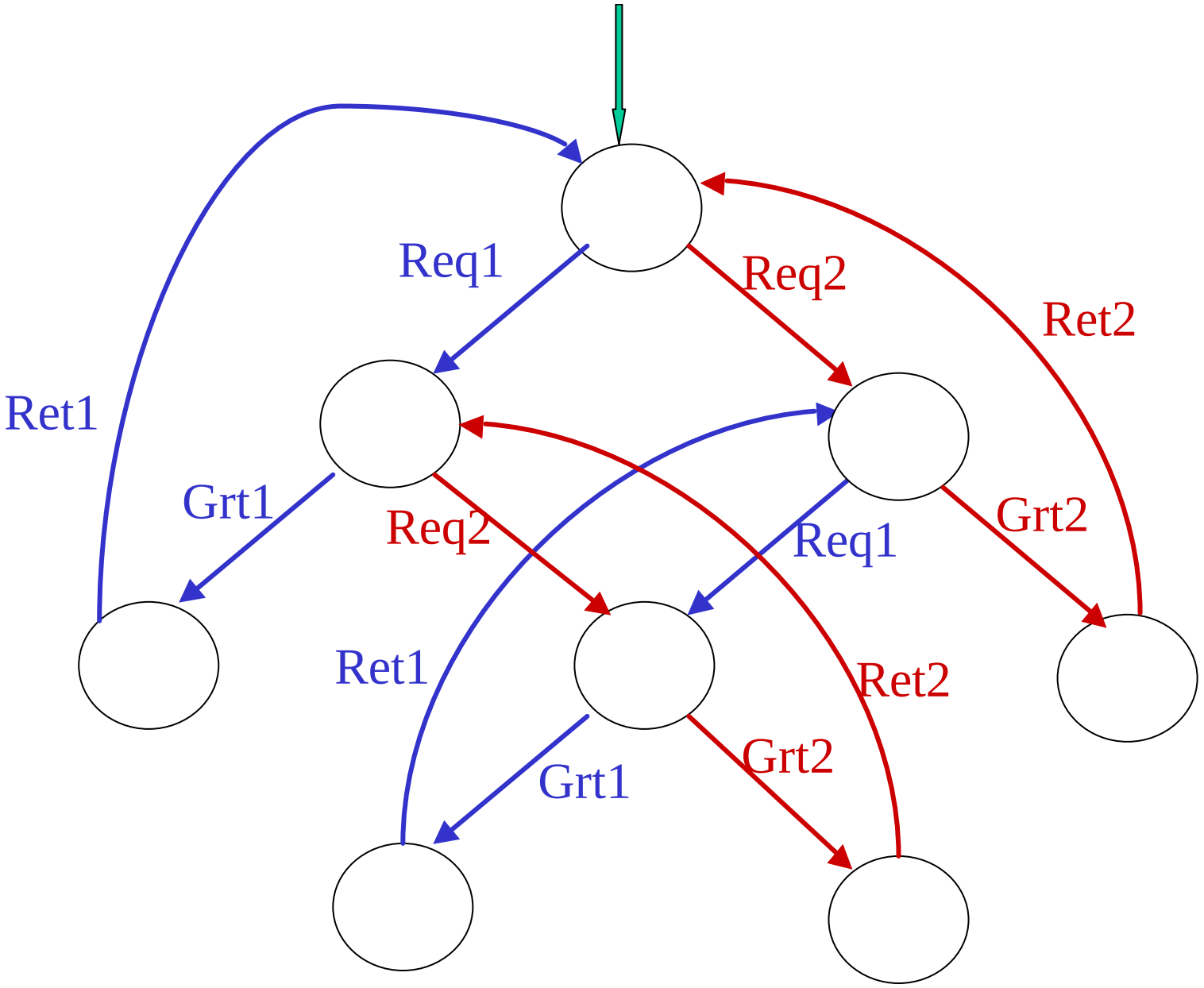
Req1
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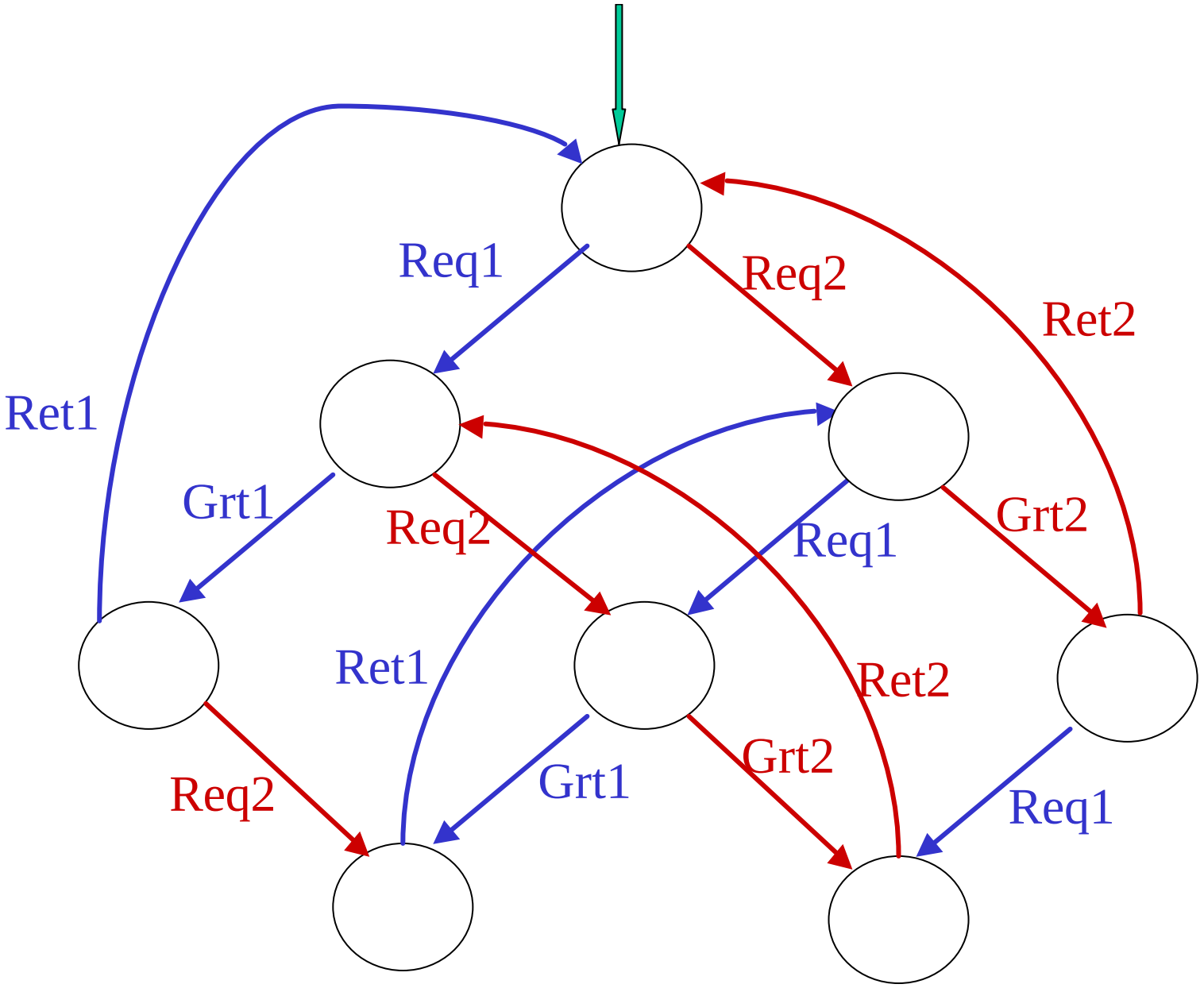
Req1
Req2
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Req2
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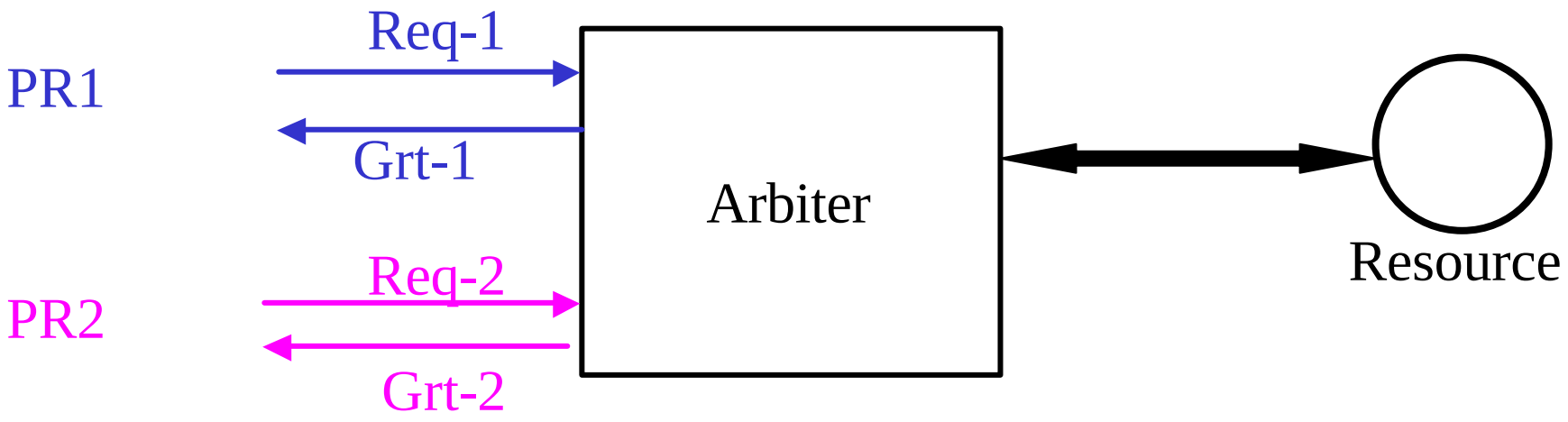
Req1

