Università degli Studi di Napoli “Federico II”

SCUOLA POLITECNICA E DELLE SCIENZE DI BASE

DIPARTIMENTO DI INGEGNERIA INDUSTRIALE

CORSO DI LAUREA IN INGEGNERIA AEROSPAZIALE

CLASSE DELLE LAUREE IN INGEGNERIA INDUSTRIALE (L-9)

Elaborato di laurea in Meccanica del Volo

Airplane static stability and control forces in MATLAB live script

Relatore:
Prof. Danilo Ciliberti

Candidato:
Sabrina Amuro
Matr. N35002994

ANNO ACCADEMICO 2023 – 2024

A mia mamma,
la mia supereroina.

A Cristina,
il mio punto di riferimento.

Abstract

The aim of this project is to describe the MATLAB code related to aircraft stability and control analysis. More specifically the work aims to implement the calculation of some specific parameters of the code. The purpose is to improve the accuracy of the hinge moment formula and develop the MATLAB functions for stick forces evaluation, which must be within specific limits given by the airworthiness regulations. The work will then be applied to a series of aircraft to verify their correctness and observe their behavior under particular flight conditions.

Sommario

Lo scopo di questo progetto è descrivere il codice MATLAB relativo all'analisi di stabilità e controllo di aerei. Più nello specifico il lavoro mira ad implementare il calcolo di alcuni parametri specifici del codice. L'obiettivo sarà migliorare la precisione della formula del momento di cerniera e sviluppare il codice MATLAB relativo agli sforzi di barra, che devono essere entro dei limiti prescritti dalle normative di aeronavigabilità. Il lavoro sarà poi applicato ad una serie di velivoli per verificarne la correttezza e osservare il loro comportamento in particolari condizioni di volo.

Table of contents

List of figures

1. Introduction

Stability of an aircraft refers to its movement in returning to a state of equilibrium. This condition is usually called trim. For this reason, we define static stability of an airplane under steady conditions its propensity to return to a condition called trimmed condition. The aim of the study is examining the forces to understand if they bring back the airplane to its equilibrium conditions. Only in this case we can say that the aircraft is statically stable.

For equilibrium along any flight path (in an unaccelerated flight) we have to satisfy the equilibrium equations applied to each degree of freedom.

The symmetric degrees of freedom include all that is called longitudinal motion of the airplane. Conversely the asymmetric degrees of freedom include all that is called lateral motion of the airplane. To study the motion of the system, we can commute the system into a condition of equilibrium. As already mentioned, for the equilibrium to be statically stable, it is necessary that a perturbation of the equilibrium generates a force or moment that tend to move the system back to its initial equilibrium condition.

To study the static stability is necessary to investigate two different types of stability:

- Longitudinal stability
- Lateral-Directional stability

The problems of airplane longitudinal static stability can be solved by observing the moments of the airplane's y-axis. Longitudinal equilibrium requires that the summation of these moments equal to zero.

In order to analyze the lateral and directional static stability we pay attention to the forces and moments that bring the airplane to rotate about its x-axis and z-axis or translate along its y-axis.

The software used in this work, a MATLAB live script developed in [5], after doing some preliminary calculations, knowing the value of geometry, aerodynamics and engine data, is able to calculate all the equations related to the above-cited longitudinal and lateral-directional equilibrium.

Preliminary calculations

Preliminary calculations are made in this section using a dedicated MATLAB function:

```
AllData= P2_aeroFun_preCalc(AllData);
```

The following input data are extracted from the AllData structure:

```
% INPUT DATA: tip chord, root chord, semi-wingspan, air density, speed,
% weight, load factor, semi-horizontal tail span, horizontal tail
% surface, 2D lift coefficient curve slope, Oswald wing factor, wing
% incidence angle, wing alpha angle at 0 lift, horizontal tail lift coeff.,
% horizontal tail Oswald coeff.
```

Figure 1.1 - Input Data in preliminary calculation function

to obtain the output data written below

```
% OUTPUT DATA: wing area, dynamic pressure, mean aerodynamic chord, lift coefficient
% from aircraft weight, wing and horizontal tail aspect ratios, wing and
% horizontal tail lift curve slope, wing lift coeff. at alpha_body = 0,
% downwash at alpha_body = 0, downwash gradient
```

Figure 1.2 - Output Data in preliminary calculation function

All the output data are collocated in the same structure. Using the above function in the live script you can view the preliminary calculations data.

1.1 Longitudinal equilibrium, stability and control

In this section of the code the longitudinal equilibrium, stability and control are considered. The study of stability and control depends on the angle of attack α_B and the equilibrium elevator angle δ_e. These two values are calculated by a system of two equations, considering the assigned horizontal tail incidence angle i_H.

The equations are:

- Vertical Translational Equilibrium;
- Rotational Equilibrium.

1.1.1 Vertical translation equilibrium

The Vertical Translation Equilibrium equation is:

$$C_L = C_{L_0} + C_{L_\alpha}\alpha_B + C_{L_{\delta_e}}\delta_e + C_{L_{i_H}} i_H = \frac{nW}{\bar{q}_\infty S} \tag{1.1}$$

where

$$C_{L_0} = C_{L_{0,W}} - \eta_H \frac{S_H}{S} C_{L_{\alpha,H}} \varepsilon_0 \tag{1.2}$$

This contribute comes from the effect of the wing and the horizontal tailplane.

The following equation is the stability derivative;

$$C_{L_\alpha} = C_{L_{\alpha,W}} + \eta_H \frac{S_H}{S} C_{L_{\alpha,H}} \left[1 - \left(\frac{\mathrm{d}}{\mathrm{d}\alpha}\varepsilon\right)_H\right] \tag{1.3}$$

The following equation is a control derivative that comes from the effect of the horizontal tailplane elevator:

$$C_{L_{\delta_e}} = \eta_H \frac{S_H}{S} C_{L_{\alpha,H}} \tau_e \tag{1.4}$$

The parameter

$$\tau_e = \frac{d}{d\delta_e} |\alpha_{0L_H}| \tag{1.5}$$

is the elevator effectiveness and it represents the horizontal tailplane α_{0L_H} variation for each δ_e degree.

$$C_{L_{i_H}} = \eta_H \frac{S_H}{S} C_{L_{\alpha,H}} \tag{1.6}$$

it is a control derivative that comes from the effect of the horizontal tailplane deflection i_H.

1.1.2 Rotational equilibrium

The equation is:

$$C_M = C_{M_0} + C_{M_\alpha}\alpha_B + C_{M_{\delta_e}}\delta_e + C_{M_{i_H}} i_H + C_{M_q}\hat{q} + C_{M_{mot}} = 0 \tag{1.7}$$

where

$$C_{M_0} = C_{M,ac_{WB}} + C_{L_{0,W}}(\bar{x}_G - \bar{x}_{ac_{WB}}) + \eta_H \frac{S_H}{S}(\bar{x}_{ac_H} - \bar{x}_G) C_{L\alpha_H} \varepsilon_0 \tag{1.8}$$

is a global contribution. Through the equilibrability criterion:

$$C_M(C_L = 0) > 0 \tag{1.9}$$

it is possible to study if the aircraft is longitudinally stable and trimmable.

