Tecniche di Specifica e di Verifica

Linear Time Temporal Logic I

Temporal Logics: The context

- ***Kripke Structures*** model systems.
- ***Temporal logics*** model dynamic behavioral properties of systems.
  - **Linear Time**
  - Branching Time
- ***Model checking*** can be used to determine if a system has the desired behavioral property.

Linear time temporal logics.

- ***LTL (Linear Time Temporal Logic)***
  - **Syntax**
  - **Semantics**
  - The Model Checking Problem.
  - Its solution.

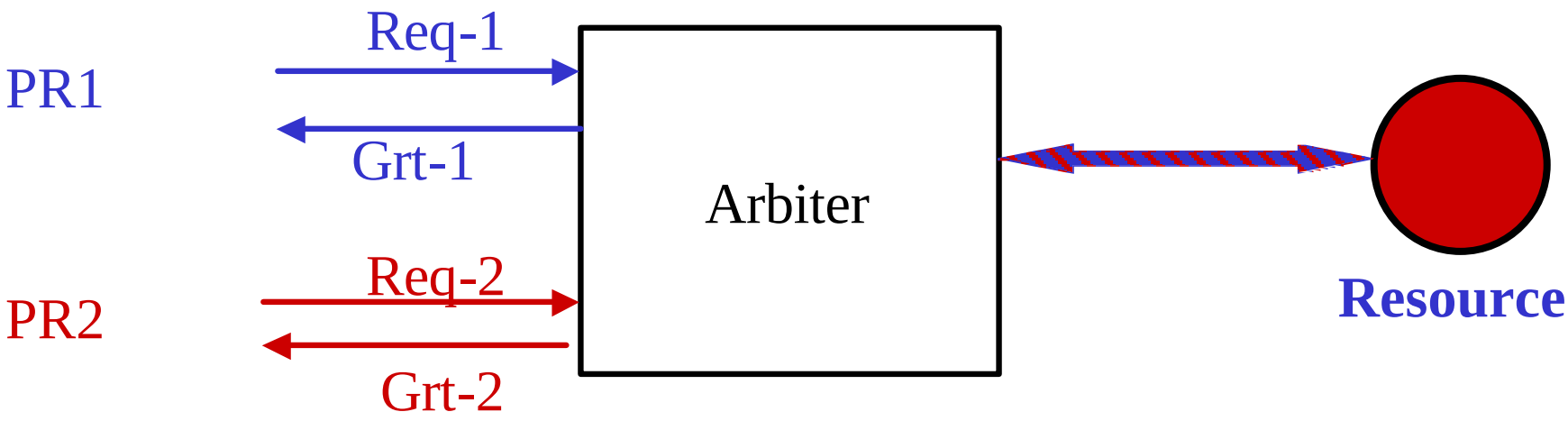
The Application

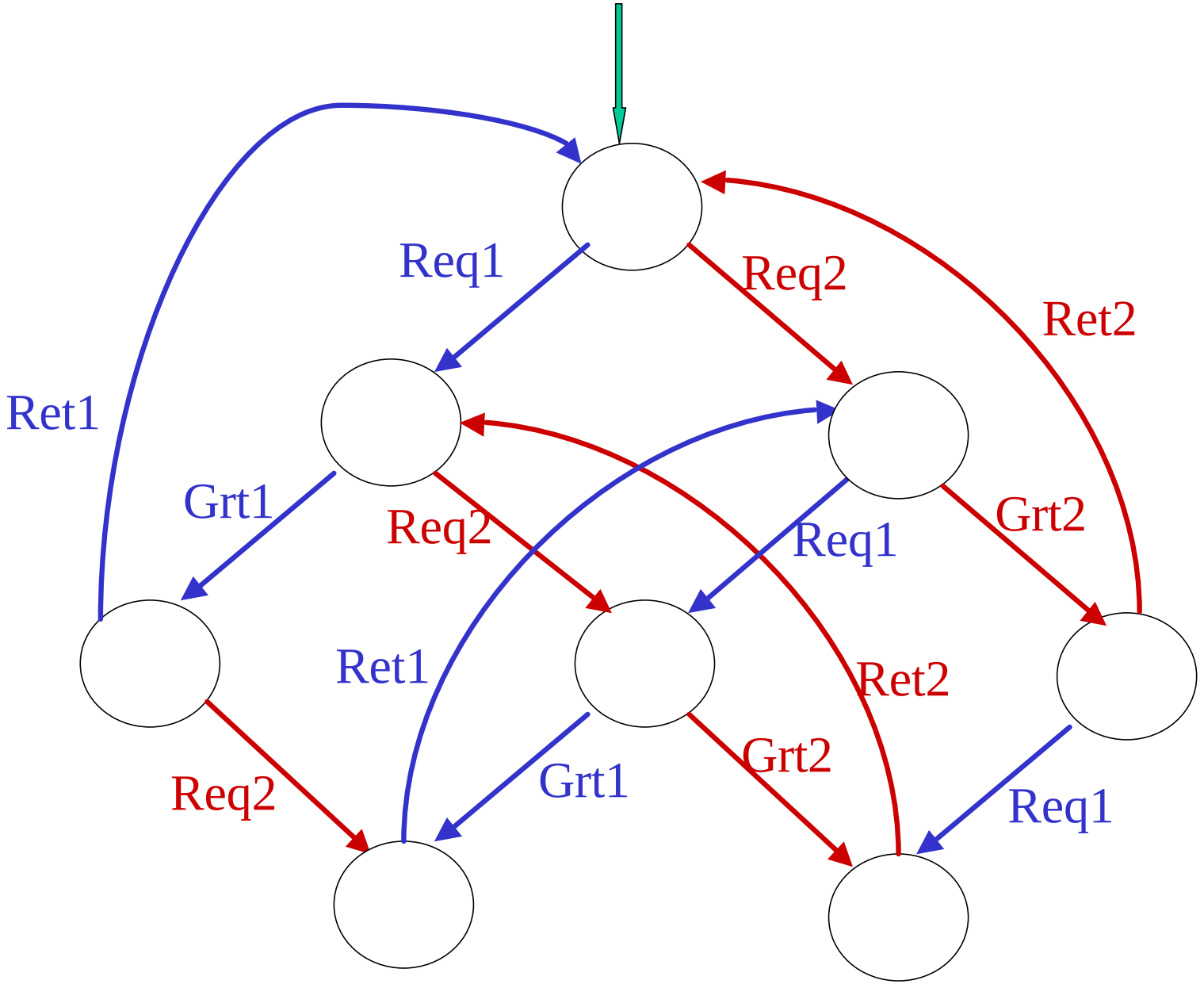
- Model a system to be verified as a Kripke structure:
  - Transition system **TS = (S, S_0, R)**
  - **AP** = A finite set of atomic propositions.
    - Basic assertions about the system
  - **L : S $\longrightarrow$ 2^{AP}** = The set of subsets of **AP**.
  - **p Î L(s)** ---- **p** is true at **s**.
  - **p Ï L(s)** ---- **p** is not true at **s**.
- **K = (S, S_0, R, AP, L)** ---- Kripke structure

The Application

- ***The computations*** of the Kripke structure **K** will be the ***models*** for **LTL** formulas.
- ***The property*** to be verified is captured as an LTL formula φ.
- The modeled system **K** has the property φ ***iff every computation of*** **K** ***is a model of*** φ.
- We need to verify (***model check***) whether:
  - $\mathbf{K} \models \varphi$

An Example




Req1
Req2
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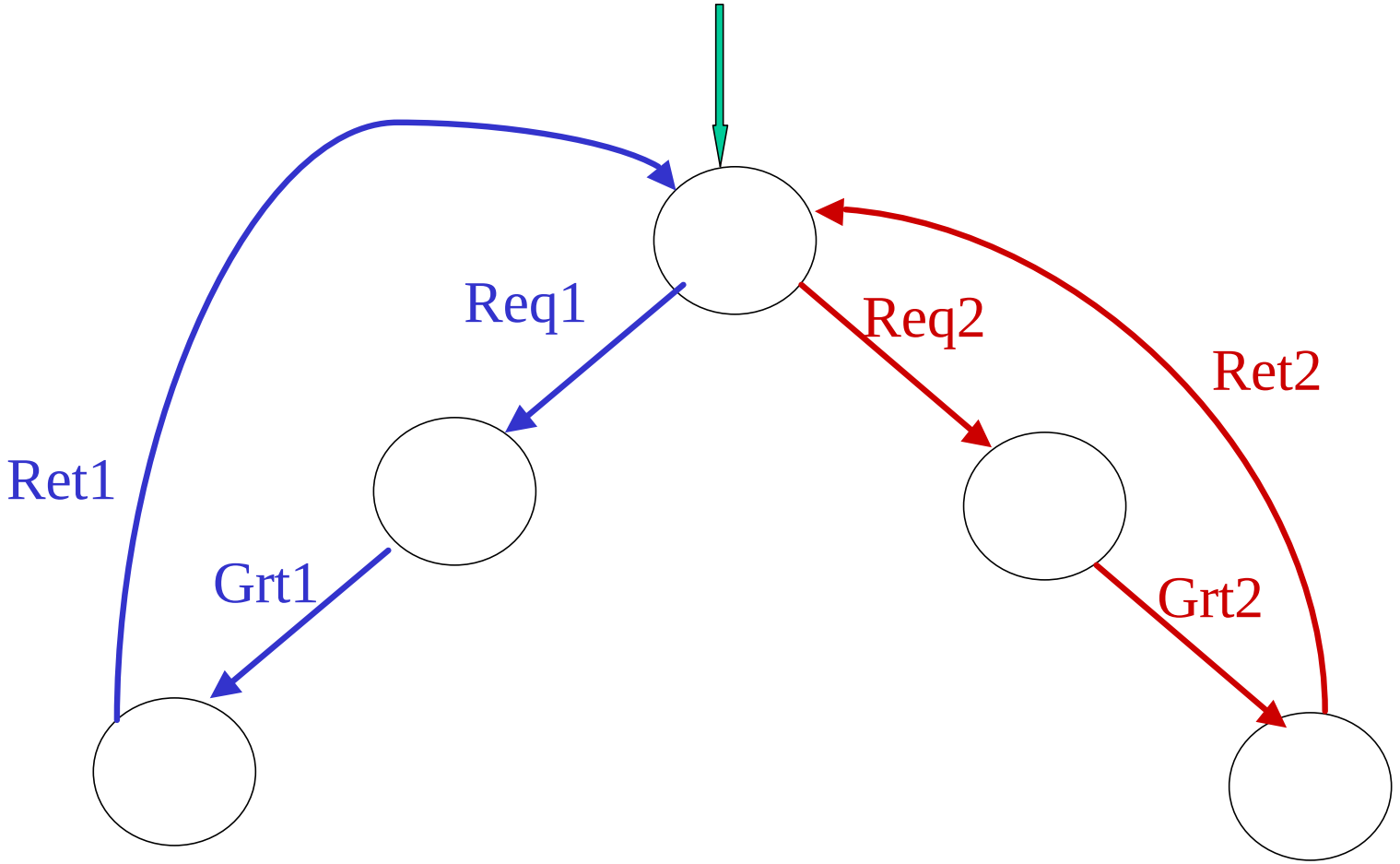
Grt2
Req1