A set of Atomic Propositions



R1 – Process 1 is ***idle***

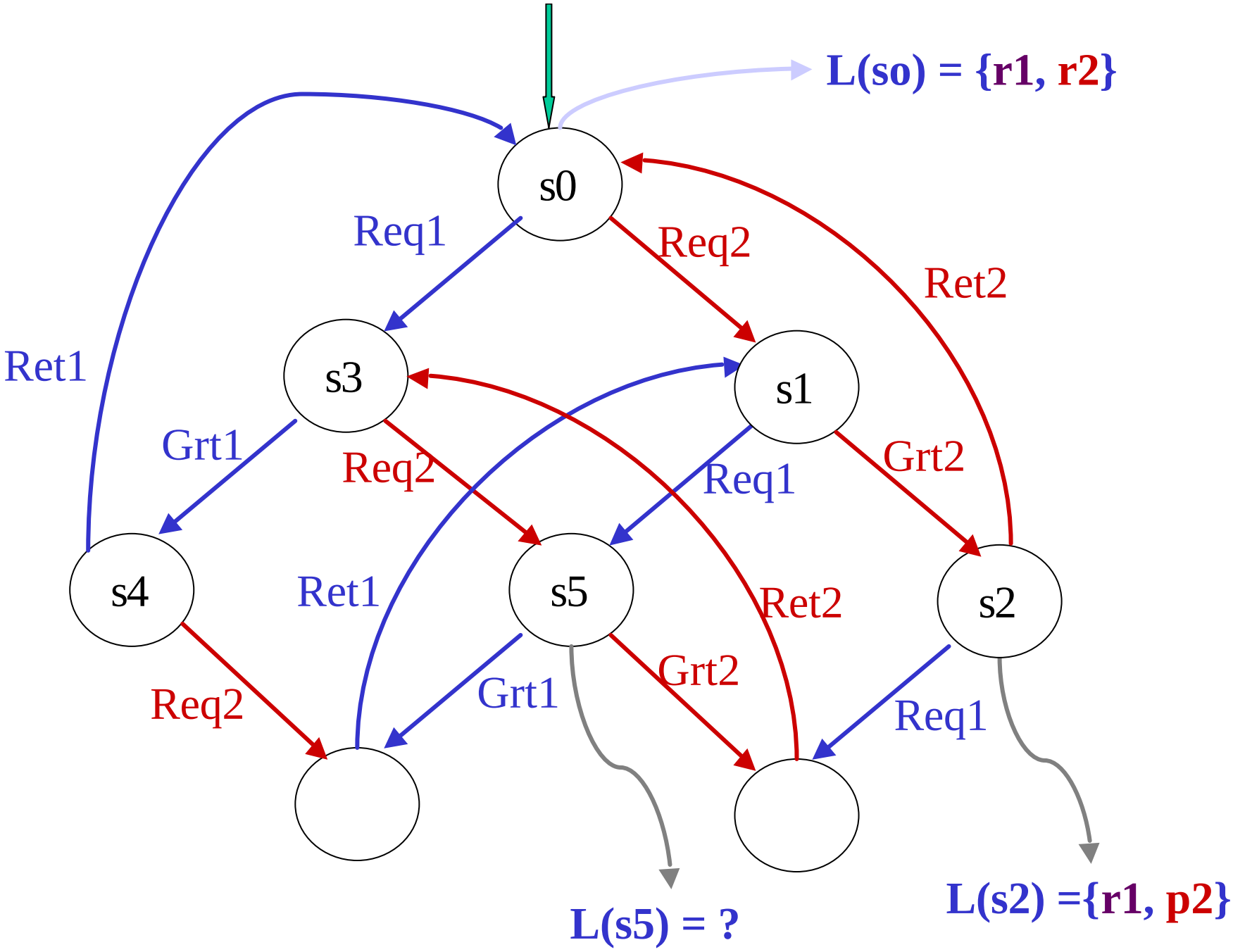
W1– Process 1 is ***waiting***

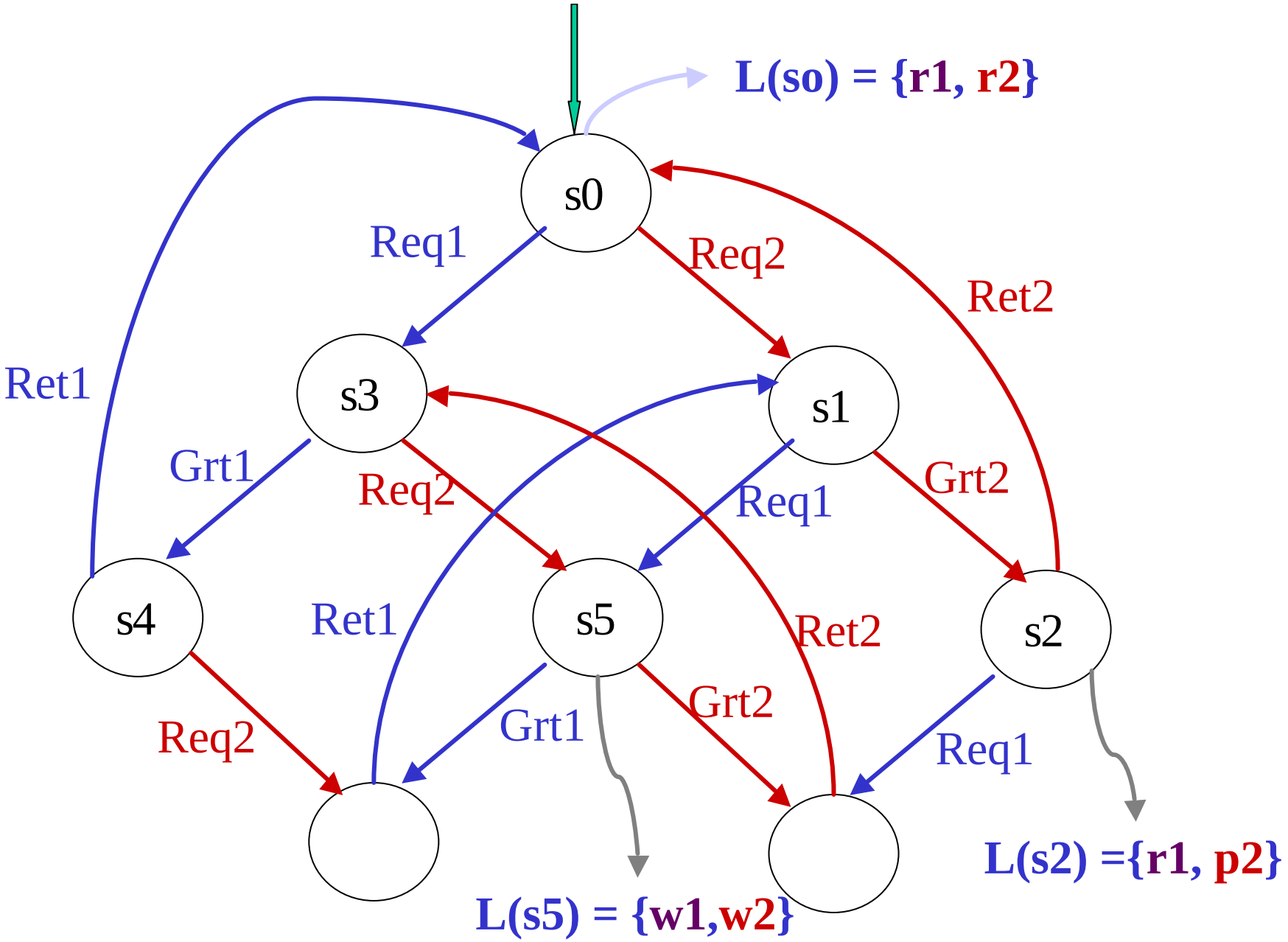
P1 – Process 1 is ***using*** the resource.

