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Stability and control analysis of aircraft with MATLAB live script

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Abstract

The purpose of this work is to perform stability and control analysis of different Aircraft with MATLAB live script. MATLAB live script is a file which contains numerous sequential lines of commands and function calls. Aircraft with different geometric and propulsive characteristics are taken into consideration. After cataloguing the data and defining the functions, analyses were carried out concerning: longitudinal and lateral-directional equilibrium, stability, and control. The chosen aircraft are: Cessna 210, Tecnam P92, Tecnam P-2012, Boeing 737, Boeing 777, Beechcraft KingAir, Dornier Do 328. The final results are presented as summary data.

Sommario

Il lavoro di tesi è finalizzato a studiare l'analisi di stabilità e controllo di aerei tramite l'utilizzo del software MATLAB live script, un file che contiene numerose righe sequenziali di comandi e richiami di funzioni di MATLAB. Sono presi in considerazione aerei con caratteristiche geometriche e propulsive differenti. Dopo aver catalogato i dati e definito le funzioni, sono state svolte le analisi riguardanti: equilibrio, stabilità e controllo longitudinale e latero-direzionale. Gli aerei selezionati per tale analisi sono: Cessna 210, Tecnam P92, Tecnam P-2012, Boeing 737, Boeing 777, Beechcraft King Air, Dornier Do 328. I risultati finali sono presentati come sommario dati.

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Introduction

The analysis of aircraft stability and control is done with MATLAB. MATLAB is a versatile platform widely used in engineering and scientific fields for system analysis, design, and numerical computations. Within MATLAB, you have the choice between two primary project formats: script files and live scripts.

1. **Scripts:** These are sequences of MATLAB commands saved in a file. They are used for running a series of calculations or processes.

2. **Live Scripts:** These provide an interactive environment where you can combine code with formatted text, equations, images, and more. They offer a more narrative way to present and document your analysis, making it easier for others to follow your work.

Each chapter focuses on specific aspects like:

- **Longitudinal Stability**
- **Lateral-Directional Stability**

The aircraft selected and analysed are grouped by engine type:

- Cessna 210 } Pistons
- Tecnam P92 } Pistons

- Tecnam P-2012 → Turbocharged

- Boeing 737 } Turbofan
- Boeing 777 } Turbofan

- Beechcraft King Air } Turboprop
- Dornier Do 328 } Turboprop

The user is allowed to change in the live script only the initial parameters as: speed, load factor, pitch rate, beta, roll rate, yaw rate. The live script contains MATLAB call-up functions for specific calculations such as of aerodynamic moments, coefficients or derivatives.

Chapter 1 – MATLAB live script code

1. Code description

The code that analyses the stability and the control of the chosen aircraft has been divided in different parts. The first step is to choose the type of aircraft powerplant:

- Pistons
- Turbocharged
- Turbofan
- Turboprop

After that, the user must choose a specific aircraft among those available:

- Cessna 210
- Tecnam P92
- Tecnam P-2012
- Boeing 737
- Boeing 777
- Beechcraft King Air
- Dornier Do 328

The interesting part of the code is that you can select a specific parameter:

1. **Speed** to study the aircraft stability and control. You can change the speed if you want to compare different flight phases.

```
AllData.Selected.Selected_Speed.Speed.V.value=100;
AllData.Selected.Selected_Speed.Speed.V.Attribute.unit="m/s";
```

2. **Load factor** for longitudinal equilibrium, stability and control. It changes the CL value and so the deflections will change.

```
AllData.Selected.Selected_Load_Factor.value= 1;
AllData.Selected.Selected_Load_Factor.Attribute.unit=" ";
```

3. **Pitch rate** to study dynamic and unsteady longitudinal derivatives.

```
AllData.Selected.Selected_Speed.Rates.Selected_q.value=0;
```

```
AllData.Selected.Selected_Speed.Rates.Selected_q.Attribute.unit="deg/s";
```

4. **Beta**: it's the sideslip angle to study lateral-directional stability system.

```
AllData.Selected.Selected_Beta.value=-3;
AllData.Selected.Selected_Beta.Attribute.unit="deg";
```

5. **Roll rate** to study dynamic and unsteady lateral derivatives.

```
AllData.Selected.Selected_Speed.Rates.Selected_p.value=0;
AllData.Selected.Selected_Speed.Rates.Selected_p.Attribute.unit="deg/s";
```

6. **Yaw rate** to study dynamic and unsteady directional derivatives.

```
AllData.Selected.Selected_Speed.Rates.Selected_r.value=0;
AllData.Selected.Selected_Speed.Rates.Selected_r.Attribute.unit="deg/s";
```

7. **OEI**: if there are two ore more engines, it is assumed that one engine fails. Choose which engine has to fail to study lateral-directional stability in one engine inoperative condition. It is important to choose because it will change the engine contributes to the equations.

```
AllData.Selected.Selected_Engine.value= '0';
AllData.Selected.Selected_Engine.Attribute.unit=" ";
```

8. **Propeller rotation direction** for engine contributes (if exists).

```
switch AllData.Aircraft.Data.Powerplant.Type.value
   case 'TP'
      propeller_rot=["clockwise","anti-clockwise"];
   case 'TC'
      propeller_rot=["clockwise","anti-clockwise"];
   case 'P'
      propeller_rot=["clockwise","anti-clockwise"];
   otherwise
      propeller_rot=["NO REQUIRED"," "];
end

AllData.Selected.Selected_Propeller.value= 'propeller_rot(1)';
AllData.Selected.Selected_Propeller.Attribute.unit=" ";
```

1.1 Data input

There is a summary of the data input:

- Geometry
- Aerodynamics
- Powerplant

In order to have correct values of stability and control analysis, it is important the data collection.

Data such as wing span, height, weight are easily found on the web or books such as “Jane's-All the World's Aircraft”. To collect more precise geometric data, one can use software such as OpenVSP, open-source software used to create detailed 3D models of aircraft, assisting engineers in analysing their designs. To also have Aerodynamics Data, you can use one of the tools provided by the software, specifically the VSPAERO tool, it utilizes a swift and linear lattice-based vortex solver to simulate airflow around the aircraft, providing crucial information about lift, drag, and other aerodynamic forces. This combined use of OpenVSP and VSPAERO allows engineers to refine and optimize aircraft designs for enhanced performance and efficiency through iterative modelling and analysis. To make sure the data was correct, the following were also taken into account: the notes from the Flight Mechanics course.

