Università degli Studi di Napoli “Federico II”

SCUOLA POLITECNICA E DELLE SCIENZE DI BASE

DIPARTIMENTO DI INGEGNERIA INDUSTRIALE

CORSO DI LAUREA IN INGEGNERIA AEROSPAZIALE

CLASSE DELLE LAUREE IN INGEGNERIA INDUSTRIALE (L-9)

Elaborato di laurea in

MATLAB live script for aircraft stability and control

Relatore:
Prof. Ing. Pierluigi Della Vecchia

Correlatore:
Dott. Danilo Ciliberti

Candidato:
Simona Giorgio
Matr. N35003825

ANNO ACCADEMICO 2022 – 2023

Abstract

In this report the MATLAB live script code about the stability and control analyses of an aircraft is described. In the code the user can choose an aircraft and select some inputs to calculate the output data for: longitudinal equilibrium, stability and control, lateral-directional stability and lateral-directional stability in One Engine Inoperative (OEI) condition.
The available engine types are:

- Turbofan;
- Turboprop;
- Turbocharged;
- Pistons.

The output data are:

- Contributions to the vertical translational and rotational equations system;
- Contributions to the lateral-directional equations system;
- Angle of attack;
- Equilibrium elevator angle;
- Aileron deflection;
- Rudder deflection;
- Neutral point and *SM* for stick-fixed and stick-free;
- Contributions to the lateral-directional equations system in OEI condition;
- Aileron deflection in OEI condition;
- Rudder deflection in OEI condition.

The live script code is composed by:

- Functions;
- Data structure that contains all the aircraft data (inputs and outputs);
- Images and plot.
- Final report written in Word, that changes in order to the new inputs.

The code was written by consulting Mathworks [1] website, the course notes in Aicraft Stability and Control [2] and the book Airplane Dynamics and Automatic Flight Controls, written by Jan Roskam [3].

Table of Contents

List of Figures

Introduction

The aircraft and stability code is written in MATLAB. It is a programming platform designed for engineers and scientists to analyze and design systems by introducing functions and algorithms. On this platform you can choose between two different kind of projects: script and live script. The second one allows to write the code, and also to add equations, images, descriptions and table of contents. The code is divided into chapters in order to the kind of analyses done.

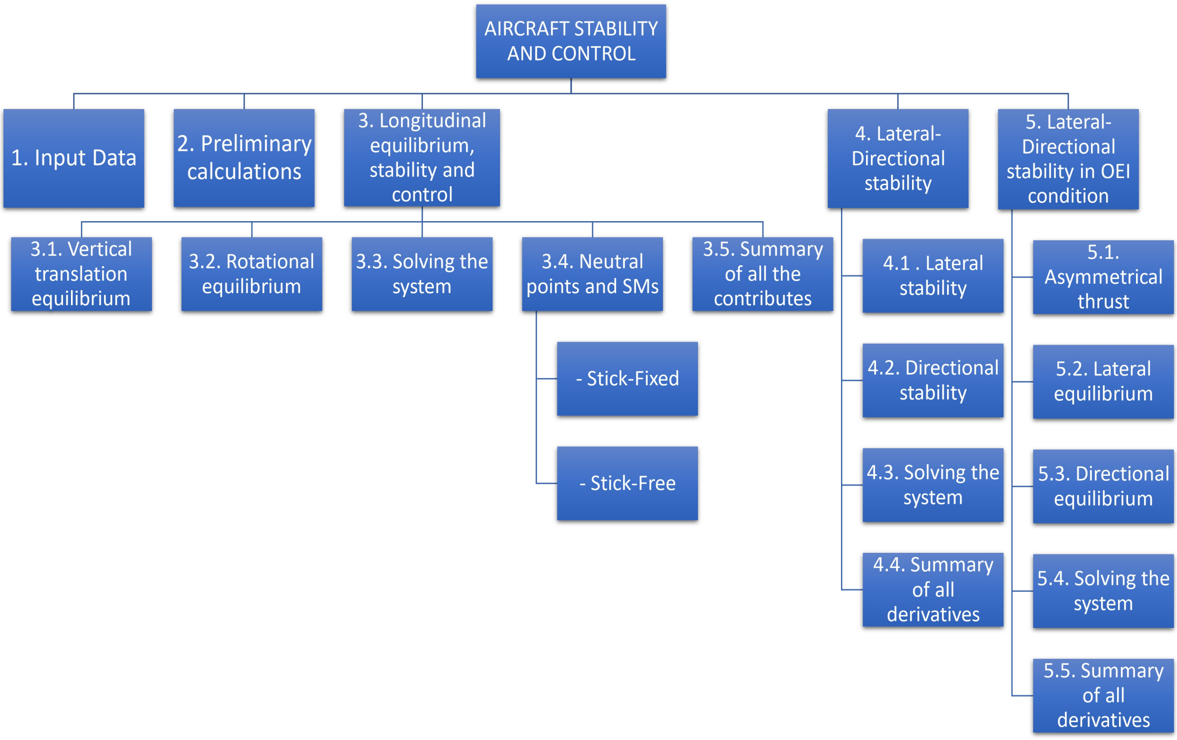


Figure 1: Chapters of the code

First, you have to choose which aircraft you want to study. Every available aircraft has a folder called Aircraft Data, where there are all the aircraft data needed by the code. When you open the chosen aircraft folder there is the following function with the name of the aircraft:

```
function AllData = Data_A320(AllData)
%% Function for aircraft input data
% Flight Mechanics course - Stability and control
% Year 2022-2023
% Authors: Prof. Pierluigi Della Vecchia, Danilo Ciliberti, Simona Giorgio
% pierluigi.dellavecchia@unina.it
% danilo.ciliberti@unina.it

% AllData >> Aircraft >> Name %
AllData.Aircraft.Name.value = 'Airbus A320 NEO';
```

Figure 2: Aircraft Data

The data are divided in section in order to the typology:

- Name of the chosen aircraft;

- Image;
- Weight;
- Geometry: there are all the geometry data divided by component. The components are wing, horizontal tail plane, vertical tail plane and engine;
- Aerodynamics: there are all the aerodynamic data divided by component. The first kind of data are called *global* , because they come from the contribution of the entire aircraft. Other data are divided by component. The components are wing, horizontal tail plane, vertical tail plane and fuselage. In this section it's possible to find contributions to the derivatives, Oswald factor and effectiveness factor;
- Powerplant: for example type and number of engines and force arms of the propeller, if exists;
- Atmosphere: density.

The code is easy to use: all you need is to change the parameters and see the results.

1 Code Analysis

1.1 Input Data

First required data at the beginning of the code is the powerplant aircraft, and after the aircraft. The choice is made through an edit field.

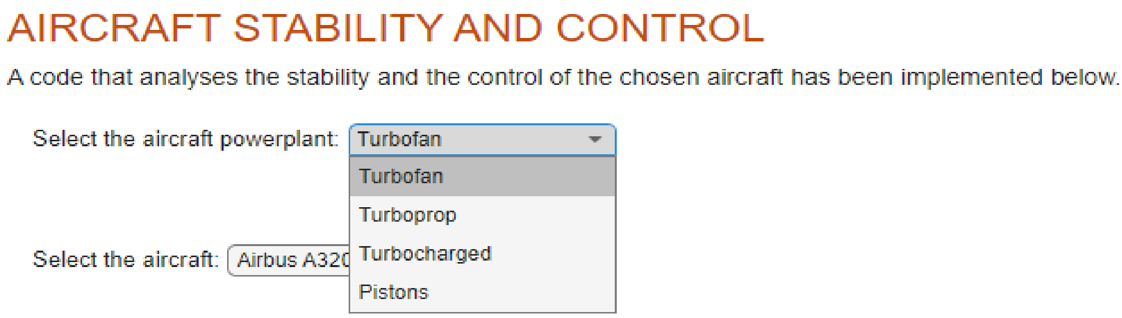


Figure 3: Selected aircraft

The aircraft I've selected are:

- Airbus A320 NEO;
- ATR72;
- Tecnam P2012;
- Tecnam P2006T.

Once you've chosen the aircraft, you have to select some parameters:

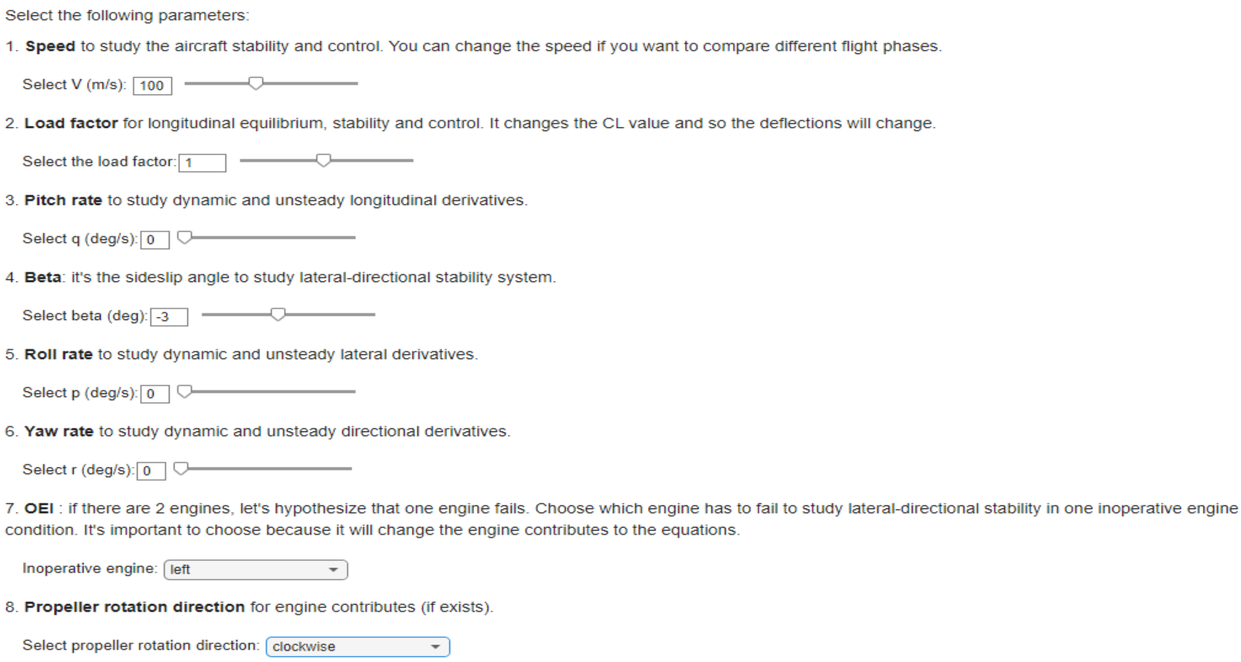


Figure 4: Selected parameters

Then all the aircraft input data are shown to the user.

1.2 Preliminary calculations

After viewing the aircraft and selecting the conditions under which to study it, there's the first part of the code. In this part some preliminary calculations are performed, because they will be necessary for all subsequent operations.
All the preliminary calculations are made in the following function:

```
function AllData = P2_aeroFun_preCalc(AllData)
%% Flight Mechanics course - Stability and control
% Year 2022-2023
% Authors: Prof. Danilo Ciliberti, Simona Giorgio
% danilo.ciliberti@unina.it

%% Preliminary calculations for aircraft stability and control

% INPUT DATA: tip chord, root chord, semi-wingspan, air density, speed,
% weight, load factor, semi-horizontal tail span, horizontal tail
% surface, 2D lift coefficient curve slope, Oswald wing factor, wing
% incidence angle, wing alpha angle at 0 lift, horizontal tail lift coeff.,
% horizontal tail Oswald coeff.
%
% OUTPUT DATA: wing area, dynamic pressure, mean aerodynamic chord, lift coefficient
% from aircraft weight, wing and horizontal tail aspect ratios, wing and
% horizontal tail lift curve slope, wing lift coeff. at alpha_body = 0,
% downwash at alpha_body = 0, downwash gradient

% AIRCRAFT must be a MATLAB structure.
% The wing is supposed to be made of a single, straight-tapered panel.
% All units must be given in SI.
```

Figure 5: Preliminary calculations function

As we can see in the function description, the input and output data are specified. Every function used in the code works with some data that are extracted from the aircraft *AllData* structure, where the input data are stored. At the end of the function all the output are saved in the same structure: all you need is to specify where they have to be and what kind of data they are.
It is possible to use the function by simply invoking it in the 'main', as shown below.

```
AllData= P2_aeroFun_preCalc(AllData);
```

Figure 6: Preliminary calculations instruction

With this instruction, you can thus update the *AllData* structure, inserting the new aircraft data.

1.3 Longitudinal equilibrium, stability and control

In *Longitudinal equilibrium, stability and control* section of the code the longitudinal equilibrium, stability and control are analysed. To study longitudinal stability and control of the aircraft, two values are calculated by a system of two equations. The values are the angle of attack α_{B} and the equilbrium elevator angle δ_{e} by considering the assigned horizontal tail incidence angle i_{H}. The equations are:

- Vertical Translational Equilibrium;
- Rotational Equilibrium.

Let's analyse the equation system.

1.3.1 Vertical translation equilibrium

The Vertical Translation Equilibrium equation is:

$$C_L = C_{L_0} + C_{L_\alpha}\alpha_\mathrm{B} + C_{L_{\delta_\mathrm{e}}}\delta_\mathrm{e} + C_{L_{i_\mathrm{H}}}i_\mathrm{H} = \frac{W}{\bar{q}_\infty S} \tag{1}$$

where

$$C_{L_0} = C_{L_{0\mathrm{W}}} - \eta_\mathrm{H}\frac{S_\mathrm{H}}{S}C_{L_{\alpha,\mathrm{H}}}\varepsilon_0 \tag{2}$$

This contribute comes from the effect of the wing and the horizontal tailplane

$$C_{L_\alpha} = C_{L_{\alpha,\mathrm{W}}} + \eta_\mathrm{H}\frac{S_\mathrm{H}}{S}C_{L_{\alpha,\mathrm{H}}}\left[1 - \left(\frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right)_\mathrm{H}\right] \tag{3}$$

it is the stability derivative;

$$C_{L_{\delta_\mathrm{e}}} = \eta_\mathrm{H}\frac{S_\mathrm{H}}{S}C_{L_{\alpha,\mathrm{H}}}\tau_\mathrm{e} \tag{4}$$

it is a control derivative that comes from the effect of the horizontal tailplane elevator. And

$$\tau_\mathrm{e} = \frac{\mathrm{d}|\alpha_{0L_H}|}{\mathrm{d}\delta_\mathrm{e}} \tag{5}$$

is the elevator effectiveness and it represents the horizontal tailplane alpha-zero-lift variation for each elevator deflection degree;

$$C_{L_{i_\mathrm{H}}} = \eta_\mathrm{H}\frac{S_\mathrm{H}}{S}C_{L_{\alpha,\mathrm{H}}} \tag{6}$$

it is a control derivative that comes from the effect of the horizontal tailplane deflection.

