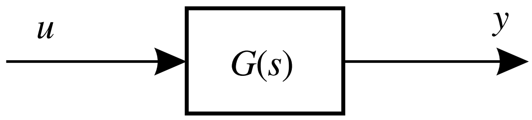
SCHEMI A BLOCCHI

Componenti di uno schema a blocchi

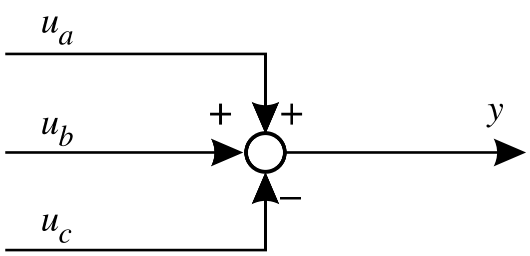
Regole di elaborazione

COMPONENTI DI UNO SCHEMA A BLOCCHI

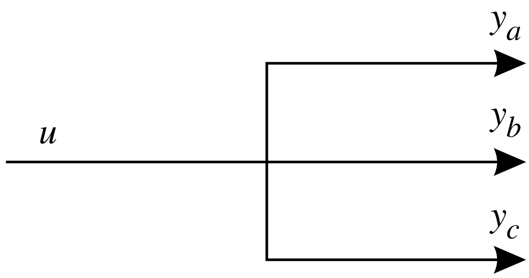
- Rappresentazione di un sistema dinamico



- Nodo sommatore
  - ⋆ $y(t) = u_a(t) + u_b(t) - u_c(t)$



- Punto di diramazione
  - ⋆ $y_a(t) = y_b(t) = y_c(t) = u(t)$



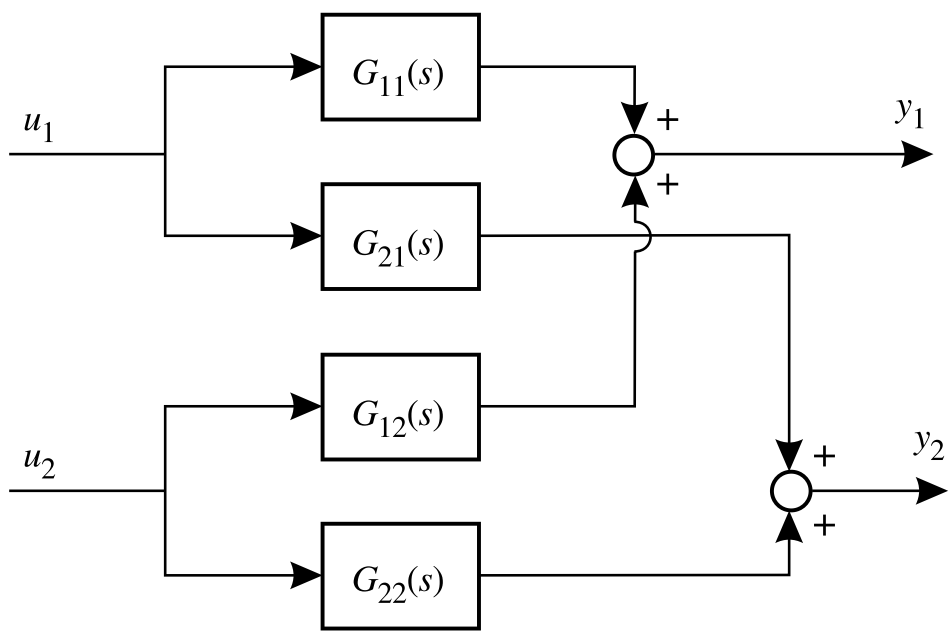
- Esempio

$$u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \qquad y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}$$