All data were collected in a separate folder: **Aircraft_Data**

In which folder there are four other folders:

- Pistons
- Turbocharged
- Turbofan
- Turboprop

In each folder are the selected aircraft related to that specific category.
The MATLAB code in which the data is written is implemented directly in the live script.

1.2 Preliminary calculations

For preliminary calculations, you can use a previously written MATLAB code.

```
AllData= P2_aeroFun_preCalc(AllData);
```

The aim is to give the following output:

- Taper ratio
- Wing area
- Dynamic pressure
- Mean aerodynamic chord
- Lift coefficient
- Aspect ratio (wing and horizontal tail, respectively)
- Wing lift curve slope
- Wing-Body’s lift coefficient at zero lift
- Horizontal tail lift curve slope
- Mean value of downwash at $\alpha_{\mathrm{B}} = 0$
- Downwash gradient

1.3 Longitudinal equilibrium, stability and control

Studying the longitudinal stability and control of an aircraft involves assessing how it behaves in pitch, specifically its tendency to return to a stable position after a disturbance. The angle of attack and the equilibrium elevator angle play crucial roles in this analysis. The equilibrium elevator angle is the angle at which the elevator needs to be positioned to maintain a steady flight at a particular angle of attack. This angle is essential for balancing the forces acting on the aircraft to maintain its desired flight path. The horizontal tail incidence angle is also significant. It affects the aircraft's stability by influencing the relationship between the main wing and the horizontal stabilizer. To calculate these values, a system of equations is typically used, considering various aerodynamic and design parameters of the aircraft. These equations are derived from the aircraft's aerodynamic properties, control surfaces, and the desired flight characteristics. The equations are:

1. Vertical Translational Equilibrium
2. Rotational Equilibrium

Recall a function of MATLAB, previously written:

```
AllData= P3_aeroFun_longEqComplete(AllData);
```

1.3.1 Vertical Translational Equilibrium

The equation is:

$$C_L = C_{L_0} + C_{L_\alpha}\alpha_{\mathrm{B}} + C_{L_{\delta_{\mathrm{e}}}}\delta_{\mathrm{e}} + C_{L_{i_{\mathrm{H}}}} i_{\mathrm{H}} = \frac{W}{\bar{q}_\infty S} \tag{1.1}$$

where

$$C_{L_0} = C_{\mathrm{L0},W} - \eta_H \frac{S_H}{S} C_{L_{\alpha,H}} \varepsilon_0 \tag{1.2}$$

This contribution comes from the effects of the wing and the horizontal tailplane.

$$C_{L_\alpha} = C_{L_{\alpha,W}} + \eta_H \frac{S_H}{S} C_{L_{\alpha,H}} \left[1 - \left(\frac{\mathrm{d}}{\mathrm{d}\alpha}\varepsilon\right)_H\right] \tag{1.3}$$

The above is the lift curve slope of the airplane.

$$C_{L_{\delta e}} = \eta_H \frac{S_H}{S} C_{L_{\alpha,H}} \tau_e \tag{1.4}$$

The above is a control derivative that comes from the effect of the horizontal tailplane elevator.

$$\tau_e = \frac{\mathrm{d}}{\mathrm{d}\delta_e} |\alpha_{0L_H}| \tag{1.5}$$

The above is the elevator effectiveness and it represents the α_{0L_H} variation for each δ_e degree.

$$C_{L_{\mathrm{iH}}} = \eta_H \frac{S_H}{S} C_{L_{\alpha,H}} \tag{1.6}$$

The last one is a control derivative that comes from the horizontal stabilizer incidence i_H.

1.3.2 Rotational equilibrium

The equation is:

$$C_M = C_{M_0} + C_{M_\alpha}\alpha_B + C_{M_{\delta_e}}\delta_e + C_{M_{i_H}} i_H + C_{M_q}\hat{q} + C_{M_{mot}} = 0 \quad (1.7)$$

where

$$C_{M_0} = C_{M,ac_{WB}} + C_{L0_W}\left(\bar{x}_G - \bar{x}_{ac_{WB}}\right) + \eta_H \frac{S_H}{S}\left(\bar{x}_{ac_H} - \bar{x}_G\right) C_{L\alpha_H}\varepsilon_0 \quad (1.8)$$

it is a global contribution. Through the equilibrability criterion:

$$C_{M_{|C_L=0}} > 0 \quad (1.9)$$

it is possible to study the aircraft is balanceable.

$$C_{M_\alpha} = C_{L\alpha_W}\left(\bar{x}_G - \bar{x}_{ac_{WB}}\right) - \eta_H \frac{S_H}{S}\left(\bar{x}_{ac_H} - \bar{x}_G\right)\left[1 - \left(\frac{d}{d\alpha}\varepsilon\right)_H\right] C_{L\alpha_H} \quad (1.10)$$

The above is a stability derivative and it is called pitch stiffness. Its sign is analysed to have stable static equilibrium in pitch. For an airplane to be statically stable in rotation any disturbances in pitch must be defeated by the production of a restoring moment that will restore the equilibrium condition. Thus, the mathematical criterion for pitch stability is:

$$\frac{\partial}{\partial\alpha} C_M < 0 \quad (1.11)$$

$$C_{M_{\delta e}} = -\eta_H \frac{S_H}{S}\left(\bar{x}_{ac_H} - \bar{x}_G\right) C_{L\alpha_H}\tau_e \quad (1.12)$$

The above is a control derivative that comes from the effect of the elevator deflection. It is also called *elevator control power*.

$$C_{M_{iH}} = -\eta_H \frac{S_H}{S}\left(\bar{x}_{ac_H} - \bar{x}_G\right) C_{L\alpha_H} \quad (1.13)$$

The above is a control derivative that comes from the horizontal stabilizer incidence i_H.

$$C_{M_q} = -2\eta_H \frac{S_H}{S}\left(\bar{x}_{ac_H} - \bar{x}_G\right) C_{L\alpha_H}\left(\bar{x}_{ac_H} - \bar{x}_G\right) \quad (1.14)$$

The above is the pitch damping derivative. When $C_{M_q} < 0$, the aircraft damps out the oscillations in pitch caused by $q \neq 0$. The following are the propulsive effects on longitudinal rotational equilibrium:

$$C_{M_{mot}} = C_{M_{mot_{Np}}} + C_{M_{mot_T}} \quad (1.15)$$

$$C_{M_{mot_{Np}}} = \frac{N_p X_T}{q_\infty S \bar{c}} \quad (1.16)$$

$$C_{M_{\mathrm{mot}_T}} = \frac{TZ_T}{q_\infty S\bar{c}} \tag{1.17}$$

where N_p is the propeller normal force (if exists), X_T is the force arm relative to the center of gravity, T is the thrust and Z_T is the force arm relative to the center of gravity.