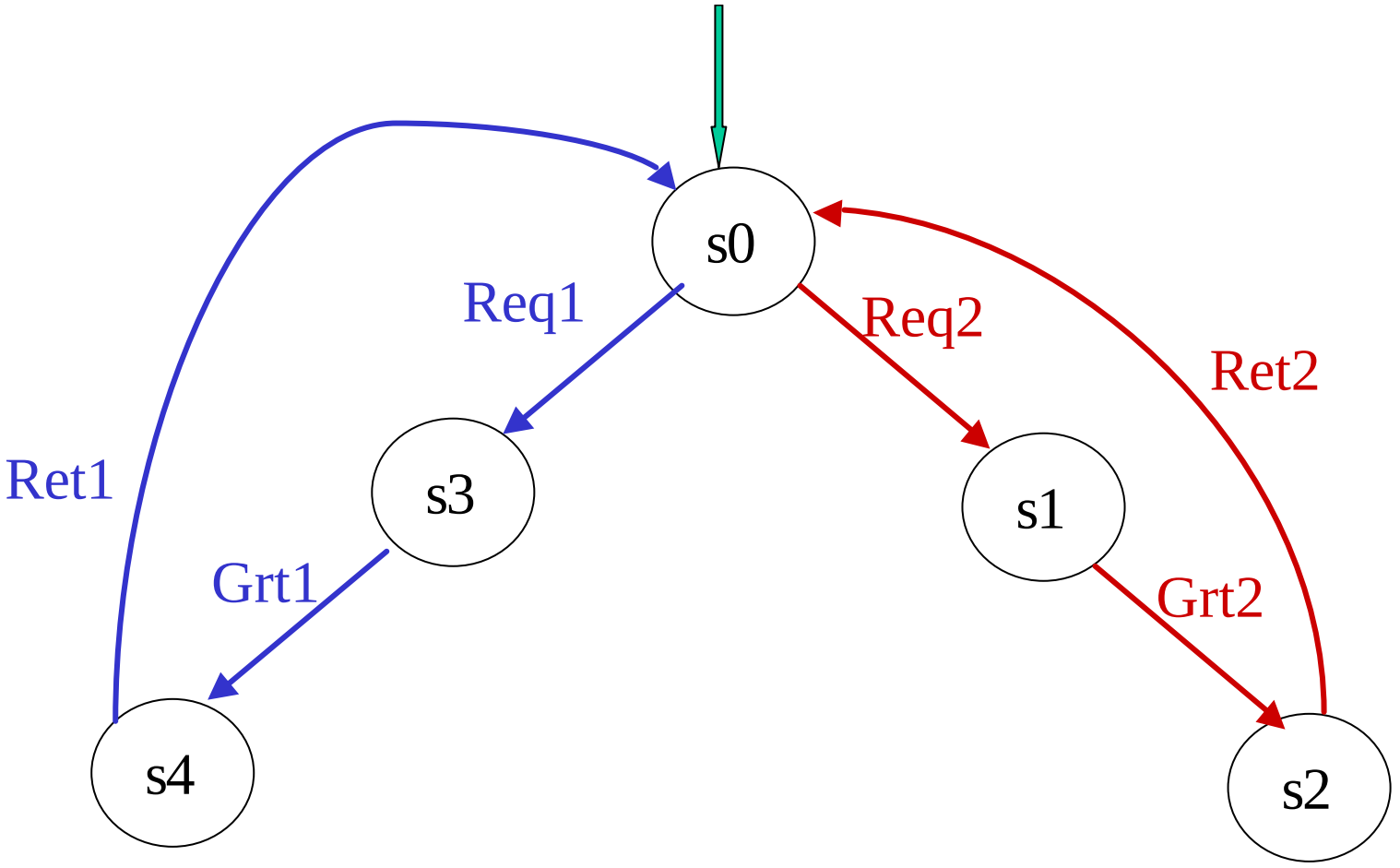
AP = { R1, W1, P1, R2, W2, P2}

The context

- Model a system to be verified as a Kripke structure:
  - Transition system **TS = (S, S_0, R)**
  - AP = A finite set of atomic propositions.
    - Basic assertions about the system
  - **L : S** $\longrightarrow$ $\mathbf{2^{AP}}$ = The set of subsets of AP.
  - **p** Î **L(s)** ---- **p** is true at **s**
  - **p** Ï **L(s)** ---- **p** is not true at **s**.
- **K = (S, S_0, R, AP, L)** ---- Kripke structure


L(so) = {r1, r2}
s0
Req1
Req2
Ret2
Ret1
s3
s1
Grt1
Req2
Req1
Grt2
s4
Ret1
s5
Ret2
s2
Req2
Grt1
Grt2
Req1
L(s5) = ?
L(s2) ={r1, p2}


L(so) = {r1, r2}
s0
Req1
Req2
Ret2
Ret1
s3
s1
Grt1
Req2
Req1
Grt2
s4
Ret1
s5
Ret2
s2
Req2
Grt1
Grt2
Req1
L(s2) ={r1, p2}
L(s5) = {w1,w2}



s0 s3 s4 s0 s1 s2 s0 s3 …

{r1, r2} {w1, r2} {u1,r2} {r1, r2} {r1,w2} {r1,p2} {r1,r2} {w1,r2}...

Assertions about a computation

s0	s3	s4	s0	s1	s2	s0	s3 …
{r1, r2}	**{w1, r2}**	**{p1,r2}**	**{r1, r2}**	**{r1,w2}**	**{r1,p2}**	**{r1,r2}**	**{w1,r2}..**

- If at some stage Process 1 is **waiting** then at some later stage it is **printing** (i.e. using the resource).
- **At no stage** are both processes using the resource.
- If a process is waiting then it does so **until** it starts to use the resource.
- **There is a stage** at which both processes are waiting.

The Application

- **K = (S, S_0, R, AP, L)**
- Every computation (sequence of states) can be viewed as a sequence of subsets of **AP**.
- **s_0 s_1 s_2 ….** **---- L(s_0) L(s_1) L(s_2)** ….
- These **AP-computations** will be the models for the formulas of LTL.

- Verification :
  - Every **AP**-computation of **K** is a model of j

Linear Time Temporal Logic (LTL)

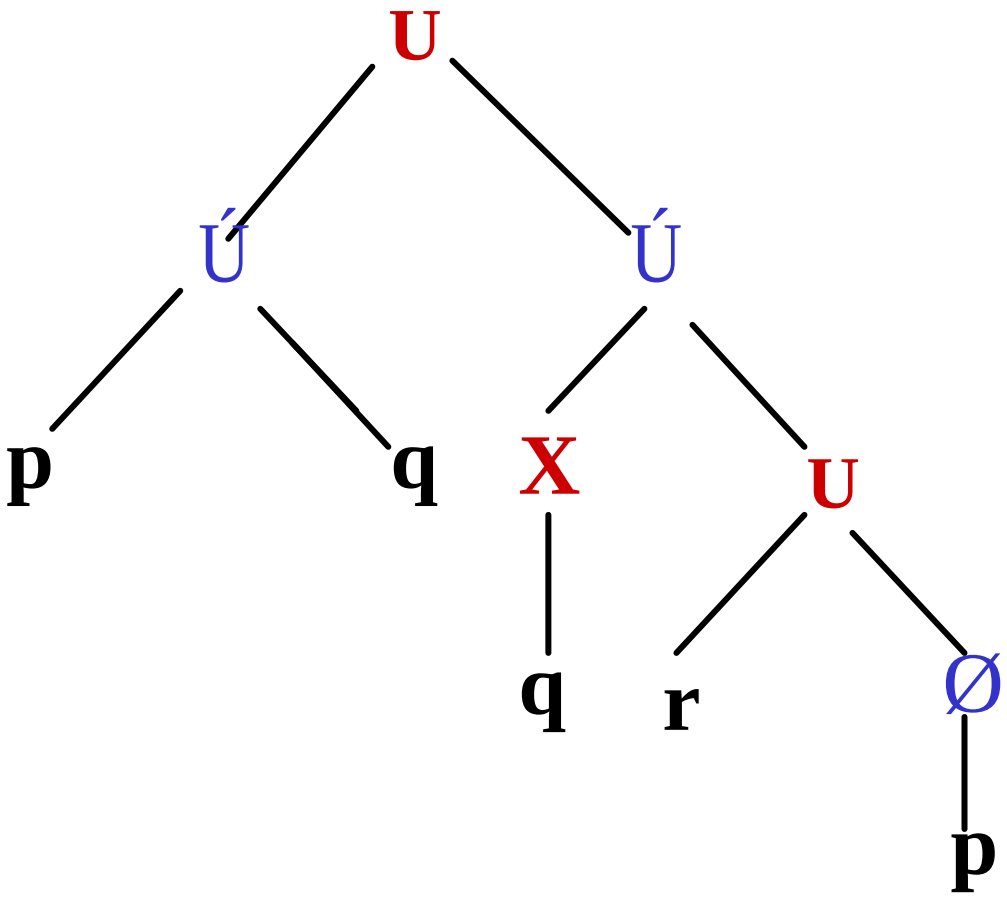
- Syntax :
  - **AP = {p_0, p_1, …p_n}**, a finite set of Atomic Propositions.
- Formulas :
  - Every **p_i** in AP is a ***LTL formula***.
  - If φ is a formula then $\neg \varphi$ is a ***LTL formula***.
  - If φ_1 and φ_2 are formulas then **$(\varphi_1 \vee \varphi_2)$** is a ***LTL formula***.
  - If φ is a formula then **X** φ, **F** φ and **G** φ are ***LTL formulae*** (**Next**, **Eventually**, **Always**).
  - If φ_1 and φ_2 are formulas then **$(\varphi_1$ U $\varphi_2)$** is a ***LTL formula*** (Until).