1.3.2 Rotational equilibrium

The rotational equilibrium equation is:

$$C_M = C_{M_0} + C_{M_\alpha}\alpha_\mathrm{B} + C_{M_{\delta_\mathrm{e}}}\delta_\mathrm{e} + C_{M_{i_\mathrm{H}}}i_\mathrm{H} + C_{M_q}\hat{q} + C_{M_\mathrm{mot}} = 0 \tag{7}$$

where

$$C_{M_0} = C_{M_{\mathrm{ac,WB}}} + C_{L_{0,\mathrm{W}}}(\bar{x}_\mathrm{G} - \bar{x}_\mathrm{ac,WB}) + \eta_\mathrm{H}\frac{S_\mathrm{H}}{S}(\bar{x}_\mathrm{ac,H} - \bar{x}_\mathrm{G})C_{L_{\alpha,\mathrm{H}}}\varepsilon_0 \tag{8}$$

it is a global contribution. Through the equilibrabability criterion

$$C_M(C_L = 0) > 0 \tag{9}$$

it is possible to study if the aircraft is equilibrable.

$$C_{M_\alpha} = C_{L_{\alpha,\mathrm{W}}}(\overline{x}_\mathrm{G} - \overline{x}_\mathrm{ac,WB}) - \eta_\mathrm{H}\frac{S_\mathrm{H}}{S}(\overline{x}_\mathrm{ac,H} - \overline{x}_\mathrm{G})\left[1 - \left(\frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right)_\mathrm{H}\right]C_{L_{\alpha,\mathrm{H}}} \tag{10}$$

It's a stability derivative and it's called *pitch stiffness*. Its sign is analysed to have stable static equilibrium in pitch. For an airplane to be statically stable in rotation any disturbances in pitch must be defeated by the production of a restoring moment that will restore the equilibrium condition. So, the mathematical criterion for pitch stability is

$$\frac{\partial C_M}{\partial \alpha} < 0 \tag{11}$$

$$C_{M_{\delta_\mathrm{e}}} = -\eta_\mathrm{H}\frac{S_\mathrm{H}}{S}(\overline{x}_\mathrm{ac,H} - \overline{x}_\mathrm{G})C_{L_{\alpha,\mathrm{H}}}\tau_\mathrm{e} \tag{12}$$

It's a control derivative that comes from the effect of the elevator deflection.

$$C_{M_{i_\mathrm{H}}} = -\eta_\mathrm{H}\frac{S_\mathrm{H}}{S}(\overline{x}_\mathrm{ac,H} - \overline{x}_\mathrm{G})C_{L_{\alpha,\mathrm{H}}} \tag{13}$$

It's a control derivative that comes from the effect of the horizontal tailplane deflection.

$$C_{M_q} = -2\eta_\mathrm{H}\frac{S_\mathrm{H}}{S}(\overline{x}_\mathrm{ac,H} - \overline{x}_\mathrm{G})C_{L_{\alpha,\mathrm{H}}}(\overline{x}_\mathrm{ac,H} - \overline{x}_\mathrm{G}) \tag{14}$$

It's the pitch damping derivative. When this contribution is < 0, the aircraft damps out the oscillations in pitch caused by a non-zero pitch rate value.

$$C_{M_\mathrm{mot}} = C_{M_{\mathrm{mot}_{N_\mathrm{p}}}} + C_{M_{\mathrm{mot}_T}} \tag{15}$$

where

$$C_{M_{\mathrm{mot}_{N_\mathrm{p}}}} = \frac{N_\mathrm{p}X_T}{\overline{q}_\infty S\overline{c}} \tag{16}$$

N_p is the propeller normal force (if exists) and X_T is the force arm relative to the center of gravity.

$$C_{M_{\mathrm{mot}_T}} = \frac{TZ_T}{\overline{q}_\infty S\overline{c}} \tag{17}$$

T is the thrust and Z_T is the force arm relative to the center of gravity.

1.3.3 Solving the system

The system of equations in matrix form

$$\begin{bmatrix} C_{L_\alpha} & C_{L_{\delta_\mathrm{e}}} \\ C_{M_\alpha} & C_{M_{\delta_\mathrm{e}}} \end{bmatrix} \begin{bmatrix} \alpha_\mathrm{B} \\ \delta_\mathrm{e} \end{bmatrix} = \begin{bmatrix} C_L - C_{L0} - C_{L_{i_\mathrm{H}}} i_\mathrm{H} \\ -C_{M_0} - C_{M_\mathrm{mot}} - C_{M_{i_\mathrm{H}}} i_\mathrm{H} - C_{M_q} \hat{q} \end{bmatrix} \tag{18}$$

is solved with the following function:

```
function AllData = P3_aeroFun_longEqComplete(AllData)
%% Flight Mechanics course - Stability and control
% Year 2022-2023
% Authors: Prof. Danilo Ciliberti, Simona Giorgio
% danilo.ciliberti@unina.it

%% Solving the 2x2 system of the static longitudinal equilibrium.
% Needs the preliminary execution of PRECALC function.

% INPUT: wing lift coefficient at 0 lift, horizontal tail dynamic pressure
% ratio, horizontal tail area, horizontal tail lift curve slope, downwash
% gradient, wing surface, wing lift curve slope, downwash at
% alpha_body = 0, wing ac pitching moment coeff., wing 0 pitching moment
% coeff., wing ac, wing pitching moment curve slope, longitudinal distance
% between horizontal tail ac and aircraft cg, mean aerodynamic chord, centre
% of gravity, elevator effectiveness, pitch rate, speed, area ratio, Oswald
% factor, air density, 0 drag coeff., lift coeff., shaft power, horizontal
% tail angle of incidence, dynamic pressure, Xt and Zt respectively
% horizontal and vertical distances between propeller force and cg,
% propeller force, propeller diameter, two dericatives due to the
% propeller.

% OUTPUT: Through this function the longitudinal equilibrium and stability
% equations system is solved. The two equations are:
% - Vertical Translational Equilibrium;
% - Rotational Equilibrium.
% The solutions are the angle of attack and the equilbrium elevator angle
% by considering the assigned horizontal tail incidence angle iH.
```

Figure 7: Longitudinal equilibrium, stability and control function

Once the solutions of the system are known, it is possible to graphically determine the trim point.

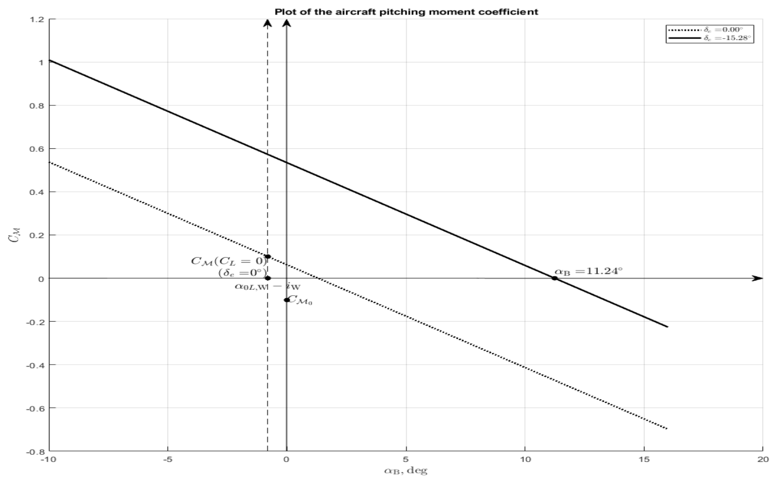


Figure 8: C_M plot

The trim point is the point at which the C_M curve intersects the α_B-axis.

Finally, the horizontal tailplane load

$$L_{\mathrm{H}} = \eta_{\mathrm{H}} \overline{q}_{\infty} S_{\mathrm{H}} C_{L_{\alpha,\mathrm{H}}} \alpha_{\mathrm{a,H}} \tag{19}$$

is obtained using the following function:

```
function AllData = P3_aeroFun_horTailLoad(AllData)
%% Flight Mechanics course - Stability and control
% Year 2022-2023
% Authors: Prof. Danilo Ciliberti, Simona Giorgio
% danilo.ciliberti@unina.it

%% Calculation of the horizontal tail lift.
% Needs the preliminary execution of PRECALC and LONGEQCOMPLETE functions.

% INPUT DATA: pitch rate, longitudinal distance between horizontal tail ac and aircraft
% cg, speed, alpha body, downwash gradient, horizontal tail incidence
% angle, elevator effectiveness, deflection of the elevator, downwash at
% alpha_body = 0, dynamic pressure, horizontal tail surface, horizontal
% tail dynamic pressure ratio, horizontal tail lift coefficient curve
% slope.

% OUTPUT: Horizontal tail lift by calculating alpha_H.
```

Figure 9: Horizontal tailplane load function

1.3.4 Neutral points and *SMs*

In *Neutral points and SMs* it's possible to study the neutral point and the *SM* in two different conditions:

- Stick-fixed: the pilot operates the control of the elevator;
- Stick-free: the pilot doesn't operate the control of the elevator and waits for it to align itself with the current.

The neutral point is the value of the center of gravity when wing pitching moment curve slope is zero. This is a neutrally stable equilibrium state: when the aircraft is displaced from the equilibrium position, there is no force or moment imbalance produced.
The neutral point for sitck-fixed condition is the solution of the following equation with $\overline{x}_{\mathrm{G}} = \overline{x}_{\mathrm{N}}$:

$$C_{M_\alpha} = C_{L_{\alpha,\mathrm{W}}}(\overline{x}_{\mathrm{G}} - \overline{x}_{\mathrm{ac,WB}}) - \eta_{\mathrm{H}} \frac{S_{\mathrm{H}}}{S} (\overline{x}_{\mathrm{ac,H}} - \overline{x}_{\mathrm{G}}) \left[1 - \left(\frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha} \right)_{\mathrm{H}} \right] C_{L_{\alpha,\mathrm{H}}} = 0 \tag{20}$$

Once known $\overline{x}_{\mathrm{N}}$, the Static Margin is obtained as follows:

$$SM = \overline{x}_{\mathrm{G}} - \overline{x}_{\mathrm{N}} \tag{21}$$

When $SM < 0$ the aircraft is stable.
Eventually, it's possible to use the approximated formula where $\overline{V}_{\mathrm{H}} = \frac{S_{\mathrm{H}}}{S}(\overline{x}_{\mathrm{ac,H}} - \overline{x}_{\mathrm{G}}) = \mathrm{const.}$

In stick-free condition we introduce the *free elevator factor*:

$$F = 1 - \tau_{\mathrm{e}} \frac{C_{\mathcal{H}\alpha_{\mathrm{e}}}}{C_{\mathcal{H}_{\delta_{\mathrm{e,e}}}}} \tag{22}$$

The tailplane contribution is scaled by $F < 1$, so the stick-free stability is less than the stick-fixed stability.
The neutral point for stick-free condition is the solution of the following equation:

$$C'_{M_\alpha} = C_{L_{\alpha,\mathrm{W}}}(\overline{x}_{\mathrm{G}} - \overline{x}_{\mathrm{ac,WB}}) - \eta_{\mathrm{H}} \frac{S_{\mathrm{H}}}{S}(\overline{x}_{\mathrm{ac,H}} - \overline{x}_{\mathrm{G}}) F \left[1 - \left(\frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right)_{\mathrm{H}}\right] C_{L_{\alpha,\mathrm{H}}} = 0 \tag{23}$$

And then, it's possible to study the *SM*′. The functions are:

```
function AllData = P3_aeroFun_neutralPoint(AllData)
```

Figure 10: Stick-fixed neutral point function

```
function AllData = P3_aeroFun_neutralPointFree(AllData)
```

Figure 11: Stick-free neutral point function

1.4 Lateral-Directional stability

In *Lateral-Directional stability* section of the code the lateral-directional stability is analysed. To study lateral-directional stability of the aircraft, two values are calculated by a system of two equations. The values are aileron deflection δ_{a} and rudder deflection δ_{e} by considering the assigned side-slip angle β. The two equations are:

- Lateral stability;
- Directional stability.

Let's analyse the equation system.

1.4.1 Lateral stability

The lateral stability equation is:

$$C_{\mathcal{L}} = C_{\mathcal{L}0} + C_{\mathcal{L}\beta}\beta + C_{\mathcal{L}\delta a}\delta_{\mathrm{a}} + C_{\mathcal{L}\delta_{\mathrm{r}}}\delta_{\mathrm{r}} + C_{\mathcal{L}p}\hat{p} + C_{\mathcal{L}r}\hat{r} + C_{\mathcal{L}\mathrm{mot}} \tag{24}$$

where
$C_{\mathcal{L}0}$ is a contribution related to the aircraft's asymmetry; therefore, for symmetric aircraft, it is zero.

$$C_{\mathcal{L}\beta} = C_{\mathcal{L}\beta_{\Gamma}} + C_{\mathcal{L}\beta_{\Lambda}} + C_{\mathcal{L}\beta_{\mathrm{W,pos}}} + C_{\mathcal{L}\beta_{\mathrm{V}}} \tag{25}$$

It's the stability derivative caused by the dihedral effect. $C_{\mathcal{L}\beta} < 0$ is the condition for lateral stability. Its contributions are as follows:

$$C_{\mathcal{L}\beta_\Gamma} = -\frac{\Gamma}{6} C_{L\alpha\mathrm{W}} \frac{1+2\lambda}{1+\lambda} \tag{26}$$

It's the effect of the wing dihedral angle. It's usually < 0.

$$C_{\mathcal{L}\beta_\Lambda} = -\frac{1}{3} \sin\left(2\Lambda\right) C_L \frac{1+2\lambda}{1+\lambda} \tag{27}$$

It's the effect of the wing sweep angle. It's usually < 0.

$$C_{\mathcal{L}\beta_\mathrm{V}} = -C_{L\alpha\mathrm{V}} \left(1 - \frac{\mathrm{d}}{\mathrm{d}\beta}\sigma\right) \eta_\mathrm{V} \frac{S_\mathrm{V}}{S} \frac{h_\mathrm{V}}{b} \tag{28}$$

It's the effect of the vertical tailplane, usually < 0. Meanwhile, the effect of the horizontal tailplane is not considered, because it's close to zero.

$$C_{\mathcal{L}\delta a} = -\frac{2C_{L\alpha W}\tau_a 0.\,90}{\mathrm{Sb}} \int_{\mathrm{yi}}^{\mathrm{yf}} \frac{c\left(y\right)}{\frac{b}{2}} \frac{y}{\frac{b}{2}} \frac{\mathrm{d}y}{\frac{b}{2}} \tag{29}$$

with y_i, y_f respectively start and end point of the ailerons. This derivative is called *aileron control power*. Usually $C_{\mathcal{L}\delta a} < 0$.

$$C_{\mathcal{L}\delta_\mathrm{r}} = C_{L\alpha V} \tau_r \eta_V \frac{S_\mathrm{V}}{S} \frac{h_\mathrm{V}}{b} \tag{30}$$

It's an indirect effect due to the rudder deflection that causes roll.

$$C_{\mathcal{L}p} = C_{\mathcal{L}pW} + C_{\mathcal{L}pV} \tag{31}$$

It's the unsteady derivative. When $C_{\mathcal{L}p} < 0$ the aircraft damps out the oscillations in roll caused by $p \neq 0$. Its contributions are as follows:

$$C_{\mathcal{L}pW} = -\frac{4}{Sb^2} C_{L\alpha W} \int_{\mathrm{yi}}^{\mathrm{yf}} \frac{c\left(y\right)}{\frac{b}{2}} \frac{y}{\frac{b}{2}} \frac{\mathrm{dy}}{\frac{b}{2}} \tag{32}$$

It's the wing effect.

$$C_{\mathcal{L}pV} = -2\eta_V \frac{S_\mathrm{V}}{S} C_{L\alpha V} \left(\frac{h_\mathrm{V}}{b}\right)^2 \tag{33}$$

It's the vertical tailplane effect.

$$C_{\mathcal{L}r} = C_{\mathcal{L}rW} + C_{\mathcal{L}rV} \tag{34}$$

where the vertical tailplane effect is

$$C_{\mathcal{L}rV} = C_{L\alpha V} \left(2\frac{l_V}{b} - \frac{\mathrm{d}}{\mathrm{d}r}\sigma\right) \eta_V \frac{S_\mathrm{V}}{S} \frac{h_\mathrm{V}}{b} \tag{35}$$

it's the unsteady derivative. Usually $C_{\mathcal{L}r} > 0$.

$$C_{\mathcal{L}\text{mot}} = \frac{-Q}{q_\infty \text{Sb}} \tag{36}$$

It's the engine contribution to the lateral stability equation. Q is the torque (couple) acting on the air due to the engine. $C_{\mathcal{L}\text{mot}} \neq 0$ when there's the propeller, and according to the convention it's < 0 when the propeller rotation is clockwise.