1.3.3 Solving the system

The values of the angle of attack and the deflection of the elevator are the solution of the following system:

$$\begin{bmatrix} C_{L_\alpha} & C_{L_{\delta e}} \\ C_{M_\alpha} & C_{M_{\delta e}} \end{bmatrix} \begin{bmatrix} \alpha_B \\ \delta_e \end{bmatrix} = \begin{bmatrix} C_L - C_{\mathrm{L0}} - C_{L_{\mathrm{iH}}} i_H \\ -C_{\mathrm{M0}} - C_{M_{\mathrm{mot}}} - C_{M_{\mathrm{iH}}} i_H - C_{M_q}\hat{q} \end{bmatrix} \tag{1.18}$$

Now it is possible to represent the evolution of the pitching moment coefficient vs the body angle of attack. Calling the MATLAB function:

```
P3_aeroFun_cmPlot(AllData);
```

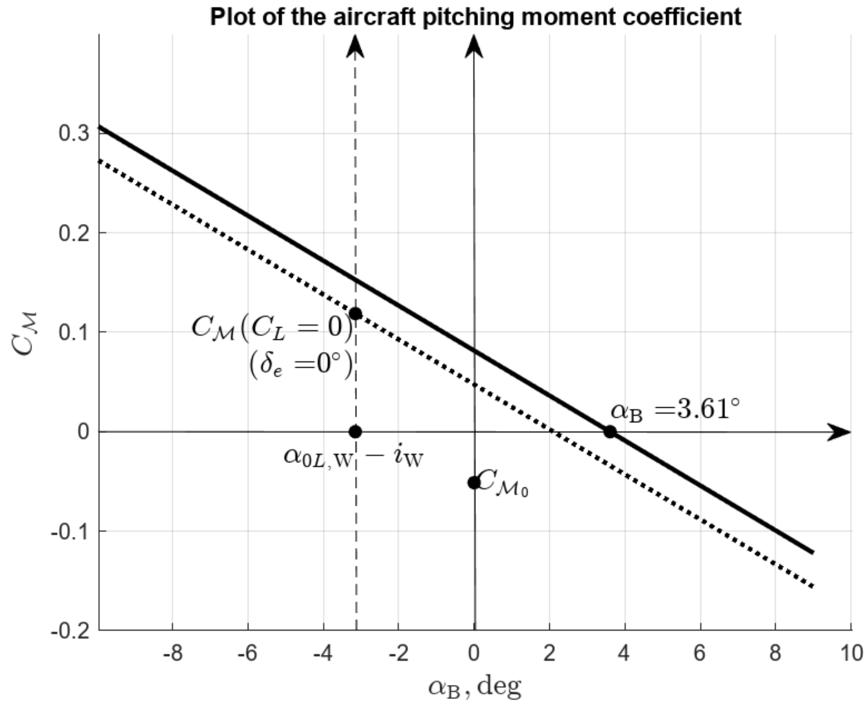

Figure 1 – Plot of the aircraft pitching moment coefficient

The horizontal tailplane load comes from the following equation:

$$L_H = \eta_H q_\infty S_H C_{L\alpha_H} \alpha_{a_H} \tag{1.19}$$

where

$$\alpha_{a_H} = \alpha_H + \tau_e \delta_e \tag{1.19}$$

is the horizontal tail absolute angle of attack, given as sum of its geometric angle of attack and the effect of the elevator deflection.

1.3.4 Hinge Moments

Longitudinal stability refers to the aircraft's tendency to return to its trimmed condition following a disturbance. The pilot measures this stability by sensing the stick-force required to change the speed from a particular trim setting. If the aircraft exhibits strong longitudinal stability, it will demand more force to deviate from its trimmed speed, whereas weak stability will result in less force needed for the same change. This feedback is crucial for pilots to maintain control and understand the aircraft's behavior during flight. The aircraft is less stable with free elevator than with locked elevator. [6] The change in stability after freeing the elevators predominantly involves the contribution of the tail (horizontal stabilizer and elevator) and is influenced by the characteristics of the elevator, particularly the elevator's hinge moments. The tail of an aircraft plays a crucial role in its longitudinal stability. It consists of the horizontal stabilizer and the elevator. The elevator is the movable surface on the tail that controls the aircraft's pitch attitude. Hinge moments refer to the aerodynamic forces acting around the hinge of the elevator. These forces are generated due to the elevator's deflection from its neutral position.

The pressure distribution over an airfoil section is significantly influenced by two primary variables:

1. **Angle of Attack (AOA):** This refers to the angle between the chord line of the airfoil and the oncoming airflow. A change in the angle of attack directly impacts the lift and pressure distribution over the airfoil. Increasing the angle of attack typically results in increased lift up to a certain point, after which the airfoil might experience stall conditions where lift decreases and drag increases abruptly.

2. **Flap Deflection:** The deflection of the flap chord, altering the geometry of the airfoil's trailing edge, affects the camber and shape of the airfoil. Flaps, being movable surfaces on the trailing edge of the wing, are primarily used to increase lift by changing the wing's camber. The deflection of flaps modifies the pressure distribution, increasing lift and altering the stall characteristics of the wing.

To write the hinge moment coefficient, you can call the MATLAB function:

```
function AllData = P6_aeroFun_hingemoment(AllData)
```

The hinge moment coefficient is obtained by dividing by the pressure, the area of the control surface and the root mean square chord of the control surface:

$$C_h = \frac{H}{q S_c c_c} \tag{1.20}$$

A different form to write the hinge moment coefficient is:

$$C_h = C_{h_\alpha}\alpha + C_{h_\delta}\delta \tag{1.21}$$

The estimation of the hinge moment is made by curves that determine: the rate of change of elevator hinge moment of angle of attack and the rate of change of elevator hinge moment with elevator angle.