Req1
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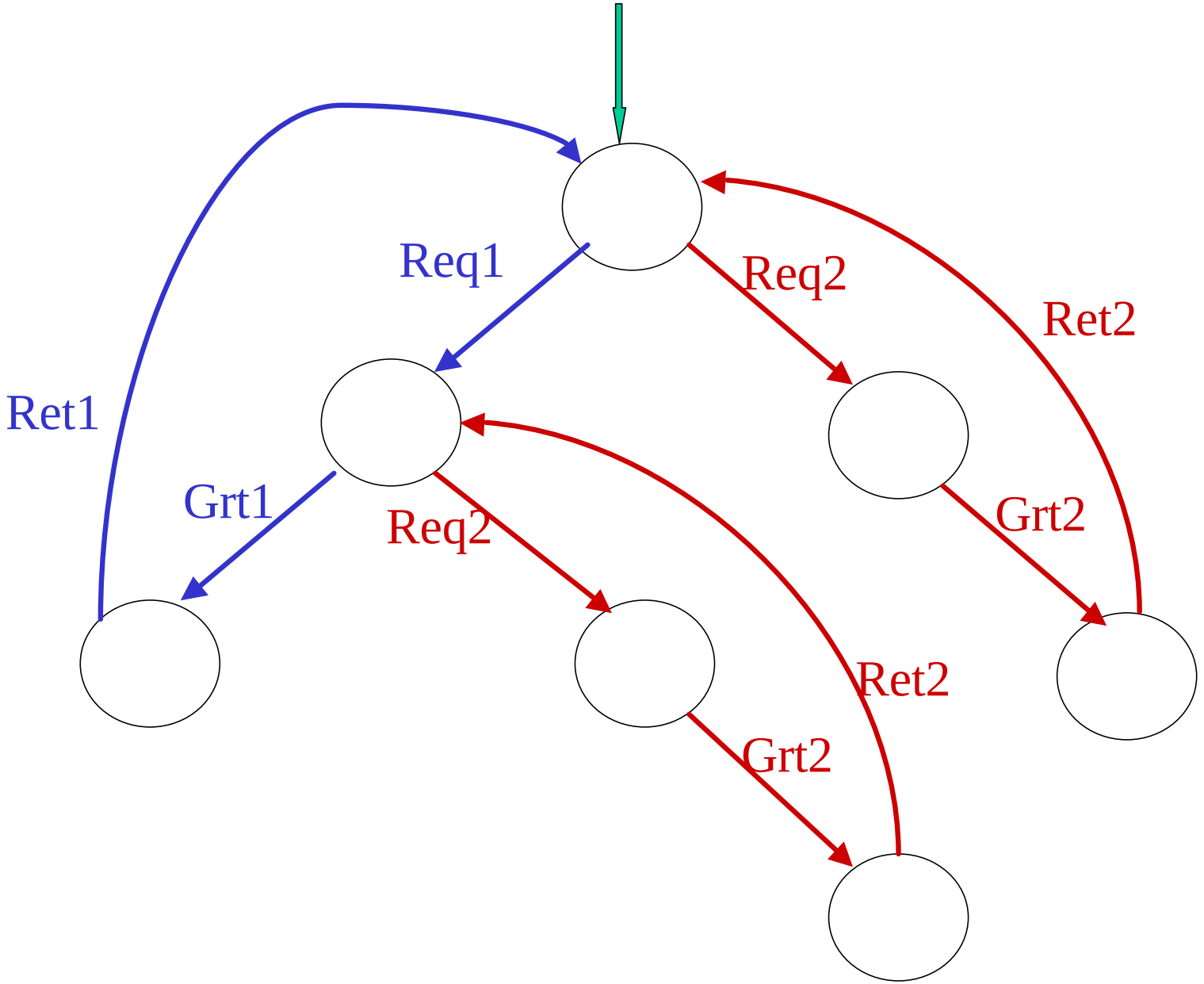
Req1
Req2
Ret2
Ret1
Grt1
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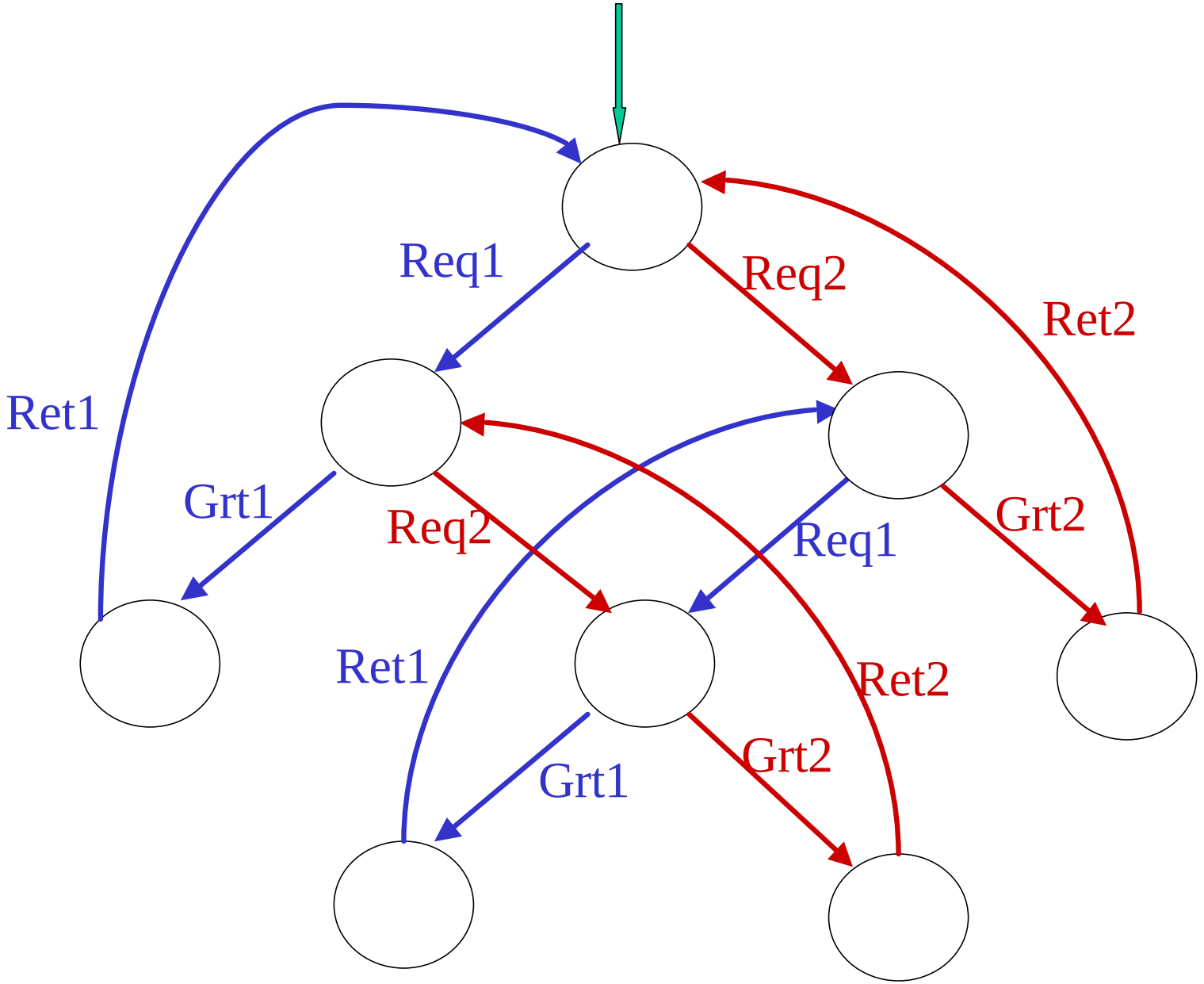
Grt2
Ret2
Grt2


Req1
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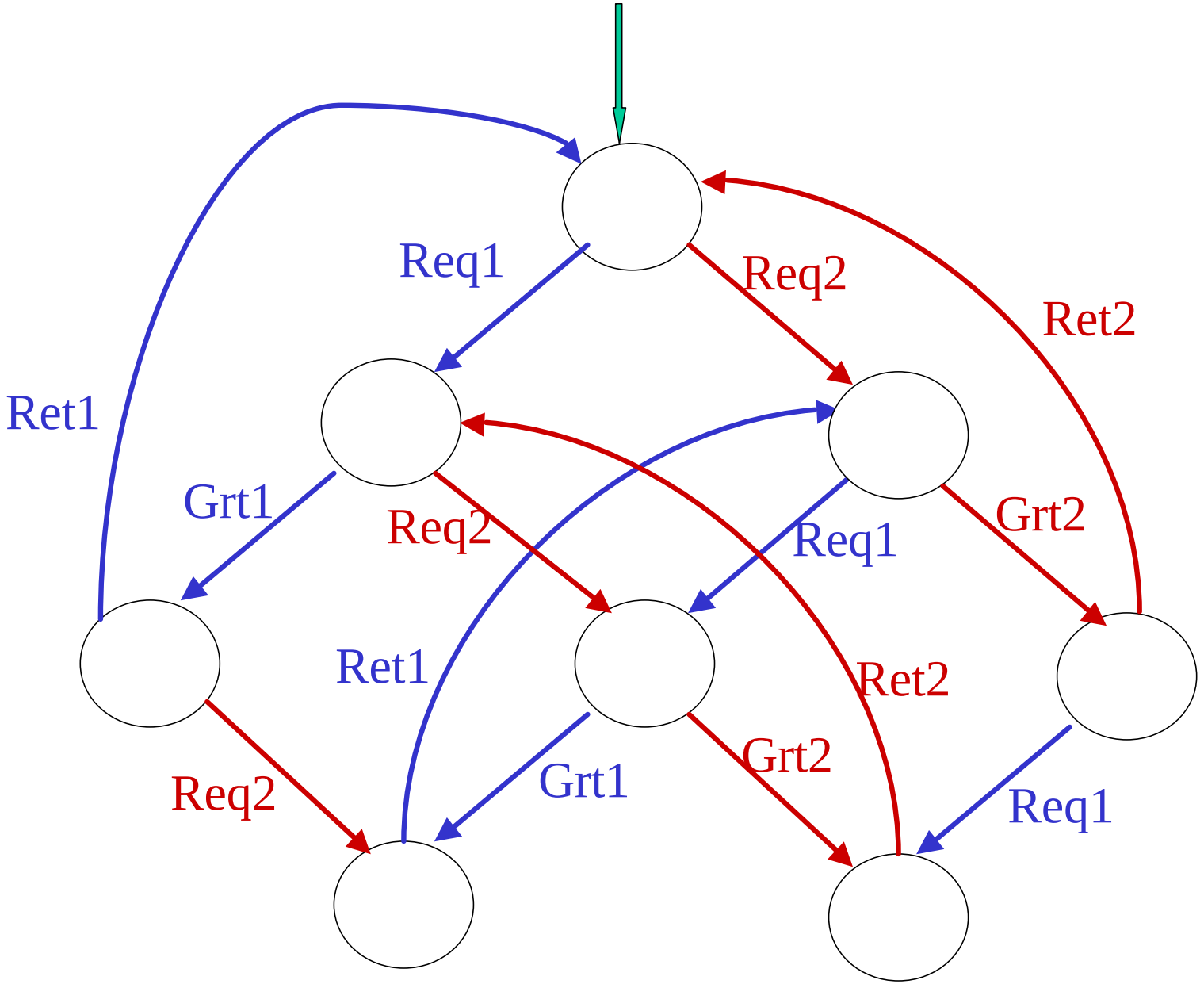
Grt1
Req2
Req1
Grt2
Ret1
Ret2
Grt2
Grt1