The expression:

$$C_{M_\alpha} = C_{L_{\alpha,W}}\left(\bar{x}_G - \bar{x}_{ac_{WB}}\right) - \eta_H \frac{S_H}{S}\left(\bar{x}_{ac_H} - \bar{x}_G\right)\left[1 - \left(\frac{\mathrm{d}}{\mathrm{d\alpha}}\varepsilon\right)_H\right] C_{L\alpha_H} \tag{1.10}$$

is a stability derivative and it is called *pitch stiffness*. Its sign is analysed to have stable static equilibrium in pitch. For an airplane to be statically stable in rotation any disturbances in pitch must be counteracted by a restoring moment that will tend the aircraft to rotate back to the equilibrium condition. Thus, in a body reference frame, the mathematical criterion for pitch stability is:

$$\frac{\partial}{\partial \alpha} C_M < 0 \tag{1.11}$$

The term:

$$C_{M_{\delta_e}} = -\eta_H \frac{S_H}{S}\left(\bar{x}_{ac_H} - \bar{x}_G\right) C_{L\alpha_H} \tau_e \tag{1.12}$$

is a control derivative due to a unit elevator deflection δ_e. The term:

$$C_{M_{i_H}} = -\eta_H \frac{S_H}{S}\left(\bar{x}_{ac_H} - \bar{x}_G\right) C_{L\alpha_H} \tag{1.13}$$

is another control derivative due to a unit horizontal tailplane incidence i_H. The term:

$$C_{M_q} = -2\eta_H \frac{S_H}{S}\left(\bar{x}_{ac_H} - \bar{x}_G\right) C_{L\alpha_H}\left(\bar{x}_{ac_H} - \bar{x}_G\right) \tag{1.14}$$

is the pitch damping derivative. When $C_{M_q} < 0$ the aircraft damps out the oscillations in pitch caused by $q \neq 0$. The following are the propulsive effects on the longitudinal rotational equilibrium:

$$C_{M_{mot}} = C_{M_{mot_{Np}}} + C_{M_{mot_T}} \tag{1.15}$$

where

$$C_{M_{mot_{Np}}} = \frac{N_p X_T}{q_\infty S \bar{c}} \tag{1.16}$$

N_p is the propeller normal force (if exists) and X_T is the longitudinal force arm relative to the center of gravity; also

$$C_{M_{mot_T}} = \frac{T Z_T}{q_\infty S \bar{c}} \tag{1.17}$$

T is the thrust and Z_T is the vertical force arm relative to the center of gravity.

1.1.3 Solving the system

By solving the following linear system:

$$\begin{bmatrix} C_{L_\alpha} & C_{L_{\delta_e}} \\ C_{M_\alpha} & C_{M_{\delta_e}} \end{bmatrix} \begin{bmatrix} \alpha_B \\ \delta_e \end{bmatrix} = \begin{bmatrix} C_L - C_{L_0} - C_{L_{i_H}} i_H \\ -C_{M_0} - C_{M_{mot}} - C_{M_{i_H}} i_H - C_{M_q} \hat{q} \end{bmatrix} \tag{1.18}$$

the values of the angle of attack and the deflection of the elevator are evaluated. It is possible to represent the evolution of the pitching moment coefficient vs the body angle of attack, calling the MATLAB function:

```
P3_aeroFun_cmPlot(AllData);
```

The horizontal tailplane load comes from the following equation:

$$L_H = \eta_H q_\infty S_H C_{L_{\alpha_H}} \alpha_{a_H} \quad (1.19)$$

1.1.4 Hinge moment

The tail of an aircraft is critical for its longitudinal stability. It consists of the horizontal stabilizer and the elevator. The elevator controls the aircraft's pitch attitude. It is the movable surface on the tail. Hinge moments refer to the aerodynamic forces acting about the hinge of the elevator. This is the moment that must be overcome to move the control surface when a pilot exerts force command on the control stick.

The hinge moment coefficient is obtained by dividing by the pressure, the area of the control surface and the root mean square chord of the control surface:

$$C_H = \frac{H}{q S_e c_e} \quad (1.20)$$

Where S_e refers to the area and c_e refers to the mean chord of the control surface behind the hinge line, according to the usual convention [Perkins].

The hinge moment coefficient can also be expressed as the linear combination of the variables angle of attack and control surface deflection:

$$C_H = C_{H_\alpha} \alpha + C_{H_\delta} \delta \quad (1.21)$$

In our work, the control surface is the elevator. The value of C_{H_α} and C_{H_δ} are obtained using the following graphs [3]:

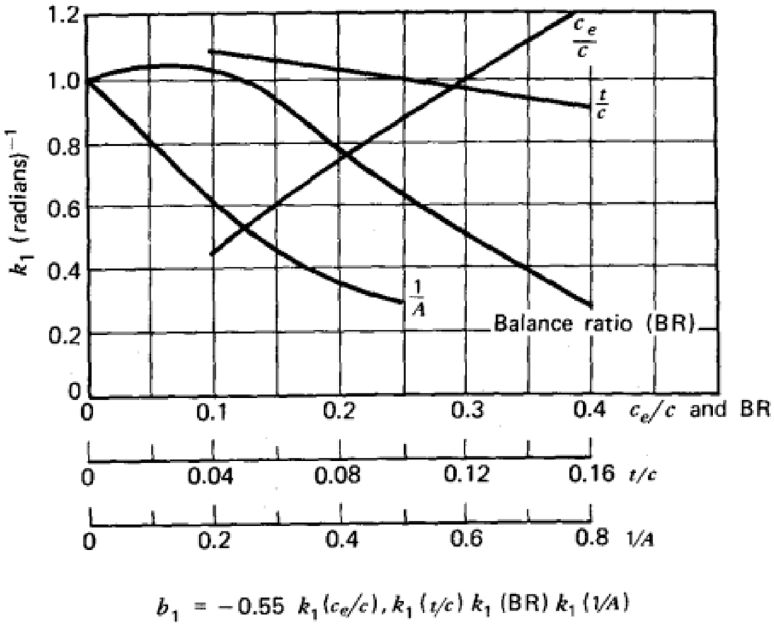

Figure 1.3 – Factors for the rate of change of elevator hinge moment with angle of attack

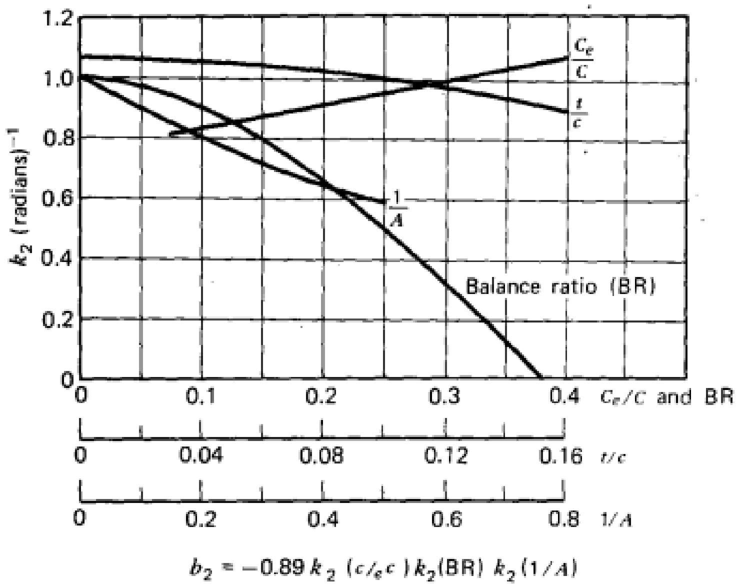

Figure 1.4 - Factors for the rate of change of elevator hinge moment with elevator angle