According to a semi-empirical methods reported in McCormick [7], for the nominal case:

$$C_{h_\alpha} = -0.55 \tag{1.22}$$
$$C_{h_\delta} = -0.89$$

while more generally, the following equations hold:

$$C_{h_\alpha} = -0.55k_1\left(\frac{c_e}{c}\right)k_1\left(\frac{t}{c}\right)k_1(\mathrm{BR})k_1\left(\frac{1}{A}\right) \tag{1.23}$$

$$C_{h_\delta} = -0.89k_2\left(\frac{c_e}{c}\right)k_2\left(\frac{t}{c}\right)k_2(\mathrm{BR})k_2\left(\frac{1}{A}\right) \tag{1.24}$$

where c_e/c is the flap chord ratio (with c_e defined as the mean chord of the elevator), t/c is the airfoil thickness ratio, BR is the balance ratio (i.e., a flap chord ratio where the numerator is evaluated from the airfoil leading edge to the hinge line), A is the horizontal tailplane aspect ratio.

1.3.5 Neutral points and SMs

Stick-Fixed

The neutral point for stick-fixed condition is the solution of the following equation:

$$C_{M_\alpha} = C_{L_{\alpha,W}}(\bar{x}_G - \bar{x}_{\mathrm{ac,WB}}) - \eta_H \frac{S_H}{S}(\bar{x}_{\mathrm{ac},H} - \bar{x}_G)C_{L_{\alpha,H}}\left[1 - \left(\frac{d\varepsilon}{d\alpha}\right)_H\right] = 0 \tag{1.25}$$

by the position $\bar{x}_G = \bar{x}_N$

The static margin is defined as:

$$\mathrm{SM} = \bar{x}_G - \bar{x}_N \tag{1.26}$$

When SM<0 the aircraft is longitudinally stable.

Stick-Free

In this case it is introduced the free-elevator factor:

$$F = 1 - \tau_e \frac{C_{H\alpha_e}}{C_{H\delta e,e}} \tag{1.27}$$

The neutral point for stick-free condition is the solution of the following equation:

$$C_{M_\alpha}{}' = C_{L_{\alpha,W}}(\bar{x}_G - \bar{x}_{\mathrm{ac,WB}}) - \eta_H \frac{S_H}{S}(\bar{x}_{\mathrm{ac},H} - \bar{x}_G)C_{L_{\alpha,H}}\left[1 - \left(\frac{d\varepsilon}{d\alpha}\right)_H\right]F = 0 \tag{1.28}$$

by the position $\bar{x}_G = \bar{x}_N{}'$

Again, when SM<0 the aircraft is stable. The magnitude of the stick-free static margin is less than the magnitude of the stick-fixed static margin.

1.3.6 Summary of all the contributes

To summarize all the contributes, you have to recall the function of MATLAB:

```
function AllData = P3_aeroFun_neutralPoint(AllData)
function AllData = P3_aeroFun_neutralPointFree(AllData)
```

1.4 Lateral-Directional stability

To study lateral-directional stability of the aircraft, two values are calculated by a system of two equations. The values are aileron deflection δ_a and rudder deflection δ_r by considering the assigned side-slip angle β. Ailerons are control surfaces on the wings used to create differential lift, allowing the aircraft to roll about its longitudinal axis. Deflecting ailerons upward on one wing and downward on the other generates lift differences, causing the aircraft to roll. The rudder is a vertical control surface at the tail of the aircraft used to control yaw, which is the rotation of the aircraft about its vertical axis. Deflecting the rudder left or right affects the aircraft's heading. Side-slip angle is the angle between the aircraft's flight path and its longitudinal axis. It occurs when the aircraft's heading differs from its direction of motion through the air. It is typically associated with uncoordinated flight, where the aircraft is not flying directly into the relative wind. The equations are:

1. Lateral Stability;
2. Directional Stability.

To write down the equations, recall the MATLAB code.

```
AllData=P4_aeroFun_latDirEqGivenBeta(AllData);
```

1.4.1 Lateral Stability

The equation is:

$$C_{\mathscr{L}} = C_{\mathscr{L}_0} + C_{\mathscr{L}_\beta}\beta + C_{\mathscr{L}_{\delta a}}\delta_a + C_{\mathscr{L}_{\delta r}}\delta_r + C_{\mathscr{L}_p}\hat{p} + C_{\mathscr{L}_r}\hat{r} + C_{\mathscr{L}_{\text{mot}}} \tag{1.29}$$

where: if $C_l = 0$ the aircraft is symmetric.

$$C_{\mathscr{L}_\beta} = C_{\mathscr{L}_{\beta_\Gamma}} + C_{\mathscr{L}_{\beta_\Lambda}} + C_{\mathscr{L}_{\beta_{W\,\text{POS}}}} + C_{\mathscr{L}_{\beta_V}} \tag{1.30}$$

$$C_{\mathscr{L}_{\beta_\Gamma}} = -\frac{\Gamma}{6} C_{L\alpha_W} \frac{1+2\lambda}{1+\lambda} \tag{1.31}$$

$$C_{\mathscr{L}_{\beta_\Lambda}} = -\frac{1}{3}\sin(2\Lambda) C_L \frac{1+2\lambda}{1+\lambda} \tag{1.32}$$

$$C_{\mathscr{L}_{\beta_V}} = -C_{L\alpha_V}\left(1 - \frac{\mathrm{d}}{\mathrm{d}\beta}\sigma\right)\eta_V \frac{S_V}{S}\frac{h_V}{b} \tag{1.33}$$

The above set of equations evaluates the stability derivative caused by the dihedral effect, with $C_{\mathscr{L}_\beta} < 0$ is the condition for lateral stability. The control derivative:

$$C_{\mathscr{L}_{\delta a}} = \frac{-2C_{L\alpha_W}\tau_a 0.90}{\text{Sb}} \int_{\text{yi}}^{\text{yf}} \frac{c(y)}{\frac{b}{2}} \frac{y}{\frac{b}{2}} \frac{\text{dy}}{\frac{b}{2}} \tag{1.34}$$

with yi, yf respectively start and end point of the ailerons. This derivative is called *aileron control power*. Usually $C_{\mathscr{L}_{\delta a}} < 0$.

$$C_{\mathcal{L}_{\delta r}} = C_{L\alpha_V} \tau_r \eta_V \frac{S_V}{S} \frac{h_V}{b} \tag{1.35}$$

The above is an indirect effect due to the rudder deflection that causes roll.