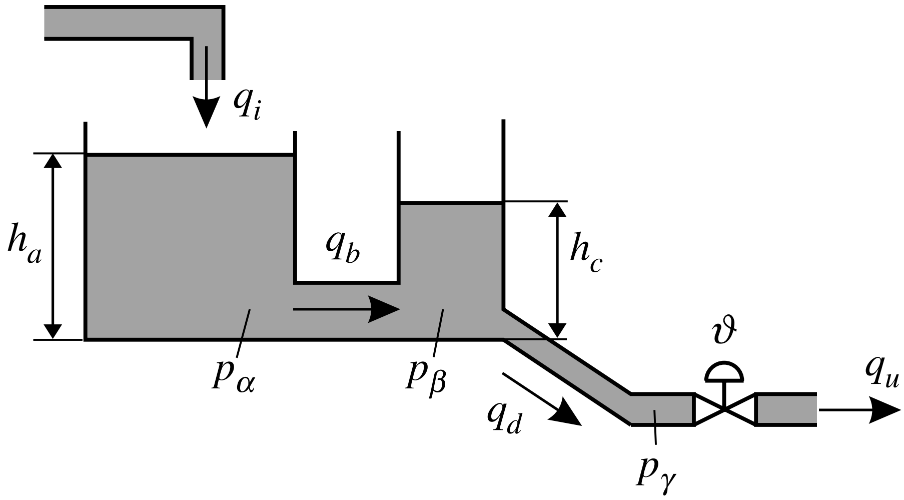
⋆ $Y(s) = G(s)U(s)$

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$$

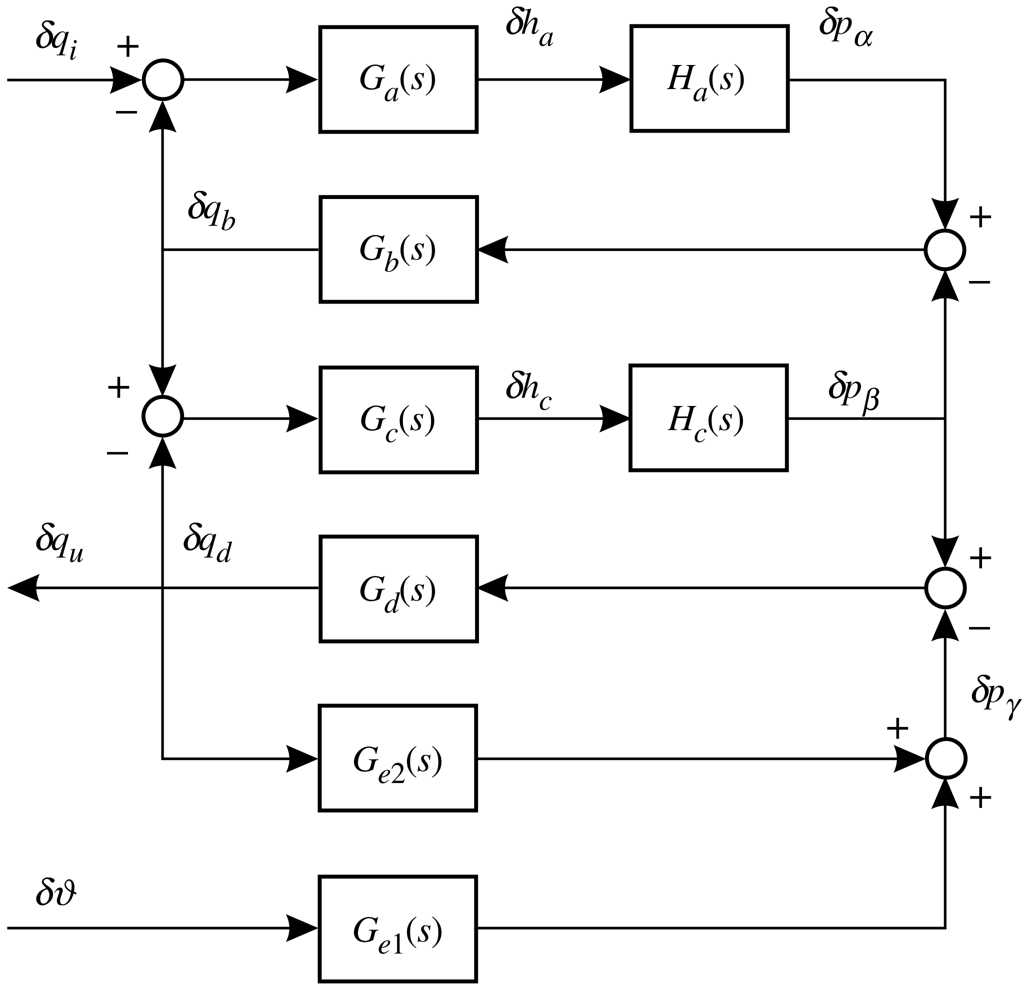
$$Y_1(s) = G_{11}(s)U_1(s) + G_{12}(s)U_2(s)$$
$$Y_2(s) = G_{21}(s)U_1(s) + G_{22}(s)U_2(s)$$