According to Ref. [3], the expression for the hinge moment derivatives are:

$$C_{H_\alpha} = -0.55k_1\left(\frac{c_e}{c}\right)k_1\left(\frac{t}{c}\right)k_1(BR)k_1\left(\frac{1}{A}\right) \tag{1.22}$$

$$C_{H\delta} = -0.89k_2\left(\frac{c_e}{c}\right)k_2\left(\frac{t}{c}\right)k_2(BR)k_2\left(\frac{1}{A}\right) \tag{1.23}$$

where c_e/c is the flap chord ratio (evaluated on the mean aerodynamic chord of the elevator), t/c is the airfoil thickness ratio, BR is the balance ratio (ratio of the chord ahead of the hinge line to the mean aerodynamic chord), A is the horizontal tailplane aspect ratio.

This is the state of the art of the hinge moment equations so far implemented in the MATLAB live script [5]. This work will expand this section and add the evaluation of control forces as function of trim airspeed, as described in the next chapters.

1.1.5 Neutral points and SMs

Stick-Fixed

The neutral point for stick-fixed condition is the solution of the following equation:

$$C_{M_\alpha} = C_{L_{\alpha,W}}(\bar{x}_G - \bar{x}_{\mathrm{ac,WB}}) - \eta_H\frac{S_H}{S}(\bar{x}_{\mathrm{ac},H} - \bar{x}_G)C_{L_{\alpha,H}}\left[1-\left(\frac{d\varepsilon}{d\alpha}\right)_H\right] = 0 \tag{1.24}$$

by the position $\bar{x}_G = \bar{x}_N$.

Now it is possible to evaluate the static margin:

$$\mathrm{SM} = \bar{x}_G - \bar{x}_N \tag{1.25}$$

When SM<0 the aircraft is longitudinally stable.

Stick-Free

In this case it is introduced the free-elevator factor:

$$F = 1 - \tau_e\frac{C_{H\alpha_e}}{C_{H\delta e,e}} \tag{1.26}$$

The neutral point for stick-free condition is the solution of the following equation:

$$C_{M_\alpha}{}' = C_{L_{\alpha,W}}(\bar{x}_G - \bar{x}_{\mathrm{ac,WB}}) - \eta_H\frac{S_H}{S}(\bar{x}_{\mathrm{ac},H} - \bar{x}_G)C_{L_{\alpha,H}}\left[1-\left(\frac{d\varepsilon}{d\alpha}\right)_H\right]F = 0 \tag{1.27}$$

by the position $\bar{x}_G = \bar{x}_N{}'$

Again, when SM<0 the aircraft is stable. The magnitude of the stick-free static margin is less than the magnitude of the stick-fixed static margin

1.2 Lateral-Directional stability

To study lateral-directional stability of the aircraft, two values are calculated by a system of two equations. The values are aileron deflection δ_a and rudder deflection δ_r by considering the assigned side-slip angle β. The ailerons are moving part on the wings of the aircraft. Their rotation is (almost) anti-symmetrical to temporarily modify the lift of the wing in which is located, in order to allow the aircraft to rotate about the roll axis (longitudinal). The rudder is the primary control surface (usually on the trailing edge of the vertical stabilizer) that controls rotation about the yaw (vertical) axis.

The equations are:

1. Lateral stability;
2. Directional stability.

The function to recall in MATLAB live script is: `AllData = P4_aeroFun_latDirEqGivenBeta(AllData);`

1.2.1 Lateral stability

The equation is:

$$C_{\mathscr{L}} = C_{\mathscr{L}_0} + C_{\mathscr{L}_\beta}\beta + C_{\mathscr{L}_{\delta a}}\delta_a + C_{\mathscr{L}_{\delta r}}\delta_r + C_{\mathscr{L}_p}\hat{p} + C_{\mathscr{L}_r}\hat{r} + C_{\mathscr{L}_{\text{mot}}} \tag{1.28}$$

where if $C_{\mathcal{L}}$= 0 the aircraft is symmetric.

$$C_{\mathscr{L}_\beta} = C_{\mathscr{L}_{\beta_\Gamma}} + C_{\mathscr{L}_{\beta_\Lambda}} + C_{\mathscr{L}_{\beta_{W\,\text{POS}}}} + C_{\mathscr{L}_{\beta_V}} \tag{1.29}$$

$$C_{\mathscr{L}_{\beta_\Gamma}} = -\frac{\Gamma}{6} C_{L\alpha_W} \frac{1+2\lambda}{1+\lambda} \tag{1.30}$$

$$C_{\mathscr{L}_{\beta_\Lambda}} = -\frac{1}{3}\sin(2\Lambda) C_L \frac{1+2\lambda}{1+\lambda} \tag{1.31}$$

$$C_{\mathscr{L}_{\beta_V}} = -C_{L\alpha_V}\left(1 - \frac{\mathrm{d}}{\mathrm{d}\beta}\sigma\right)\eta_V \frac{S_V}{S}\frac{h_V}{b} \tag{1.32}$$

This set of equations represent the stability derivative caused by the dihedral effect. $C_{\mathscr{L}_\beta} < 0$ is the condition for lateral stability.

$$C_{\mathscr{L}_{\delta a}} = \frac{-2C_{L\alpha_W}\tau_a 0.90}{\mathrm{Sb}}\int_{\mathrm{yi}}^{\mathrm{yf}} \frac{c(y)}{\frac{b}{2}}\frac{y}{\frac{b}{2}}\frac{\mathrm{dy}}{\frac{b}{2}} \tag{1.33}$$

with y_i, y_f respectively start and end point of the ailerons. This derivative is called *aileron control power*. Usually $C_{\mathscr{L}_{\delta a}} < 0$

$$C_{\mathscr{L}_{\delta r}} = C_{L\alpha_V}\tau_r\eta_V\frac{S_V}{S}\frac{h_V}{b} \tag{1.34}$$

The above is an indirect effect due to the rudder deflection that causes roll.

$$C_{\mathscr{L}_p} = C_{\mathscr{L}_{p_W}} + C_{\mathscr{L}_{p_V}} \tag{1.35}$$

$$C_{\mathscr{L}_{p_W}} = -\frac{4}{Sb^2}C_{L\alpha_W}\int_{\mathrm{yi}}^{\mathrm{yf}} \frac{c(y)}{\frac{b}{2}}\frac{y}{\frac{b}{2}}\frac{\mathrm{dy}}{\frac{b}{2}} \tag{1.36}$$

$$C_{\mathscr{L}_{p_V}} = -2\eta_V\frac{S_V}{S}C_{L\alpha_V}\left(\frac{h_V}{b}\right)^2 \tag{1.37}$$

This set of equations is a dynamic derivative of the rolling moment. It is the dynamic derivative. When $C_{\mathscr{L}_p} < 0$ the aircraft damps out the oscillations in roll caused by $p \neq 0$. The terms:

$$C_{\mathscr{L}_r} = C_{\mathscr{L}_{r_W}} + C_{\mathscr{L}_{r_V}} \tag{1.38}$$

$$C_{\mathscr{L}_{r_V}} = C_{L\alpha_V}\left(2\frac{l_V}{b} - \frac{\mathrm{d}}{\mathrm{dr}}\sigma\right)\eta_V\frac{S_V}{S}\frac{h_V}{b} \tag{1.39}$$

represent the cross-effect of aircraft rolling moment due to a yawing motion. Usually $C_{\mathscr{L}_r} > 0$

$$C_{\mathscr{L}_{\mathrm{mot}}} = \frac{-Q}{q_\infty \mathrm{Sb}} \tag{1.40}$$

This is the engine contribution to the lateral stability equation. $C_{\mathscr{L}_{\mathrm{mot}}} \neq 0$ when there is one or more propellers.