1.4.2 Directional stability

The directional stability equation is:

$$C_{\mathcal{N}} = C_{\mathcal{N}0} + C_{\mathcal{N}\beta}\beta + C_{\mathcal{N}\delta a}\delta_\text{a} + C_{\mathcal{N}\delta_\text{r}}\delta_\text{r} + C_{\mathcal{N}p}\hat{p} + C_{\mathcal{N}r}\hat{r} + C_{\mathcal{N}\text{mot}} \tag{37}$$

where
$C_{\mathcal{N}0}$ is a contribution related to the aircraft's asymmetry; therefore, for symmetric aircraft, it is zero.

$$C_{\mathcal{N}\beta} = C_{\mathcal{N}\beta_W} + C_{\mathcal{N}\beta_F} + C_{\mathcal{N}\beta_V} \tag{38}$$

where $C_{\mathcal{N}\beta_W}$ is the wing effect, $C_{\mathcal{N}\beta_F}$ is the fuselage effect and

$$C_{\mathcal{N}\beta_V} = C_{L\alpha V}\left(1 - \frac{\text{d}}{\text{d}\beta}\sigma\right)\eta_V \frac{S_\text{V}}{S}\frac{l_V}{b} \tag{39}$$

is the vertical tailplane effect.
It's a stability derivative. To have stability we need $C_{\mathcal{N}\beta} > 0$, because for $\beta > 0$ there will be a restoring moment that will restore the equilibrium condition.
$C_{\mathcal{N}\delta a}$ is a side effect, caused by the ailerons deflection. Then the induced drag causes adverse yaw. Usually we try to defeat it.

$$C_{\mathcal{N}\delta_\text{r}} = -C_{L\alpha V}\eta_V \frac{S_\text{V}}{S}\frac{h_\text{V}}{b}\tau_r \tag{40}$$

It's the *rudder control power*. Usually $C_{\mathcal{N}\delta_\text{r}} < 0$.

$$C_{\mathcal{N}p} = C_{\mathcal{N}p_W} + C_{\mathcal{N}p_V} \tag{41}$$

It's the unsteady derivative. With this term we study how the aircraft damps out the oscillations in yaw caused by $p \neq 0$. The effect of the wing is as follows:

$$C_{\mathcal{N}p_W} = C_{\mathcal{N}p_{W\text{drag}}} + C_{\mathcal{N}p_{W\text{tilt}}} + C_{\mathcal{N}p_{W\text{tipsuc}}} \tag{42}$$

$C_{\mathcal{N}p_{W\text{drag}}} = \frac{4}{Sb^2}C_{D\alpha}\int_0^{\frac{b}{2}} cy^2\text{dy} \quad > 0$ caused by the wings induced drag.
$C_{\mathcal{N}p_{W\text{tilt}}} < 0$ is caused by the lift tilt $\neq 90°$. So, there will be an adverse yaw.
$C_{\mathcal{N}p_{W\text{tipsuc}}}$ depends on the tip vortices caused by the pressure difference between back and belly of the wing.

The effect of the vertical tailplane is as follows:

$$C_{\mathcal{N}pV} = C_{L\alpha V}\left(2\frac{h_V}{b} - \frac{\mathrm{d}}{\mathrm{dp}}\sigma\right)\eta_V \frac{S_V}{S}\frac{l_V}{b} \tag{43}$$

$$C_{\mathcal{N}r} = C_{\mathcal{N}rW} + C_{\mathcal{N}rV} \tag{44}$$

where usually $C_{\mathcal{N}rW} < 0$ and

$$C_{\mathcal{N}rV} = -C_{L\alpha V}\left(2\frac{l_V}{b} - \frac{\mathrm{d}}{\mathrm{dr}}\sigma\right)\eta_V \frac{S_V}{S}\frac{l_V}{b} < 0 \tag{45}$$

$C_{\mathcal{N}r} < 0$ represents a contribution due to $r \neq 0$ restoring moment.

$$C_{\mathcal{N}\mathrm{mot}} = \frac{T d_{\mathrm{mot}}}{q_\infty \mathrm{Sb}} \tag{46}$$

This contribution is $C_{\mathcal{N}\mathrm{mot}} \neq 0$ when there's an asymmetric thrust, especially in case of engine plant. There will be a yaw moment caused by T with d_{mot} as its arm.

1.4.3 Solving the system

The system of equations in matrix form

$$\begin{bmatrix} C_{\mathcal{L}\delta a} & C_{\mathcal{L}\delta_r} \\ C_{\mathcal{N}\delta a} & C_{\mathcal{N}\delta_r} \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} = \begin{bmatrix} -(C_{\mathcal{L}0} + C_{\mathcal{L}\beta}\beta + C_{\mathcal{L}p}\hat{p} + C_{\mathcal{L}r}\hat{r} + C_{\mathcal{L}\mathrm{mot}}) \\ -(C_{\mathcal{N}0} + C_{\mathcal{N}\beta}\beta + C_{\mathcal{N}p}\hat{p} + C_{\mathcal{N}r}\hat{r} + C_{\mathcal{N}\mathrm{mot}}) \end{bmatrix} \tag{47}$$

is solved with the following function:

```
function AllData = P4_aeroFun_latDirEqGivenBeta(AllData)
%% Flight Mechanics course - Stability and control
% Year 2022-2023
% Authors: Prof. Danilo Ciliberti, Simona Giorgio
% danilo.ciliberti@unina.it

%% Solving the 2x2 system of the static lateral-directional equilibrium.
% Needs the preliminary execution of PRECALC function.

% INPUT: dihedral angle, wing lift curve slope, taper ratio, sweep angle,
% lift coeff., vertical tail lift curve slope, lift curve slope, side-slip,
% vertical tail dynamic pressure ratio, vertical tail area, vertical
% distance between vertical tail ac and aircraft cg, wing area, semi-wing
% span, contributes to roll coeff., root chord, tip chord, ailerons data,
% ailerons effectiveness, rudder effectiveness, longitudinal distance
% between vertical tail ac and aircraft cg, dynamic pressure, drag coeff.,
% speed, roll rate, yaw rate, side-slip angle beta, contributes to yaw
% coeff., distance between propeller force and cg, propeller data, shaft
% power, area ratio, Oswald factor, mean aerodynamic chord, engines number.

% OUTPUT: Through this function the lateral-directional equilibrium equations
% system is solved. The two equations are:
% - Lateral stability;
% - Directional stability.
% The solutions are the deflection of the aileron and the deflection of the rudder
% by considering the assigned the side-slip angle beta.
```

Figure 12: Lateral-Directional stability function

1.5 Lateral-Directional stability in One-Engine-Inoperative condition

In *Lateral-Directional stability in One-Engine-Inoperative condition* section of the code it's analysed the lateral-directional stability of the aircraft when one of its engines has fallen, in case of the number of engines is > 1. The user chooses which engine has to fall at the beginning of the code. The system made by the lateral and directional equilibrium equations is solved by considering the asymmetrical thrust that acts on the working engine. While the lateral equilibrium equation remains unchanged, the directional equilibrium equation changes.

1.5.1 Asymmetrical thrust

The asymmetrical thrust is:

$$T = q_\infty S \mathrm{C}_D \tag{48}$$

where $C_D = C_{\mathrm{D0}} + kC_L{}^2$ is the drag coefficient.

1.5.2 Directional equilibrium

In *OEI* condition the engine contribution to the yaw moment is

$$C_{\mathcal{N}\mathrm{mot}} = \frac{T d_{\mathrm{mot}}}{q_\infty \mathrm{Sb}} \neq 0 \tag{49}$$

because there's a yaw moment due to the asymmetric thrust with its arm d_{mot}. The sign depends on which engine fails. According to the convention when the left engine fails $C_{\mathcal{N}\mathrm{mot}} < 0$, because T causes $N < 0$. Instead, when the right engine fails $C_{\mathcal{N}\mathrm{mot}} > 0$, because T causes $N > 0$.

1.5.3 Solving the system

The system of equations in matrix form is solved with the following function:

```
function AllData = P5_aeroFun_latDirEqGivenBetaOEI(AllData)
%% Flight Mechanics course - Stability and control
% Year 2022-2023
% Authors: Prof. Danilo Ciliberti, Simona Giorgio
% danilo.ciliberti@unina.it

%% Solving the 2x2 system of the static lateral-directional equilibrium for One Engine Inoperative condition.
% Needs the preliminary execution of PRECALC function.

% INPUT: dihedral angle, wing lift curve slope, taper ratio, sweep angle,
% lift coeff., vertical tail lift curve slope, lift curve slope, side-slip,
% vertical tail dynamic pressure ratio, vertical tail area, vertical
% distance between vertical tail ac and aircraft cg, wing area, semi-wing
% span, contributes to roll coeff., root chord, tip chord, ailerons data,
% ailerons effectiveness, rudder effectiveness, longitudinal distance
% between vertical tail ac and aircraft cg, dynamic pressure, drag coeff.,
% speed, roll rate, yaw rate, side-slip angle beta, contributes to yaw
% coeff., distance between propeller force and cg, propeller data, shaft
% power, area ratio, Oswald factor, mean aerodynamic chord, engines number.

% OUTPUT: Through this function the lateral-directional equilibrium equations
% system for OEI is solved. The two equations are:
% - Lateral stability;
% - Directional stability.
% The solutions are the deflection of the aileron and the deflection of the rudder
% by considering the assigned the side-slip angle beta and which engine fails (when
% engines number >1).
```

Figure 13: Lateral-Directional stability in OEI condition function

2 Data Structure

2.1 Description

The Data Structure is a database composed by all the aircraft data: input and output. The user can access it through the workspace.
Once opened the structure, there are three fields:

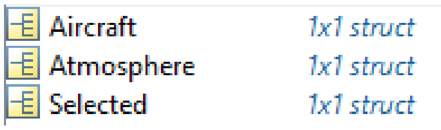


Figure 14: AllData

2.1.1 Aircraft

In Aircraft field the user can find:

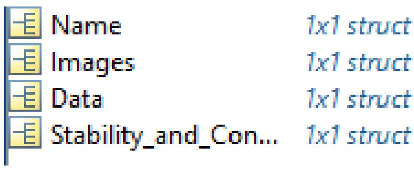


Figure 15: AllData.Aircraft

Let's analyse the last two.
In Data field there are four fields:

- Weight;
- Geometry: it contains all the geometric data of the aircraft, divided by the aircraft component.

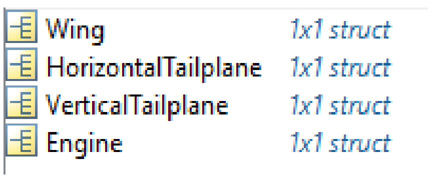


Figure 16: AllData.Aircraft.Data.Geometry

- Aerodynamics: it contains all the aerodynamic data of the aircraft, divided by the aircraft component.

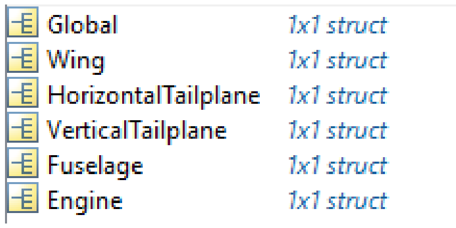


Figure 17: AllData.Aircraft.Data.Aerodynamics

- Powerplant: it contains engine data, especially most of them refer to the propeller.

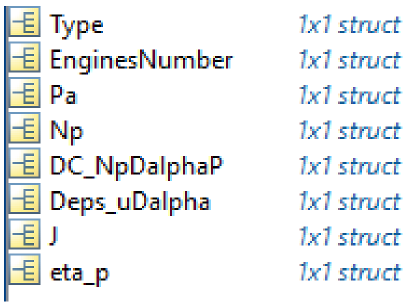


Figure 18: AllData.Aircraft.Data.Powerplant

In Stability and Control field the user can find the following fields:

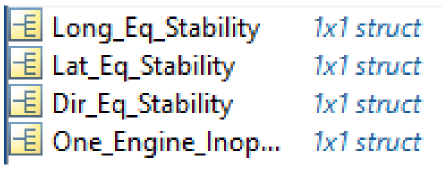
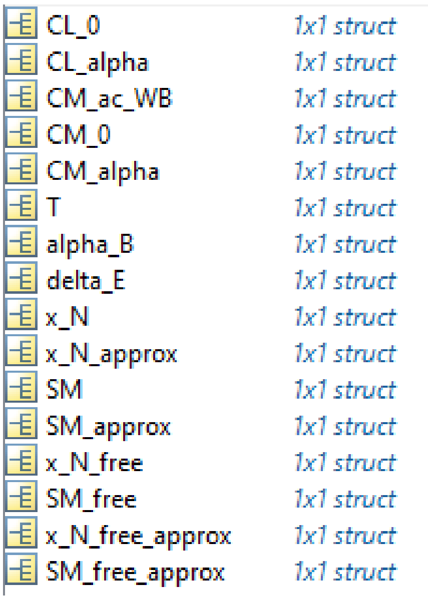


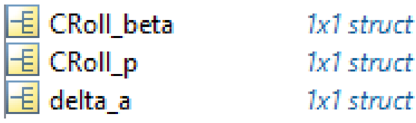
Figure 19: AllData.Aircraft.StabilityandControl

In each field there are the output data divided by the different kind of stability analysis:

- Longitudinal equilibrium stability;
- Lateral equilibrium stability;
- Directional equilibrium stability;
- One-Engine-Inoperative condition: there are the outputs of lateral-directional equilibrium stability in case an engine is faulty.



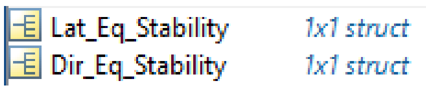
(a) AllData.Aircraft.StabilityandControl.LongEqStability



(b) AllData.Aircraft.StabilityandControl.LatEqStability

Figure 20: AllData.Aircraft.StabilityandControl (1)

(a) AllData.Aircraft.StabilityandControl.DirEqStability



(b) AllData.Aircraft.StabilityandControl.OneEngineInoperativeCo

Figure 21: AllData.Aircraft.StabilityandControl (2)

2.1.2 Selected

In Selected field the user can find all the data required at the beginning of the code.

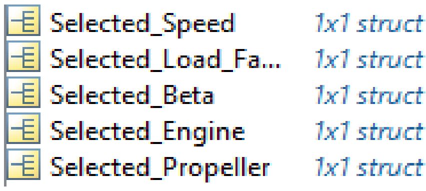


Figure 22: AllData.Selected

In particular, in Selected Speed there are:

- V: speed of the aircraft in m/s;
- Rates: pitch, roll and yaw rates in deg/s.

2.2 How the data structure is built in live script

Building a data structure is simple. All you need to do is:

- Choose the fields;
- Use the the correct procedure in the live script and functions.

Once the fields are chosen, each field is built with its respective subfields of the structure, by simply proceeding with the definition of each piece of information that needs to be inserted. Specifically, for each piece of information, all the fields in which it is entered must be indicated, from the outermost to the innermost.
For example:

```
AllData.Aircraft.Data.Geometry.Wing.b.value = 11.4;
AllData.Aircraft.Data.Geometry.Wing.b.Attribute.unit = 'm';
```

Figure 23: AllData piece of information

In this figure the wing span is defined. From these instructions it is possible to see that b is a geometric wing piece of information, based in Data, that is an Aircraft field in AllData.
To see the complete structure, it's necessary to Run the code and see the result.

3 Application

In this section the MATLAB outputs are shown. The available aircraft are:

1. Turbofan: Airbus A320 NEO;
2. Turboprop: ATR72;
3. Turbocharged: Tecnam P2012;
4. Pistons: Tecnam P2006T.

3.1 Airbus A320 NEO

AIRCRAFT STABILITY AND CONTROL

A code that analyses the stability and the control of the chosen aircraft has been implemented below.

3.1.1 Input Data

Figure 24: Airbus A320 NEO

Select the following parameters:

1. **Speed**.

2. **Load factor**.

3. **Pitch rate**.

4. **Beta**.

5. **Roll rate**.
6. **Yaw rate**.
7. **OEI** : which engine failure to study.
8. **Propeller rotation direction** (if exists).