- Esempio: sistema idraulico



⋆ schema a blocchi

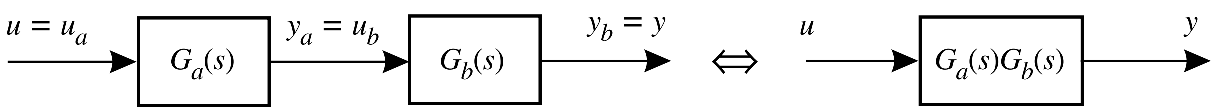


REGOLE DI ELABORAZIONE

- Connessione di due sistemi

$$Y_a(s) = G_a(s)U_a(s)$$
$$Y_b(s) = G_b(s)U_b(s)$$

Sistemi in serie (o in cascata)



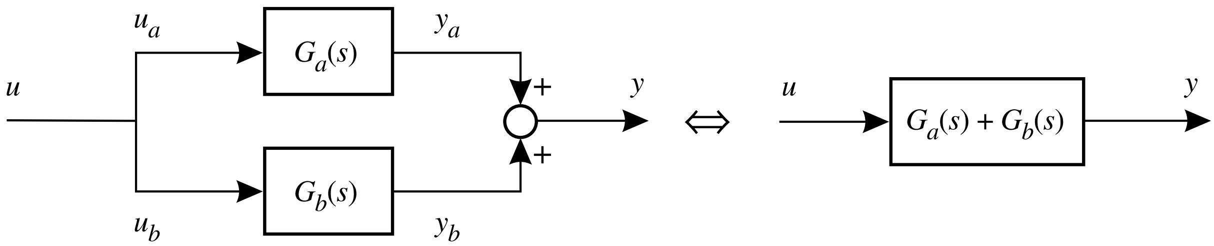
- Sistema complessivo

$$Y(s) = G_b(s)Y_a(s) = G_b(s)G_a(s)U(s)$$

$$\Downarrow$$

$$G(s) = \frac{Y(s)}{U(s)} = G_a(s)G_b(s)$$

Sistemi in parallelo



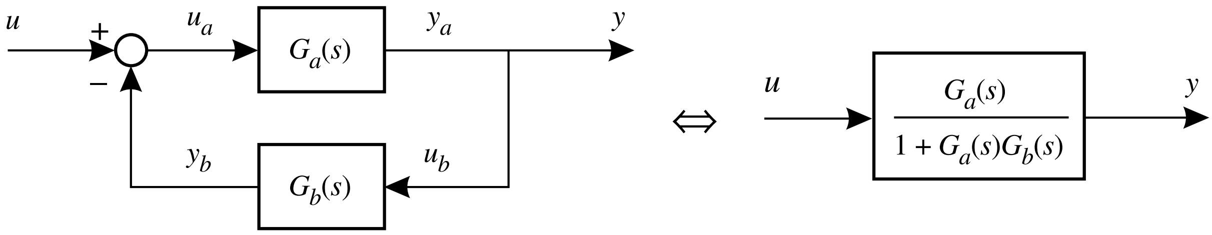
- Sistema complessivo

$$
\begin{aligned}
Y(s) &= Y_a(s) + Y_b(s) = G_a(s)U_a(s) + G_b(s)U_b(s) \\
&= (G_a(s) + G_b(s))\,U(s)
\end{aligned}
$$

$$\Downarrow$$

$$G(s) = \frac{Y(s)}{U(s)} = G_a(s) + G_b(s)$$

Sistemi in retroazione negativa (o controreazione)