CONVENTION: The contribution is less than zero when the propeller rotation is clockwise from behind (positive rotation), because the reaction torque has opposite sign to the rotation.

1.2.2 Directional stability

The equation is:

$$C_{\mathcal{N}} = C_{\mathcal{N}_0} + C_{\mathcal{N}_\beta}\beta + C_{\mathcal{N}_{\delta a}}\delta_a + C_{\mathcal{N}_{\delta r}}\delta_r + C_{\mathcal{N}_p}\hat{p} + C_{\mathcal{N}_r}\hat{r} + C_{\mathcal{N}_{\mathrm{mot}}} \tag{1.41}$$

where if $C_{\mathcal{N}} = 0$ the aircraft is symmetric.

$$C_{\mathcal{N}_\beta} = C_{\mathcal{N}_{\beta_W}} + C_{\mathcal{N}_{\beta_F}} + C_{\mathcal{N}_{\beta_V}} \tag{1.42}$$

$$C_{\mathcal{N}_{\beta_V}} = C_{L\alpha_V}\left(1 - \frac{\mathrm{d}}{\mathrm{d}\beta}\sigma\right)\eta_V \frac{S_V}{S}\frac{l_V}{b} \tag{1.43}$$

These two equations represent the directional stability derivative. To have directional stability, $C_{\mathcal{N}_\beta} > 0$ because for $\beta > 0$ there will be a restoring moment that will restore the equilibrium condition. $C_{\mathcal{N}_{\delta a}}$ is a side effect, caused by the ailerons deflection (adverse yaw). The term:

$$C_{\mathcal{N}_{\delta r}} = -C_{L\alpha_V}\eta_V \frac{S_V}{S}\frac{h_V}{b}\tau_r \tag{1.44}$$

is the rudder control power. Usually, $C_{\mathcal{N}_{\delta r}} < 0$. The following contributions are due to a rolling motion causing yaw (cross-effect):

$$C_{\mathcal{N}_p} = C_{\mathcal{N}_{p_W}} + C_{\mathcal{N}_{p_V}} \tag{1.45}$$

$$C_{\mathcal{N}_{p_W}} = C_{\mathcal{N}_{p_{W_{\mathrm{drag}}}}} + C_{\mathcal{N}_{p_{W_{\mathrm{tilt}}}}} + C_{\mathcal{N}_{p_{W_{\mathrm{tipsuc}}}}} \tag{1.46}$$

$$C_{\mathcal{N}_{p_{W_{\mathrm{drag}}}}} = \frac{4}{Sb^2} C_{D_\alpha} \int_0^{\frac{b}{2}} c y^2 \mathrm{dy} \quad > 0 \tag{1.47}$$

where the last contribution is caused by the wings induced drag, whereas $C_{\mathcal{N}_{p_{W_{\mathrm{tilt}}}}} < 0$ is caused by the lift vectors not being normal to the wing-plane. Finally, $C_{\mathcal{N}_{p_{W_{\mathrm{tipsuc}}}}}$ is a term depending on the wingtip suction by the asymmetric pressure differential during a roll maneuver. The last term:

$$C_{\mathcal{N}_{p_V}} = C_{L\alpha_V}\left(2\frac{h_V}{b} - \frac{\mathrm{d}}{\mathrm{dp}}\sigma\right)\eta_V \frac{S_V}{S}\frac{l_V}{b} \tag{1.48}$$

is the vertical tail contribution to the yawing damping derivative due to roll. With this term we study how the aircraft damps out the oscillations in yaw caused by $p \neq 0$. Finally, the dynamic term yawing moment coefficient due to a yawing motion:

$$C_{\mathscr{N}_r} = C_{\mathscr{N}_{r_W}} + C_{\mathscr{N}_{r_V}} \tag{1.49}$$

Usually, $C_{\mathscr{N}_{r_W}} < 0$ and

$$C_{\mathscr{N}_{r_V}} = -C_{L\alpha_V}\left(2\frac{l_V}{b} - \frac{\mathrm{d}}{\mathrm{dr}}\sigma\right)\eta_V \frac{S_V}{S}\frac{l_V}{b} < 0 \tag{1.50}$$

with $C_{\mathscr{N}_r} < 0$ represents a contribution due to $r \neq 0$ restoring moment. The propulsive effects on equilibrium are evaluated as:

$$C_{\mathscr{N}_{\text{mot}}} = \frac{T d_{\text{mot}}}{q_\infty \text{Sb}} \tag{1.51}$$

This contribution is $C_{\mathscr{N}_{\text{mot}}} \neq 0$ when there is an asymmetric thrust, especially in case of engine failure. There will be a yawing moment caused by T with d_{mot} as its arm. The p-factor (due to propeller rotation in non-axial flow) is always ignored.

1.2.3 Solving the system

The deflection values of the aileron and the rudder are the unknown values of the following linear system:

$$\begin{bmatrix} C_{\mathscr{L}_{\delta a}} & C_{\mathscr{L}_{\delta r}} \\ C_{\mathscr{N}_{\delta a}} & C_{\mathscr{N}_{\delta r}} \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} = \begin{bmatrix} -\left(C_{\mathscr{L}_0} + C_{\mathscr{L}_\beta}\beta + C_{\mathscr{L}_p}\hat{p} + C_{\mathscr{L}_r}\hat{r} + C_{\mathscr{L}_{\text{mot}}}\right) \\ -\left(C_{\mathscr{N}_0} + C_{\mathscr{N}_\beta}\beta + C_{\mathscr{N}_p}\hat{p} + C_{\mathscr{N}_r}\hat{r} + C_{\mathscr{N}_{\text{mot}}}\right) \end{bmatrix} \tag{1.52}$$

1.2.4 One-Engine-Inoperative conditions

This section executes calculations if the aircraft has more than one engine, simulating the failure of one of them. If so, the asymmetrical thrust still has to counteract the aerodynamic drag:

$T = q_\infty S C_D$ where $C_D = C_{D_0} + kC_L^2$

Ther are two possibilities in this section:

1. The engine that you have chosen fails. The lateral-directional equilibrium will be updated considering the asymmetrical thrust generated by the operating engine.
2. The aircraft is a single-engine type. The engine does not fail. The lateral-directional equilibrium will not be updated.