Formulas

LTL ::= *p* ½ Ø j ½ j_1 Ú j_2 ½X j ½ F j ½ G j ½ j_1 U j_2

- **p ; p Ú q ; (Ø p Ú q) Ú Ø (r Ú q)**
- **X q ; X (p Ú q) ; X ((Ø p Ú q) Ú XØ(r Ú q))**
- **(p Ú q) U (X r Ú (Ø q U (X Ø p)))**

$$(p \acute{U} q) \; U \; (X \, q \acute{U} (r \, U \, \emptyset p))$$



Semantics

- **AP** = A finite set of atomic propositions.
- $S = 2^{AP}$ = The set of subsets of **AP**
- **AP = { p, q, r }**
- **S = { f, {p}, {q}, {r}, {p,q}, {p,r}, {q,r}, {p,q,r} }**

- S^{w} = The set of ***infinite sequences*** over S.

Semantics

- **AP = {p, q, r}** S = 2^{AP}
- S = {f, **{ p }, { q }, ….,{ p, q, r } }**

s : **{ p,r } { q }** Æ **{ p, q, r } { r }**…

path: 0 → 1 → 2 → 3 → 4 …

- At stage **0** of s, **p** and **r** are true but not **q**; at stage **2** of s no member of **AP** is true….

Semantics

- S^w = The set of infinite sequences over Σ.
- $s \in S^w$ --- A model
- $s(i)$ ---- **i**-th position of s
- **{p} {q,r} Æ {r, q} {p, q, r}……..**

  | | | | | |

- **0 1 2 3 4 ……**
- $s(0) = \{p\}$ $s(2) = Æ$ $s(3) = ?$

Semantics

- **AP** $S = 2^{AP}$
- S^w = The set of infinite sequences over S.
- $s \hat{I} S^w$ --- A model
- $s(i)$ ---- **i**-th position of s
- j, a formula.
- $s(i) \models j$
  - $s(i)$ ***satisfies*** j
  - j is true in the **i**-th position of s

Semantics

LTL ::= ***p*** ½ Ø j ½ j $_1$ Ú j $_2$ ½**X** j ½ **F** j ½ **G** j ½ j $_1$ **U** j $_2$

- s = G_0 G_1 G_2 ….. G_i G_{i+1} ……..
- Each G_j is a subset of **AP**.
- **s(i)⊨ p *iff* p** Î G_i

Semantics

LTL ::= ***p*** ½ Ø j ½ j **1** Ú j **2** ½**X** j ½ **F** j ½ **G** j ½ j **1** **U** j **2**

- **AP = {p, q, r}**
- s = **{p,q} {r} Æ {q, r} {p, q, r} …..**

0 1 2 3 4

- s **(0)** ***satisfies*** **q**
- s **(1)** ***satisfies*** **r**
- s **(2)** does ***not*** *satisfy* **q !**

Semantics

LTL ::= ***p*** ½ Ø j ½ j_1 Ú j_2 ½**X** j ½ **F** j ½ **G** j ½ j_1 **U** j_2

σ = G_0 G_1 G_2 ….. G_i G_{i+1} ……

Each G_j is a subset of **AP**.

- s**(i)**⊨ Ø j ***iff*** s**(i)**⊭ j

Semantics

LTL ::= ***p*** ½ Ø j ½ j_1 Ú j_2 ½**X** j ½ **F** j ½ **G** j ½ j_1 **U** j_2

s = G_0 G_1 G_2 ….. G_i G_{i+1} ……

Each G_j is a subset of **AP**.

- **s(i) ⊨ j_1 Ú j_2** ***iff*** **s(i)⊨ j_1** ***OR*** **s(i)⊨ j_2**

Semantics

LTL ::= *p* ½ Ø j ½ j $_1$ Ú j $_2$ ½X j ½ F j ½ G j ½ j $_1$ U j $_2$

AP = {p, q, r}

σ = {p, q} {r} Æ {q, r} {p, q, r} ….

0 1 2 3 4

- s**(0)** ***satisfies*** Ø**r** ; s**(0)** does ***not satisfy*** **r**
- s**(1)** ***satisfies*** **p** Ú **r** ; s**(1)** ***satisfies*** **r**
- s**(2)** ***satisfies*** Ø**(p** Ú **r) ?**

Semantics

LTL ::= ***p*** ½ Ø j ½ j **1** Ú j **2** ½**X** j ½ **F** j ½ **G** j ½ j **1** **U** j **2**

AP = {p, q, r}

s = **{p, q} {r}** Æ **{q, r} {p, q, r) …..**

0 1 2 3 4

- s**(2)** ***satisfies*** Ø**(p** Ú **r)** ? ***Yes!***
- s**(2)** does ***not satisfy*** **p** Ú **r**

Semantics

LTL ::= ***p*** ½ Ø j ½ j $_1$ Ú j $_2$ ½**X** j ½ **F** j ½ **G** j ½ j $_1$ **U** j $_2$

- s = G_0 G_1 G_2 ….. G_i G_{i+1} ……..

  X j j

- s**(i)** ⊨ **X** j ***iff*** s**(i+1)** ⊨ j

Semantics

LTL ::= *p* ½ Ø j ½ j 1 Ú j 2 ½X j ½ F j ½ G j ½ j 1 U j 2

AP = {p, q, r}

s = **{p,q} {r} Æ {q,r} {p,q,r} …..**

0 1 2 3 4

- s**(2)** ***satisfies*** **X r** ; s**(3)** ***satisfies*** **r**
- s**(0)** ***satisfies*** **X(p Ú r)** ; s**(1)** ***satisfies*** **r**
- s**(1)** does ***not satisfy*** **X(p Ú r)**
  - s**(2)** does ***not satisfy*** **p Ú r**

Semantics

LTL ::= *p* ½ Ø j ½ j 1 Ú j 2 ½**X** j ½ **F** j ½ **G** j ½ j 1 **U** j 2

AP = {p, q, r}

s = **{p,q} {r}** Æ **{q,r} {p,q,r) …..**

0 1 2 3 4

- s **(1)** ***satisfies*** **X(X** Ø **p)** ***iff***
  - s **(2)** ***satisfies*** **X** Ø **p** ***iff***
  - s **(3)** ***satisfies*** Ø **p** ***iff***
  - s **(3)** does ***not satisfy*** **p**

Semantics

LTL ::= ***p*** ½ Ø j ½ j $_1$ Ú j $_2$ ½**X** j ½ **F** j ½ **G** j ½ j $_1$ **U** j $_2$

- s = G_0 G_1 G_2 ….. G_i ….. G_{j-1} G_j ……..
  F j at G_i; j at G_j

- **s(i)** ⊨ **F** j *iff* **s(j)** ⊨ j *for some* **j** ³ **i**

Semantics

LTL ::= *p* ½ Ø j ½ j 1 Ú j 2 ½**X** j ½ **F** j ½ **G** j ½ j 1 **U** j 2

AP = {p, q, r}

s **= {p,q} {r}** Æ **{q,r} {p,q,r) …..**

0 1 2 3 4

- s **(0)** ***satisfies*** **F(X p)** this is true since
  - s **(3)** ***satisfies*** **X p** ***iff***
  - s **(4)** ***satisfies*** **p** is true since

Semantics

LTL ::= ***p*** ½ Ø j ½ j **1** Ú j **2** ½**X** j ½ **F** j ½ **G** j ½ j **1** **U** j **2**