Req1
Req2
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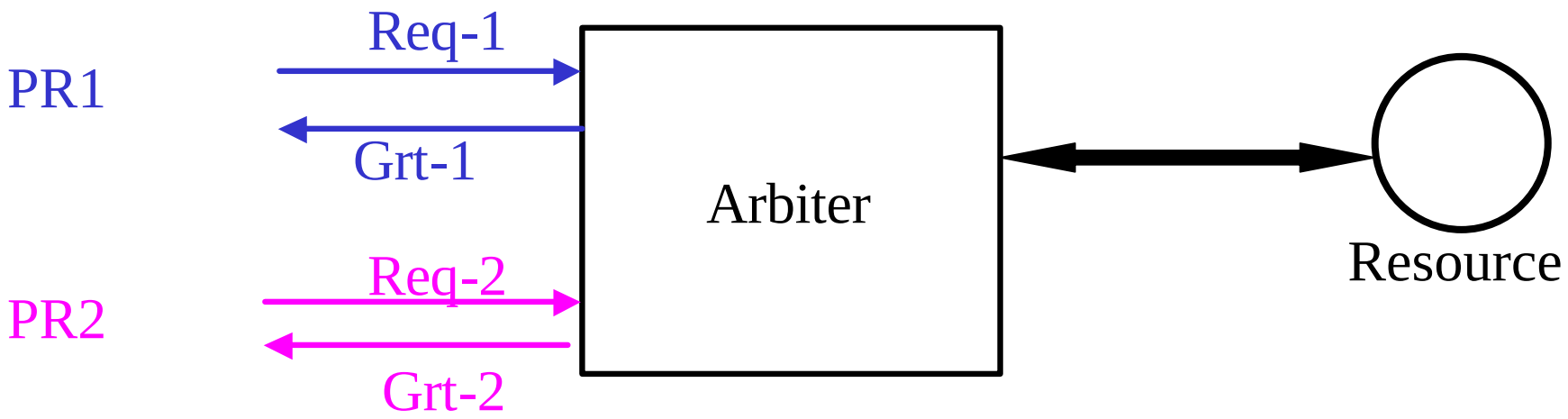
Req2
Req1
Grt2
Ret1
Ret2
Req2
Grt1
Grt2
Req1

A set of Atomic Propositions



R1 – Process 1 is ***idle***

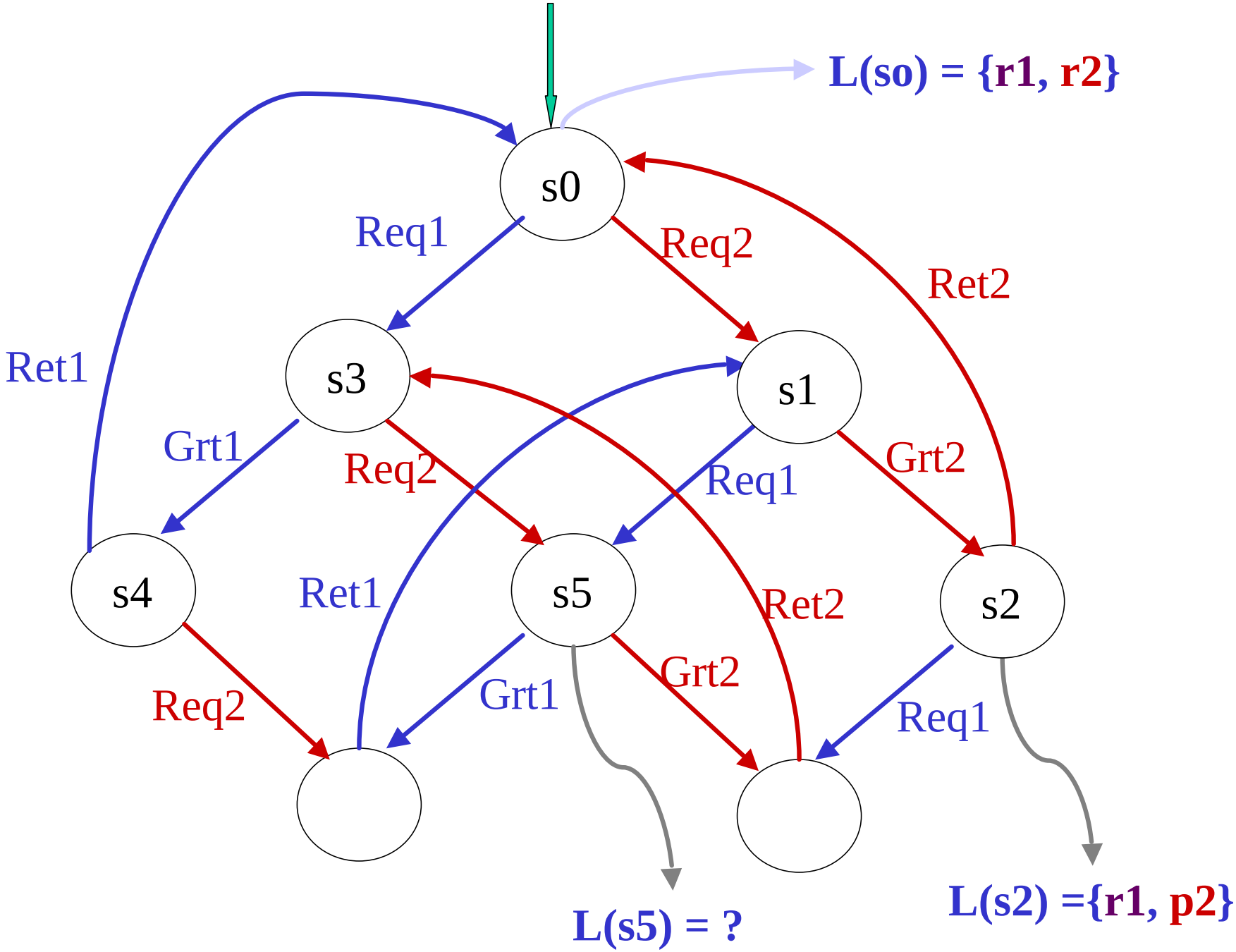
W1– Process 1 is ***waiting***

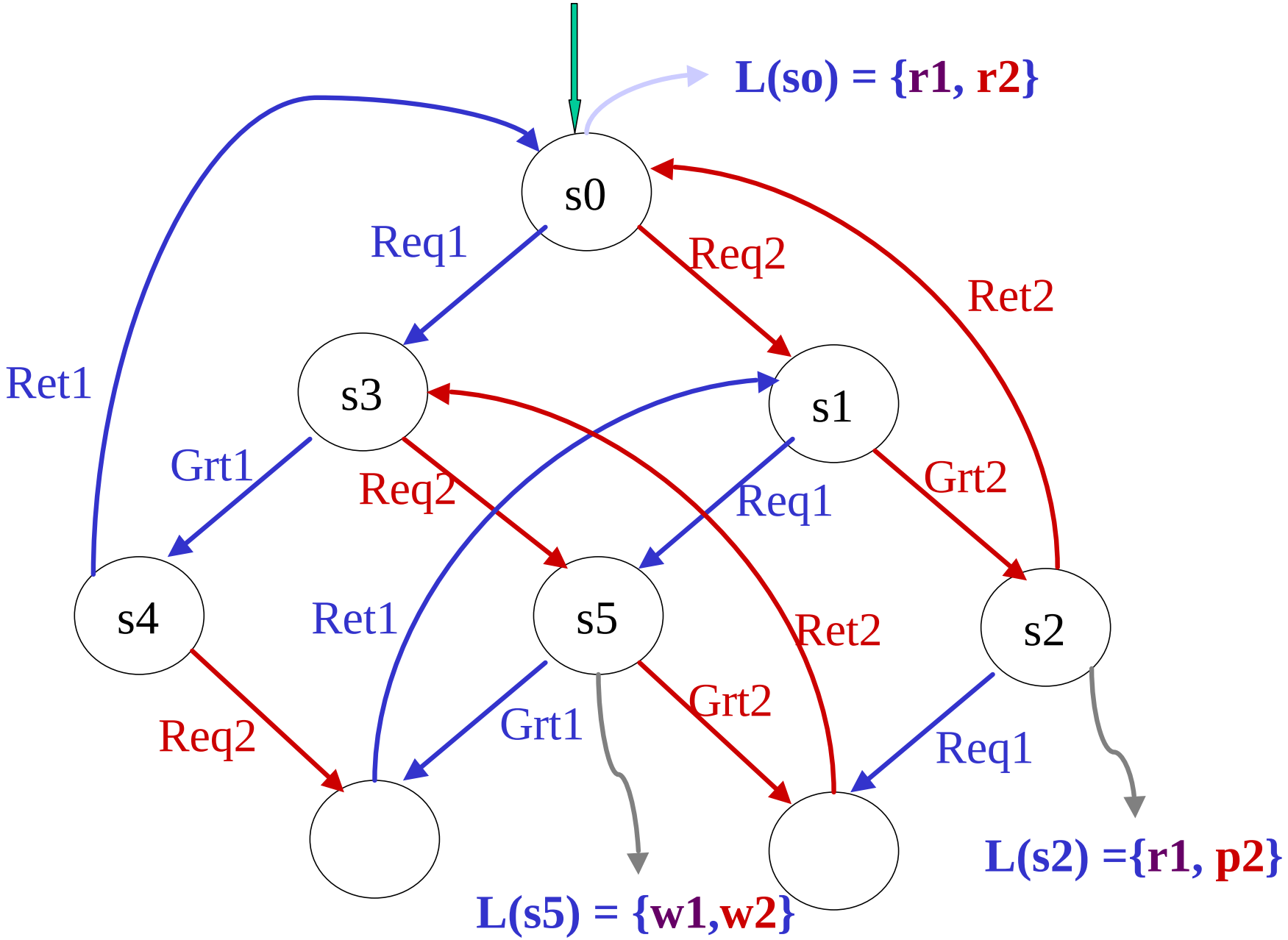
P1 – Process 1 is ***using*** the resource.