2. Methods

2.1 MATLAB live script

MATLAB live script is an interactive document that combines MATLAB code with formatted text, equations and images. In live scripts, you can write your code and view the generated output and graphics along with the code that produced it. Recalling a series of functions previously written in MATLAB it is possible to view the result of our calculations in the live script referring to a specific aircraft selected by us.

The project implements the hinge moment and stick force formulas, referring to the previous results already obtained and shown in the first chapter of this work.

2.2 Hinge moment

The implementation of hinge moment calculation is based on the introduction of the hinge moment derivative due to the tab.

Hinge moments refer to the aerodynamic forces generating moments abound the hinge of the control surface. This is the moment that must be overcome to move the control surface when a pilot exerts force command on the control stick.

We can implement the calculation writing the hinge moment coefficient including the contribution of the trim tab:

$$C_H = C_{H_\alpha}\alpha + C_{H_\delta}\delta + C_{H_{\delta_t}}\delta_t \tag{2.1}$$

The last term is estimated from the following graph:

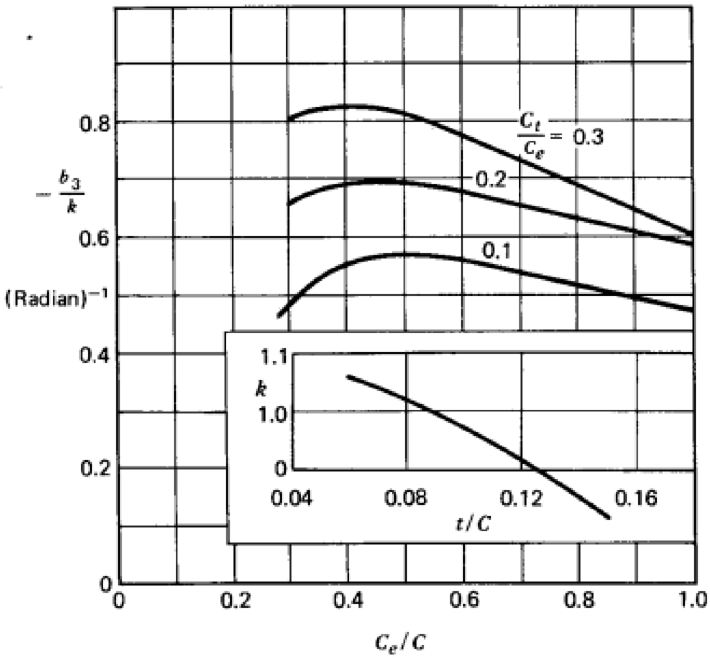

Figure 2.1 – Rate of change of elevator hinge moment coefficient whit trim tab [3]

Figure 2.1 shows two graphs at the same time. At first, we have to focus on the bottom one. The first step is using a software to digitize the point of the curve reported in McCormick [3]. Using a function that returns the coefficient of a polynomial that is the best fit to the data, we can track the curve we were looking for. We can use another function in MATLAB to plot these curves to show the success of the procedure. In this way we get the curve which, when the parameter $\frac{t}{c}$ change, returns the value of k. Extrapolation is not allowed, the fitting is trimmed at the ends of the data range.

```
k3t_c_valore=polyval(k3t_c_pol,t_c);
if t_c<=0.06
    k3t_c_valore=k3t_c(1,2);
end
if t_c>=0.15
    k3t_c_valore=k3t_c(end,2);
end
```

Figure 2.2 – Procedure of calculation of values outside the curve

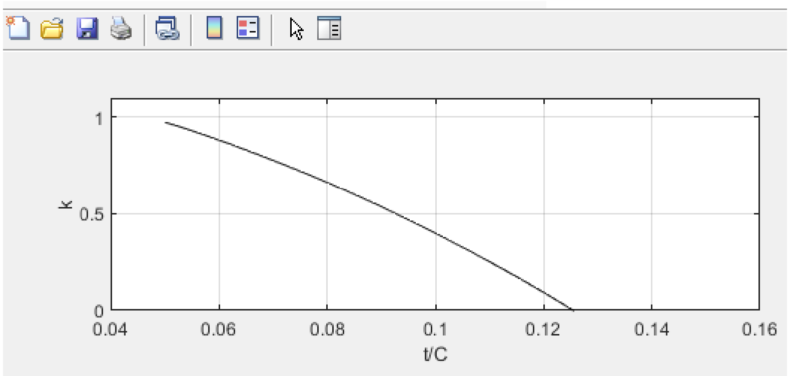

Figure 2.3 - Result obtained in MATLAB

We now repeat the same process for the curves in the top graph. It shows the curves of the parameter $-\frac{b_3}{k}$ as a function of $\frac{c_e}{c}$, the flap chord ratio (specific for each aircraft). In this graph are represented only three curves, plotted referring to three exact values of the term $\frac{c_t}{c_e}$, 0.1, 0.2 and 0.3. In this case we have to define the values at the extremes of our curves too. We do the same process for each of the three curves. We will obtain three different values of the parameter, based on the chosen curve.

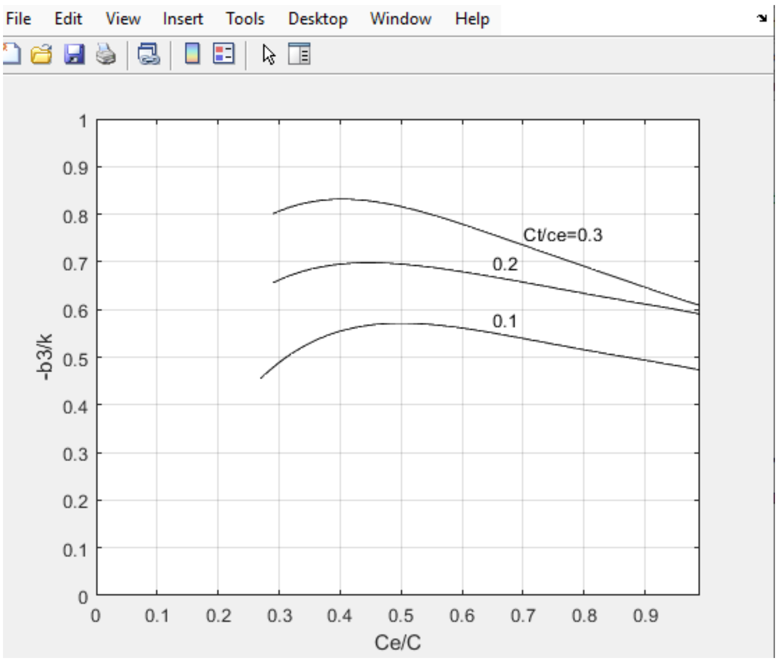

Figure 2.4 - Results obtained in MATLAB

The Figure 2.4 show the results of our work. In this way is possible to use these curves, knowing the necessary parameters of each aircraft, to calculate the missing coefficient to complete the writing of the hinge moment coefficient.