- Sistema complessivo (in anello chiuso o retroazionato)

$$Y(s) = G_a(s)\left(U(s) - Y_b(s)\right) = G_a(s)\left(U(s) - G_b(s)Y(s)\right)$$

$$\Downarrow$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{G_a(s)}{1 + G_a(s)G_b(s)}$$

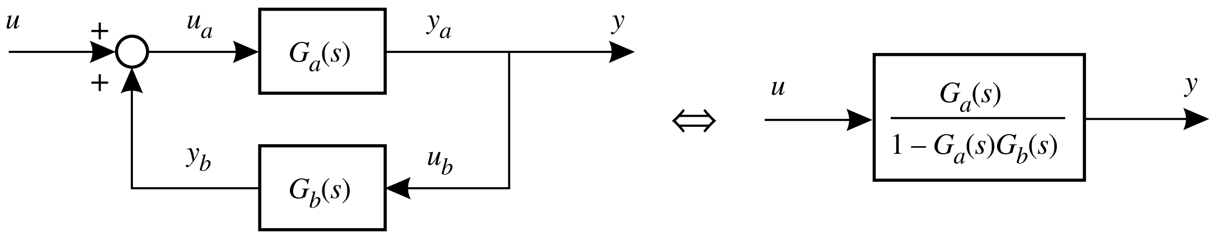
⋆ funzione di trasferimento d'anello

$$L(s) = G_a(s)G_b(s)$$

⋆ anello algebrico

$$\lim_{s\to\infty} L(s) = -1 \quad \Longrightarrow \quad \lim_{s\to\infty} G(s) = \infty$$

- Sistemi in retroazione positiva



- Sistema complessivo

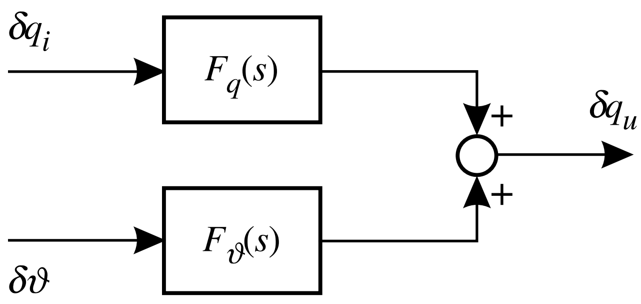
$$G(s) = \frac{Y(s)}{U(s)} = \frac{G_a(s)}{1 - G_a(s)G_b(s)}$$