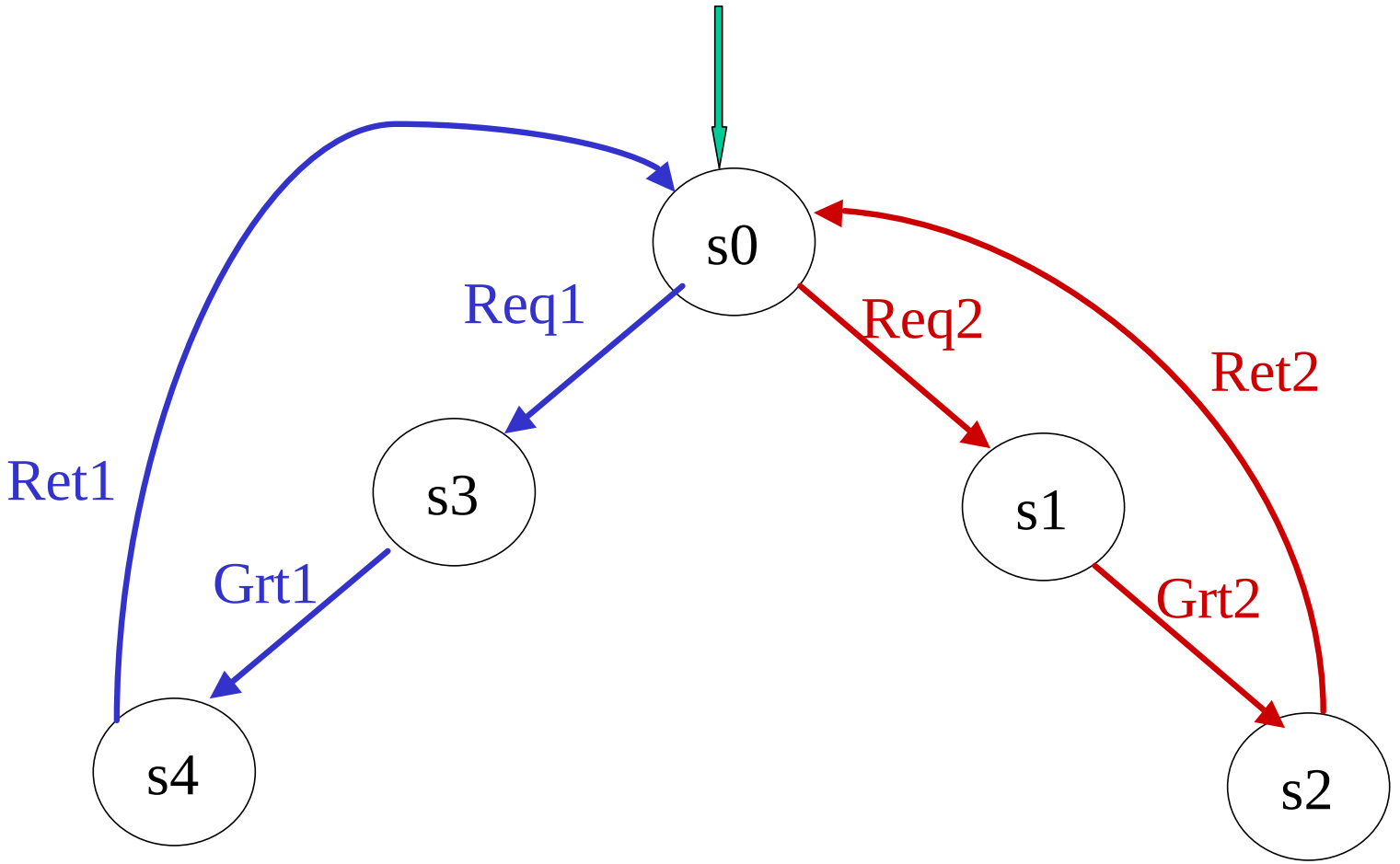
AP = { R1, W1, P1, R2, W2, P2}

The context

- Model a system to be verified as a Kripke structure:
  - Transition system **TS = (S, S_0, R)**
  - AP = A finite set of atomic propositions.
    - Basic assertions about the system
  - **L : S** $\longrightarrow$ $\mathbf{2^{AP}}$ = The set of subsets of AP.
  - **p** Î **L(s)** ---- **p** is true at **s**
  - **p** Ï **L(s)** ---- **p** is not true at **s**.
- **K = (S, S_0, R, AP, L)** ---- Kripke structure


L(so) = {r1, r2}
s0
Req1
Req2
Ret2
Ret1
s3
s1
Grt1
Req2
Req1
Grt2
s4
Ret1
s5
Ret2
s2
Req2
Grt1
Grt2
Req1
L(s5) = ?
L(s2) ={r1, p2}


L(so) = {r1, r2}
s0
Req1
Req2
Ret2
Ret1
s3
s1
Grt1
Req2
Req1
Grt2
s4
Ret1
s5
Ret2
s2
Req2
Grt1
Grt2
Req1
L(s2) ={r1, p2}
L(s5) = {w1,w2}



s0 s3 s4 s0 s1 s2 s0 s3 …

{r1, r2} {w1, r2} {u1,r2} {r1, r2} {r1,w2} {r1,p2} {r1,r2} {w1,r2}...

Assertions about a computation

s0	s3	s4	s0	s1	s2	s0	s3 …
{r1, r2}	**{w1, r2}**	**{p1,r2}**	**{r1, r2}**	**{r1,w2}**	**{r1,p2}**	**{r1,r2}**	**{w1,r2}..**

- If at some stage Process 1 is **waiting** then at some later stage it is **printing** (i.e. using the resource).
- **At no stage** are both processes using the resource.
- If a process is waiting then it does so **until** it starts to use the resource.
- **There is a stage** at which both processes are waiting.

The Application

- **K = (S, S_0, R, AP, L)**
- Every computation (sequence of states) can be viewed as a sequence of subsets of **AP**.
- **s_0 s_1 s_2 ….** **---- L(s_0) L(s_1) L(s_2)** ….
- These **AP-computations** will be the models for the formulas of LTL.

- Verification :
  - Every **AP**-computation of **K** is a model of j

Linear Time Temporal Logic (LTL)

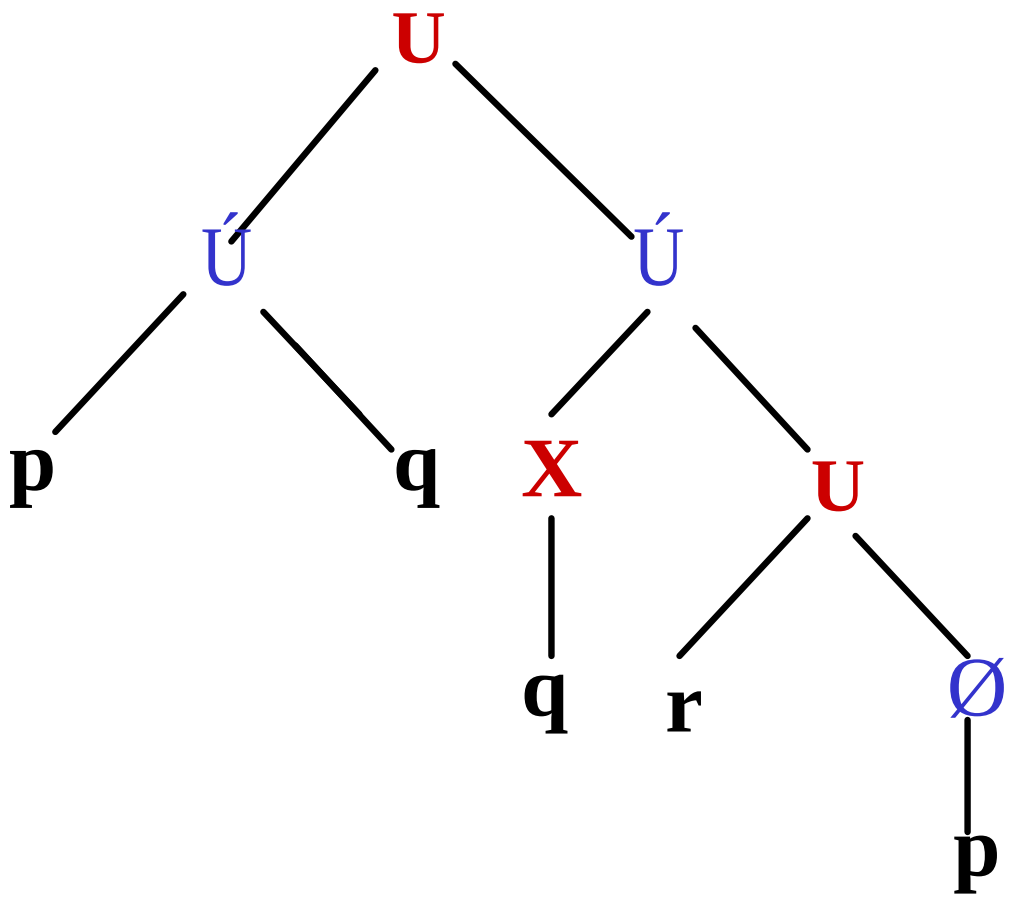
- Syntax :
  - **AP = {p_0, p_1, …p_n}**, a finite set of atomic propositions.
- Formulas :
  - Every **p_i** in AP is a ***LTL formula***.
  - If j is a formula then Ø j is a ***LTL formula***.
  - If j_1 and j_2 are formulas then **(j_1 Ú j_2)** is a ***LTL formula***.
  - If j is a formula then **X** j is a ***LTL formula*** (Next).
  - If j_1 and j_2 are formulas then **(j_1 U j_2)** is a ***LTL formula*** (Until).