An interpolation was realized in order to generalize the use of this method. The function allows you to insert a different value of $\frac{c_t}{c_e}$, between 0.1 and 0.3 and gives you a result due to the interpolation of the values on the initial curves.

```
if ct_ce == 0.1
    b3_kce_c_valore = b3_kce_c1_valore;
end
if ct_ce > 0.1 && ct_ce < 0.2
    b3_kce_c_valore = (b3_kce_c2_valore - b3_kce_c1_valore)/(0.2-0.1) ...
        * (ct_ce-0.1) + b3_kce_c1_valore;
end
if ct_ce == 0.2
    b3_kce_c_valore = b3_kce_c2_valore ;
end
if ct_ce > 0.2 && ct_ce < 0.3
    b3_kce_c_valore = (b3_kce_c3_valore - b3_kce_c2_valore)/(0.3-0.2) ...
        * (ct_ce-0.2) + b3_kce_c2_valore;
end
if ct_ce == 0.3
    b3_kce_c_valore = b3_kce_c3_valore;
end
```

Figure 2.5 – Interpolation process

2.3 Stick force

2.3.1 Unaccelerated flight

The implementation of stick force calculation is based on the writing of the formula reported in Perkins and Hage [4]:

$$F_s = K \frac{W}{S} \frac{C_{h_\delta}}{C_{m_\delta}} \left(\frac{dC_m}{dC_L}\right)_{free} \left(\frac{V^2}{V_{Trim}^2} - 1\right) \tag{2.2}$$

Is necessary to calculate or extract the variables form the aircraft data.

The value of $\left(\frac{dC_m}{dC_L}\right)_{free}$, the stick-free stability criterion, is equal to the static margin in the stick-free case. The static margin is calculated with the following function:

```
AllData= P3_aeroFun_neutralPointFree(AllData);
```

When SM<0 the aircraft is longitudinally stable.

The term $\left(\frac{dC_m}{dC_L}\right)_{free}$ plays an important role in establishment of the flight condition of a stable stick force variation with speed.

The value of K is obtained from the following formula:

$$K = -G\,S_e\,c_e\eta_t \tag{2.3}$$

G is referred to as the gearing and it depends on the geometry of the control stick or wheel and the linkages to the control surface. The value of G has been approximated to the value 0.5. The ratio $\frac{W}{S}$ represents the wing loading of the aircraft.

In the following figure, shown in Perkins and Hage [4], it is represented a plot of stick force versus velocity, for different tab angles.

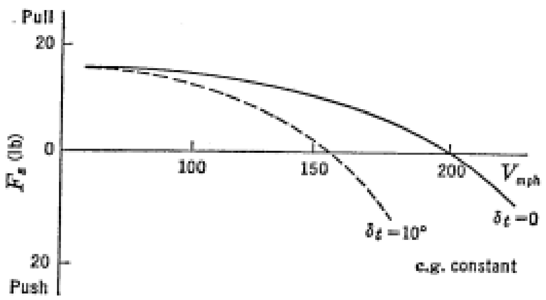

Figure 2.6 – Stick force versus velocity

For a given trim speed as the speed of aircraft change, we can obtain the values of the stick force corresponding.

Trim speed refers to the airspeed at which an aircraft naturally seeks equilibrium when you release the controls, i.e. $C_M = 0$.

In our case the trim speed is set equal to the velocity that the user can choose at the beginning of the live script. Creating the following function `AllData = P6_aeroFun_Control_Forces(AllData);` we can realize a plot that represent the same situation of the below graph.

After the writing of the formula, is possible to generate a plot in which on the x-axis we have the velocity of our airplane, on the y-axis the resulting stick force. The velocity is given as an array to allow the creation of the chart.

The following figure represents a particular graph for the stick force obtained in MATLAB live script, inserting all the data necessary to write the formula. This plot is a particular one; the y-axis is reversed. This is because in the original graph the upper part represents the "pull" situation: a pull on the control should result in a decrease in the speed.

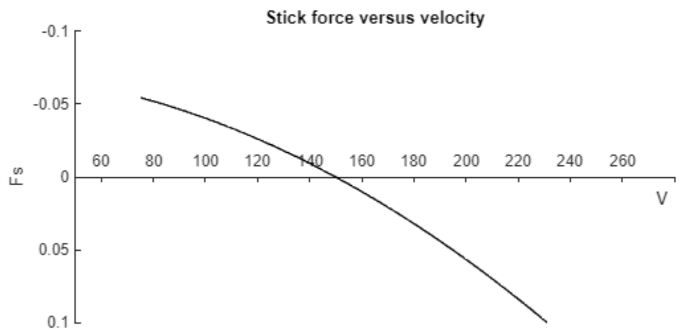

Figure 2.7 - Results obtained in MATLAB

When V is equal to the V_{trim} we always obtain a stick force equal to zero.

The graph shown above may be different from the graph in figure 2.6: this is because in Figure 2.6 the units of measure are different from the one used by us, which are SI units.

2.3.2 Accelerated flight

The maneuvering flight is also called accelerated flight and it is the condition in which the aircraft is subject to out of trim accelerations due to the moving of the controls relative to their trim setting. In this section we are going to analyze the stick force necessary to accelerate the airplane in pull-ups and turns maneuvers.

- **Pull-ups**

If the angle of attack of an airplane rapidly changes and consequentially the lift coefficient changes, the airplane's lift will be larger than its weight, generating an unbalanced force in the vertical plane. This force curves upward the trajectory of aircraft. This maneuver is used for pulling airplanes out of dives and is called a pull-up.

Referring to the treatment in Perkins and Hage [4], we can define the stick force equation for pull-up maneuvers as:

$$F_s = \frac{-G\eta_t S_e c_e \left(\frac{W}{S}\right) C_{h_\delta}}{C_{m_\delta}} \left(\frac{dC_m}{dC_L}\right)_{free} \left(\frac{V^2}{V_{Trim}^2} - n\right) - G\eta_t S_e c_e g l_t \frac{\rho}{2}(n-1)\left(C_{h_\alpha} - \frac{1.1 C_{h_\delta}}{\tau}\right) \quad (2.4)$$

It's easy to see that if $n = 1$, the equation became the (2.2), the variation of stick force with speed in unaccelerated flight.

Here are summarized the factors that compose this formula:

- ρ is the air density (corresponding to several altitudes, this value is given in the data of each analyzed aircraft)
- n is the load factor (lift over weight)
- $\left(\frac{dC_m}{dC_L}\right)_{free}$, the stick-free stability criterion, which equals to the static margin in stick-free conditions
- G is referred to as the gearing
- $\frac{W}{S}$ represent the wing loading of the aircraft
- η_t is the ratio of dynamic pressure at horizontal tail
- $C_{H\delta}$ represents the restoring tendency (hinge moment coefficient derivative due to unit elevator deflection)
- $C_{H\alpha}$is the floating tendency (hinge moment coefficient derivative due to unit angle of attack change)
- l_t is the distance between the aircraft center of gravity and horizontal tail aerodynamic center
- $C_{m\delta}$ is the elevator control power
- g is the acceleration of gravity
- S_e and c_e are respectively the elevator area and the elevator chord. The elevator area, in a first writing of the formula is approximate to the area of the horizontal tail while the elevator chord is calculated with the following approximation: $c_e = \frac{c_e}{c}\ c_H$
- τ is the elevator effectiveness parameter

- **Coordinated turns**

The method used by the pilot for the coordinated turn is another method for creating unbalanced forces perpendicular to the flight path of the airplane. During this maneuver, to curve the flight path in the horizontal plane the wings of airplane are banked over, rotating the resultant lift vector out of the vertical and creating an unbalanced component, in the horizontal direction. This component curves the airplane's flight path in the unbalanced force's direction.