$$C_{\mathcal{L}_p} = C_{\mathcal{L}_{p_W}} + C_{\mathcal{L}_{p_V}} \tag{1.36}$$

$$C_{\mathcal{L}_{p_W}} = -\frac{4}{Sb^2} C_{L\alpha_W} \int_{\mathrm{yi}}^{\mathrm{yf}} \frac{c(y)}{\frac{b}{2}} \frac{y}{\frac{b}{2}} \frac{\mathrm{dy}}{\frac{b}{2}} \tag{1.37}$$

$$C_{\mathcal{L}_{p_V}} = -2\eta_V \frac{S_V}{S} C_{L\alpha_V} \left(\frac{h_V}{b} \right)^2 \tag{1.38}$$

The above set of equations is a dynamic derivative of the rolling moment due to a rolling motion. When $C_{\mathcal{L}_p} < 0$ the aircraft damps out the oscillations in roll caused by $p \neq 0$.

$$C_{\mathcal{L}_r} = C_{\mathcal{L}_{r_W}} + C_{\mathcal{L}_{r_V}} \tag{1.39}$$

$$C_{\mathcal{L}_{r_V}} = C_{L\alpha_V} \left(2\frac{l_V}{b} - \frac{\mathrm{d}}{\mathrm{d}r}\sigma \right) \eta_V \frac{S_V}{S} \frac{h_V}{b} \tag{1.40}$$

The above equations, with the first contribution of difficult estimation, represent the dynamic derivative of aircraft rolling moment due to a yawing motion. Usually $C_{\mathcal{L}_r} > 0$.

$$C_{\mathcal{L}_{\mathrm{mot}}} = \frac{-Q}{q_\infty \mathrm{Sb}} \tag{1.41}$$

The above is the engine contribution to the lateral stability equation. $C_{\mathcal{L}_{\mathrm{mot}}} \neq 0$ when there is one or more propellers.
<u>CONVENTION</u>**:** The contribution is less than zero when the propeller rotation is clockwise in front view.

1.4.2 Directional stability

The equation is:

$$C_{\mathcal{N}} = C_{\mathcal{N}_0} + C_{\mathcal{N}_\beta}\beta + C_{\mathcal{N}_{\delta a}}\delta_a + C_{\mathcal{N}_{\delta r}}\delta_r + C_{\mathcal{N}_p}\widehat{p} + C_{\mathcal{N}_r}\widehat{r} + C_{\mathcal{N}_{\mathrm{mot}}} \tag{1.42}$$

where: if $C_N = 0$ the aircraft is symmetric.

$$C_{\mathcal{N}_\beta} = C_{\mathcal{N}_{\beta_W}} + C_{\mathcal{N}_{\beta_F}} + C_{\mathcal{N}_{\beta_V}} \tag{1.43}$$

$$C_{\mathcal{N}_{\beta_V}} = C_{L\alpha_V} \left(1 - \frac{\mathrm{d}}{\mathrm{d}\beta}\sigma \right) \eta_V \frac{S_V}{S} \frac{l_V}{b} \tag{1.44}$$

The above set of equations is the directional stability derivative, with the contribution of the vertical tail of easy estimation. To have directional stability $C_{\mathcal{N}_\beta} > 0$, because for $\beta > 0$ there will be a restoring moment that will restore the equilibrium condition.

The term $C_{\mathcal{N}_{\delta a}}$ is a side effect, caused by the ailerons’ deflection, with the induced drag causing adverse yaw. Usually, the aircraft are designed to minimize it.

$$C_{\mathcal{N}_{\delta r}} = -C_{L\alpha_V}\eta_V \frac{S_V}{S}\frac{h_V}{b}\tau_r \quad (1.45)$$

The above term is the rudder control power. Usually $C_{\mathcal{N}_{\delta r}} < 0$.

$$C_{\mathcal{N}_p} = C_{\mathcal{N}_{p_W}} + C_{\mathcal{N}_{p_V}} \quad (1.46)$$

$$C_{\mathcal{N}_{p_W}} = C_{\mathcal{N}_{p_{W_{\text{drag}}}}} + C_{\mathcal{N}_{p_{W_{\text{tilt}}}}} + C_{\mathcal{N}_{p_{W_{\text{tipsuc}}}}} \quad (1.47)$$

$$C_{\mathcal{N}_{p_{W_{\text{drag}}}}} = \frac{4}{Sb^2} C_{D_\alpha} \int_0^{\frac{b}{2}} c y^2 \text{dy} \quad > 0 \quad (1.48)$$

The above set of equations represent a yawing moment due to a rolling motion. The last contribution is caused by the wings induced drag, while

$C_{\mathcal{N}_{p_{W_{\text{tilt}}}}} < 0$ is caused by the lift tilt $\neq 90^\circ$. So, there will be an adverse yaw.

$C_{\mathcal{N}_{p_{W_{\text{tipsuc}}}}}$ depends on the tip vortices caused by the pressure difference between back and belly of the wing.

$$C_{\mathcal{N}_{p_V}} = C_{L\alpha_V}\left(2\frac{h_V}{b} - \frac{\text{d}}{\text{dp}}\sigma\right)\eta_V \frac{S_V}{S}\frac{l_V}{b} \quad (1.49)$$

The above is the vertical tail contribution to the yawing damping derivative due to roll. With this term we study how the aircraft damps out the oscillations in yaw caused by $p \neq 0$.

Finally, the yawing moment due to yawing motion is:

$$C_{\mathcal{N}_r} = C_{\mathcal{N}_{r_W}} + C_{\mathcal{N}_{r_V}} \quad (1.50)$$

with $C_{\mathcal{N}_{r_W}} < 0$ usually and

$$C_{\mathcal{N}_{r_V}} = -C_{L\alpha_V}\left(2\frac{l_V}{b} - \frac{\text{d}}{\text{dr}}\sigma\right)\eta_V \frac{S_V}{S}\frac{l_V}{b} \quad < 0 \quad (1.51)$$

with $C_{\mathcal{N}_r} < 0$ represents a contribution due to $r \neq 0$ restoring moment. The propulsive effects on equilibrium are evaluated as:

$$C_{\mathcal{N}_{\text{mot}}} = \frac{T d_{\text{mot}}}{q_\infty \text{Sb}} \quad (1.52)$$

This contribution is $C_{\mathcal{N}_{\text{mot}}} \neq 0$ when there is an asymmetric thrust, especially in case of engine failure. There will be a yaw moment caused by T with d_{mot} as its arm. See Section 4.4. The p-factor (due to propeller rotation in non-axial flow) is always ignored.