Formulas

- **LTL ::= *p*** ½ Ø j ½ j_1 Ú j_2 ½**X** j ½ j_1 **U** j_2

- **p ; p**Ú**q ; (**Ø**p**Ú**q)**Ú Ø**(r**Ú**q)**
- **X q ; X(p**Ú**q) ; X((**Ø**p**Ú**q)** Ú **X**Ø**(r**Ú**q))**
- **(p**Ú**q)U(X r**Ú **(**Ø**q U (X**Ø**p)))**

(p Ú q) U (X q Ú (r U Ø p))



Semantics

- **AP** = A finite set of atomic propositions.
- $S = 2^{AP}$ = The set of subsets of **AP**
- **AP = { p, q, r }**
- S = { f, **{p}, {q}, {r}, {p,q}, {p,r}, {p,r}, {p,q,r}** }

- S^{w} = The set of infinite sequences over S.

Semantics

- **AP = {p, q, r}** S = $\mathbf{2^{AP}}$
- S = **{** f, **{ p }, { q }, ….,{ p, q, r } }**

s : **{ p,r } { q }** Æ **{ p, q, r } { r }**…

path: 0 → 1 → 2 → 3 → 4 …

- At stage **0** of s, **p** and **r** are true but not **q**; at stage **2** of s no member of **AP** is true….

Semantics

- S^w = The set of infinite sequences over Σ.
- $s \in S^w$ --- A model
- $s(i)$ ---- **i**-th position of s
- **{p} {q,r} Æ {r, q} {p, q, r}……..**

  | | | | | |

- **0 1 2 3 4 ……**
- **$s(0) = \{p\}$ $s(2) = Æ$ $s(3) = ?$**

Semantics

- **AP** $S = 2^{AP}$
- S^w = The set of infinite sequences over S.
- $s \in S^w$ --- A model
- $s(i)$ ---- **i**-th position of s
- j , a formula.
- $s(i) \models j$
  - $s(i)$ ***satisfies*** j
  - j is true in the **i**-th position of s

Semantics

- **LTL ::= *p*** ½Ø j ½ j_1 Ú j_2 ½**X** j ½ j_1 **U** j_2
- s = G_0 G_1 G_2 ….. G_i G_{i+1} ……
- Each G_j is a subset of **AP**.
- **s(i)⊨p *iff* p** Î G_i

Semantics

- **LTL ::= *p*** ½Ø j ½ j $_1$ Ú j $_2$ ½**X** j ½ j $_1$ **U** j $_2$
- **AP = {p, q, r}**
- s = **{p,q} {r}** Æ **{q, r} {p, q, r} …..**

0 1 2 3 4

- s**(0)** ***satisfies*** **q**
- s**(1)** ***satisfies*** **r**
- s**(2)** does ***not*** *satisfy* **q !**

Semantics

- **LTL ::= *p*** ½Ø j ½ j_1 Ú j_2 ½**X** j ½ j_1 **U** j_2
- s = G_0 G_1 G_2 ….. G_i G_{i+1} ……
- Each G_j is a subset of **AP**.
- **s(i) ⊨ Ø j** ***iff*** **s(i) ⊭ j**

Semantics

- **LTL ::= *p*** ½Ø j ½ j_1 Ú j_2 ½**X** j ½ j_1 **U** j_2
- s = G_0 G_1 G_2 ….. G_i G_{i+1} ……
- Each G_j is a subset of **AP**.
- **s(i) ⊨ j_1 Ú j_2** ***iff*** **s(i) ⊨ j_1** ***OR*** **s(i) ⊨ j_2**

Semantics

- **LTL ::= *p*** ½Ø j ½ **j** $_1$ Ú **j** $_2$ ½**X** j ½ **j** $_1$ **U j** $_2$
- **AP = {p, q, r}**
- s = **{p, q} {r}** Æ **{q, r} {p, q, r} …..**

  0 1 2 3 4
- s**(0)** ***satisfies*** Ø**r** ; s**(0)** does ***not satisfy*** **r**
- s**(1)** ***satisfies*** **p** Ú **r** ; s**(1)** ***satisfies*** **r**
- s**(2)** ***satisfies*** Ø**(p** Ú **r) ?**

Semantics

- **LTL ::= *p*** ½Ø j ½ j $_1$ Ú j $_2$ ½**X** j ½ j $_1$ **U** j $_2$
- **AP = {p, q, r}**
- s = **{p, q} {r}** Æ **{q, r} {p, q, r) …..**

  0 1 2 3 4
- s**(2)** ***satisfies*** Ø**(p** Ú **r)** ? ***Yes!***
- s**(2)** does ***not satisfy*** **p** Ú **r**

Semantics

- **LTL ::= *p*** ½Ø j ½ j $_1$ Ú j $_2$ ½**X** j ½ j $_1$ **U** j $_2$
- s = G_0 G_1 G_2 ….. G_i G_{i+1} ……..
  X j j

- **s(i) ⊨ X** j ***iff*** **s(i+1) ⊨** j

Semantics

- **LTL ::= *p*** ½Ø j ½ j $_1$ Ú j $_2$ ½**X** j ½ j $_1$ **U** j $_2$
- **AP = {p, q, r}**
- s = **{p,q} {r}** Æ **{q,r} {p,q,r) …..**

  0 1 2 3 4
- s**(2)** ***satisfies*** **X r** ; s**(3)** ***satisfies*** **r**
- s**(0)** ***satisfies*** **X(p** Ú **r)** ; s**(1)** ***satisfies*** **r**
- s**(1)** does ***not satisfy*** **X(p** Ú **r)**
  - s**(2)** does ***not satisfy*** **p** Ú **r**

Semantics

- **LTL ::= *p* ½Ø j ½ j $_1$ Ú j $_2$ ½**X** j ½ j $_1$ **U** j $_2$**
- **AP = {p, q, r}**
- s = **{p,q} {r} Æ {q,r} {p,q,r} …..**

  0 1 2 3 4
- s**(1)** ***satisfies*** **X(X Ø p)** ***iff***
  - s**(2)** ***satisfies*** **X Ø p** ***iff***
  - s**(3)** ***satisfies*** **Ø p** ***iff***
  - s**(3)** does ***not satisfy*** **p**

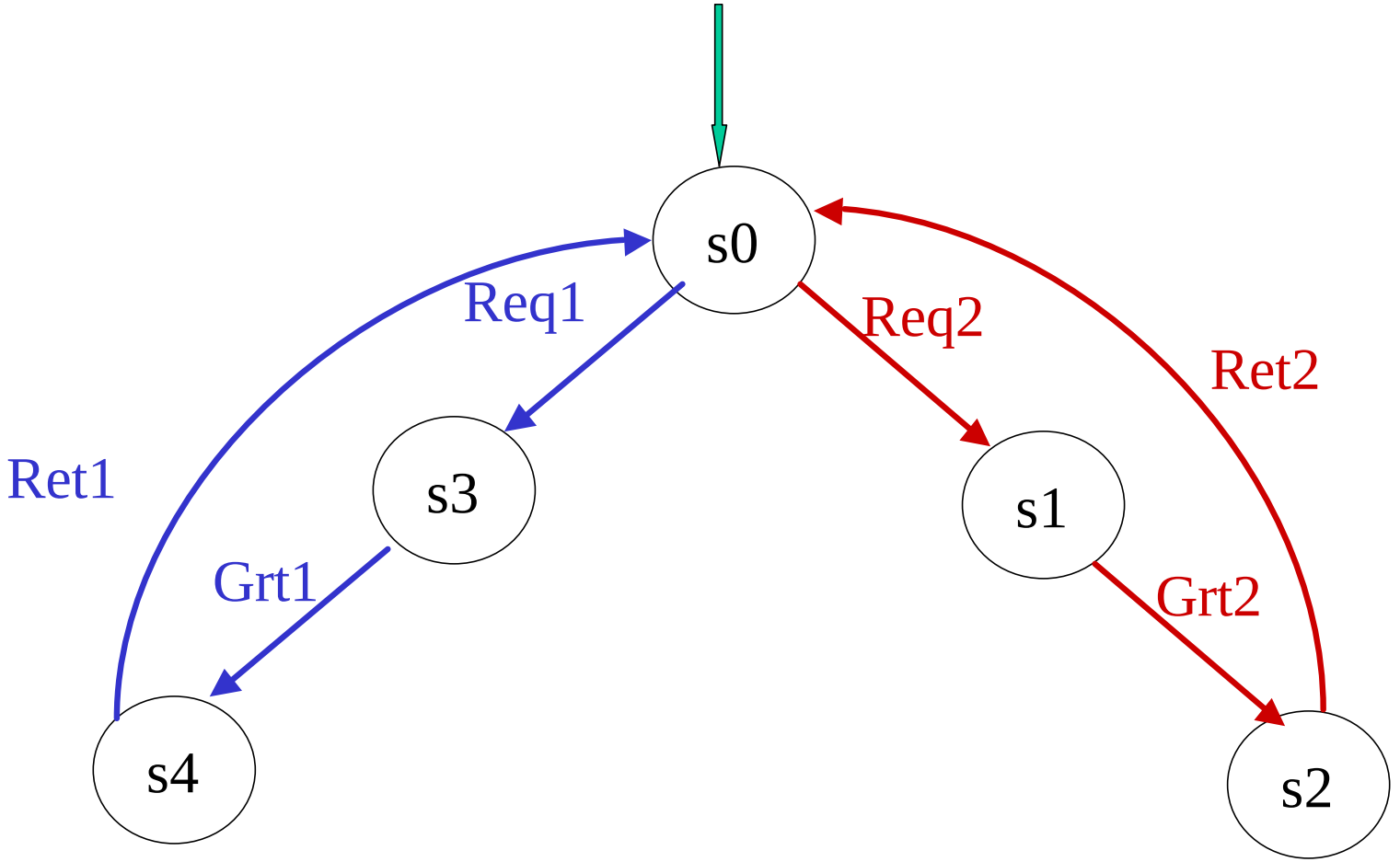
Semantics

- **LTL ::= *p*** ½Ø j ½ j_1 Ú j_2 ½**X** j ½ j_1 **U** j_2
- s = G_0 G_1 … G_i G_{i+1} …. G_{k-1} G_k …
  j_1… j_1 … j_1 j_2

- **s(i)** ⊨ j_1 **U** j_2 ***iff*** there exists **k** ³ **i** s.t.
  - **s(k)** ⊨ j_2
  - **s(j)** ⊨ j_1 for every **i £ j < k**

Semantics

- **LTL ::= *p*** ½Ø j ½ j **1** Ú j **2** ½**X** j ½ j **1** **U** j **2**
- **k** could be arbitrarily greater than **i**.
- **k = i** is allowed and there is **no i** £ **j** < **k**
- s**(i)**⊨ j **1** **U** j **2** ***iff*** there exists **k** ³ **i** s.t.
  - s**(k)**⊨ j **2**
  - s**(j)**⊨ j **1** for every **i** £ **j** < **k**



s0 s3 s4 s0 s1 s2 s0 s3 …

{r1,r2} {w1,r2} {p1,r2} {r1,r2} {r1,w2} {r1,p2} {r1,r2} {w1,r2}...