Input data are here below summarized:

```
AIRCRAFT NAME: Airbus A320 NEO
```

Geometry

```
WING: b = 35.80 m | cr = 5.82 m | ct = 1.40 m | iW = 0.000 rad |
      Λ = 0.493 rad | Γ = 0.079 rad

HORIZONTAL TAILPLANE: bH = 12.45 m | SH = 31.00 m2 | lH = 18.10 m |
                      iH = -0.035 rad

VERTICAL TAILPLANE: SV = 23.50 m2 | lV = 18.10 m | hV = 6.26 m

ENGINE: d_mot= Yt = 5.73 m | Xt = 0.00 m | Zt = 0.00 m | D = 0.00 m
```

Aerodynamics

```
GLOBAL: CD0 = 0.020 | e = 0.84 | xcg = 0.33 | CRoll_0 = 0.00 | CN_0 = 0.00

WING: xac_W = 0.29 | Cl_alphaW = 5.70 rad-1 | alpha_0l_W = -0.01 rad |
      eW = 0.88 | CM_ac_W = -0.068 rad-1 | CRoll_p_W = -0.013 |
      CRoll_delta_a = -0.005 rad-1 | CRoll_beta_wpos = -0.001 rad-1 |
      CN_delta_a = 0.000 rad-1 | CN_r_W = 0.000 | τa = 0.40 |
      CN_beta_W = 0.000| CN_p_Wtilt = 0.000 | CN_p_W_tipsuc = 0.000 |
      CN_r_W = 0.000

HORIZONTAL TAILPLANE: Cl_alphaH = 6.300 rad-1 | eH = 0.90 | ηH = 1.00 |
                      τe = 0.38 | CH_alpha_e = -0.440 rad-1 |
                      CH_deltaE_e = -0.800 rad-1

VERTICAL TAILPLANE: CL_alphaV = 2.800 rad-1 | ηV = 1.00 | dσ/dβ = 0.130 |
                    τr = 0.45 | CN_p_V = 0.000 | CN_r_V = 0.000 |
                    dσ/dp = 0.000 | dσ/dr = 0.000

FUSELAGE: CM_0_B = -0.060 | CM_alpha_B(W) = 0.144 rad-1 |
          CN_beta_B = -0.089 rad-1
```

Powerplant

```
POWERPLANT: Engines number = 2 | Pa = 0.00 W | Np = 0.00 kgf | J = 0.00 s-1
```

3.1.2 Preliminary calculations

Preliminary calculations used in the entire code are shown below.

- Taper ratio

```
λ = 0.24
```

- Wing area

```
S = 129.24 m²
```

- Dynamic pressure

```
q_inf = 6125.00 Pa
```

- Mean aerodynamic chord

```
m.a.c. = 4.06 m
```

- Lift coefficient

```
CL = 0.98
```

- Aspect ratio (wing and horizontal tail, respectively)

```
AR = 9.92

ARH = 5.00
```

- Wing lift curve slope

```
CLα,W = 4.719 rad-1 = 0.082 deg-1
```

- Wing-Body's lift coefficient at zero lift

```
CL0,W = 0.07
```

- Horizontal tail lift curve slope

```
CLα,H = 4.358 rad-1 = 0.076 deg-1
```

- Mean value of downwash at $\alpha_{\mathrm{B}} = 0$

```
ε0 = 0.005 rad = 0.276 deg
```

- Downwash gradient

```
dε/dα = 0.344
```

3.1.3 Longitudinal equilibrium, stability and control

```
Selected input: the aircraft speed is V = 100 m/s with a load factor n = 1.
The pitch rate is q = 0 deg/s.
```

To study longitudinal stability and control of the aircraft, two values are calculated by a system of two equations. The values are the angle of attack and the equilbrium elevator angle by considering the assigned horizontal tail incidence angle i_{H}. The equations are:

1. Vertical Translational Equilibrium;
2. Rotational Equilibrium.

3.1.3.1 Vertical translation equilibrium

The equation is:

$$C_L = C_{L_0} + C_{L_\alpha}\alpha_{\mathrm{B}} + C_{L_{\delta_{\mathrm{e}}}}\delta_{\mathrm{e}} + C_{L_{i_{\mathrm{H}}}} i_{\mathrm{H}} = \frac{W}{\bar{q}_\infty S} \tag{50}$$

where

```
CL0 = 0.061

CLα = 5.404 rad-1 = 0.094 deg-1

CLδe = 0.397 rad-1 = 0.007 deg-1

CLiH = 1.045 rad-1 = 0.018 deg-1
```

3.1.3.2 Rotational equilibrium

The equation is:

$$C_M = C_{M_0} + C_{M_\alpha}\alpha_{\mathrm{B}} + C_{M_{\delta_{\mathrm{e}}}}\delta_{\mathrm{e}} + C_{M_{i_{\mathrm{H}}}} i_{\mathrm{H}} + C_{M_q}\hat{q} + C_{M_{\mathrm{mot}}} = 0 \tag{51}$$

where

```
CM0 = -0.101

CMα = -2.722 rad-1 = -0.048 deg-1

CMδe = -1.770 rad-1 = -0.031 deg-1

CMiH = -4.659 rad-1 = -0.081 deg-1

CMq = -41.532 rad-1 = -0.725 deg-1

CMmot = 0.000
```

3.1.3.3 Solving the system

The values of the angle of attack and the deflection of the elevator are the solution of the following system:

$$\begin{bmatrix} C_{L_\alpha} & C_{L_{\delta_\mathrm{e}}} \\ C_{M_\alpha} & C_{M_{\delta_\mathrm{e}}} \end{bmatrix} \begin{bmatrix} \alpha_\mathrm{B} \\ \delta_\mathrm{e} \end{bmatrix} = \begin{bmatrix} C_L - C_{L_0} - C_{L_{i_\mathrm{H}}} i_\mathrm{H} \\ -C_{M_0} - C_{M_\mathrm{mot}} - C_{M_{i_\mathrm{H}}} i_\mathrm{H} - C_{M_q} \hat{q} \end{bmatrix} \tag{52}$$

```
αB = 0.196 rad = 11.244 deg
```

```
δe = -0.267 rad = -15.278 deg
```

Now it is possible to represent the evolution of the pitching moment coefficient vs the body angle of attack.

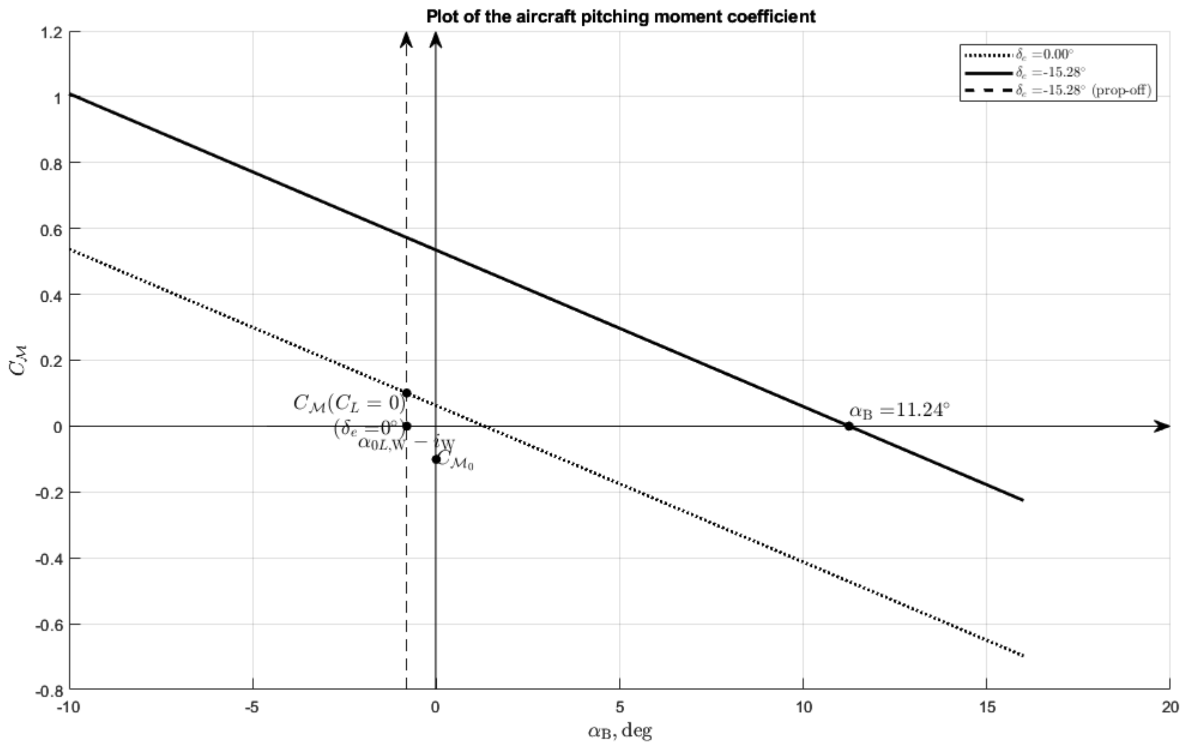


Figure 25: Airbus A320 NEO C_M plot

The horizontal tailplane load is:

```
LH = -10308.79 N = -1051.21 kgf
```

```
αH,a = -0.012 rad = -0.714 deg
```

3.1.3.4 Neutral points and SMs - Stick-Fixed

The neutral point for stick-fixed condition is the solution of the following equation:

$$C_{M_\alpha} = C_{L_{\alpha,\mathrm{W}}}(\overline{x}_\mathrm{G} - \overline{x}_\mathrm{ac,WB}) - \eta_\mathrm{H}\frac{S_\mathrm{H}}{S}(\overline{x}_\mathrm{ac,H} - \overline{x}_\mathrm{G})\left[1 - \left(\frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right)_\mathrm{H}\right] C_{L_{\alpha,\mathrm{H}}} = 0 \tag{53}$$

By the position $\overline{x}_\mathrm{G} = \overline{x}_\mathrm{N}$:

```
xN = 0.83
```

Now it's possible to study the $SM = \overline{x}_\mathrm{G} - \overline{x}_\mathrm{N}$:

```
SM = -0.50

The aircraft is stable.
```

If the approximated formula is used:

```
xN approx = 0.91

SM approx = -0.58
```

- Stick-Free

In this case it's introduced the free elevator factor:

```
F = 0.79
```

The neutral point for sitck-free condition is the solution of the following equation:

$$C'_{M_\alpha} = C_{L_{\alpha,\mathrm{W}}}(\overline{x}_\mathrm{G} - \overline{x}_\mathrm{ac,WB}) - \eta_\mathrm{H}\frac{S_\mathrm{H}}{S}(\overline{x}_\mathrm{ac,H} - \overline{x}_\mathrm{G})F\left[1 - \left(\frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right)_\mathrm{H}\right] C_{L_{\alpha,\mathrm{H}}} = 0 \tag{54}$$

By the position $\overline{x}_\mathrm{G} = \overline{x}'_\mathrm{N}$:

```
xN' = 0.73
```

Now it's possible to study the $SM' = \overline{x}_\mathrm{G} - \overline{x}'_\mathrm{N}$:

```
SM' = -0.40

The aircraft is stable.
```

If the approximated formula is used:

```
xN' approx = 0.77

SM' approx = -0.44
```

3.1.3.5 Summary of all the contributes

	Derivative	Value	Unit
1	'CL0'	0.0610	' '
2	'CLα'	5.4044	'rad-1'
3	'CLδe'	0.3972	'rad-1'
4	'CLiH'	1.0453	'rad-1'
5	'CM0'	-0.1009	' '
6	'CMα'	-2.7225	'rad-1'
7	'CMδe'	-1.7705	'rad-1'
8	'CMiH'	-4.6591	'rad-1'
9	'CMq'	-41.5319	'rad-1'
10	'CMmot'	0	' '

3.1.4 Lateral-Directional stability

```
Selected input: the sideslip angle is β = -3 deg with a roll rate
p = 0 deg/s. The yaw rate is r = 0 deg/s.
```

To study lateral-directional stability of the aircraft, two values are calculated by a system of two equations. The values are aileron deflection δ_a and rudder deflection δ_r by considering the assigned side-slip angle β. The equations are:

1. Lateral Stability;
2. Directional Stability.

3.1.4.1 Lateral stability

The equation is:

$$C_{\mathcal{L}} = C_{\mathcal{L}0} + C_{\mathcal{L}\beta}\beta + C_{\mathcal{L}\delta a}\delta_{\mathrm{a}} + C_{\mathcal{L}\delta_{\mathrm{r}}}\delta_{\mathrm{r}} + C_{\mathcal{L}p}\hat{p} + C_{\mathcal{L}r}\hat{r} + C_{\mathcal{L}\mathrm{mot}} \tag{55}$$

where

```
CRoll0 = 0.000

because the aircraft is symmetric.

CRollβ = -0.160 rad-1 = -0.003 deg-1

CRollδa = -0.104 rad-1 = -0.002 deg-1

CRollδr = 0.040 rad-1 = 0.001 deg-1
```

```
CRollp = -0.916 rad-1= -0.016 deg-1

CRollr = 0.090 rad-1= 0.002 deg-1
```

CONVENTION**:** The contribution is <0 when clockwise. In this case there is no propeller.

```
CRollmot = -0.000
```

3.1.4.2 Directional stability

The equation is:

$$C_{\mathcal{N}} = C_{\mathcal{N}0} + C_{\mathcal{N}\beta}\beta + C_{\mathcal{N}\delta a}\delta_{\mathrm{a}} + C_{\mathcal{N}\delta_{\mathrm{r}}}\delta_{\mathrm{r}} + C_{\mathcal{N}p}\hat{p} + C_{\mathcal{N}r}\hat{r} + C_{\mathcal{N}\mathrm{mot}} \tag{56}$$

where

```
CN0 = 0.000

because the aircraft is symmetric.

CNβ = 0.135 rad-1 = 0.002 deg-1

CNδa = 0.000 rad-1 = 0.000 deg-1

CNδr = -0.116 rad-1 = -0.002 deg-1

CNp = 0.166 rad-1 = 0.003 deg-1

CNr = -0.260 rad-1 = -0.005 deg-1

CNmot = 0.000
```

3.1.4.3 Solving the system

The deflection values of the aileron and the elevator are the unknown values of the following system:

$$\begin{bmatrix} C_{\mathcal{L}\delta a} & C_{\mathcal{L}\delta_{\mathrm{r}}} \\ C_{\mathcal{N}\delta a} & C_{\mathcal{N}\delta_{\mathrm{r}}} \end{bmatrix} \begin{bmatrix} \delta_{\mathrm{a}} \\ \delta_{\mathrm{r}} \end{bmatrix} = \begin{bmatrix} -(C_{\mathcal{L}0} + C_{\mathcal{L}\beta}\beta + C_{\mathcal{L}p}\hat{p} + C_{\mathcal{L}r}\hat{r} + C_{\mathcal{L}\mathrm{mot}}) \\ -(C_{\mathcal{N}0} + C_{\mathcal{N}\beta}\beta + C_{\mathcal{N}p}\hat{p} + C_{\mathcal{N}r}\hat{r} + C_{\mathcal{N}\mathrm{mot}}) \end{bmatrix} \tag{57}$$

```
δa = 0.057 rad = 3.255 deg

δr = -0.061 rad = -3.495 deg
```

3.1.4.4 Summary of all derivatives

	Derivative	Value	Unit
1	'CRoll0'	0	' '
2	'CRollβ'	-0.1596	'rad-1'
3	'CRollδa'	-0.1041	'rad-1'
4	'CRollδr'	0.0401	'rad-1'
5	'CRollp'	-0.9159	'rad-1'
6	'CRollr'	0.0900	'rad-1'
7	'CRollmot'	0	' '
8	'CN0'	0	' '
9	'CNβ'	0.1349	'rad-1'
10	'CNδa'	0	'rad-1'
11	'CNδr'	-0.1158	'rad-1'
12	'CNp'	0.1658	'rad-1'
13	'CNr'	-0.2603	'rad-1'
14	'CNmot'	0	' '

3.1.5 Lateral-Directional stability in One-Engine-Inoperative condition

The engine that you have chosen fails. The system made by the lateral and directional stability is solved by considering the asymmetrical thrust that acts on the working engine.