1.4.3 Solving the system

The deflection values of the aileron and the rudder are the unknown values of the following system:

$$\begin{bmatrix} C_{\mathcal{L}_{\delta a}} & C_{\mathcal{L}_{\delta r}} \\ C_{\mathcal{N}_{\delta a}} & C_{\mathcal{N}_{\delta r}} \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} = \begin{bmatrix} -\left(C_{\mathcal{L}_0} + C_{\mathcal{L}_\beta}\beta + C_{\mathcal{L}_p}\hat{p} + C_{\mathcal{L}_r}\hat{r} + C_{\mathcal{L}_{\text{mot}}}\right) \\ -\left(C_{\mathcal{N}_0} + C_{\mathcal{N}_\beta}\beta + C_{\mathcal{N}_p}\hat{p} + C_{\mathcal{N}_r}\hat{r} + C_{\mathcal{N}_{\text{mot}}}\right) \end{bmatrix} \quad (1.53)$$

1.4.4 One-Engine-Inoperative condition

This section executes calculations if the aircraft has more than one engine, simulating the failure of one of them. If so, the asymmetrical thrust still has to counteract the aerodynamic drag:

$$T = q_{\infty} S C_D \tag{1.54}$$

where

$$C_D = C_{D0} + k{C_L}^2 \tag{1.55}$$

Ther are two possibilities in this section:

1. The engine that you have chosen fails. The lateral-directional equilibrium will be updated considering the asymmetrical thrust generated by the operating engine.
2. The chosen aircraft have only one engine, the OEI condition is not executed.

1.4.5 Summary of all the contributes

Recall of all the contributes of previous sub-chapter.

Chapter 2 – Application to selected airplanes

2. Aircraft

The selected aircraft are 7:

- Cessna 210
- Tecnam P92
- Tecnam P-2012
- Boeing 737
- Boeing 777
- Beechcraft King Air
- Dornier Do 328

2.1 Cessna 210

2.1.1 Input Data

Figure 2 – CESSNA210

1. Speed.
2. Load
3. Pitch rate
4. Beta
5. Roll rate
6. Yaw rate

7. OEI
8. Propeller rotation direction

Input data are here below summarized:

```
AIRCRAFT NAME: CESSNA 210
```

Geometry

```
WING: b = 11.00 m | cr = 1.63 m | ct = 1.13 m | iW = 0.035 rad | Λ = 0.000 rad | Γ = 0.000 rad
HORIZONTAL TAILPLANE: bH = 3.30 m | SH = 3.14 m2 | lH = 4.20 m | iH = -0.034 rad | ce_c = 0.25 |
BR = 0.02 | t_c = 0.10
VERTICAL TAILPLANE: SV = 3.50 m2 | lV = 7.00 m | hV = 1.20 m
ENGINE: d_mot = 0.000 m | Xt = 2.500 m | Zt = 0.030 m | D = 2.690 m
```

Aerodynamics

```
GLOBAL: CD0 = 0.03 | e = 0.80 | xcg = 0.30 | CRoll_0 = 0.000 | CN_0 = 0.000
WING: xac_W = 0.28 | Cl_alphaW = 6.40 rad-1 | alpha_0l_W = -0.01 rad | eW = 0.90 | CM_ac_W = -
0.070 rad-1 | CRoll_p_W = -0.693 | CRoll_delta_a = -0.220 rad-1 | CRoll_beta_wpos = 0.000 rad-1 |
CN_delta_a = 0.000 rad-1  CN_r_W = 0.000 | τa = 0.40 | CN_beta_W = 0.000 rad-1 | CN_p_Wtilt =
0.000 rad-1 | CN_p_W_tipsuc = 0.000 rad-1 | CN_r_W = 0.000 rad-1
HORIZONTAL TAILPLANE: Cl_alphaH = 6.400 rad-1 | eH = 0.90 | ηH = 0.95 | τe = 0.38 | CH_alpha_e =
-0.004 rad-1 | CH_deltaE_e = -0.011 rad-1
VERTICAL TAILPLANE: CL_alphaV = 3.000 rad-1 | ηV = 1.00 | dσ/dβ = 0.150 | τr = 0.45 | CN_p_V =
0.000 | CN_r_V = 0.000 | dσ/dp = 0.000 | dσ/dr = 0.000
FUSELAGE: CM_0_B = -0.060 | CM_alpha_B(W) = 0.300 rad-1 | CN_beta_B = -0.100 rad-1
```

Powerplant

```
POWERPLANT: Engines number = 1 | Pa = 230 kW | Np = 220.0 kgf | J = 0.00
```

2.1.2 Preliminary calculations

Preliminary calculations used in the entire code are shown below.

- Taper ratio

```
λ = 0.69
```

- Wing area

```
S = 15.18 m²
```

- Dynamic pressure

```
q∞ = 6125.0 Pa
```

- Mean aerodynamic chord

```
m.a.c. = 1.40 m
```

- Lift coefficient

```
CL = 0.19
```

- **Aspect ratio** (wing and horizontal tail, respectively)

```
AR = 7.97
ARH = 3.47
```

- **Wing lift curve slope**

CLα,W = 4.985 rad-1 = 0.087 deg-1

- **Wing-Body's lift coefficient at zero lift**

CL0,W = 0.23

- **Horizontal tail lift curve slope**

CLα,H = 3.873 rad-1 = 0.068 deg-1

- **Mean value of downwash at** $\alpha_{\mathrm{B}} = 0$

ε0 = 0.021 rad = 1.2 deg

- **Downwash gradient**

dε/dα = 0.442

2.1.3 Longitudinal equilibrium, stability and control

Selected input: the aircraft speed is V = 80 m/s with a load factor n = 1. The pitch rate is q = 0 deg/s.