- s = G_0 G_1 G_2 ….. G_i G_{i+1} …….. G_j ……

G j j … j ….. j ……

- **s(i)** ⊨ **G** j ***iff*** **s(j)** ⊨ j ***for all*** **j** ³ **i**

Semantics

LTL ::= ***p*** ½ Ø j ½ j_1 Ú j_2 ½**X** j ½ **F** j ½ **G** j ½ j_1 **U** j_2

$s = G_0 \quad G_1 \cdots G_i \quad G_{i+1} \cdots G_{k-1} \quad G_k \cdots$

$j_1 \cdots \quad j_1 \quad \cdots \quad j_1 \quad j_2$

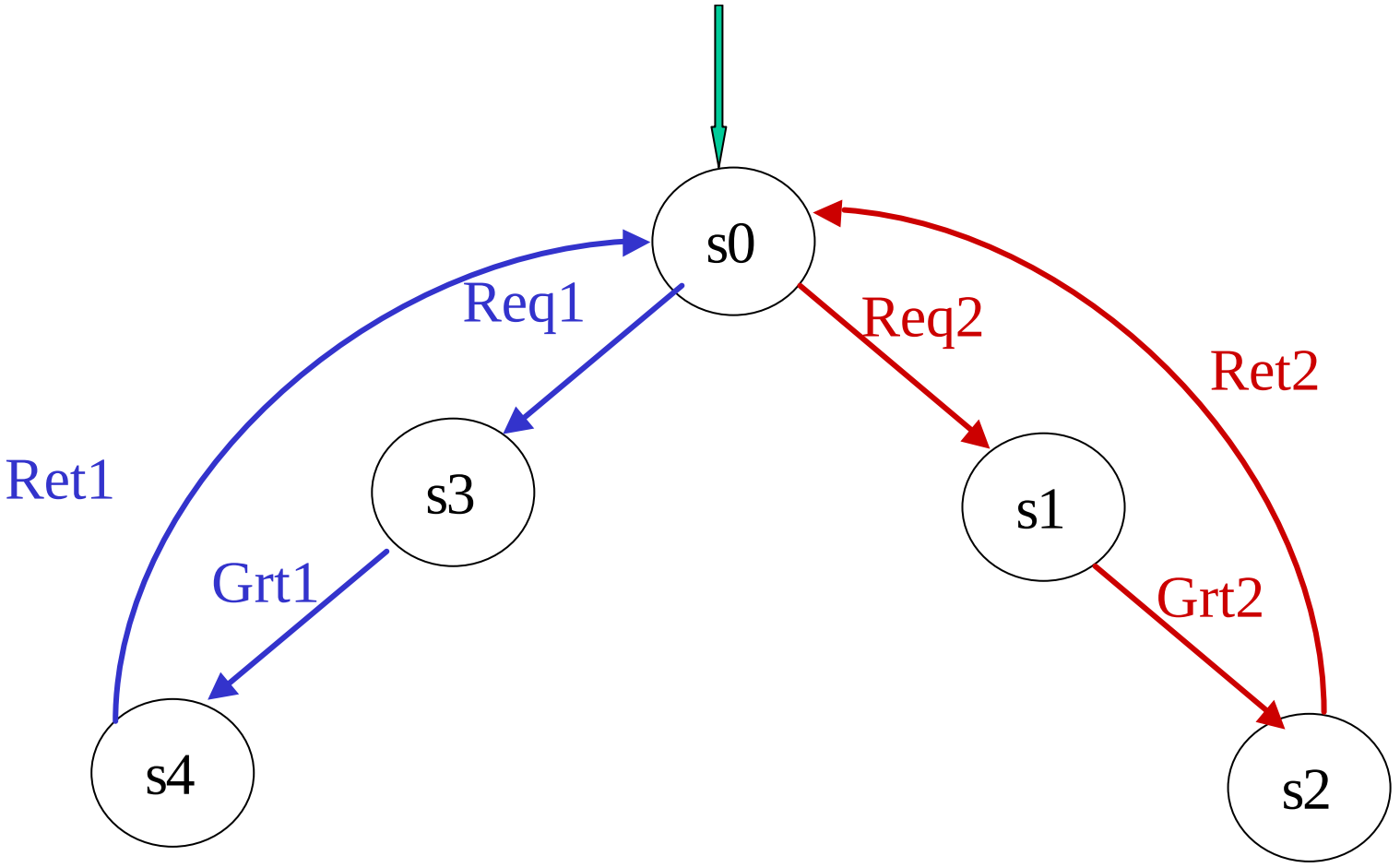
- $s(i) \models j_1$ **U** j_2 ***iff*** there exists **k** ³ **i** s.t.
  - $s(k) \models j_2$
  - $s(j) \models j_1$ for every **i** £ **j** < **k**

Semantics

LTL ::= ***p*** ½ Ø j ½ j_1 Ú j_2 ½**X** j ½ **F** j ½ **G** j ½ j_1 **U** j_2

- **k** could be arbitrarily greater than **i**.
- **k = i** is allowed and there is **no i £ j < k**

- s**(i)**⊨ j_1 **U** j_2 ***iff*** there exists **k** ³ **i** s.t.
  - s**(k)**⊨ j_2
  - s**(j)**⊨ j_1 for every **i £ j < k**



s0 s3 s4 s0 s1 s2 s0 s3 …

{r1,r2} {w1,r2} {p1,r2} {r1,r2} {r1,w2} {r1,p2} {r1,r2} {w1,r2}...

An Example

AP = {r1, w1, p1, r2, w2, p2}

{r1,r2}	**{w1,r2}**	**{p1,r2}**	**{r1,r2}**	**{r1,w2}**	**{r1,p2}**	**{r1,r2}**	**{w1,r2}...**
0	1	2	3	4	5	6	7

- s **(1)** ***satisfies*** **(r2 U w2)** ;
  - s **(4)** ***satisfies*** **w2** and
  - s **(1),** s **(2),** s **(3)** ***satisfy*** **r2**.

An Example

AP = {r1, w1, p1, r2, w2, p2}

{r1,r2}	{w1,r2}	{p1,r2}	{r1,r2}	{r1,w2}	{r1,p2}	{r1,r2}	{w1,r2}...
0	1	2	3	4	5	6	7

- s**(1)** does ***not satisfy*** **(r2 U p2)** ;
  - s**(5)** ***satisfies*** **p2** and
  - s**(1),** s**(2),** s**(3)** ***satisfy*** **r2**.
  - but s**(4)** does ***not satisfy*** **r2 !**

An Example

AP = {r1, w1, p1, r2, w2, p2}

{r1,r2}	{w1,r2}	{p1,r2}	{r1,r2}	{r1,w2}	{r1,p2}	{r1,r2}	{w1,r2}...
0	1	2	3	4	5	6	7

- s **(1)** ***does satisfy*** **((r2 Ú w2) U p2)** ;
  - s **(5)** ***satisfies*** **p2** and
  - s **(1),** s **(2),** s **(3)** ***satisfy*** **r2, hence also (r2 Ú w2).**
  - s **(4)** ***satisfies*** **w2, hence also (r2 Ú w2) !**

Models

- **AP** $S^{AP} = 2$
- S^{ω} = The set of infinite sequences over S.
- $s \in S^{\omega}$
- φ an **LTL** formula.

- A path s is a ***model*** of φ **(**$s \models \varphi$ **)** ***iff***
  - s**(0)** $\models \varphi$

Validity in LTL

- **AP** $S^{AP} = 2$
- S^{ω} = The set of infinite sequences over S.
- $s \in S^{\omega}$
- φ an **LTL** formula.

- φ is ***LTL-valid*** **(** $\models \varphi$ **)** ***iff for every*** $s \in S^{\omega}$ $-$ $s \models \varphi$

Basic LTL Language

We will use the *reduced LTL language*

LTL ::= *p* ½Ø j ½ j $_1$ Ú j $_2$ ½X** j ½ j $_1$ **U** j $_2$**

- j $_1$ Ù j $_2$ --- Ø (Ø j $_1$ Ú Ø j $_2$) (**And**)
- j $_1$ É j $_2$ --- Ø j $_1$ Ú j $_2$ (**Implies**)
- j $_1$ ° j $_2$ ---- (j $_1$ É j $_2$) Ù (j $_2$ É j $_1$) (**Iff**)
- **AP = {p_1, p_2, …,p_n}**
- ⊤ ---- **p_1** Ú Ø**p_1** (**true**)

- **Fact : In *every model*** s **, *at every* i,**
  - s**(i)**⊨⊤

Derived Operators

- **LTL ::= *p* ½Ø j ½ j 1 Ú j 2 ½X j ½ j 1 U j 2**
- **F** j ° **(⊤ U** j **)** (future ; diamond: ◇)

- **We gave the following semantics** :
  - s **(i)** ⊨ **F** j *iff* **there exists k** ³ **i** such that s **(k)** ⊨ j .

Derived Operators

- **We gave the following semantics :**
  - $s(i) \models F\, j$ *iff* **there exists k** $\geq$ **i** such that $s(k) \models j$.

Proof of $F\, j \equiv (\top\, U\, j)$

$s(i) \models (\top\, U\, j)$ *iff*

$\exists\, j \geq i,\ s(j) \models j$ and $\forall\, i \leq k < j,\ s(k) \models \top$ *iff*

$\exists\, j \geq i,\ s(j) \models j$ *iff*

$s(i) \models F\, j$

Derived Operators

- **LTL ::= *p*** ½Ø j ½ j $_1$ Ú j $_2$ ½**X** j ½ j $_1$ **U** j $_2$
- **F** j º (⊤ **U** j)
- **G** j º Ø**F**Ø j (invariant; box: □)

- **We gave the following semantics** :
  - s**(i)** ⊨ **G** j *iff* **for every k** ³ **i** ,
    s**(k)** ⊨ j .