3.1.5.1 Asymmetrical thrust

```
T = 44824.50 N = 4569.27 kg
```

3.1.5.2 Lateral equilibrium

The equation is:

$$C_{\mathcal{L}} = C_{\mathcal{L}0} + C_{\mathcal{L}\beta}\beta + C_{\mathcal{L}\delta a}\delta_{\mathrm{a}} + C_{\mathcal{L}\delta_{\mathrm{r}}}\delta_{\mathrm{r}} + C_{\mathcal{L}p}\hat{p} + C_{\mathcal{L}r}\hat{r} + C_{\mathcal{L}\mathrm{mot}} \tag{58}$$

where

```
CRoll0 = 0.000

because the aircraft is symmetric.

CRollβ = -0.160 rad-1 = -0.003 deg-1

CRollδa = -0.104 rad-1 = -0.002 deg-1
```

```
CRollδr = 0.040 rad-1 = 0.001 deg-1

CRollp = -0.916 rad-1 = -0.016 deg-1

CRollr = 0.090 rad-1 = 0.002 deg-1

CRollmot = -0.000
```

3.1.5.3 Directional equilibrium

The equation is:

$$C_{\mathcal{N}} = C_{\mathcal{N}0} + C_{\mathcal{N}\beta}\beta + C_{\mathcal{N}\delta a}\delta_{\text{a}} + C_{\mathcal{N}\delta_{\text{r}}}\delta_{\text{r}} + C_{\mathcal{N}p}\hat{p} + C_{\mathcal{N}r}\hat{r} + C_{\mathcal{N}\text{mot}} \tag{59}$$

where

```
CN0 = 0.000

because the aircraft is symmetric.

CNβ = 0.135 rad-1 = 0.002 deg-1

CNδa = 0.000 rad-1 = 0.000 deg-1

CNδr = -0.116 rad-1 = -0.002 deg-1

CNp = 0.166 rad-1 = 0.003 deg-1

CNr = -0.260 rad-1 = -0.005 deg-1
```

$C_{\mathcal{N}\text{mot}} = \frac{T d_{\text{mot}}}{q_\infty \text{Sb}} \neq 0$ because there's a yaw moment due to the asymmetric thrust with its arm d_{mot}. The sign depends on which engine fails. Usually when the left engine fails $C_{\mathcal{N}\text{mot}} < 0$, because T causes $N < 0$. Instead when the right engine fails $C_{\mathcal{N}\text{mot}} > 0$, because T causes $N > 0$.

```
CNmot = -0.009
```

3.1.5.4 Solving the system

The deflection values of the aileron and the elevator are the unknown values of the following system:

$$\begin{bmatrix} C_{\mathcal{L}\delta a} & C_{\mathcal{L}\delta_{\text{r}}} \\ C_{\mathcal{N}\delta a} & C_{\mathcal{N}\delta_{\text{r}}} \end{bmatrix} \begin{bmatrix} \delta_{\text{a}} \\ \delta_{\text{r}} \end{bmatrix} = \begin{bmatrix} -(C_{\mathcal{L}0} + C_{\mathcal{L}\beta}\beta + C_{\mathcal{L}p}\hat{p} + C_{\mathcal{L}r}\hat{r} + C_{\mathcal{L}\text{mot}}) \\ -(C_{\mathcal{N}0} + C_{\mathcal{N}\beta}\beta + C_{\mathcal{N}p}\hat{p} + C_{\mathcal{N}r}\hat{r} + C_{\mathcal{N}\text{mot}}) \end{bmatrix} \tag{60}$$

```
δa = 0.027 rad = 1.530 deg

δr = -0.139 rad = -7.978 deg
```

3.1.5.5 Summary of all derivatives

	Derivative	Value	Unit
1	'CRoll0'	0	' '
2	'CRollβ'	-0.1596	'rad-1'
3	'CRollδa'	-0.1041	'rad-1'
4	'CRollδr'	0.0401	'rad-1'
5	'CRollp'	-0.9159	'rad-1'
6	'CRollr'	0.0900	'rad-1'
7	'CRollmot'	0	' '
8	'CN0'	0	' '
9	'CNβ'	0.1349	'rad-1'
10	'CNδa'	0	'rad-1'
11	'CNδr'	-0.1158	'rad-1'
12	'CNp'	0.1658	'rad-1'
13	'CNr'	-0.2603	'rad-1'
14	'CNmot'	-0.0091	' '

3.2 ATR72

AIRCRAFT STABILITY AND CONTROL

A code that analyses the stability and the control of the chosen aircraft has been implemented below.

3.2.1 Input Data

Select the following parameters:

1. **Speed**.

2. **Load factor**.

3. **Pitch rate**.

4. **Beta**.

5. **Roll rate**.

6. **Yaw rate**.

7. **OEI** : which engine failure to study.

8. **Propeller rotation direction** (if exists).

Input data are here below summarized:

```
AIRCRAFT NAME: ATR72
```

Figure 26: ATR72

Geometry

```
WING: b = 27.05 m | cr = 2.83 m | ct = 1.67 m | iW = 0.035 rad |
      Λ = 0.520 rad | Γ = 0.087 rad

HORIZONTAL TAILPLANE: bH = 7.50 m | SH = 11.70 m2 | lH = 13.00 m |
                      iH = -0.052 rad

VERTICAL TAILPLANE: SV = 12.50 m2 | lV = 14.43 m | hV = 3.20 m

ENGINE: d_mot = 4.00 m | Xt = 4.00 m | Zt = 4.00 m | D = 3.93 m
```

Aerodynamics

```
GLOBAL: CD0 = 0.03 | e = 0.82 | xcg = 0.40 | CRoll_0 = 0.00 | CN_0 = 0.00

WING: xac_W = 0.24 | Cl_alphaW = 6.19 rad-1 | alpha_0l_W = -0.01 rad |
      eW = 0.85 | CM_ac_W = -0.068 rad-1 | CRoll_p_W = -0.013 |
      CRoll_delta_a = -0.160 rad-1 | CRoll_beta_wpos = -0.001 rad-1 |
      CN_delta_a = 0.000 rad-1  CN_r_W = 0.000 | τa = 0.40 |
      CN_beta_W = 0.000 rad-1 | CN_p_Wtilt = 0.000 rad-1 |
      CN_p_W_tipsuc = 0.000 rad-1 | CN_r_W = 0.000 rad-1
```

```
HORIZONTAL TAILPLANE: Cl_alphaH = 6.190 rad-1 | eH = 0.85 | ηH = 1.00 |
                      τe = 0.38 | CH_alpha_e = -0.400 rad-1 |
                      CH_deltaE_e = -0.750 rad-1

VERTICAL TAILPLANE: CL_alphaV = 2.800 rad-1 | ηV = 1.00 |
                    dσ/dβ = 0.100 | τr = 0.45 | CN_p_V = 0.000 |
                    CN_r_V = 0.000 | dσ/dp = 0.000 | dσ/dr = 0.000

FUSELAGE: CM_0_B = -0.060 | CM_alpha_B(W) = 0.573 rad-1 |
          CN_beta_B = -0.132 rad-1
```

Powerplant

```
POWERPLANT: Engines number = 2 | Pa = 2050674.65 W | Np = 11.00 kgf |
            J = 1.00 s-1
```

3.2.2 Preliminary calculations

Preliminary calculations used in the entire code are shown below.

- Taper ratio

```
λ = 0.59
```

- Wing area

```
S = 60.86 m²
```

- Dynamic pressure

```
qinf = 6125.00 Pa
```

- Mean aerodynamic chord

```
m.a.c. = 2.30 m
```

- Lift coefficient

```
CL = 0.58
```

- Aspect ratio (wing and horizontal tail, respectively)

```
AR = 12.02
```

```
ARH = 4.81
```

- Wing lift curve slope

```
CLα,W = 5.189 rad-1 = 0.091 deg-1
```

- Wing-Body's lift coefficient at zero lift

```
CL0,W = 0.23
```

- Horizontal tail lift curve slope

```
CLα,H = 4.176 rad-1 = 0.073 deg-1
```

- Mean value of downwash at $\alpha_{\mathrm{B}} = 0$

```
ε0 = 0.014 rad = 0.815 deg
```

- Downwash gradient

```
dε/dα = 0.323
```

3.2.3 Longitudinal equilibrium, stability and control

```
Selected input: the aircraft speed is V = 100 m/s with a load factor n = 1.
The pitch rate is q = 0 deg/s.
```

To study longitudinal stability and control of the aircraft, two values are calculated by a system of two equations. The values are the angle of attack and the equilbrium elevator angle by considering the assigned horizontal tail incidence angle i_{H}. The equations are:

1. Vertical Translational Equilibrium;
2. Rotational Equilibrium.

3.2.3.1 Vertical translation equilibrium

The equation is:

$$C_L = C_{L_0} + C_{L_\alpha}\alpha_{\mathrm{B}} + C_{L_{\delta_{\mathrm{e}}}}\delta_{\mathrm{e}} + C_{L_{i_{\mathrm{H}}}} i_{\mathrm{H}} = \frac{W}{\bar{q}_\infty S} \tag{61}$$

where

```
CL0 = 0.217

CLα = 5.733 rad-1 = 0.100 deg-1

CLδe = 0.305 rad-1 = 0.005 deg-1

CLiH = 0.803 rad-1 = 0.014 deg-1
```

3.2.3.2 Rotational equilibrium

The equation is:

$$C_M = C_{M_0} + C_{M_\alpha}\alpha_\mathrm{B} + C_{M_{\delta_\mathrm{e}}}\delta_\mathrm{e} + C_{M_{i_\mathrm{H}}}i_\mathrm{H} + C_{M_q}\hat{q} + C_{M_\mathrm{mot}} = 0 \tag{62}$$

where

```
CM0 = -0.002

CMα = -1.668 rad-1 = -0.029 deg-1

CMδe = -1.725 rad-1 = -0.030 deg-1

CMiH = -4.538 rad-1 = -0.079 deg-1

CMq = -51.305 rad-1 = -0.895 deg-1

CMmot = 0.077
```

3.2.3.3 Solving the system

The values of the angle of attack and the deflection of the elevator are the solution of the following system:

$$\begin{bmatrix} C_{L_\alpha} & C_{L_{\delta_\mathrm{e}}} \\ C_{M_\alpha} & C_{M_{\delta_\mathrm{e}}} \end{bmatrix} \begin{bmatrix} \alpha_\mathrm{B} \\ \delta_\mathrm{e} \end{bmatrix} = \begin{bmatrix} C_L - C_{L0} - C_{L_{i_\mathrm{H}}}i_\mathrm{H} \\ -C_{M_0} - C_{M_\mathrm{mot}} - C_{M_{i_\mathrm{H}}}i_\mathrm{H} - C_{M_q}\hat{q} \end{bmatrix} \tag{63}$$

```
αB = 0.064 rad = 3.675 deg

δe = 0.118 rad = 6.780 deg
```

Now it is possible to represent the evolution of the pitching moment coefficient vs the body angle of attack.

The horizontal tailplane load is:

```
LH = 6626.00 N = 675.67 kgf

αH,a = 0.022 rad = 1.269 deg
```

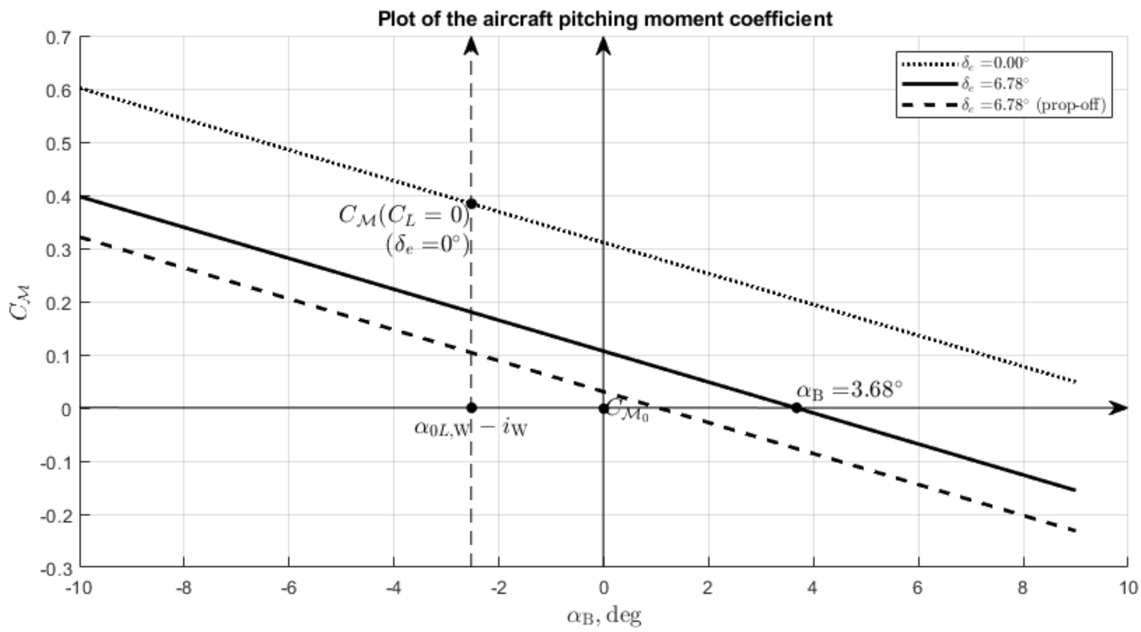


Figure 27: ATR72 C_M plot

3.2.3.4 Neutral points and SMs - Stick-Fixed

The neutral point for stick-fixed condition is the solution of the following equation:

$$C_{M_\alpha} = C_{L_{\alpha,\mathrm{W}}}(\overline{x}_{\mathrm{G}} - \overline{x}_{\mathrm{ac,WB}}) - \eta_{\mathrm{H}}\frac{S_{\mathrm{H}}}{S}(\overline{x}_{\mathrm{ac,H}} - \overline{x}_{\mathrm{G}})\left[1 - \left(\frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right)_{\mathrm{H}}\right] C_{L_{\alpha,\mathrm{H}}} = 0 \tag{64}$$

By the position $\overline{x}_{\mathrm{G}} = \overline{x}_{\mathrm{N}}$:

```
xN = 0.69
```

Now it's possible to study the $SM = \overline{x}_{\mathrm{G}} - \overline{x}_{\mathrm{N}}$:

```
SM = -0.29
```

```
The aircraft is stable.
```

If the approximated formula is used:

```
xN approx = 0.72
```

```
SM approx = -0.32
```

- Stick-Free

In this case it's introduced the free elevator factor:

```
F = 0.80
```

The neutral point for sitck-free condition is the solution of the following equation:

$$C'_{M_\alpha} = C_{L_{\alpha,\mathrm{W}}}(\overline{x}_{\mathrm{G}} - \overline{x}_{\mathrm{ac,WB}}) - \eta_{\mathrm{H}}\frac{S_{\mathrm{H}}}{S}(\overline{x}_{\mathrm{ac,H}} - \overline{x}_{\mathrm{G}})F\left[1 - \left(\frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right)_{\mathrm{H}}\right]C_{L_{\alpha,\mathrm{H}}} = 0 \quad (65)$$

By the position $\overline{x}_{\mathrm{G}} = \overline{x}_{\mathrm{N}}$:

```
xN' = 0.59
```

Now it's possible to study the $SM' = \overline{x}_{\mathrm{G}} - \overline{x}'_{\mathrm{N}}$:

```
SM' = -0.19

The aircraft is stable.
```

If the approximated formula is used:

```
xN' approx = 0.60

SM' approx = -0.20
```

3.2.3.5 Summary of all the contributes

	Derivative	Value	Unit
1	'CL0'	0.2169	' '
2	'CLα'	5.7327	'rad-1'
3	'CLδe'	0.3051	'rad-1'
4	'CLiH'	0.8028	'rad-1'
5	'CM0'	-0.0017	' '
6	'CMα'	-1.6677	'rad-1'
7	'CMδe'	-1.7245	'rad-1'
8	'CMiH'	-4.5382	'rad-1'
9	'CMq'	-51.3046	'rad-1'
10	'CMmot'	0.0767	' '

3.2.4 Lateral-Directional stability

```
Selected input: the sideslip angle is Î² = -3 deg with a roll rate p = 0 deg/s.
The yaw rate is r = 0 deg/s.
```

To study lateral-directional stability of the aircraft, two values are calculated by a system of two equations. The values are aileron deflection δ_a and rudder deflection δ_r by considering the assigned side-slip angle β. The equations are:

1. Lateral Stability;
2. Directional Stability.

3.2.4.1 Lateral stability

The equation is:

$$C_{\mathcal{L}} = C_{\mathcal{L}0} + C_{\mathcal{L}\beta}\beta + C_{\mathcal{L}\delta a}\delta_{\mathrm{a}} + C_{\mathcal{L}\delta_{\mathrm{r}}}\delta_{\mathrm{r}} + C_{\mathcal{L}p}\hat{p} + C_{\mathcal{L}r}\hat{r} + C_{\mathcal{L}\mathrm{mot}} \quad (66)$$

where

```
CRoll0 = 0.000

because the aircraft is symmetric.

CRollβ = -0.170 rad-1 = -0.003 deg-1

CRollδa = -0.160 rad-1 = -0.003 deg-1

CRollδr = 0.031 rad-1 = 0.001 deg-1

CRollp = -0.900 rad-1= -0.016 deg-1

CRollr = 0.073 rad-1= 0.001 deg-1

CRollmot = -0.001
```

CONVENTION**:** The contribution is <0 when clockwise.