To study longitudinal stability and control of the aircraft, two values are calculated by a system of two equations. The values are the angle of attack and the equilibrium elevator angle by considering the assigned horizontal tail incidence angle i_{H}. The equations are:

1. Vertical Translational Equilibrium;
2. Rotational Equilibrium.

Vertical translation equilibrium

	Derivative	Value	Unit
1	'CL0'	0.2185	' '
2	'CLα'	5.4089	'rad-1'
3	'CLδe'	0.2892	'rad-1'
4	'CLiH'	0.7610	'rad-1'

Rotational equilibrium

	Derivative	Value	Unit
5	'CM0'	-0.0636	' '
6	'CMα'	-0.8779	'rad-1'

	Derivative	Value	Unit
7	'CMδe'	-0.8706	'rad-1'
8	'CMiH'	-2.2910	'rad-1'
9	'CMq'	-13.7942	'rad-1'
10	'CMmot'	0.0127	' '

Solving the system

The values of the angle of attack and the deflection of the elevator are the solution of the following system:

$$\begin{bmatrix} C_{L_\alpha} & C_{L_{\delta e}} \\ C_{M_\alpha} & C_{M_{\delta e}} \end{bmatrix} \begin{bmatrix} \alpha_B \\ \delta_e \end{bmatrix} = \begin{bmatrix} C_L - C_{\mathrm{L0}} - C_{L_{\mathrm{iH}}} i_H \\ -C_{\mathrm{M0}} - C_{M_{\mathrm{mot}}} - C_{M_{\mathrm{iH}}} i_H - C_{M_q} \hat{q} \end{bmatrix} \tag{1.18}$$

```
αB = 0.064 rad = 3.7 deg

δe = -0.003 rad = -1.9 deg
```

Now it is possible to represent the evolution of the pitching moment coefficient vs the body angle of attack.

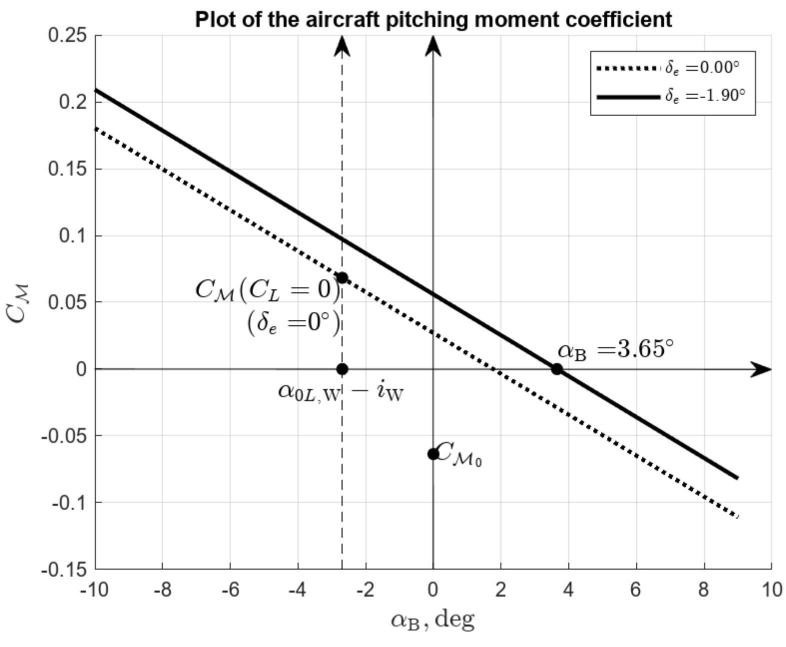

Figure 3 – Plot of the aircraft pitching moment coefficient

```
LH = -812 N = -82.8 kgf
```

Hinge Moment

The estimation of the hinge moment

$$C_{h_\alpha} = -0.55k_1\left(\frac{c_e}{c}\right)k_1\left(\frac{t}{c}\right)k_1(\mathrm{BR})k_1\left(\frac{1}{A}\right) \tag{1.23}$$

```
Cha = -0.146 rad^-1
```

$$C_{h_\delta} = -0.89k_2\left(\frac{c_e}{c}\right)k_2\left(\frac{t}{c}\right)k_2(\mathrm{BR})k_2\left(\frac{1}{A}\right) \tag{1.24}$$

```
Chd = -0.489 rad^-1
```

Neutral points and SMs

Stick-Fixed

The neutral point for stick-fixed condition is the solution of the following equation:

$$C_{M_\alpha} = C_{L_{\alpha,W}}(\bar{x_G} - \bar{x}_{\mathrm{ac,WB}}) - \eta_H \frac{S_H}{S}(\bar{x}_{\mathrm{ac},H} - \bar{x_G})C_{L_{\alpha,H}}\left[1 - \left(\frac{d\varepsilon}{d\alpha}\right)_H\right] = 0 \tag{1.25}$$

By the position $\bar{x_G} = \bar{x_N}$:

```
xN = 0.46
```

Now it's possible to study the $\mathrm{SM} = \bar{x_G} - \bar{x_N}$:

```
SM = -0.16
```

```
The aircraft is stable.
```

If the approximated formula is used:

```
xN approx = 0.48
```

```
SM approx = -0.18
```

- Stick-Free

```
F = 0.871
```

The neutral point for stick-free condition is the solution of the following equation:

$$C_{M_\alpha}{}' = C_{L_{\alpha,W}}(\bar{x_G} - \bar{x}_{\mathrm{ac,WB}}) - \eta_H \frac{S_H}{S}(\bar{x}_{\mathrm{ac},H} - \bar{x_G})C_{L_{\alpha,H}}\left[1 - \left(\frac{d\varepsilon}{d\alpha}\right)_H\right]F = 0 \tag{1.26}$$

By the position $\bar{x_G} = \bar{x_N}'$:

```
xN' = 0.43
```

Now it's possible to study the $\mathrm{SM} = \bar{x_G} - \bar{x_N}$:

```
SM' = -0.13

The aircraft is stable.
```

If the approximated formula is used:

```
xN' approx = 0.44

SM' approx = -0.14
```

2.1.4 Lateral-Directional stability

```
Selected input: the sideslip angle is β = -3 deg with a roll rate p = 0 deg/s. The yaw rate is r = 0 deg/s.
```

To study lateral-directional stability of the aircraft, two values are calculated by a system of two equations. The values are aileron deflection δ_a and rudder deflection δ_r by considering the assigned side-slip angle β. The equations are:

1. Lateral Stability;
2. Directional Stability.

Lateral stability

	Derivative	Value	Unit
1	'CL0'	0.000	''
2	'CLβ'	-0.064	'rad-1'
3	'CLδa'	-0.163	'rad-1'
4	'C$\mathcal{L}$δr'	0.0340	'rad-1'
5	'C$\mathcal{L}$p'	-0.8563	'rad-1'
6	'C$\mathcal{L}$r'	0.0960	'rad-1'
7	'C$\mathcal{L}$mot'	0	''

Directional stability

	Derivative	Value	Unit
8	'C$\mathcal{N}$0'	0	''
9	'C$\mathcal{N}$β'	0.2741	'rad-1'