Derived Operators

- **LTL ::= *p* ½Ø j ½ j 1 Ú j 2 ½X j ½ j 1 U j 2**
- **(y R j) º Ø (Øy U Ø j)** (Releases)
- **G j º (^ R j)**

- **Fact:**
  - s**(i)** ⊨ (y **R** j) ***iff*** **for each k** ³ **i *either***
    - s**(k)** ⊨ j ***or***
    - **for some i** £ **j < k,** s**(j)** ⊨ y

Derived Operators

- **LTL ::= *p*** ½Ø j ½ j $_1$ Ú j $_2$ ½**X** j ½ j $_1$ **U** j $_2$
- **(**y **W** j **)** (Unless)

Give the semantics according to the following intuition:

- **(**y **W** j **)**: if y must be true unless j occurs.

Show that **(**y **W** j **)** º **G** y Ú **(**y **U** j **)**

Derived Operators

- **LTL ::= *p*** ½Ø j ½ j **1** Ú j **2** ½**X** j ½ j **1** **U** j **2**
- **(**y **B** j **)** (Before)

Give the semantics according to the following intuition:

- **(**y **B** j **)**: if j ever occurs, then y must occur before j .

Show that **(**y **B** j **)** º Ø **(** Øy **U** j **)**

Tableau rules

- **(**y **U** j**)** ≡ j Ú (y Ù **X (**y **U** j**)** **)**
- **(**y **R** j**)** ≡ j Ù (y Ú **X (**y **R** j**)** **)** ≡
  ≡ (j Ù y) Ú (j Ù **X (**y **R** j**)** **)**
- **F** j ≡ j Ú **X F** j
- **G** j ≡ j Ù **X G** j

LTL: Some examples

- **Safety**: “it *never happens* that both A and B are print at the same time”

$$\mathbf{G(\varnothing\ (P_A \wedge P_B))}$$

- **Liveness:** “*whenever* A waiting, it will *eventually* print in the future”

$$\mathbf{G(W_A \Rightarrow F\ P_A)}$$

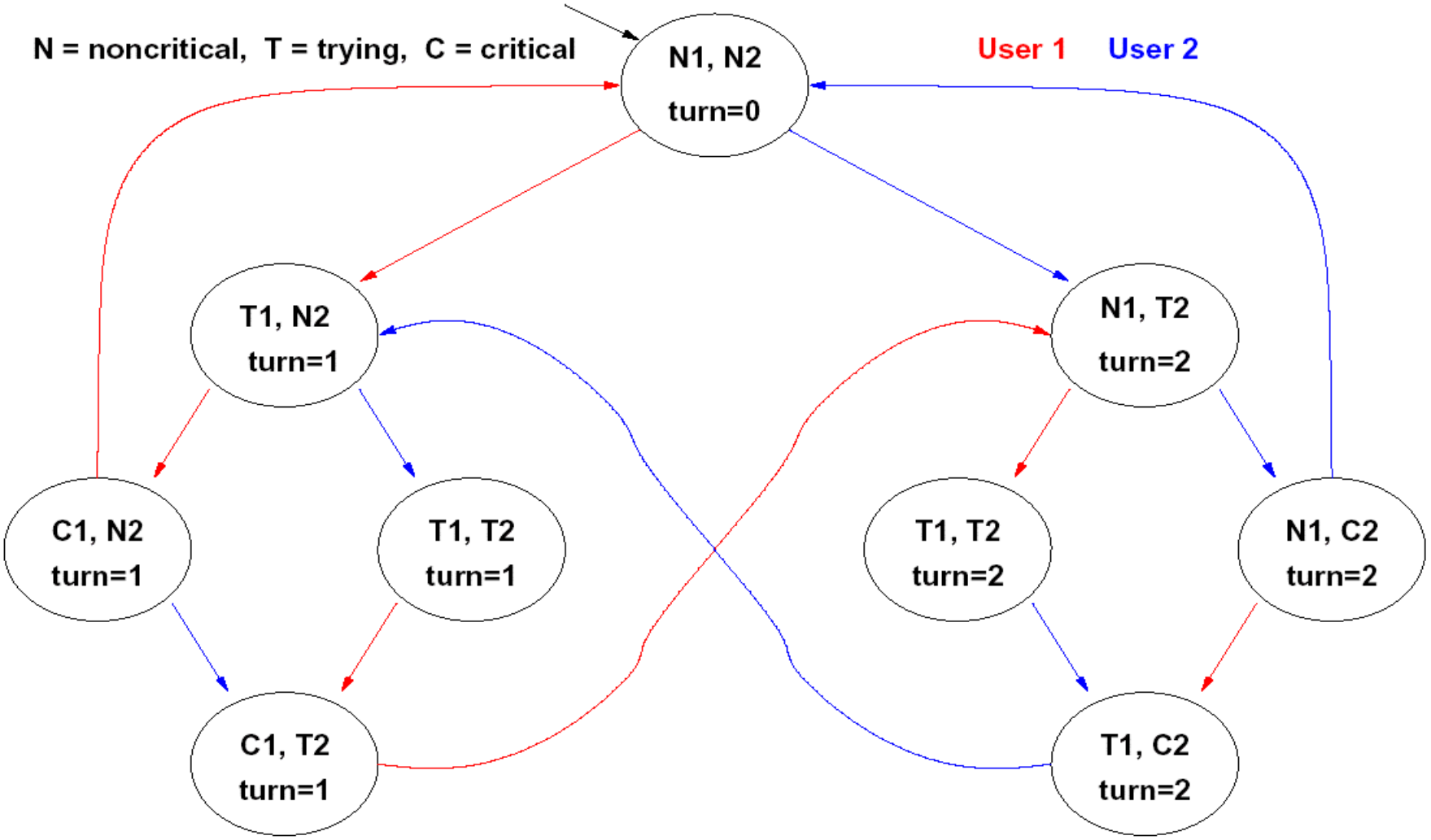
- **Fairness:** “A *infinitely often* idle”

$$\mathbf{GF\ R_A}$$

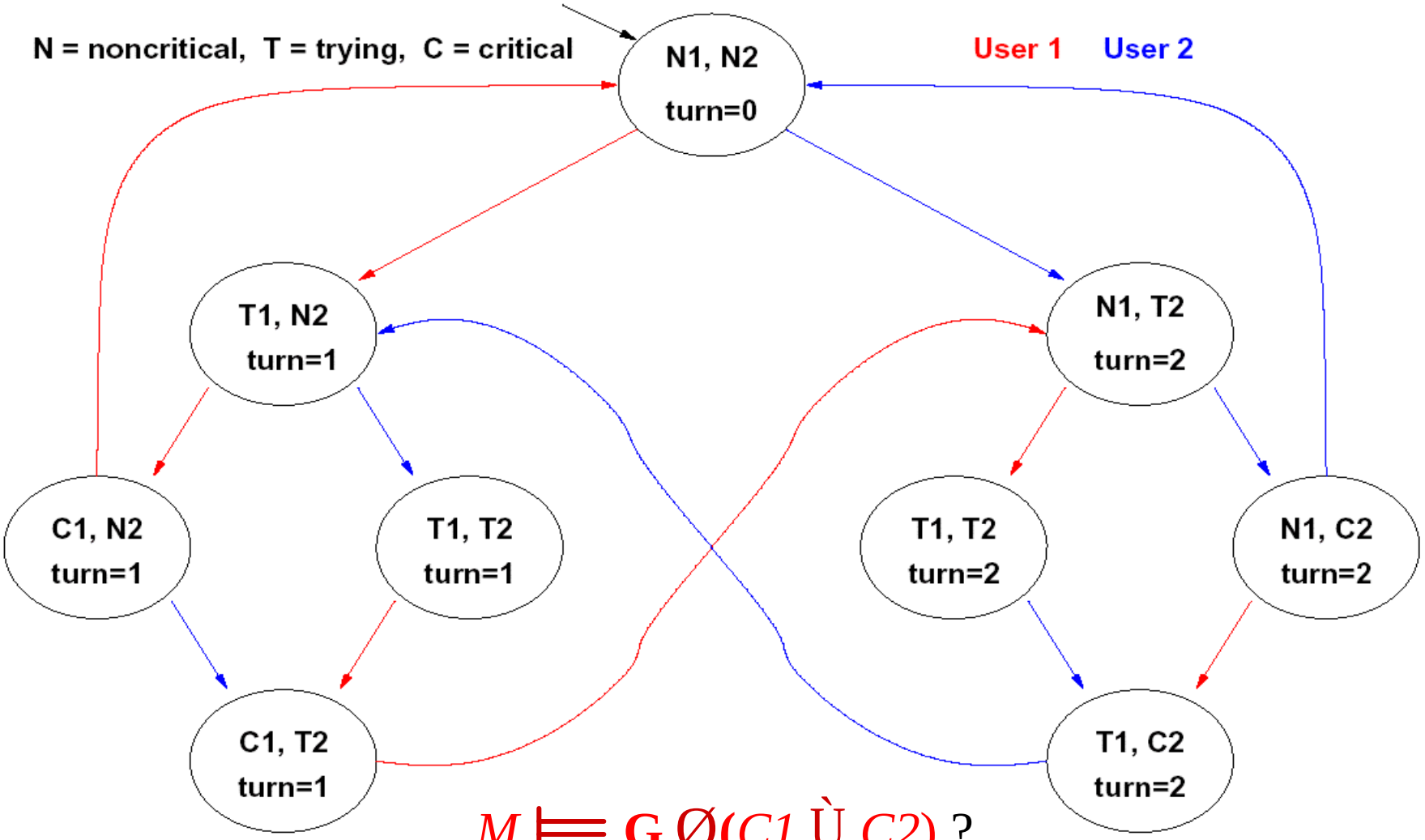
- **Strong fairness:** “if A *infinitely often* waiting, then it will *infinitely often* printing”