An Example

AP = {r1, w1, p1, r2, w2, p2}

{r1,r2}	**{w1,r2}**	**{p1,r2}**	**{r1,r2}**	**{r1,w2}**	**{r1,p2}**	**{r1,r2}**	**{w1,r2}...**
0	1	2	3	4	5	6	7

- s**(1)** ***satisfies*** **(r2 U w2)** ;
  - s**(4)** ***satisfies*** **w2** and
  - s**(1),** s**(2),** s**(3)** ***satisfy*** **r2**.

An Example

AP = {r1, w1, p1, r2, w2, p2}

{r1,r2}	**{w1,r2}**	**{p1,r2}**	**{r1,r2}**	**{r1,w2}**	**{r1,p2}**	**{r1,r2}**	**{w1,r2}...**
0	1	2	3	4	5	6	7

- s**(1)** does ***not satisfy*** **(r2 U p2)** ;
  - s**(5)** ***satisfies*** **p2** and
  - s**(1),** s**(2),** s**(3)** ***satisfy*** **r2**.
  - but s**(4)** does ***not satisfy*** **r2 !**

An Example

AP = {r1, w1, p1, r2, w2, p2}

{r1,r2} {w1,r2} {p1,r2} {r1,r2} {r1,w2} {r1,p2} {r1,r2} {w1,r2}...

0 1 2 3 4 5 6 7

- s**(1)** ***does satisfy*** **((r2** Ú **w2) U p2)** ;
  - s**(5)** ***satisfies*** **p2** and
  - s**(1),** s**(2),** s**(3)** ***satisfy*** **r2, hence also (r2** Ú **w2).**
  - s**(4)** ***satisfies*** **w2, hence also (r2** Ú **w2) !**

Models

- **AP** $S^{AP} = 2$
- S^{ω} = The set of infinite sequences over S.
- $s \hat{I} S^{\omega}$
- j an **LTL** formula.

- A path s is a ***model*** of j **(** $s \models j$ **)** ***iff***
  - s**(0)** $\models$ j

Validity in LTL

- **AP** $S^{AP} = 2$
- S^{ω} = The set of infinite sequences over S.
- $s \in S^{\omega}$
- φ an **LTL** formula.

- φ is ***LTL-valid*** **(** $\models \varphi$ **)** ***iff for every*** $s \in S^{\omega}$ $-$ $s \models \varphi$

Derived Operators

- **LTL ::= *p*** ½Ø j ½ j_1 Ú j_2 ½**X** j ½ j_1 **U** j_2
- j_1 Ù j_2 --- Ø (Ø j_1 Ú Ø j_2) (**And**)
- j_1 É j_2 --- Ø j_1 Ú j_2 (**Implies**)
- j_1 ° j_2 ---- (j_1 É j_2) Ù (j_2 É j_1) (**Iff**)
- **AP = {p_1, p_2, …,p_n}**
- T ---- p_1 Ú Øp_1 (**true**)

- **Fact : In *every model*** s, ***at every*** **i**,
  - s**(i)** ⊨ T

Derived Operators

- **LTL ::= *p*** ½Ø j ½ j $_1$ Ú j $_2$ ½**X** j ½ j $_1$ **U** j $_2$
- **F** j **---- (**⊤ **U** j **)** (future ; diamond: ◇)

- **Fact** :
  - s**(i)** ⊨ **F** j *iff* **there exists k** ³ **i** such that s**(k)** ⊨ j .

Derived Operators

- **Fact** :
  - $s(i) \models F\, j$ *iff* **there exists k** $\geq$ **i** such that $s(k) \models j$.

Proof:

$s(i) \models F\, j$ *iff*

$s(i) \models (\top\ U\ j)$ *iff*

$\exists\, k \geq i,\ s(j) \models j$ and $\forall i \leq j \leq k,\ s(k) \models \top$ *iff*

$\exists\, k \geq i,\ s(j) \models j$

Derived Operators

- **LTL ::= *p*** ½Ø j ½ j **1** Ú j **2** ½**X** j ½ j **1** **U** j **2**
- **F** j **----** **(**⊤ **U** j **)**
- **G** j **----** Ø**F**Ø j (invariant; box: □)

- **Fact** :
  - s**(i)** ⊨ **G** j *iff* **for every k** ³ **i** ,
    s**(k)** ⊨ j .

Derived Operators

- **LTL ::= *p* ½Ø j ½ j 1 Ú j 2 ½X j ½ j 1 U j 2**
- **(y R j) ---- Ø (Ø y U Ø j)** (Releases)
- **G j ---- (^ R j)**

- **Fact** :
  - s**(i)** ⊨ (y **R** j) ***iff*** **for every k** ³ **i**
    - s**(k)** ⊨ j ***or***
    - **for some i** £ **j < k,** s**(j)** ⊨ y

Derived Operators

- **LTL ::= *p*** ½Ø j ½ j $_1$ Ú j $_2$ ½**X** j ½ j $_1$ **U** j $_2$
- **(** y **W** j **)** **----** **G** y Ú **(** y **U** j **)** (Unless)

Derived Operators

- **LTL ::= *p*** ½Ø j ½ j $_1$ Ú j $_2$ ½**X** j ½ j $_1$ **U** j $_2$
- **(y B j)** **----** **?????** (Before)

Derived Operators

- **LTL ::= *p*** ½Ø j ½ j **1** Ú j **2** ½**X** j ½ j **1** **U** j **2**
- **(y B j)** **----** Ø **(** Ø y **U** j **)** (Before)
- Why???? Prove that it matches the intuition!

Tableau rules

- **(**y **U** j**)** ≡ j Ú (y Ù **X (**y **U** j**)** **)**
- **(**y **R** j**)** ≡ j Ù (y Ú **X (**y **R** j**)** **)** ≡
  ≡ (j Ù y) Ú (j Ù **X (**y **R** j**)** **)**
- **F** j ≡ j Ú **X F** j
- **G** j ≡ j Ù **X G** j

LTL: Some examples

- **Safety**: “it never happens that both A and B are print at the same time”

$$\mathbf{G}(\varnothing\ (\mathbf{P}_A \wedge \mathbf{P}_B))$$

- **Liveness:** “if A waiting, then it will eventually print”

$$\mathbf{G}(\mathbf{W}_A \rightarrow \mathbf{F}\ \mathbf{P}_A)$$

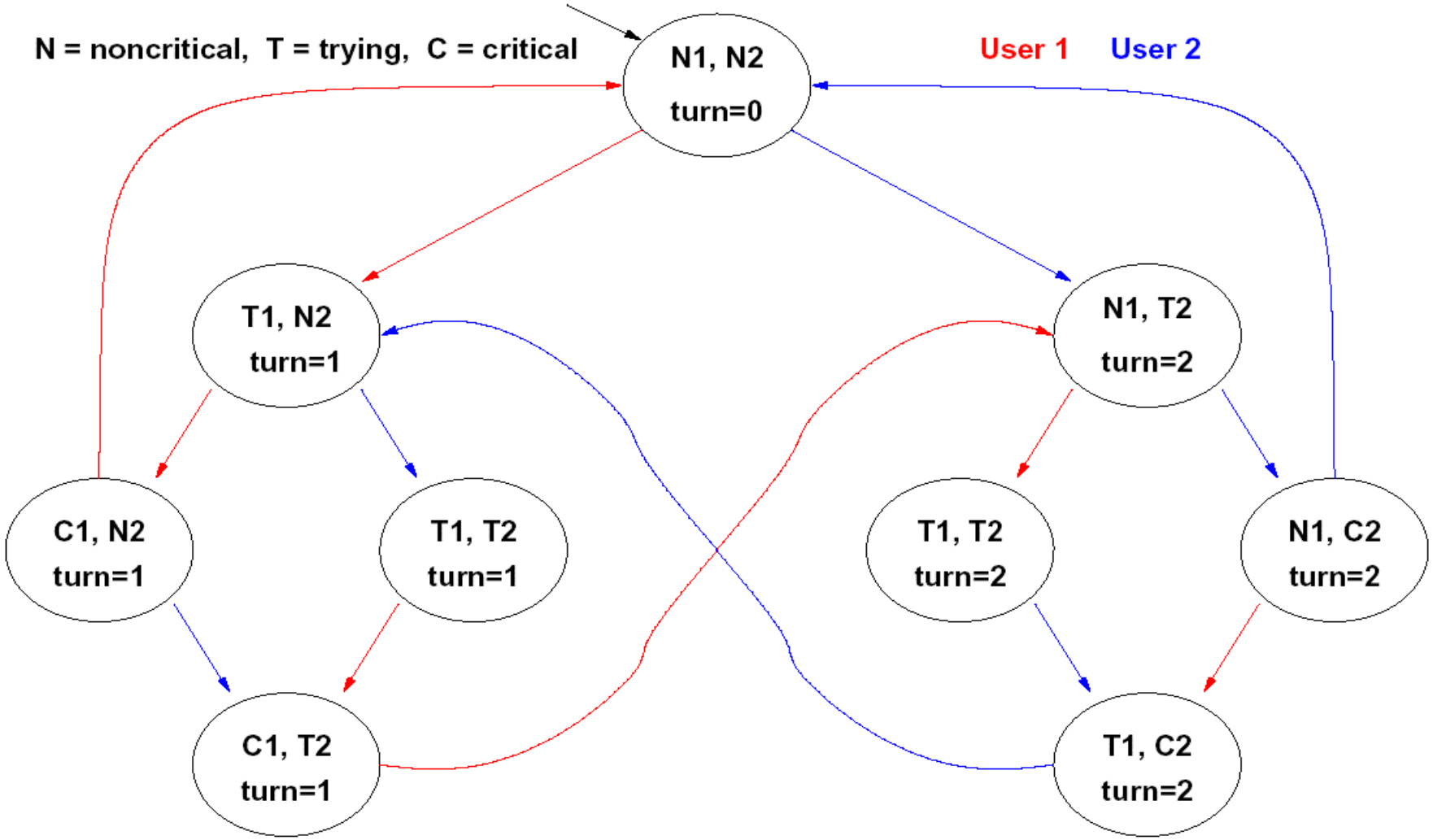
- **Fairness:** “A infinitely often idle”