  ⋆ condizione di congruenza

$$\lim_{s \to \infty} L(s) \neq 1$$

Riduzione di schemi a blocchi

- Esempio (precedente)
  - ⋆ schema equivalente



- ⋆ successive elaborazioni

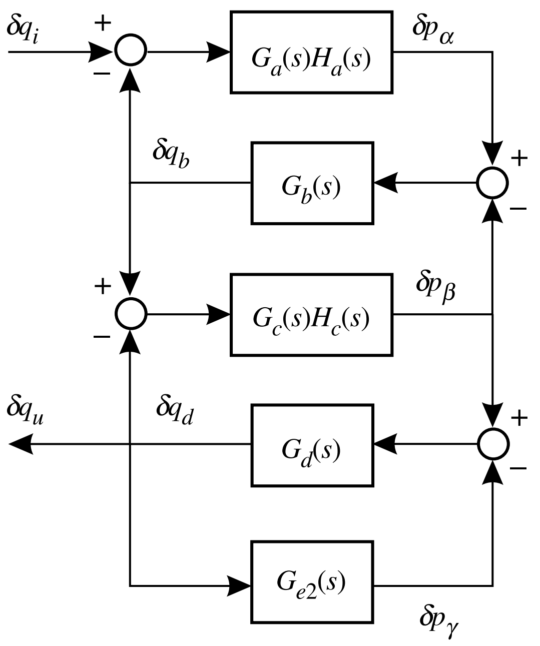
$$M_1(s) = \frac{G_d(s)}{1 + G_d(s)G_{e2}(s)}$$

$$\begin{aligned} M_2(s) &= \frac{G_c(s)H_c(s)}{1 + G_c(s)H_c(s)M_1(s)} \\ &= \frac{G_c(s)H_c(s)(1 + G_d(s)G_{e2}(s))}{1 + G_d(s)G_{e2}(s) + G_c(s)H_c(s)G_d(s)} \end{aligned}$$

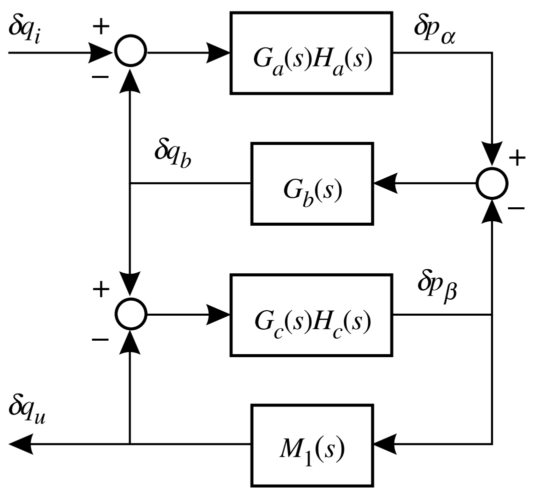
$$M_3(s) = \frac{G_b(s)}{1 + G_b(s)M_2(s)}$$

$$\begin{aligned} M_4(s) &= M_2(s)M_1(s) \\ &= \frac{G_c(s)H_c(s)G_d(S)}{1 + G_b(s)G_{e2}(s) + G_c(s)H_c(s)G_d(s)} \end{aligned}$$

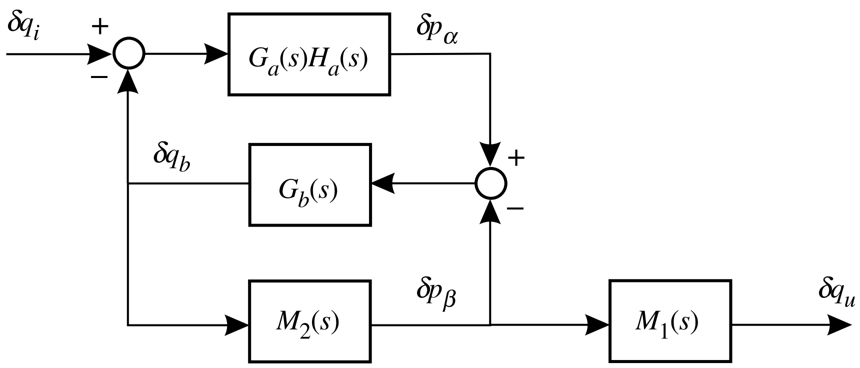
$$\begin{aligned} M_5(s) &= \frac{G_a(s)H_a(s)M_3(s)}{1 + G_a(s)H_a(s)M_3(s)} \\ &= \frac{G_a(s)H_a(s)G_b(s)}{1 + G_b(s)M_2(s) + G_a(s)H_a(s)G_b(s)} \end{aligned}$$


δq_i
+
−
G_a(s)H_a(s)
δp_α
δq_b
G_b(s)
+
−
+
−
G_c(s)H_c(s)
δp_β
δq_u
δq_d
G_d(s)
+
−
G_e2(s)
δp_γ