$$\mathbf{GF\ W_A \Rightarrow GF\ P_A}$$

Example: mutual esclusion



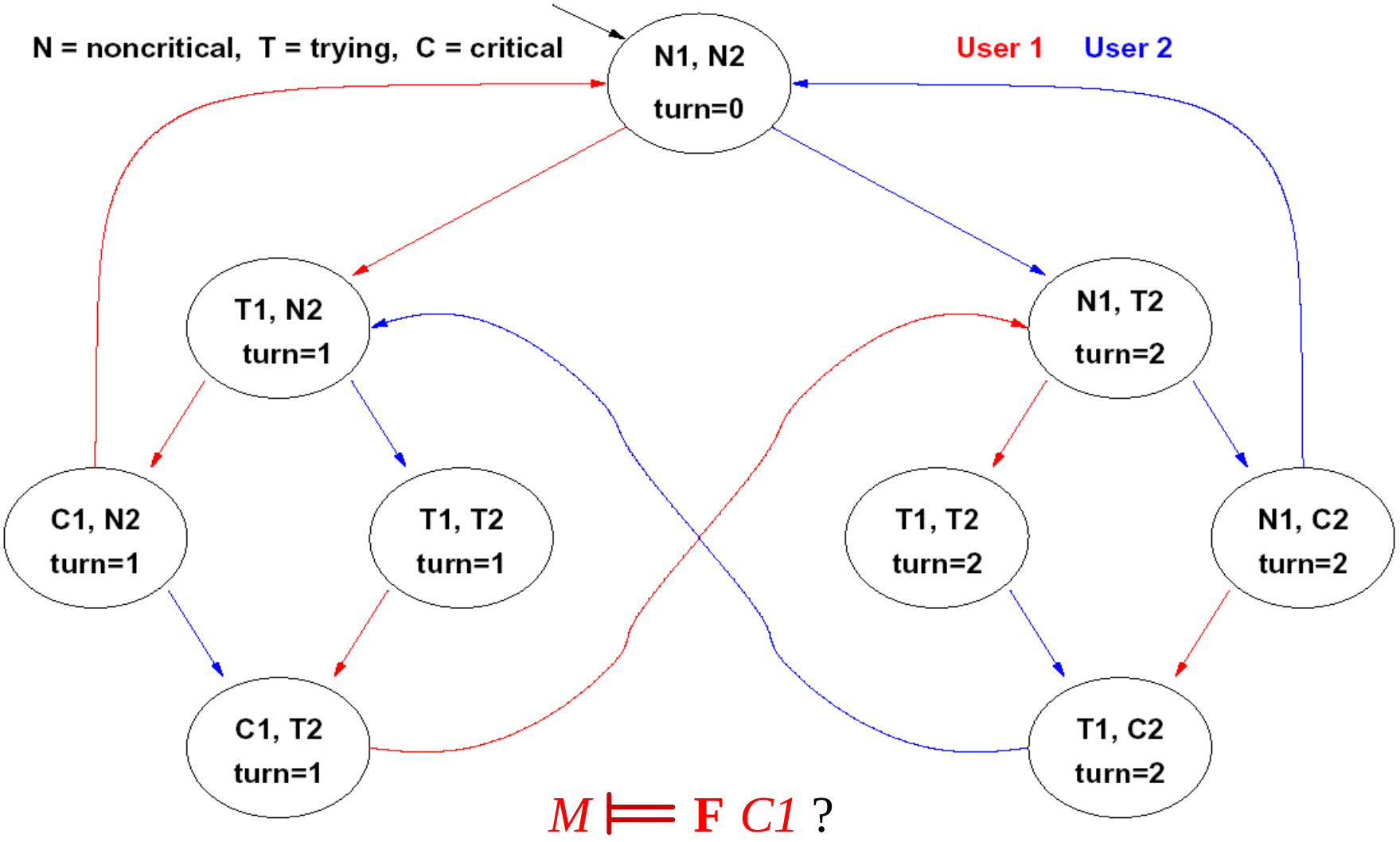
Example: mutual esclusion (safety)



$M \models \mathbf{G}\,\emptyset(C1 \wedge C2)$?

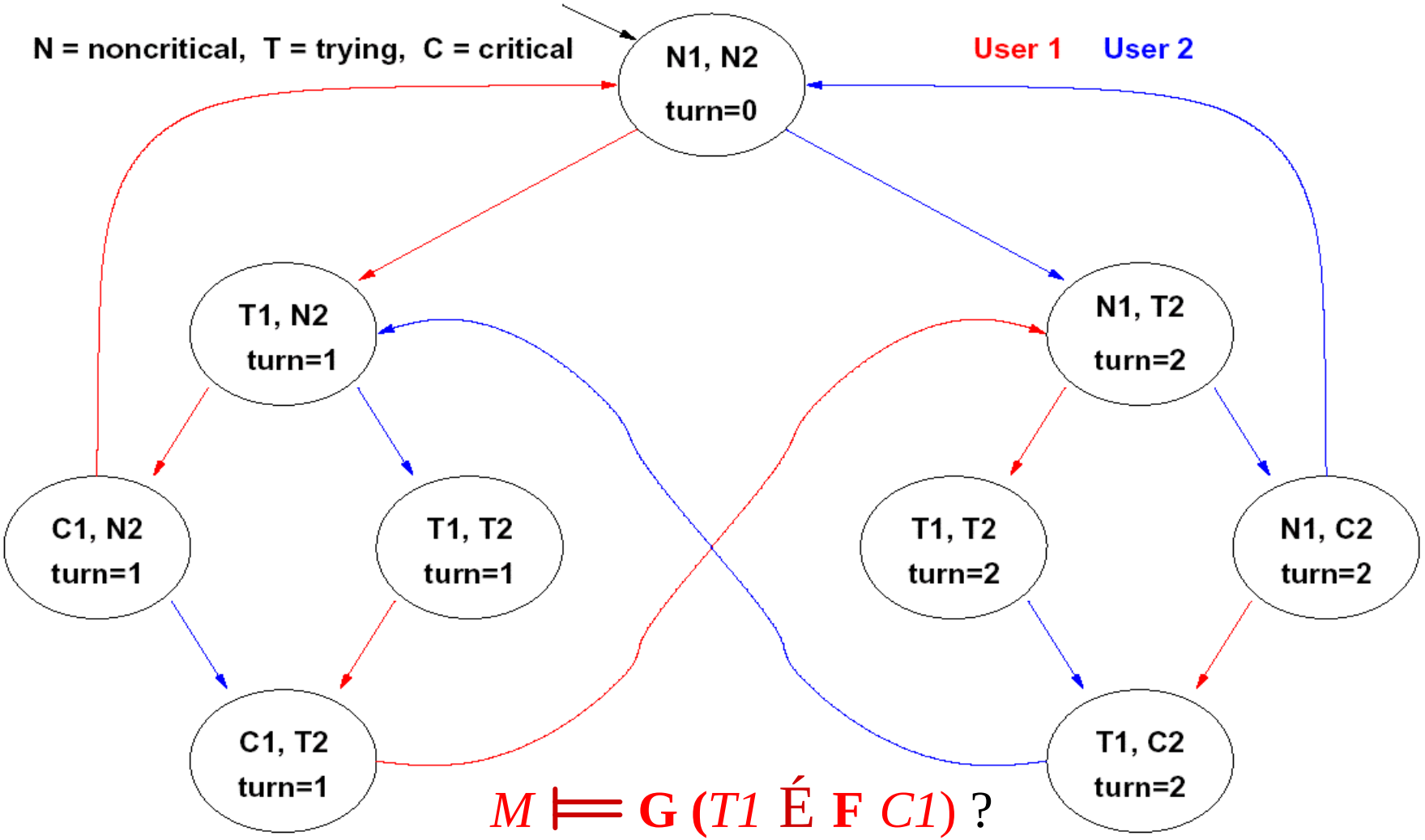
YES: There is no reachable state in which both C1 and C2 hold!