3.2.4.2 Directional stability

The equation is:

$$C_{\mathcal{N}} = C_{\mathcal{N}0} + C_{\mathcal{N}\beta}\beta + C_{\mathcal{N}\delta a}\delta_{\mathrm{a}} + C_{\mathcal{N}\delta_{\mathrm{r}}}\delta_{\mathrm{r}} + C_{\mathcal{N}p}\hat{p} + C_{\mathcal{N}r}\hat{r} + C_{\mathcal{N}\mathrm{mot}} \quad (67)$$

where

```
CN0 = 0.000

because the aircraft is symmetric.

CNβ = 0.144 rad-1 = 0.003 deg-1

CNδa = 0.000 rad-1 = 0.000 deg-1

CNδr = -0.138 rad-1 = -0.002 deg-1

CNp = 0.109 rad-1 = 0.002 deg-1

CNr = -0.327 rad-1 = -0.006 deg-1

CNmot = 0.000
```

3.2.4.3 Solving the system

The deflection values of the aileron and the elevator are the unknown values of the following system:

$$\begin{bmatrix} C_{\mathcal{L}\delta a} & C_{\mathcal{L}\delta_r} \\ C_{\mathcal{N}\delta a} & C_{\mathcal{N}\delta_r} \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} = \begin{bmatrix} -(C_{\mathcal{L}0} + C_{\mathcal{L}\beta}\beta + C_{\mathcal{L}p}\hat{p} + C_{\mathcal{L}r}\hat{r} + C_{\mathcal{L}\mathrm{mot}}) \\ -(C_{\mathcal{N}0} + C_{\mathcal{N}\beta}\beta + C_{\mathcal{N}p}\hat{p} + C_{\mathcal{N}r}\hat{r} + C_{\mathcal{N}\mathrm{mot}}) \end{bmatrix} \tag{68}$$

```
δa = 0.037 rad = 2.132 deg

δr = -0.055 rad = -3.131 deg
```

3.2.4.4 Summary of all derivatives

	Derivative	Value	Unit
1	'CRoll0'	0	' '
2	'CRollβ'	-0.1698	'rad-1'
3	'CRollδa'	-0.1597	'rad-1'
4	'CRollδr'	0.0306	'rad-1'
5	'CRollp'	-0.9002	'rad-1'
6	'CRollr'	0.0726	'rad-1'
7	'CRollmot'	-0.0013	' '
8	'CN0'	0	' '
9	'CNβ'	0.1441	'rad-1'
10	'CNδa'	0	'rad-1'
11	'CNr'	-0.1380	'rad-1'
12	'CNp'	0.1091	'rad-1'
13	'CNδr'	-0.3273	'rad-1'
14	'CNmot'	0	' '

3.2.5 Lateral-Directional stability in One-Engine-Inoperative condition

The engine that you have chosen fails. The system made by the lateral and directional stability is solved by considering the asymmetrical thrust that acts on the working engine.

3.2.5.1 Asymmetrical thrust

```
T = 16336.23 N = 1665.26 kg
```

3.2.5.2 Lateral equilibrium

The equation is:

$$C_{\mathcal{L}} = C_{\mathcal{L}0} + C_{\mathcal{L}\beta}\beta + C_{\mathcal{L}\delta a}\delta_a + C_{\mathcal{L}\delta_r}\delta_r + C_{\mathcal{L}p}\hat{p} + C_{\mathcal{L}r}\hat{r} + C_{\mathcal{L}\mathrm{mot}} \tag{69}$$

where

```
CRoll0 = 0.000

because the aircraft is symmetric.

CRollβ = -0.170 rad-1 = -0.003 deg-1

CRollδa = -0.160 rad-1 = -0.003 deg-1

CRollδr = 0.031 rad-1 = 0.001 deg-1

CRollp = -0.900 rad-1 = -0.016 deg-1

CRollr = 0.073 rad-1 = 0.001 deg-1

CRollmot = -0.001
```

3.2.5.3 Directional equilibrium

The equation is:

$$C_{\mathcal{N}} = C_{\mathcal{N}0} + C_{\mathcal{N}\beta}\beta + C_{\mathcal{N}\delta a}\delta_{\mathrm{a}} + C_{\mathcal{N}\delta_{\mathrm{r}}}\delta_{\mathrm{r}} + C_{\mathcal{N}p}\hat{p} + C_{\mathcal{N}r}\hat{r} + C_{\mathcal{N}\mathrm{mot}} \tag{70}$$

where

```
CN0 = 0.000

because the aircraft is symmetric.

CNβ = 0.144 rad-1 = 0.003 deg-1

CNδa = 0.000 rad-1 = 0.000 deg-1

CNδr = -0.138 rad-1 = -0.002 deg-1

CNp = 0.109 rad-1 = 0.002 deg-1

CNr = -0.327 rad-1 = -0.006 deg-1
```

$C_{\mathcal{N}\mathrm{mot}} = \frac{T d_{\mathrm{mot}}}{q_\infty \mathrm{Sb}} \neq 0$ because there's a yaw moment due to the asymmetric thrust with its arm d_{mot}. The sign depends on which engine fails. Usually when the left engine fails $C_{\mathcal{N}\mathrm{mot}} < 0$, because T causes $N < 0$. Instead when the right engine fails $C_{\mathcal{N}\mathrm{mot}} > 0$, because T causes $N > 0$.

```
CNmot = -0.006
```

3.2.5.4 Solving the system

The deflection values of the aileron and the elevator are the unknown values of the following system:

$$\begin{bmatrix} C_{\mathcal{L}\delta a} & C_{\mathcal{L}\delta_{\mathrm{r}}} \\ C_{\mathcal{N}\delta a} & C_{\mathcal{N}\delta_{\mathrm{r}}} \end{bmatrix} \begin{bmatrix} \delta_{\mathrm{a}} \\ \delta_{\mathrm{r}} \end{bmatrix} = \begin{bmatrix} -(C_{\mathcal{L}0} + C_{\mathcal{L}\beta}\beta + C_{\mathcal{L}p}\hat{p} + C_{\mathcal{L}r}\hat{r} + C_{\mathcal{L}\mathrm{mot}}) \\ -(C_{\mathcal{N}0} + C_{\mathcal{N}\beta}\beta + C_{\mathcal{N}p}\hat{p} + C_{\mathcal{N}r}\hat{r} + C_{\mathcal{N}\mathrm{mot}}) \end{bmatrix} \tag{71}$$

```
δa = 0.028 rad = 1.617 deg
```

```
δr = -0.102 rad = -5.821 deg
```

3.2.5.5 Summary of all derivatives

	Derivative	Value	Unit
1	'CRoll0'	0	' '
2	'CRollβ'	-0.1698	'rad-1'
3	'CRollδa'	-0.1597	'rad-1'
4	'CRollδr'	0.0306	'rad-1'
5	'CRollp'	-0.9002	'rad-1'
6	'CRollr'	0.0726	'rad-1'
7	'CRollmot'	-0.0013	' '
8	'CN0'	0	' '
9	'CNβ'	0.1441	'rad-1'
10	'CNδa'	0	'rad-1'
11	'CNδr'	-0.1380	'rad-1'
12	'CNp'	0.1091	'rad-1'
13	'CNr'	-0.3273	'rad-1'
14	'CNmot'	-0.0065	' '

3.3 Tecnam P2012

AIRCRAFT STABILITY AND CONTROL

A code that analyses the stability and the control of the chosen aircraft has been implemented below.

3.3.1 Input Data

Select the following parameters:

1. **Speed**.

2. **Load factor**.

3. **Pitch rate**.

Figure 28: Tecnam P2012

4. **Beta**.

5. **Roll rate**.

6. **Yaw rate**.

7. **OEI** : which engine failure to study.

8. **Propeller rotation direction** (if exists).

Input data are here below summarized:

```
AIRCRAFT NAME: Tecnam P2012
```

Geometry

```
WING: b = 14.00 m | cr = 2.30 m | ct = 1.38 m | iW = 0.035 rad |
      Λ = 0.493 rad | Γ = 0.052 rad

HORIZONTAL TAILPLANE: bH = 5.30 m | SH = 7.20 m2 | lH = 5.40 m |
                      iH = -0.052 rad

VERTICAL TAILPLANE: SV = 3.50 m2 | lV = 7.00 m | hV = 1.20 m

ENGINE: d_mot = 1.50 m | Xt = 1.50 m | Zt = 1.50 m | D = 2.50 m
```

Aerodynamics

```
GLOBAL: CD0 = 0.03 | e = 0.80 | xcg = 0.30 | CRoll_0 = 0.00 | CN_0 = 0.00

WING: xac_W = 0.25 | Cl_alphaW = 6.30 rad-1 | alpha_0l_W = -0.01 rad |
      eW = 0.90 | CM_ac_W = -0.070 rad-1 | CRoll_p_W = -0.693 |
      CRoll_delta_a = -0.212 rad-1 | CRoll_beta_wpos = -0.001 rad-1 |
      CN_delta_a = 0.000 rad-1  CN_r_W = 0.000 | τa = 0.40 |
      CN_beta_W = 0.000 rad-1 | CN_p_Wtilt = 0.000 rad-1 |
      CN_p_W_tipsuc = 0.000 rad-1 | CN_r_W = 0.000 rad-1

HORIZONTAL TAILPLANE: Cl_alphaH = 6.300 rad-1 | eH = 0.90 | ηH = 1.00 |
                      τe = 0.45 | CH_alpha_e = -0.400 rad-1 |
                      CH_deltaE_e = -0.750 rad-1

VERTICAL TAILPLANE: CL_alphaV = 3.000 rad-1 | Î·V = 1.00 | dσ/dβ = 0.150 |
                    τr = 0.45 | CN_p_V = 0.000 | CN_r_V = 0.000 |
                    dσ/dp = 0.000 | dσ/dr = 0.000

FUSELAGE: CM_0_B = -0.060 | CM_alpha_B(W) = 0.340 rad-1 |
          CN_beta_B = -0.089 rad-1
```

Powerplant

```
POWERPLANT: Engines number = 2 | Pa = 279637.45 W | Np = 11.00 kgf |
            J = 1.00 s-1
```

3.3.2 Preliminary calculations

Preliminary calculations used in the entire code are shown below.

- Taper ratio

```
λ = 0.60
```

- Wing area

```
S = 25.76 m²
```

- Dynamic pressure

```
qinf = 6125.00 Pa
```

- Mean aerodynamic chord

```
m.a.c. = 1.88 m
```

- Lift coefficient

```
CL = 0.23
```

- Aspect ratio (wing and horizontal tail, respectively)

```
AR = 7.61

ARH = 3.90
```

- Wing lift curve slope

```
CLα,W = 4.873 rad-1 = 0.085 deg-1
```

- Wing-Body's lift coefficient at zero lift

```
CL0,W = 0.21
```

- Horizontal tail lift curve slope

```
CLα,H = 4.010 rad-1 = 0.070 deg-1
```

- Mean value of downwash at $\alpha_{\mathrm{B}} = 0$

```
ε0 = 0.020 rad = 1.142 deg
```

- Downwash gradient

```
dε/dα = 0.453
```

3.3.3 Longitudinal equilibrium, stability and control

```
Selected input: the aircraft speed is V = 100 m/s with a load factor n = 1.
The pitch rate is q = 0 deg/s.
```

To study longitudinal stability and control of the aircraft, two values are calculated by a system of two equations. The values are the angle of attack and the equilbrium elevator angle by considering the assigned horizontal tail incidence angle i_{H}. The equations are:

1. Vertical Translational Equilibrium;
2. Rotational Equilibrium.

3.3.3.1 Vertical translation equilibrium

The equation is:

$$C_L = C_{L_0} + C_{L_\alpha}\alpha_{\mathrm{B}} + C_{L_{\delta_{\mathrm{e}}}}\delta_{\mathrm{e}} + C_{L_{i_{\mathrm{H}}}} i_{\mathrm{H}} = \frac{W}{\bar{q}_\infty S} \tag{72}$$

where

```
CL0 = 0.192

CLα = 5.486 rad-1 = 0.096 deg-1

CLδe = 0.504 rad-1 = 0.009 deg-1

CLiH = 1.121 rad-1 = 0.020 deg-1
```

3.3.3.2 Rotational equilibrium

The equation is:

$$C_M = C_{M_0} + C_{M_\alpha}\alpha_{\mathrm{B}} + C_{M_{\delta_{\mathrm{e}}}}\delta_{\mathrm{e}} + C_{M_{i_{\mathrm{H}}}} i_{\mathrm{H}} + C_{M_q}\hat{q} + C_{M_{\mathrm{mot}}} = 0 \tag{73}$$

where

```
CM0 = -0.040

CMα = -1.179 rad-1 = -0.021 deg-1

CMδe = -1.450 rad-1 = -0.025 deg-1

CMiH = -3.222 rad-1 = -0.056 deg-1

CMq = -18.526 rad-1 = -0.323 deg-1

CMmot = 0.026
```

3.3.3.3 Solving the system

The values of the angle of attack and the deflection of the elevator are the solution of the following system:

$$\begin{bmatrix} C_{L_\alpha} & C_{L_{\delta_e}} \\ C_{M_\alpha} & C_{M_{\delta_e}} \end{bmatrix} \begin{bmatrix} \alpha_B \\ \delta_e \end{bmatrix} = \begin{bmatrix} C_L - C_{L_0} - C_{L_{i_H}} i_H \\ -C_{M_0} - C_{M_{mot}} - C_{M_{i_H}} i_H - C_{M_q} \hat{q} \end{bmatrix} \tag{74}$$

```
αB = 0.008 rad = 0.469 deg
```

```
δe = 0.099 rad = 5.691 deg
```

Now it is possible to represent the evolution of the pitching moment coefficient vs the body angle of attack.

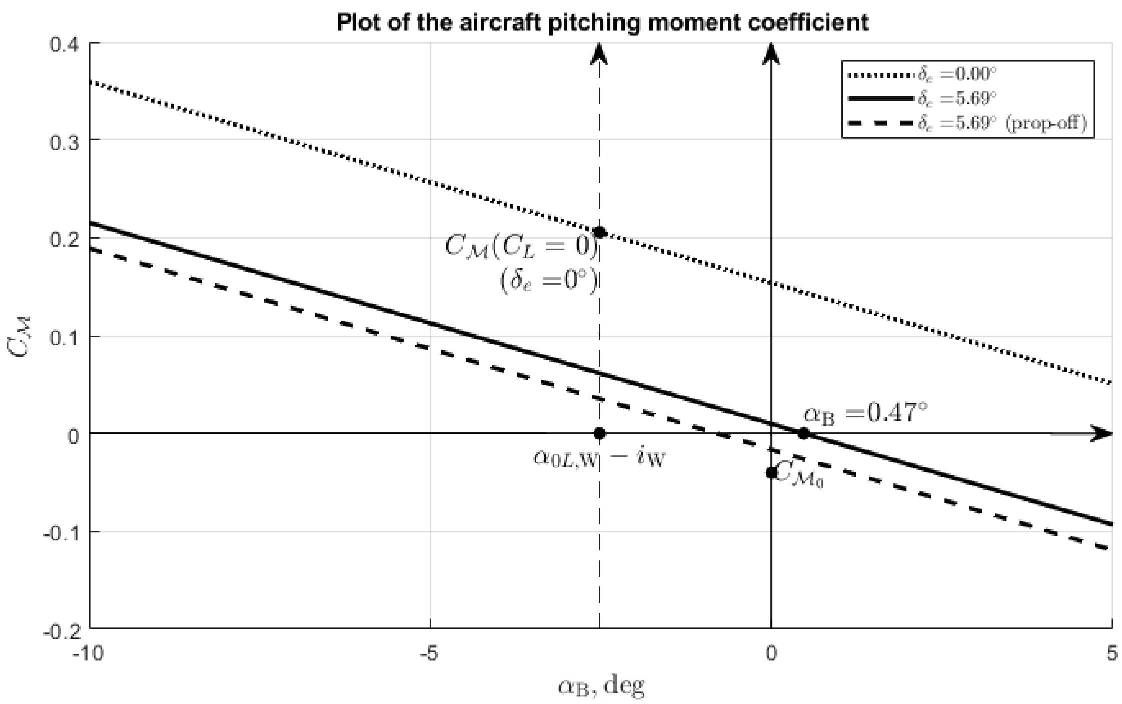


Figure 29: Tecnam P2012 C_M plot

The horizontal tailplane load is:

```
LH = -4025.09 N = -410.45 kgf
```

```
αH,a = -0.023 rad = -1.304 deg
```