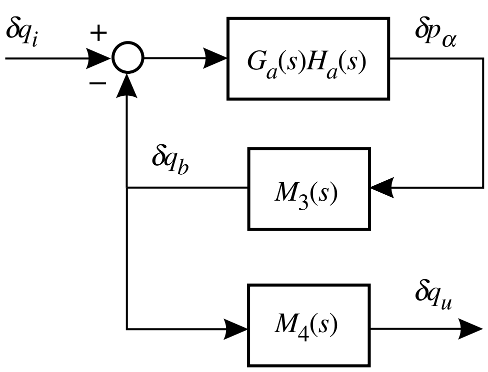
a)


δq_i
+
−
G_a(s)H_a(s)
δp_α
δq_b
G_b(s)
+
−
+
−
G_c(s)H_c(s)
δp_β
δq_u
M_1(s)

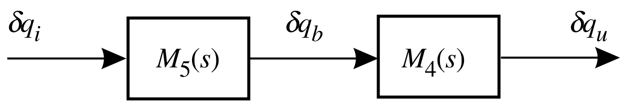
b)


δq_i
+
−
G_a(s)H_a(s)
δp_α
δq_b
G_b(s)
+
−
M_2(s)
δp_β
M_1(s)
δq_u

c)


δq_i
+
−
G_a(s)H_a(s)
δp_α
δq_b
M_3(s)
M_4(s)
δq_u

d)


δq_i
M_5(s)
δq_b
M_4(s)
δq_u

e)

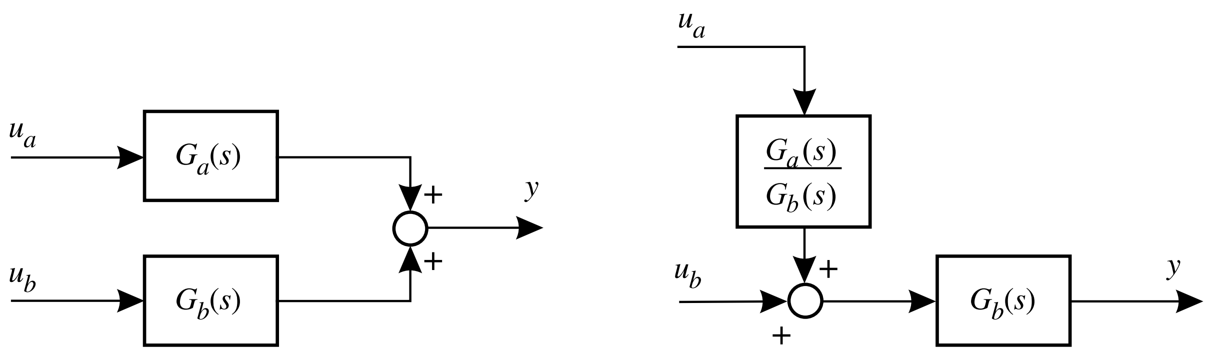
⋆ in conclusione

$$K(s) = 1 + G_a(s)H_a(s)G_b(s) + G_b(s)G_c(s)H_c(s)$$

$$\begin{aligned} F_q(s) &= M_4(s)M_5(s) \\ &= \frac{G_aH_aG_bG_cH_cG_d}{[1+G_dG_{e2}]K + [1+G_aH_aG_b]G_cH_cG_d} \end{aligned}$$

$$F_\vartheta(s) = \frac{-G_dG_{e1}K}{[1+G_dG_{e2}]K + [1+G_aH_aG_b]G_cH_cG_d}$$

- Due schemi equivalenti



⋆ $G_a(s)/G_b(s)$

potrebbe avere un polinomio a numeratore di grado maggiore di quello del polinomio a denominatore

potrebbe contenere termini di anticipo qualora $G_b(s)$ contenga un ritardo