$$\mathbf{GF}\ \mathbf{R}_A$$

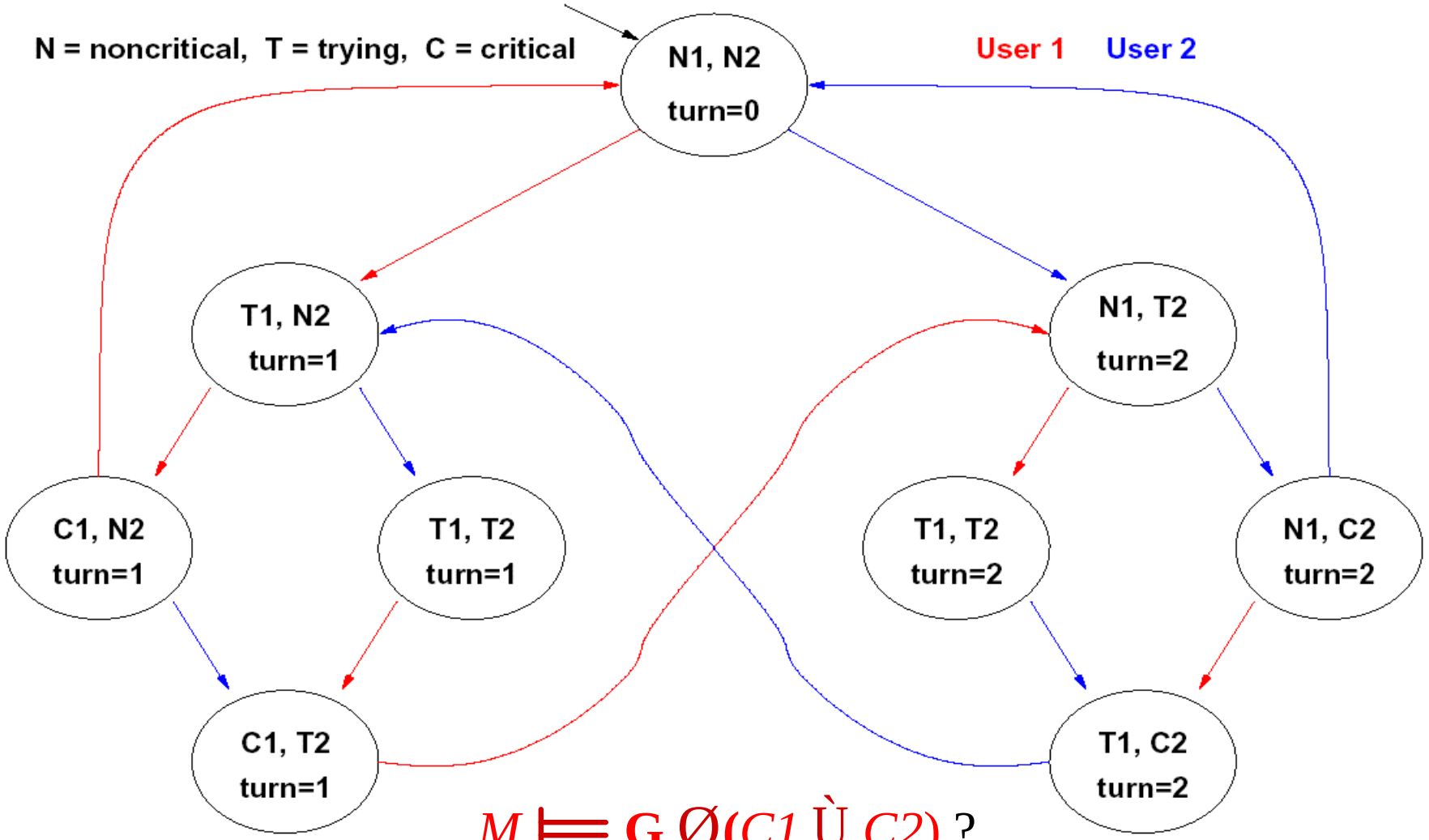
- **Strong fairness:** “if A infinitely often waiting, then it will infinitely often printing”

$$\mathbf{GF}\ \mathbf{W}_A \rightarrow \mathbf{GF}\ \mathbf{P}_A$$

Example: mutual esclusion



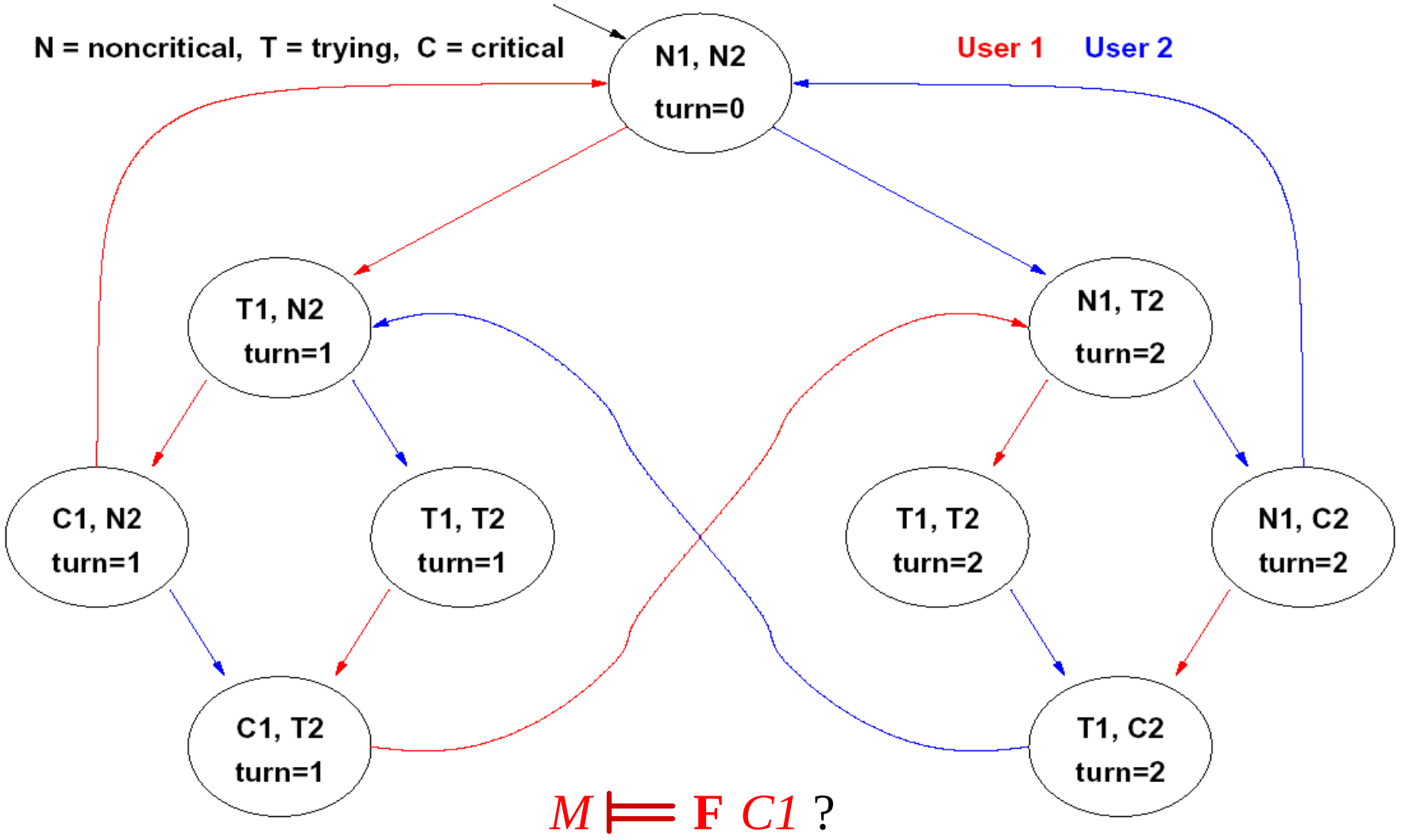
Example: mutual esclusion (safety)



$M \models \mathbf{G}\,\emptyset(C1 \wedge C2)$?