3.3.3.4 Neutral points and SMs - Stick-Fixed

The neutral point for stick-fixed condition is the solution of the following equation:

$$C_{M_\alpha} = C_{L_{\alpha,\mathrm{W}}}(\overline{x}_\mathrm{G} - \overline{x}_\mathrm{ac,WB}) - \eta_\mathrm{H}\frac{S_\mathrm{H}}{S}(\overline{x}_\mathrm{ac,H} - \overline{x}_\mathrm{G})\left[1 - \left(\frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right)_\mathrm{H}\right] C_{L_{\alpha,\mathrm{H}}} = 0 \tag{75}$$

By the position $\overline{x}_\mathrm{G} = \overline{x}_\mathrm{N}$:

```
xN = 0.51
```

Now it's possible to study the $SM = \overline{x}_\mathrm{G} - \overline{x}_\mathrm{N}$:

```
SM = -0.21

The aircraft is stable.
```

If the approximated formula is used:

```
xN approx = 0.54

SM approx = -0.24
```

- Stick-Free

In this case it's introduced the free elevator factor:

```
F = 0.76
```

The neutral point for sitck-free condition is the solution of the following equation:

$$C'_{M_\alpha} = C_{L_{\alpha,\mathrm{W}}}(\overline{x}_\mathrm{G} - \overline{x}_\mathrm{ac,WB}) - \eta_\mathrm{H}\frac{S_\mathrm{H}}{S}(\overline{x}_\mathrm{ac,H} - \overline{x}_\mathrm{G})F\left[1 - \left(\frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right)_\mathrm{H}\right] C_{L_{\alpha,\mathrm{H}}} = 0 \tag{76}$$

By the position $\overline{x}_\mathrm{G} = \overline{x}_\mathrm{N}$:

```
xN' = 0.44
```

Now it's possible to study the $SM' = \overline{x}_\mathrm{G} - \overline{x}'_\mathrm{N}$:

```
SM' = -0.14

The aircraft is stable.
```

If the approximated formula is used:

```
xN' approx = 0.46

SM' approx = -0.16
```

3.3.3.5 Summary of all the contributes

	Derivative	Value	Unit
1	'CL0'	0.1921	' '
2	'CLα'	5.4860	'rad-1'
3	'CLδe'	0.5043	'rad-1'
4	'CLiH'	1.1208	'rad-1'
5	'CM0'	-0.0401	' '
6	'CMα'	-1.1788	'rad-1'
7	'CMδe'	-1.4499	'rad-1'
8	'CMiH'	-3.2221	'rad-1'
9	'CMq'	-18.5263	'rad-1'
10	'CMmot'	0.0262	' '

3.3.4 Lateral-Directional stability

```
Selected input: the sideslip angle is Î² = -3 deg with a roll rate p = 0 deg/s.
The yaw rate is r = 0 deg/s.
```

To study lateral-directional stability of the aircraft, two values are calculated by a system of two equations. The values are aileron deflection δ_a and rudder deflection δ_r by considering the assigned side-slip angle β. The equations are:

1. Lateral Stability;
2. Directional Stability.

3.3.4.1 Lateral stability

The equation is:

$$C_{\mathcal{L}} = C_{\mathcal{L}0} + C_{\mathcal{L}\beta}\beta + C_{\mathcal{L}\delta a}\delta_{\mathrm{a}} + C_{\mathcal{L}\delta_{\mathrm{r}}}\delta_{\mathrm{r}} + C_{\mathcal{L}p}\hat{p} + C_{\mathcal{L}r}\hat{r} + C_{\mathcal{L}\mathrm{mot}} \tag{77}$$

where

```
CRoll0 = 0.000

because the aircraft is symmetric.

CRollβ = -0.091 rad-1 = -0.002 deg-1

CRollδa = -0.151 rad-1 = -0.003 deg-1

CRollδr = 0.016 rad-1 = 0.000 deg-1
```

```
CRollp = -0.835 rad-1= -0.015 deg-1

CRollr = 0.035 rad-1= 0.001 deg-1

CRollmot = -0.001
```

CONVENTION**:** The contribution is <0 when clockwise.

3.3.4.2 Directional stability

The equation is:

$$C_{\mathcal{N}} = C_{\mathcal{N}0} + C_{\mathcal{N}\beta}\beta + C_{\mathcal{N}\delta a}\delta_{\mathrm{a}} + C_{\mathcal{N}\delta_{\mathrm{r}}}\delta_{\mathrm{r}} + C_{\mathcal{N}p}\hat{p} + C_{\mathcal{N}r}\hat{r} + C_{\mathcal{N}\mathrm{mot}} \tag{78}$$

where

```
CN0 = 0.000

because the aircraft is symmetric.

CNβ = 0.084 rad-1 = 0.001 deg-1

CNδa = 0.000 rad-1 = 0.000 deg-1

CNδr = -0.092 rad-1 = -0.002 deg-1

CNp = 0.057 rad-1 = 0.001 deg-1

CNr = -0.204 rad-1 = -0.004 deg-1

CNmot = 0.000
```

3.3.4.3 Solving the system

The deflection values of the aileron and the elevator are the unknown values of the following system:

$$\begin{bmatrix} C_{\mathcal{L}\delta a} & C_{\mathcal{L}\delta_{\mathrm{r}}} \\ C_{\mathcal{N}\delta a} & C_{\mathcal{N}\delta_{\mathrm{r}}} \end{bmatrix} \begin{bmatrix} \delta_{\mathrm{a}} \\ \delta_{\mathrm{r}} \end{bmatrix} = \begin{bmatrix} -(C_{\mathcal{L}0} + C_{\mathcal{L}\beta}\beta + C_{\mathcal{L}p}\hat{p} + C_{\mathcal{L}r}\hat{r} + C_{\mathcal{L}\mathrm{mot}}) \\ -(C_{\mathcal{N}0} + C_{\mathcal{N}\beta}\beta + C_{\mathcal{N}p}\hat{p} + C_{\mathcal{N}r}\hat{r} + C_{\mathcal{N}\mathrm{mot}}) \end{bmatrix} \tag{79}$$

```
δa = 0.023 rad = 1.328 deg

δr = -0.048 rad = -2.755 deg
```

3.3.4.4 Summary of all derivatives

	Derivative	Value	Unit
1	'CRoll0'	0	' '
2	'CRollβ'	-0.0909	'rad-1'
3	'CRollδa'	-0.1509	'rad-1'
4	'CRollδr'	0.0157	'rad-1'
5	'CRollp'	-0.8351	'rad-1'
6	'CRollr'	0.0349	'rad-1'
7	'CRollmot'	-0.0005	' '
8	'CN0'	0	' '
9	'CNβ'	0.0842	'rad-1'
10	'CNδa'	0	'rad-1'
11	'CNδr'	-0.0917	'rad-1'
12	'CNp'	0.0573	'rad-1'
13	'CNr'	-0.2038	'rad-1'
14	'CNmot'	0	' '

3.3.5 Lateral-Directional stability in One-Engine-Inoperative condition

The engine that you have chosen fails. The system made by the lateral and directional stability is solved by considering the asymmetrical thrust that acts on the working engine.

3.3.5.1 Asymmetrical thrust

```
T = 5165.35 N = 526.54 kg
```

3.3.5.2 Lateral equilibrium

The equation is:

$$C_{\mathcal{L}} = C_{\mathcal{L}0} + C_{\mathcal{L}\beta}\beta + C_{\mathcal{L}\delta a}\delta_{\mathrm{a}} + C_{\mathcal{L}\delta_{\mathrm{r}}}\delta_{\mathrm{r}} + C_{\mathcal{L}p}\hat{p} + C_{\mathcal{L}r}\hat{r} + C_{\mathcal{L}\mathrm{mot}} \tag{80}$$

where

```
CRoll0 = 0.000

because the aircraft is symmetric.

CRollβ = -0.091 rad-1 = -0.002 deg-1

CRollδa = -0.151 rad-1 = -0.003 deg-1
```

```
CRollδr = 0.016 rad-1 = 0.000 deg-1

CRollp = -0.835 rad-1 = -0.015 deg-1

CRollr = 0.035 rad-1 = 0.001 deg-1

CRollmot = -0.001
```

3.3.5.3 Directional equilibrium

The equation is:

$$C_{\mathcal{N}} = C_{\mathcal{N}0} + C_{\mathcal{N}\beta}\beta + C_{\mathcal{N}\delta a}\delta_{\mathrm{a}} + C_{\mathcal{N}\delta_{\mathrm{r}}}\delta_{\mathrm{r}} + C_{\mathcal{N}p}\hat{p} + C_{\mathcal{N}r}\hat{r} + C_{\mathcal{N}\mathrm{mot}} \tag{81}$$

where

```
CN0 = 0.000

because the aircraft is symmetric.

CNβ = 0.084 rad-1 = 0.001 deg-1

CNδa = 0.000 rad-1 = 0.000 deg-1

CNδr = -0.092 rad-1 = -0.002 deg-1

CNp = 0.057 rad-1 = 0.001 deg-1

CNr = -0.204 rad-1 = -0.004 deg-1
```

$C_{\mathcal{N}\mathrm{mot}} = \frac{T d_{\mathrm{mot}}}{q_\infty \mathrm{Sb}} \neq 0$ because there's a yaw moment due to the asymmetric thrust with its arm d_{mot}. The sign depends on which engine fails. Usually when the left engine fails $C_{\mathcal{N}\mathrm{mot}} < 0$, because T causes $N < 0$. Instead when the right engine fails $C_{\mathcal{N}\mathrm{mot}} > 0$, because T causes $N > 0$.

```
CNmot = -0.004
```

3.3.5.4 Solving the system

The deflection values of the aileron and the elevator are the unknown values of the following system:

$$\begin{bmatrix} C_{\mathcal{L}\delta a} & C_{\mathcal{L}\delta_{\mathrm{r}}} \\ C_{\mathcal{N}\delta a} & C_{\mathcal{N}\delta_{\mathrm{r}}} \end{bmatrix} \begin{bmatrix} \delta_{\mathrm{a}} \\ \delta_{\mathrm{r}} \end{bmatrix} = \begin{bmatrix} -(C_{\mathcal{L}0} + C_{\mathcal{L}\beta}\beta + C_{\mathcal{L}p}\hat{p} + C_{\mathcal{L}r}\hat{r} + C_{\mathcal{L}\mathrm{mot}}) \\ -(C_{\mathcal{N}0} + C_{\mathcal{N}\beta}\beta + C_{\mathcal{N}p}\hat{p} + C_{\mathcal{N}r}\hat{r} + C_{\mathcal{N}\mathrm{mot}}) \end{bmatrix} \tag{82}$$

```
δa = 0.019 rad = 1.100 deg

δr = -0.086 rad = -4.947 deg
```

3.3.5.5 Summary of all derivatives

	Derivative	Value	Unit
1	'CRoll0'	0	' '
2	'CRollβ'	-0.0909	'rad-1'
3	'CRollδa'	-0.1509	'rad-1'
4	'CRollδr'	0.0157	'rad-1'
5	'CRollp'	-0.8351	'rad-1'
6	'CRollr'	0.0349	'rad-1'
7	'CRollmot'	-0.0005	' '
8	'CN0'	0	' '
9	'CNβ'	0.0842	'rad-1'
10	'CNδa'	0	'rad-1'
11	'CNδr'	-0.0917	'rad-1'
12	'CNp'	0.0573	'rad-1'
13	'CNr'	-0.2038	'rad-1'
14	'CNmot'	-0.0035	' '

3.4 Tecnam P2006T

AIRCRAFT STABILITY AND CONTROL

A code that analyses the stability and the control of the chosen aircraft has been implemented below.

3.4.1 Input Data

Select the following parameters:

1. **Speed**.

2. **Load factor**.

3. **Pitch rate**.

4. **Beta**.

5. **Roll rate**.

6. **Yaw rate**.

7. **OEI** : which engine failure to study.

8. **Propeller rotation direction** (if exists).

Input data are here below summarized:

```
AIRCRAFT NAME: Tecnam P2006T
```

Figure 30: Tecnam P2006T

Geometry

```
WING: b = 11.40 m | cr = 1.44 m | ct = 1.15 m | iW = 0.026 rad |
      Λ = 0.000 rad | Γ = 0.000 rad

HORIZONTAL TAILPLANE: bH = 3.14 m | SH = 1.98 m2 | lH = 5.40 m |
                      iH = -0.035 rad

VERTICAL TAILPLANE: SV = 1.17 m2 | lV = 7.00 m | hV = 1.20 m

ENGINE: d_mot = 1.60 m | Xt = 1.60 m | Zt = 1.60 m | D = 1.78 m
```

Aerodynamics

```
GLOBAL: CD0 = 0.03 | e = 0.80 | xcg = 0.33 | CRoll_0 = 0.00 | CN_0 = 0.00

WING: xac_W = 0.29 | Cl_alphaW = 5.70 rad-1 | alpha_0l_W = -0.01 rad |
      eW = 0.88 | CM_ac_W = -0.068 rad-1 | CRoll_p_W = -0.013 |
      CRoll_delta_a = -0.005 rad-1 | CRoll_beta_wpos = 0.000 rad-1 |
      CN_delta_a = 0.000 rad-1  CN_r_W = 0.000 | τa = 0.40 |
      CN_beta_W = 0.000 rad-1 | CN_p_Wtilt = 0.000 rad-1 |
      CN_p_W_tipsuc = 0.000 rad-1 | CN_r_W = 0.000 rad-1
```

```
HORIZONTAL TAILPLANE: Cl_alphaH = 6.300 rad-1 | eH = 0.90 | ηH = 1.00 |
                      τe = 0.38 | CH_alpha_e = -0.004 rad-1 |
                      CH_deltaE_e = -0.011 rad-1

VERTICAL TAILPLANE: CL_alphaV = 2.800 rad-1 | ηV = 1.00 | dσ/dβ = 0.150 |
                    τr = 0.45 | CN_p_V = 0.000 | CN_r_V = 0.000 |
                    dσ/dp = 0.000 | dσ/dr = 0.000

FUSELAGE: CM_0_B = -0.060 | CM_alpha_B(W) = 0.144 rad-1 |
          CN_beta_B = -0.000 rad-1
```

Powerplant

```
POWERPLANT: Engines number = 2 | Pa = 74569.99 W | Np = 11.00 kgf |
            J = 1.24 s-1
```

3.4.2 Preliminary calculations

Preliminary calculations used in the entire code are shown below.