The airplane will make diving turns when the vertical component's lift is less than its weight. In the turn maneuver, as well as the pull-up one, the acceleration of the airplane manifest itself as a curvature of flight path and a rotation of the airplane about its y-axis.

Referring to the treatment in Perkins and Hage [4], we can define the stick force equation for turns maneuvers as:

$$F_s = \frac{-G\eta_t S_e c_e \left(\frac{W}{S}\right) C_{h_\delta}}{C_{m_\delta}} \left(\frac{dC_m}{dC_L}\right)_{free} \left(\frac{V^2}{V_{Trim}^2} - n\right) - G\eta_t S_e c_e g l_t \frac{\rho}{2}\left(n - \frac{1}{n}\right)\left(C_{h_\alpha} - \frac{1.1 C_{h_\delta}}{\tau}\right) \tag{2.5}$$

This formula seems the same as the pull-up maneuver's one. The difference between the two consist in the term $n - 1$ that in the above formula became $n - \frac{1}{n}$.

Unlike the pull-up case, for which the formula is reported in its entirety in the Perkins and Hage [4], this formula should be derived from other formulas in previously cited text.

Starting from the elevator angle formula (turn maneuver):

$$\delta_{e_{Turns}} = \delta_{e_0} - \frac{2n(W/S)}{\rho V^2 C_{m_\delta}}\left(\frac{dC_M}{dC_L}\right)_{Fix} - \frac{63 g l_t}{\tau V^2}\left(n - \frac{1}{n}\right) \tag{2.6}$$

and the following expression:

$$\alpha_s = \alpha_0 + \frac{2n(W/S)}{\rho V^2 a_w}\left(1 - \frac{d\epsilon}{d\alpha}\right) - i_w + i_t + \frac{g l_t}{V^2}\left(n - \frac{1}{n}\right) 57.3 \tag{2.7}$$

we can replace them in the general stick force equation.

$$F_s = -G S_e c_e 1/2 \rho V^2 \eta_t \left(C_{H_0} + C_{H_\alpha}\alpha_s + C_{H_{\delta e}}\delta_e + C_{H_{\delta t}}\delta t\right) \tag{2.8}$$

With some rearranging we can obtain the 2.5 formula.

The two equation shows that a little more up elevator is required to pull some acceleration increment in steady turns than it is in pull-ups.

In this equation if $n = 1$, it became the (2.2) too.

3. Results

Let’s consider now in details each type of aircraft selectable in our MATLAB live script. In this chapter, we are going to report the results of our implementation to each aircraft. In particular we will only analyze propeller aircraft.

The purpose of this paragraph is showing the value of hinge moment and control forces obtained through the live script.

First, we have to analyze the four engine types into which the aircraft are divided:

1. Pistons
2. Turbocharged
3. Turbofan
4. Turboprop

3.1 Pistons

This is typically a heat engine that uses one or more reciprocating pistons to convert high temperature and high pressure into a rotating motion. In this section we are going to analyze two different aircrafts models:

1. Cessna 210
2. Tecnam P92

3.1.1 Cessna 210

Figure 3.1 - Cessna 210

Selected input: the aircraft speed is V = 100 m/s with a load factor n = 1.4. The pitch rate is q = 0 deg/s.

3.1.1.1 Hinge moment

```
Cha = -0.233 rad^-1

Chd = -0.604 rad^-1

Chdt = -0.264 rad^-1
```

3.1.1.2 Control Forces

For an accelerated flight we can plot the graph of stick force versus speed:

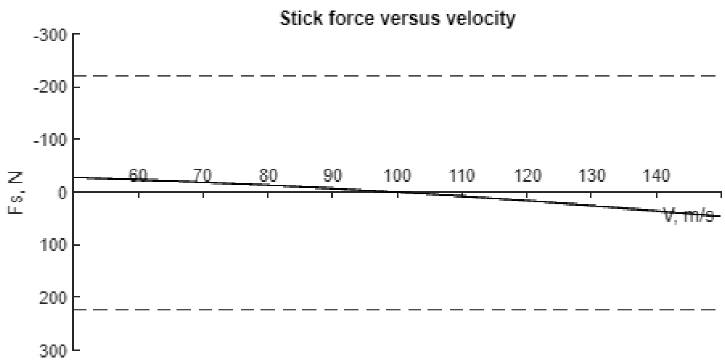

Figure 3.2 - Stick force versus velocity in an unaccelerated flight for Cessna210

For an accelerated flight we calculate the value of stick force in pull-ups and turns maneuvers:

- Pull-ups: `Fsp = -20.2 N`
- Turns: `Fst = -24.1 N`

3.1.2 Tecnam P92

Figure 3.3 - Tecnam P92

Selected input: the aircraft speed is V = 150 m/s with a load factor n = 1.4. The pitch rate is q = 0 deg/s.

3.1.2.1 Hinge moment

```
Cha = -0.146 rad^-1

Chd = -0.489 rad^-1

Chdt = -0.264 rad^-1
```

3.1.2.2 Control Forces

For an accelerated flight we can plot the graph of stick force versus speed:

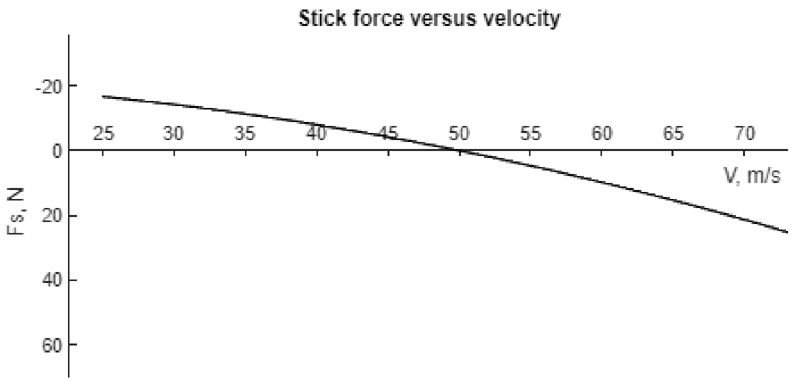

Figure 3.4 - Stick force versus velocity in an unaccelerated flight for Tecnam P92

For an accelerated flight we calculate the value of stick force in pull-ups and turns maneuvers:

- Pull-ups: `Fsp = -13.4 N`
- Turns: `Fst = -16.6 N`

3.2 Turbocharged

Turbocharged uses the outgoing gas from the engine (with a turbine) to collect a greater quantity of incoming air (with a compressor), to give a greater load to the combustion chamber

3.2.1 Tecnam P-2012

Figure 3.5 - Tecnam P-2012

Selected input: the aircraft speed is V = 95 m/s with a load factor n = 1.4. The pitch rate is q = 0 deg/s.