Example: mutual exclusion (liveness)



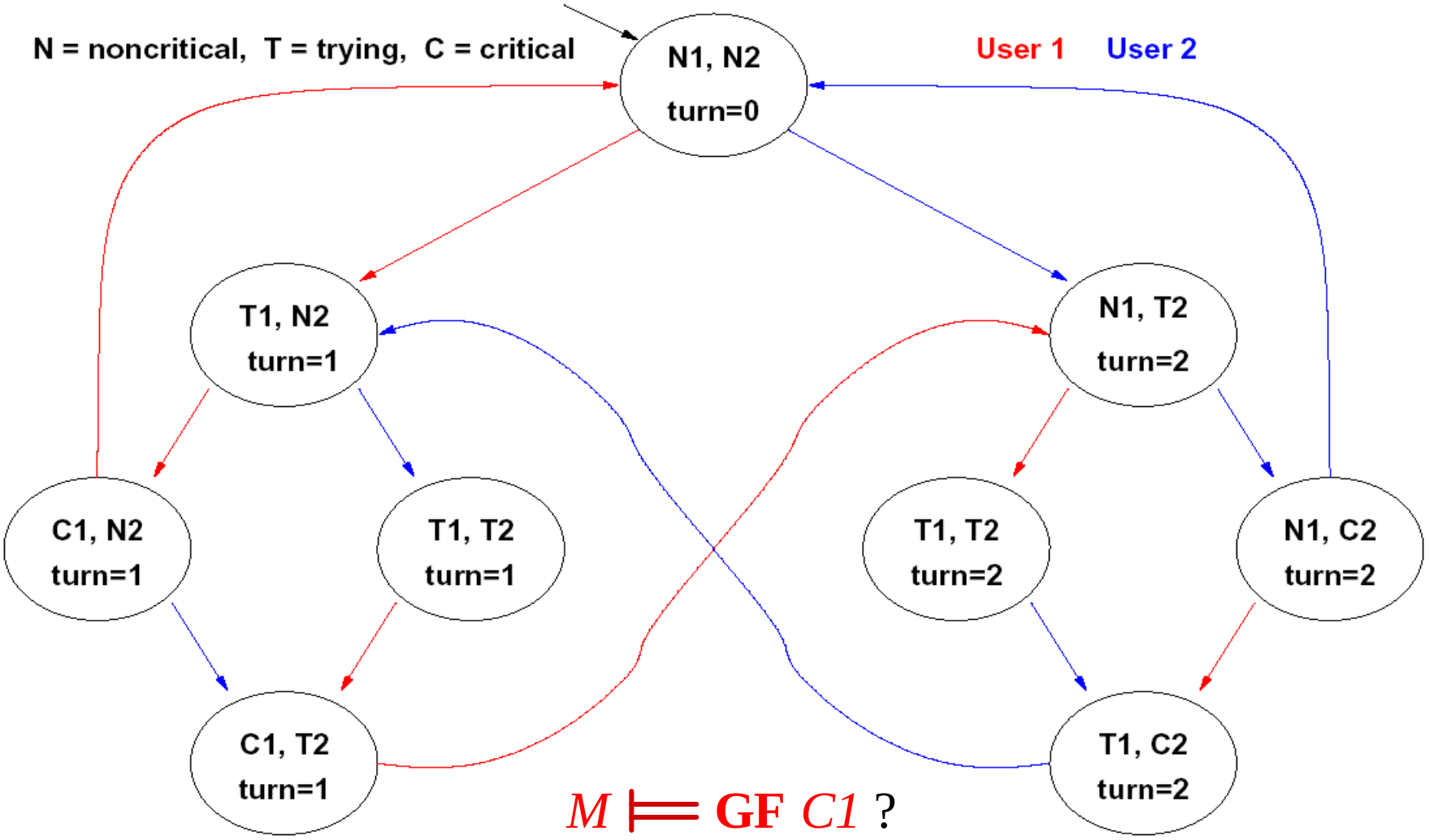
NO: there is an infinite cyclic solution in which C1 never holds!

Example: mutual exclusion (liveness)



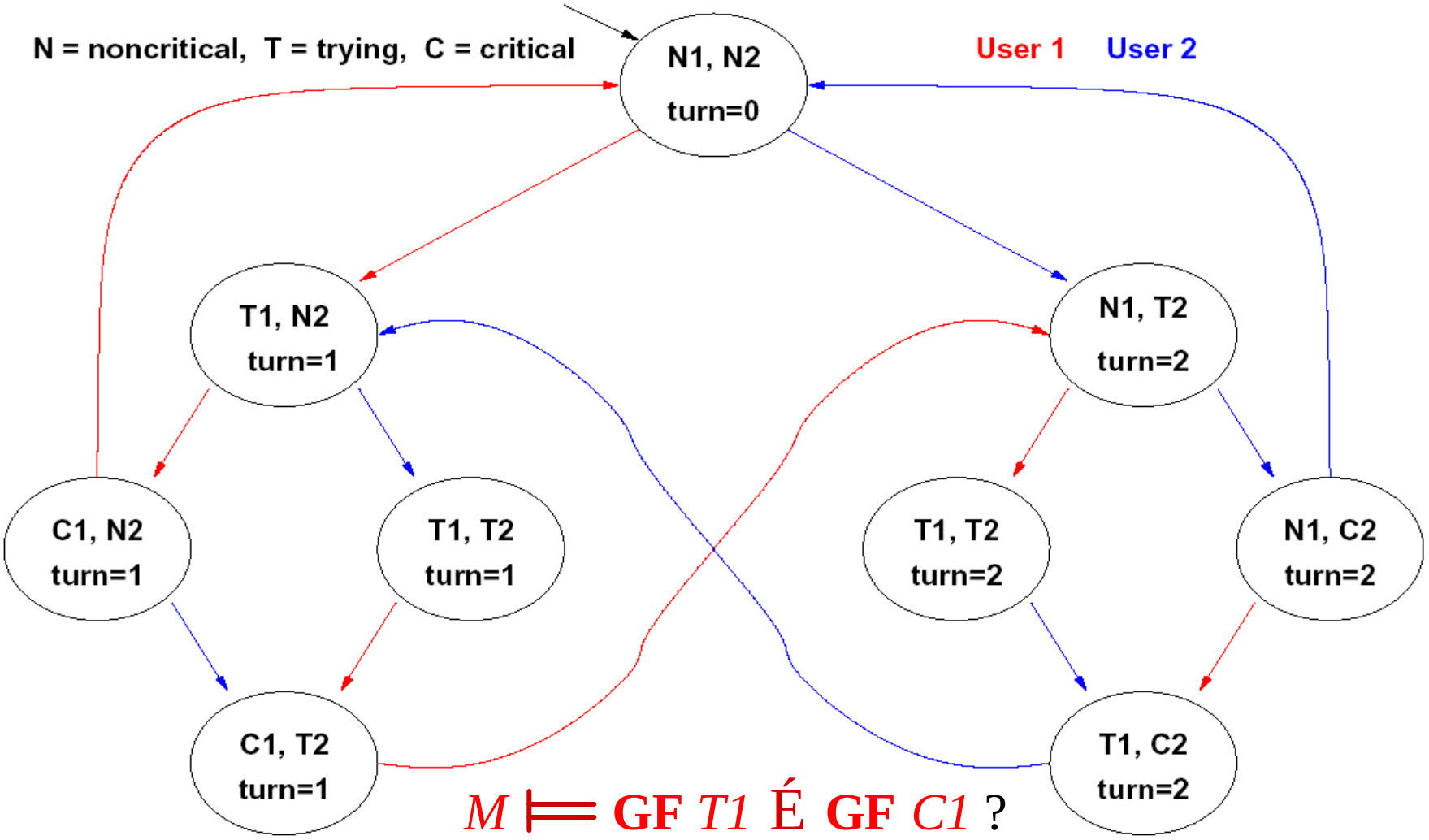
YES: every path starting from each state where *T1* holds passes through a state where *C1* holds!

Example: mutual exclusion (fairness)



NO: e.g., in the initial state, there is an infinite cyclic solution in which *C1* never holds!

Example: mutual exclusion (strong fairness)



YES: every path which visits *T1* infinitely often also visits *C1* infinitely often (see liveness prop. in previous example)!

Model Checking

- **K = (S, S_0, R, AP, L)** (the system)
- φ, an **LTL** formula. (the property)
- **K** $\models$ φ ***iff*** **every AP-computation** of **K** is a model of φ.
- Determining this is the ***model checking problem***.
- A solution to this problem can be automated!