- Taper ratio

```
λ = 0.80
```

- Wing area

```
S = 14.76 m²
```

- Dynamic pressure

```
qinf = 6125.00 Pa
```

- Mean aerodynamic chord

```
m.a.c. = 1.30 m
```

- Lift coefficient

```
CL = 0.13
```

- Aspect ratio (wing and horizontal tail, respectively)

```
AR = 8.80
```

```
ARH = 4.98
```

- Wing lift curve slope

```
CLα,W = 4.618 rad-1 = 0.081 deg-1
```

- Wing-Body's lift coefficient at zero lift

```
CL0,W = 0.18
```

- Horizontal tail lift curve slope

```
CLα,H = 4.352 rad-1 = 0.076 deg-1
```

- Mean value of downwash at $\alpha_{\mathrm{B}} = 0$

```
ε0 = 0.015 rad = 0.870 deg
```

- Downwash gradient

```
dε/dα = 0.380
```

3.4.3 Longitudinal equilibrium, stability and control

```
Selected input: the aircraft speed is V = 100 m/s with a load factor n = 1.
The pitch rate is q = 0 deg/s.
```

To study longitudinal stability and control of the aircraft, two values are calculated by a system of two equations. The values are the angle of attack and the equilbrium elevator angle by considering the assigned horizontal tail incidence angle i_{H}. The equations are:

1. Vertical Translational Equilibrium;
2. Rotational Equilibrium.

3.4.3.1 Vertical translation equilibrium

The equation is:

$$C_L = C_{L_0} + C_{L_\alpha}\alpha_{\mathrm{B}} + C_{L_{\delta_{\mathrm{e}}}}\delta_{\mathrm{e}} + C_{L_{i_{\mathrm{H}}}} i_{\mathrm{H}} = \frac{W}{\bar{q}_\infty S} \tag{83}$$

where

```
CL0 = 0.176

CLα = 4.981 rad-1 = 0.087 deg-1

CLδe = 0.222 rad-1 = 0.004 deg-1

CLiH = 0.584 rad-1 = 0.010 deg-1
```

3.4.3.2 Rotational equilibrium

The equation is:

$$C_M = C_{M_0} + C_{M_\alpha}\alpha_{\mathrm{B}} + C_{M_{\delta_\mathrm{e}}}\delta_{\mathrm{e}} + C_{M_{i_\mathrm{H}}}i_{\mathrm{H}} + C_{M_q}\hat{q} + C_{M_{\mathrm{mot}}} = 0 \tag{84}$$

where

```
CM0 = -0.078

CMα = -1.175 rad-1 = -0.021 deg-1

CMδe = -0.921 rad-1 = -0.016 deg-1

CMiH = -2.424 rad-1 = -0.042 deg-1

CMq = -20.132 rad-1 = -0.351 deg-1

CMmot = 0.034
```

3.4.3.3 Solving the system

The values of the angle of attack and the deflection of the elevator are the solution of the following system:

$$\begin{bmatrix} C_{L_\alpha} & C_{L_{\delta_\mathrm{e}}} \\ C_{M_\alpha} & C_{M_{\delta_\mathrm{e}}} \end{bmatrix} \begin{bmatrix} \alpha_\mathrm{B} \\ \delta_\mathrm{e} \end{bmatrix} = \begin{bmatrix} C_L - C_{L0} - C_{L_{i_\mathrm{H}}} i_\mathrm{H} \\ -C_{M_0} - C_{M_{\mathrm{mot}}} - C_{M_{i_\mathrm{H}}} i_\mathrm{H} - C_{M_q}\hat{q} \end{bmatrix} \tag{85}$$

```
αB = -0.007 rad = -0.389 deg

δe = 0.053 rad = 3.056 deg
```

Now it is possible to represent the evolution of the pitching moment coefficient vs the body angle of attack.

The horizontal tailplane load is:

```
LH = -1801.34 N = -183.69 kgf

αH,a = -0.034 rad = -1.955 deg
```

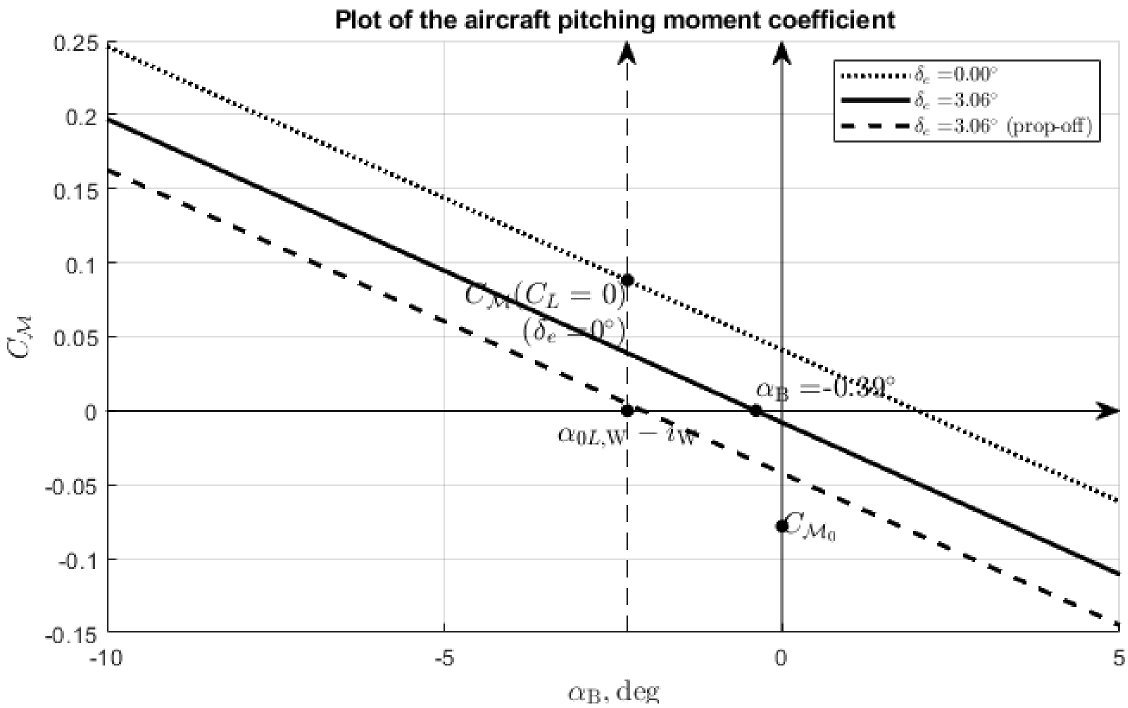


Figure 31: Tecnam P2006T C_M plot

3.4.3.4 Neutral points and SMs - Stick-Fixed

The neutral point for stick-fixed condition is the solution of the following equation:

$$C_{M_\alpha} = C_{L_{\alpha,\mathrm{W}}}(\overline{x}_\mathrm{G} - \overline{x}_\mathrm{ac,WB}) - \eta_\mathrm{H}\frac{S_\mathrm{H}}{S}(\overline{x}_\mathrm{ac,H} - \overline{x}_\mathrm{G})\left[1 - \left(\frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right)_\mathrm{H}\right]C_{L_{\alpha,\mathrm{H}}} = 0 \tag{86}$$

By the position $\overline{x}_\mathrm{G} = \overline{x}_\mathrm{N}$:

```
xN = 0.57
```

Now it's possible to study the $SM = \overline{x}_\mathrm{G} - \overline{x}_\mathrm{N}$:

```
SM = -0.24

The aircraft is stable.
```

If the approximated formula is used:

```
xN approx = 0.58

SM approx = -0.25
```

- Stick-Free

In this case it's introduced the free elevator factor:

```
F = 0.87
```

The neutral point for sitck-free condition is the solution of the following equation:

$$C'_{M_\alpha} = C_{L_{\alpha,\mathrm{W}}}(\overline{x}_\mathrm{G} - \overline{x}_\mathrm{ac,WB}) - \eta_\mathrm{H}\frac{S_\mathrm{H}}{S}(\overline{x}_\mathrm{ac,H} - \overline{x}_\mathrm{G})F\left[1 - \left(\frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right)_\mathrm{H}\right]C_{L_{\alpha,\mathrm{H}}} = 0 \qquad (87)$$

By the position $\overline{x}_\mathrm{G} = \overline{x}_\mathrm{N}$:

```
xN' = 0.53
```

Now it's possible to study the $SM' = \overline{x}_\mathrm{G} - \overline{x}'_\mathrm{N}$:

```
SM' = -0.20

The aircraft is stable.
```

If the approximated formula is used:

```
xN' approx = 0.54

SM' approx = -0.21
```

3.4.3.5 Summary of all the contributes

	Derivative	Value	Unit
1	'CL0'	0.1759	' '
2	'CLα'	4.9805	'rad-1'
3	'CLδe'	0.2218	'rad-1'
4	'CLiH'	0.5837	'rad-1'
5	'CM0'	-0.0781	' '
6	'CMα'	-1.1753	'rad-1'
7	'CMδe'	-0.9211	'rad-1'
8	'CMiH'	-2.4240	'rad-1'
9	'CMq'	-20.1317	'rad-1'
10	'CMmot'	0.0344	' '

3.4.4 Lateral-Directional stability

```
Selected input: the sideslip angle is Î² = -3 deg with a roll rate p = 0 deg/s.
The yaw rate is r = 0 deg/s.
```

To study lateral-directional stability of the aircraft, two values are calculated by a system of two equations. The values are aileron deflection δ_a and rudder deflection δ_r by considering the assigned side-slip angle β. The equations are:

1. Lateral Stability;
2. Directional Stability.

3.4.4.1 Lateral stability

The equation is:

$$C_{\mathcal{L}} = C_{\mathcal{L}0} + C_{\mathcal{L}\beta}\beta + C_{\mathcal{L}\delta a}\delta_{\mathrm{a}} + C_{\mathcal{L}\delta_{\mathrm{r}}}\delta_{\mathrm{r}} + C_{\mathcal{L}p}\hat{p} + C_{\mathcal{L}r}\hat{r} + C_{\mathcal{L}\mathrm{mot}} \tag{88}$$

where

```
CRoll0 = 0.000

because the aircraft is symmetric.

CRollβ = -0.020 rad-1 = -0.000 deg-1

CRollδa = -0.159 rad-1 = -0.003 deg-1

CRollδr = 0.011 rad-1 = 0.000 deg-1

CRollp = -0.778 rad-1= -0.014 deg-1

CRollr = 0.029 rad-1= 0.001 deg-1

CRollmot = -0.0003
```

CONVENTION**:** The contribution is <0 when clockwise.

3.4.4.2 Directional stability

The equation is:

$$C_{\mathcal{N}} = C_{\mathcal{N}0} + C_{\mathcal{N}\beta}\beta + C_{\mathcal{N}\delta a}\delta_{\mathrm{a}} + C_{\mathcal{N}\delta_{\mathrm{r}}}\delta_{\mathrm{r}} + C_{\mathcal{N}p}\hat{p} + C_{\mathcal{N}r}\hat{r} + C_{\mathcal{N}\mathrm{mot}} \tag{89}$$

where

```
CN0 = 0.000

because the aircraft is symmetric.

CNβ = 0.116 rad-1 = 0.002 deg-1

CNδa = 0.000 rad-1 = 0.000 deg-1

CNδr = -0.061 rad-1 = -0.001 deg-1

CNp = 0.039 rad-1 = 0.001 deg-1

CNr = -0.167 rad-1 = -0.003 deg-1

CNmot = 0.000
```

3.4.4.3 Solving the system

The deflection values of the aileron and the elevator are the unknown values of the following system:

$$\begin{bmatrix} C_{\mathcal{L}\delta a} & C_{\mathcal{L}\delta_{\mathrm{r}}} \\ C_{\mathcal{N}\delta a} & C_{\mathcal{N}\delta_{\mathrm{r}}} \end{bmatrix} \begin{bmatrix} \delta_{\mathrm{a}} \\ \delta_{\mathrm{r}} \end{bmatrix} = \begin{bmatrix} -(C_{\mathcal{L}0} + C_{\mathcal{L}\beta}\beta + C_{\mathcal{L}p}\hat{p} + C_{\mathcal{L}r}\hat{r} + C_{\mathcal{L}\mathrm{mot}}) \\ -(C_{\mathcal{N}0} + C_{\mathcal{N}\beta}\beta + C_{\mathcal{N}p}\hat{p} + C_{\mathcal{N}r}\hat{r} + C_{\mathcal{N}\mathrm{mot}}) \end{bmatrix} \tag{90}$$

```
δa = -0.002 rad = -0.091 deg


δr = -0.099 rad = -5.657 deg
```

3.4.4.4 Summary of all derivatives

	Derivative	Value	Unit
1	'CRoll0'	0	' '
2	'CRollβ'	-0.0199	'rad-1'
3	'CRollδa'	-0.1587	'rad-1'
4	'CRollδr'	0.0105	'rad-1'
5	'CRollp'	-0.7779	'rad-1'
6	'CRollr'	0.0287	'rad-1'
7	'CRollmot'	-0.0003	' '
8	'CN0'	0	' '
9	'CNβ'	0.1156	'rad-1'
10	'CNδa'	0	'rad-1'
11	'CNδr'	-0.0613	'rad-1'
12	'CNp'	0.0387	'rad-1'
13	'CNr'	-0.1673	'rad-1'
14	'CNmot'	0	' '

3.4.5 Lateral-Directional stability in One-Engine-Inoperative condition

The engine that you have chosen fails. The system made by the lateral and directional stability is solved by considering the asymmetrical thrust that acts on the working engine.

3.4.5.1 Asymmetrical thrust

```
T = 2514.21 N = 256.29 kg
```

3.4.5.2 Lateral equilibrium

The equation is:

$$C_{\mathcal{L}} = C_{\mathcal{L}0} + C_{\mathcal{L}\beta}\beta + C_{\mathcal{L}\delta a}\delta_{\mathrm{a}} + C_{\mathcal{L}\delta_{\mathrm{r}}}\delta_{\mathrm{r}} + C_{\mathcal{L}p}\hat{p} + C_{\mathcal{L}r}\hat{r} + C_{\mathcal{L}\mathrm{mot}} \tag{91}$$

where

```
CRoll0 = 0.000

because the aircraft is symmetric.

CRollβ = -0.020 rad-1 = -0.000 deg-1

CRollδa = -0.159 rad-1 = -0.003 deg-1

CRollδr = 0.011 rad-1 = 0.000 deg-1

CRollp = -0.778 rad-1 = -0.014 deg-1

CRollr = 0.029 rad-1 = 0.001 deg-1

CRollmot = -0.0003
```

3.4.5.3 Directional equilibrium

The equation is:

$$C_{\mathcal{N}} = C_{\mathcal{N}0} + C_{\mathcal{N}\beta}\beta + C_{\mathcal{N}\delta a}\delta_{\mathrm{a}} + C_{\mathcal{N}\delta_{\mathrm{r}}}\delta_{\mathrm{r}} + C_{\mathcal{N}p}\hat{p} + C_{\mathcal{N}r}\hat{r} + C_{\mathcal{N}\mathrm{mot}} \tag{92}$$

where

```
CN0 = 0.000

because the aircraft is symmetric.

CNβ = 0.116 rad-1 = 0.002 deg-1

CNδa = 0.000 rad-1 = 0.000 deg-1

CNδr = -0.061 rad-1 = -0.001 deg-1
```

```
CNp = 0.039 rad-1 = 0.001 deg-1

CNr = -0.167 rad-1 = -0.003 deg-1
```

$C_{\mathcal{N}\text{mot}} = \frac{T d_{\text{mot}}}{q_\infty \text{Sb}} \neq 0$ because there's a yaw moment due to the asymmetric thrust with its arm d_{mot}. The sign depends on which engine fails. Usually when the left engine fails $C_{\mathcal{N}\text{mot}} < 0$, because T causes $N < 0$. Instead when the right engine fails $C_{\mathcal{N}\text{mot}} > 0$, because T causes $N > 0$.

```
CNmot = -0.004
```

3.4.5.4 Solving the system

The deflection values of the aileron and the elevator are the unknown values of the following system:

$$\begin{bmatrix} C_{\mathcal{L}\delta a} & C_{\mathcal{L}\delta_\text{r}} \\ C_{\mathcal{N}\delta a} & C_{\mathcal{N}\delta_\text{r}} \end{bmatrix} \begin{bmatrix} \delta_\text{a} \\ \delta_\text{r} \end{bmatrix} = \begin{bmatrix} -(C_{\mathcal{L}0} + C_{\mathcal{L}\beta}\beta + C_{\mathcal{L}p}\hat{p} + C_{\mathcal{L}r}\hat{r} + C_{\mathcal{L}\text{mot}}) \\ -(C_{\mathcal{N}0} + C_{\mathcal{N}\beta}\beta + C_{\mathcal{N}p}\hat{p} + C_{\mathcal{N}r}\hat{r} + C_{\mathcal{N}\text{mot}}) \end{bmatrix} \tag{93}$$

```
δa = -0.006 rad = -0.332 deg

δr = -0.162 rad = -9.304 deg
```

3.4.5.5 Summary of all derivatives

	Derivative	Value	Unit
1	'CRoll0'	0	' '
2	'CRollβ'	-0.0199	'rad-1'
3	'CRollδa'	-0.1587	'rad-1'
4	'CRollδr'	0.0105	'rad-1'
5	'CRollp'	-0.7779	'rad-1'
6	'CRollr'	0.0287	'rad-1'
7	'CRollmot'	-0.0003	' '
8	'CN0'	0	' '
9	'CNβ'	0.1156	'rad-1'
10	'CNδa'	0	'rad-1'
11	'CNδr'	-0.0613	'rad-1'
12	'CNp'	0.0387	'rad-1'
13	'CNr'	-0.1673	'rad-1'
14	'CNmot'	-0.0039	' '

Bibliography

[1] John N. Little, Cleve Moler, Steven Bangert. Mathworks, 1984.

[2] Prof. Agostino De Marco, Dott. Danilo Ciliberti. Course notes in aircraft stability and control.

[3] Jan Roskam. *Airplane Flight Dynamics and Automatic Flight Controls, Part 1*. DARcorporation, 1995.