3.2.1.1 Hinge moment

```
Cha = -0.252 rad^-1

Chd = -0.624 rad^-1

Chdt = -0.264 rad^-1
```

3.2.1.2 Control Forces

For an accelerated flight we can plot the graph of stick force versus speed:

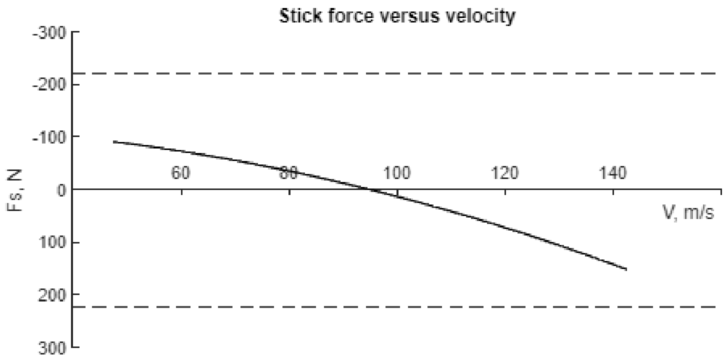

Figure 3.6 - Stick force versus velocity in an unaccelerated flight for Tecnam P-2012

For an accelerated flight we calculate the value of stick force in pull-ups and turns maneuvers:

- Pull-ups: `Fsp = -69.1 N`
- Turns: `Fst = -83.8 N`

3.3 Turboprop

A turboprop is a turbine engine that drives an aircraft propeller. Air enters the intake and is compressed by the compressor. Fuel is then added to the compressed air in the combustor, where the fuel-air mixture then combusts. The hot combustion gases expand through the turbine stages, generating power at the point of exhaust. Some of the power generated by the turbine is used to drive the compressor and electric generator. The gases are then exhausted from the turbine.

3.3.1 Beechcraft KingAir

Figure 3.7 - Beechcraft KingAir

Selected input: the aircraft speed is V = 130 m/s with a load factor n = 1.4. The pitch rate is q = 0 deg/s.

3.3.1.1 Hinge moment

```
Cha = -0.285 rad^-1

Chd = -0.655 rad^-1

Chdt = -0.264 rad^-1
```

3.3.1.2 Control Forces

For an accelerated flight we can plot the graph of stick force versus speed:

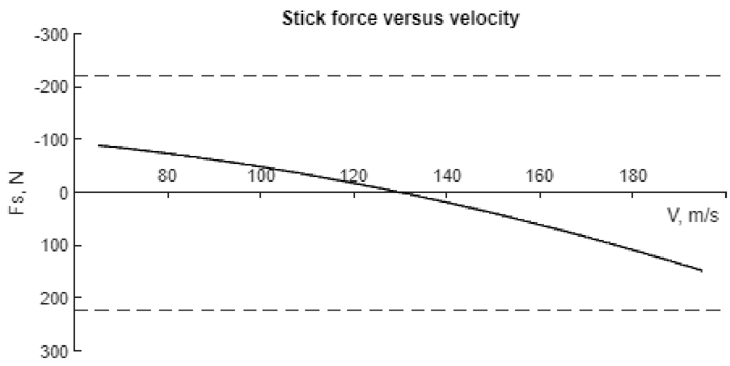

Figure 3.8 - Stick force versus velocity in an unaccelerated flight for Beechcraft KingAir

For an accelerated flight we calculate the value of stick force in pull-ups and turns maneuvers:

- Pull-ups: `Fsp = -57.2 N`
- Turns: `Fst = -64.2 N`

3.3.2 Dornier Do 328

Figure 3.9 - Dornier Do 328

Selected input: the aircraft speed is V = 170 m/s with a load factor n = 1.4. The pitch rate is q = 0 deg/s.

3.3.2.1 Hinge moment

```
Cha = -0.227 rad^-1

Chd = -0.598 rad^-1

Chdt = -0.264 rad^-1
```

3.3.2.2 Control Forces

For an accelerated flight we can plot the graph of stick force versus speed:

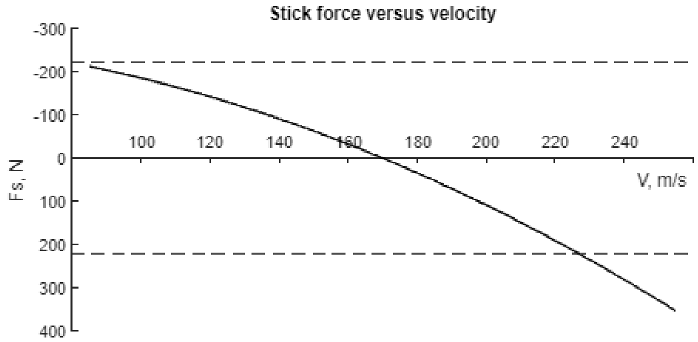

Figure 3.10 - Stick force versus velocity in an unaccelerated flight for Dornier Do 328

For an accelerated flight we calculate the value of stick force in pull-ups and turns maneuvers:

- Pull-ups: `Fsp = -134.7 N`
- Turns: `Fst = -150.1 N`

It can be seen that the value of $C_{H_{\delta t}}$ is the same for all observed aircraft. This happens because, in a first approximation, the value of c_t/c_e was chosen equal to 0.20 for all the airplane available in the live script. Since the value of t/c is equal to 0.10 in all aircraft data, we will then have the value of $C_{H_{\delta t}}$ always the same, due to the fact that the coefficient only depends on the two terms described above.

In the representation of the graphs above, two lines have been added in order to report limits in maneuvering. The values chosen refer to the standard [6] for CS-23 airplanes. In particular, in CS 23.155 section, referred to the elevator control force in manoeuvres, stick controls and wheel controls' values are defined.

4. Conclusion

The purpose of this chapter is to summarize the work and leave indication for the continuation of it. Through the work made available in the dissertation cited in bibliography [1] and [2], this thesis aimed to implement the work done and make the Aircraft Stability and Control live script more detailed and increasingly rich in information and data. Through the use of the aforementioned texts [3] and [4], the formulation of hinge moment has been improved and the control forces equations have been inserted. In this way we are able to know more information about the aircraft that we choose.

For making the formulas correct in all their parts, it is necessary to evaluate parameters that are difficult to estimate: for this reason, some factors are approximated with formulas that are not exactly accurate but gives a good approximation.

The work also involved the generation of some graphs that shown us the development of some factor using in the formulas. Graphs were realized and tested in MATLAB to verify the correct working of our scheme: they were compared with their respective graphs in texts [3] and [4].

In order to suggest the continuation of this work, it could be interesting to observe the trend of the curves as other parameters change, in addition to those already shown.

Another observation could be made about the values of $C_{H_{\delta t}}$. Changing the c_t/c_e and choosing a more specific approximation of the value for each aircraft, we can see the variation of the coefficient, based on the aircraft chose, when the flight conditions are the same.

The live script analyzed in this work aims to be able to improve and speed up the search for stability and control characteristics, bring together all the data about the selectable aircraft and give as much information as possible to the user who uses it.

Bibliography

[1] Roberta Pacelli. Stability and control analysis of aircraft with MATLAB live script (2023)

[2] Simona Giorgio. MATLAB live script for aircraft stability and control (2023)

[3] McCormick. Aerodynamics, Aeronautics and Flight Mechanics-Wiley (1994)

[4] Perkins and Hage. Airplane Performance Stability and Control (1949)

[5] Aircraft Stability and Control MATLAB live script

[6] Easy Access Rules for Normal, Utility, Aerobatic and Commuter Category Aeroplanes (CS-23). EASA