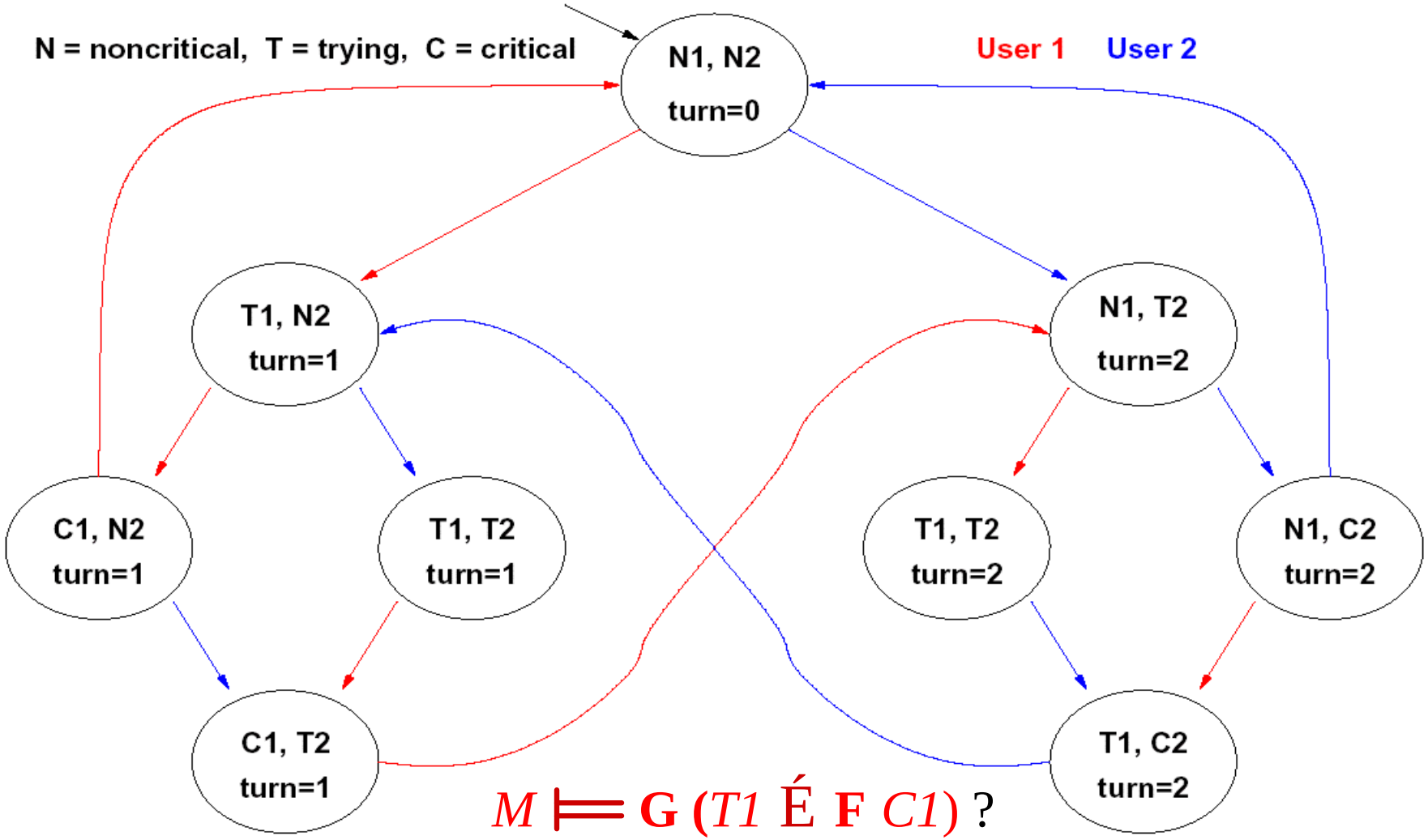
YES: There is no reachable state in which both C1 and C2 hold!

Example: mutual exclusion (liveness)



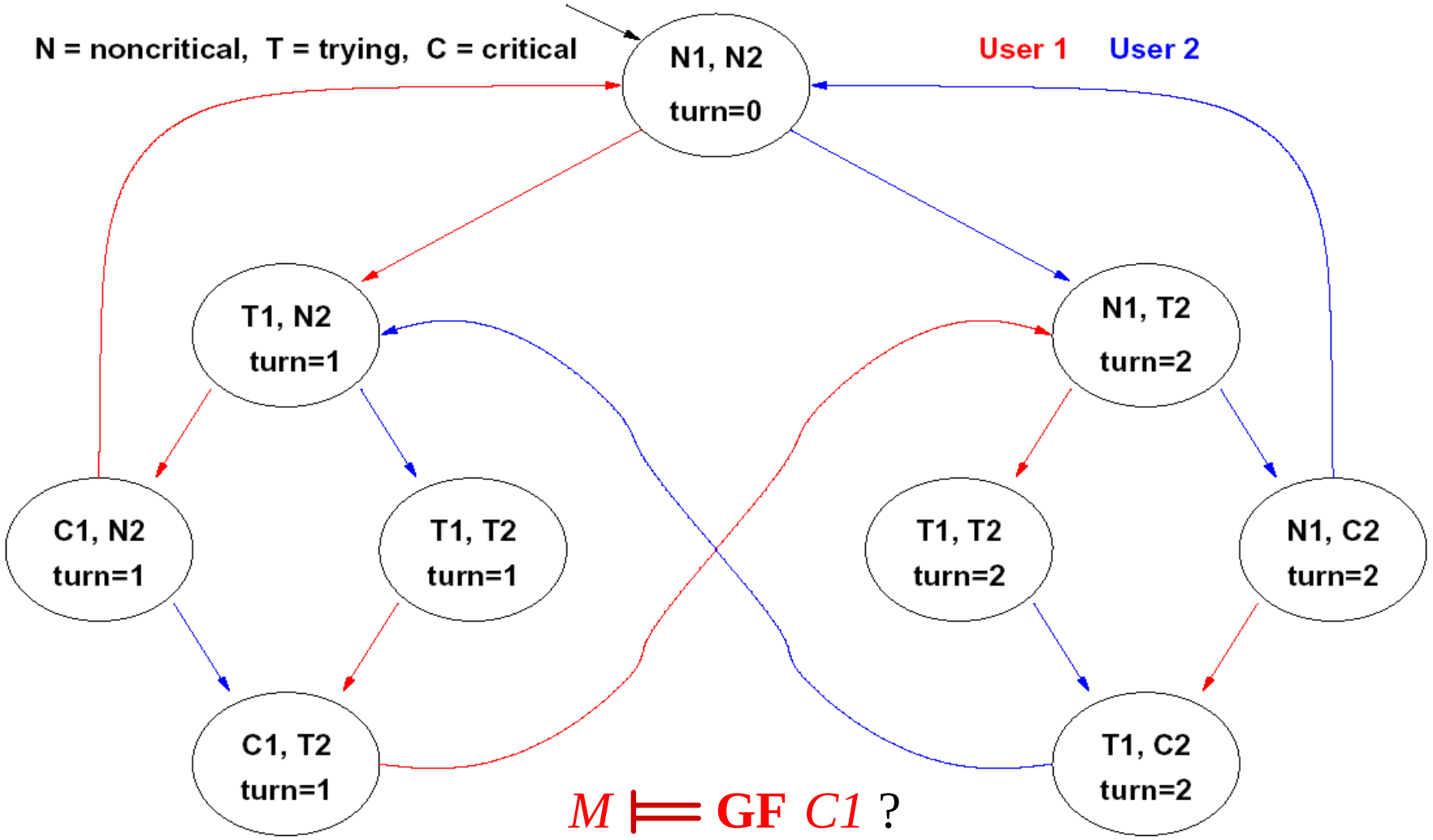
NO: there is an infinite cyclic solution in which C1 never holds!

Example: mutual exclusion (liveness)



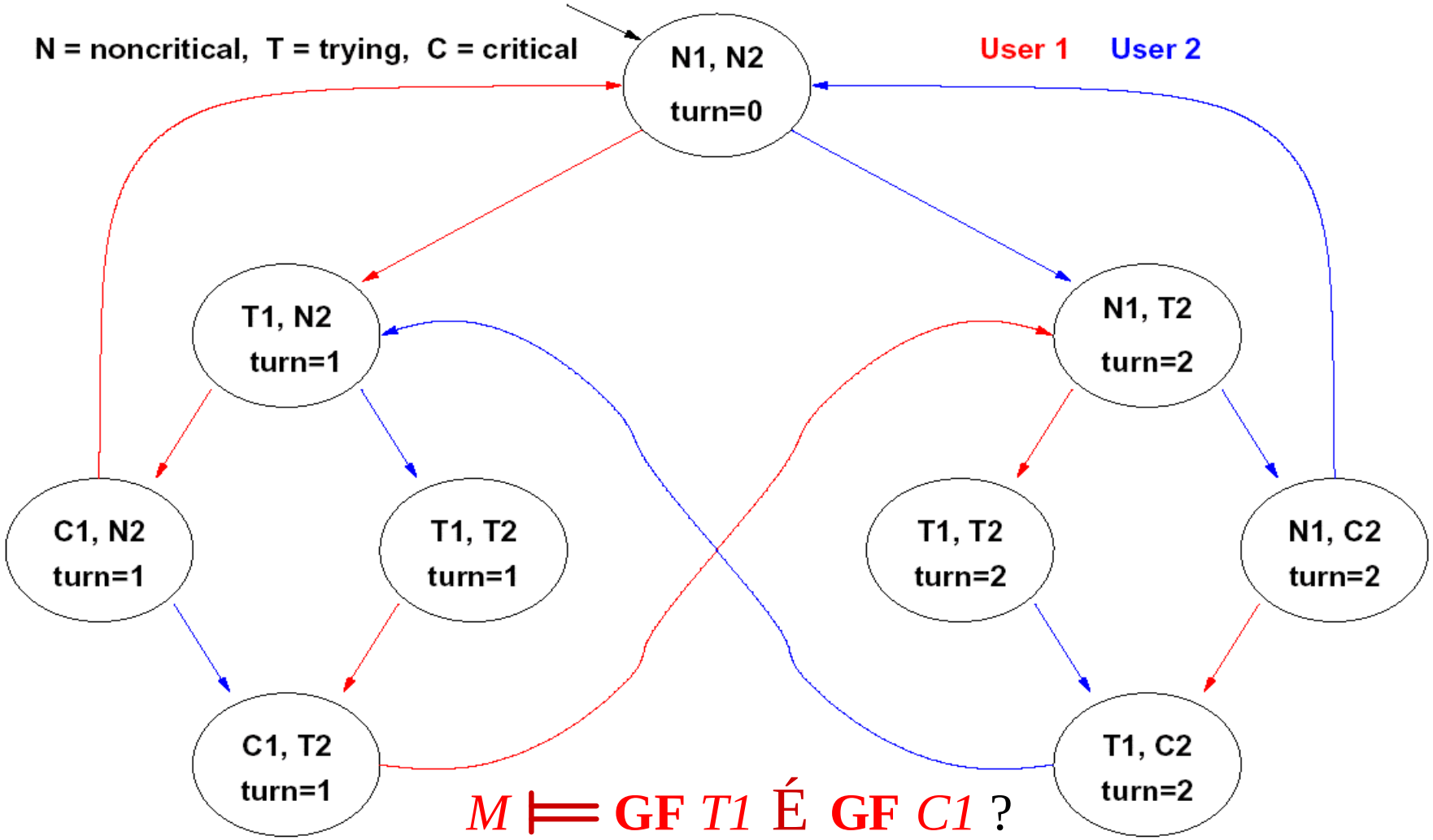
YES: every path starting from each state where *T1* holds passes through a state where *C1* holds!

Example: mutual exclusion (fairness)



NO: e.g., in the initial state, there is an infinite cyclic solution in which *C1* never holds!

Example: mutual exclusion (strong fairness)



YES: every path which visits *T1* infinitely often also visits *C1* infinitely often (see liveness prop. In previous example)!

Model Checking

- **K = (S, S_0, R, AP, L)** (the system)
- φ , an **LTL** formula. (the property)
- **K** $\models$ φ ***iff*** **every AP-computation** of **K** is a model of φ .
- Determining this is the ***model checking problem***.
- A solution to this problem can be automated!