	Derivative	Value	Unit
10	'C𝒩δa'	0	'rad-1'
11	'C𝒩δr'	-0.1981	'rad-1'
12	'C𝒩p'	0.1440	'rad-1'
13	'C𝒩r'	-0.5602	'rad-1'
14	'C𝒩mot'	0	' '

Solving the system

The deflection values of the aileron and the rudder are the unknown values of the following system:

$$\begin{bmatrix} C_{\mathscr{L}_{\delta a}} & C_{\mathscr{L}_{\delta r}} \\ C_{\mathscr{N}_{\delta a}} & C_{\mathscr{N}_{\delta r}} \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} = \begin{bmatrix} -\left(C_{\mathscr{L}_0} + C_{\mathscr{L}_\beta}\beta + C_{\mathscr{L}_p}\hat{p} + C_{\mathscr{L}_r}\hat{r} + C_{\mathscr{L}_{\text{mot}}}\right) \\ -\left(C_{\mathscr{N}_0} + C_{\mathscr{N}_\beta}\beta + C_{\mathscr{N}_p}\hat{p} + C_{\mathscr{N}_r}\hat{r} + C_{\mathscr{N}_{\text{mot}}}\right) \end{bmatrix} \tag{1.53}$$

```
δa = 0.006 rad = 0.3 deg

δr = -0.072 rad = -4.2 deg
```

One-Engine-Inoperative condition

```
The aircraft is a single-engine type. The engine does not fail. The lateral-directional
equilibrium will NOT be updated.
```

2.2 Tecnam P92

2.2.1 Input Data

Figure 4- Tecnam P92

Select the following parameters:

1. Speed.
2. Load
3. Pitch rate
4. Beta
5. Roll rate
6. Yaw rate
7. OEI
8. Propeller rotation direction

Input data are here below summarized:

```
AIRCRAFT NAME: Tecnam P92
```

Geometry

```
WING: b = 8.70 m | cr = 1.40 m | ct = 0.90 m | iW = 0.035 rad | Λ = 0.000 rad | Γ = 0.260 rad
HORIZONTAL TAILPLANE: bH = 2.40 m | SH = 3.00 m2 | lH = 3.50 m | iH = -0.035 rad | ce_c = 0.25 |
BR = 0.02 | t_c = 0.10
VERTICAL TAILPLANE: SV = 3.00 m2 | lV = 6.40 m | hV = 1.20 m
ENGINE: d_mot = 0.000 m | Xt = 2.500 m | Zt = 0.030 m | D = 0.076 m
```

Aerodynamics

```
GLOBAL: CD0 = 0.03 | e = 0.80 | xcg = 0.25 | CRoll_0 = 0.000 | CN_0 = 0.000
```

```
WING: xac_W = 0.24 | Cl_alphaW = 6.20 rad-1 | alpha_0l_W = -0.01 rad | eW = 0.90 | CM_ac_W = -
0.080 rad-1 | CRoll_p_W = -0.690 | CRoll_delta_a = -0.210 rad-1 | CRoll_beta_wpos = 0.000 rad-1 |
CN_delta_a = 0.000 rad-1  CN_r_W = 0.000 | τa = 0.40 | CN_beta_W = 0.000 rad-1 | CN_p_Wtilt =
0.000 rad-1 | CN_p_W_tipsuc = 0.000 rad-1 | CN_r_W = 0.000 rad-1
HORIZONTAL TAILPLANE: Cl_alphaH = 6.200 rad-1 | eH = 0.90 | ηH = 0.90 | τe = 0.38 | CH_alpha_e =
-0.004 rad-1 | CH_deltaE_e = -0.011 rad-1
VERTICAL TAILPLANE: CL_alphaV = 3.000 rad-1 | ηV = 1.00 | dσ/dβ = 0.150 | τr = 0.45 | CN_p_V =
0.000 | CN_r_V = 0.000 | dσ/dp = 0.000 | dσ/dr = 0.000
FUSELAGE: CM_0_B = -0.050 | CM_alpha_B(W) = 0.250 rad-1 | CN_beta_B = -0.100 rad-1
```

Powerplant

```
POWERPLANT: Engines number = 1 | Pa = 74 kW | Np = 200.0 kgf | J = 1.00
```

2.2.2 Preliminary calculations

Preliminary calculations used in the entire code are shown below.

- Taper ratio

```
λ = 0.64
```

- Wing area

```
S = 10.00 m²
```

- Dynamic pressure

```
q∞ = 6125.0 Pa
```

- Mean aerodynamic chord

```
m.a.c. = 1.17 m
```

- Lift coefficient

```
CL = 0.60
```

- Aspect ratio (wing and horizontal tail, respectively)

```
AR = 7.57
ARH = 1.92
```

- Wing lift curve slope

```
CLα,W = 4.807 rad-1 = 0.084 deg-1
```

- Wing-Body's lift coefficient at zero lift

```
CL0,W = 0.21
```

- Horizontal tail lift curve slope

```
CLα,H = 2.894 rad-1 = 0.051 deg-1
```

- Mean value of downwash at $\alpha_{\mathrm{B}} = 0$

```
ε0 = 0.020 rad = 1.1 deg
```

- **Downwash gradient**

```
dε/dα = 0.449
```

2.2.3 Longitudinal equilibrium, stability and control

```
Selected input: the aircraft speed is V = 40 m/s with a load factor n = 1. The pitch rate is q =
0 deg/s.
```

To study longitudinal stability and control of the aircraft, two values are calculated by a system of two equations. The values are the angle of attack and the equilibrium elevator angle by considering the assigned horizontal tail incidence angle i_H. Although the P-92 has a stabilator tailplane, the longitudinal equilibrium is solved as it had the conventional tailplane with stabilizer and elevator. The equations are:

1. Vertical Translational Equilibrium;
2. Rotational Equilibrium.

Vertical translation equilibrium

	Derivative	Value	Unit
1	'CL0'	0.1987	' '
2	'CLα'	5.2368	'rad-1'
3	'CLδe'	0.2968	'rad-1'
4	'CLiH'	0.7811	'rad-1'

Rotational equilibrium

	Derivative	Value	Unit
5	'CM0'	-0.0698	' '
6	'CMα'	-0.9905	'rad-1'
7	'CMδe'	-0.8893	'rad-1'
8	'CMiH'	-2.3404	'rad-1'
9	'CMq'	-14.0248	'rad-1'
10	'CMmot'	0.0450	' '

Solving the system

The values of the angle of attack and the deflection of the elevator are the solution of the following system:

$$\begin{bmatrix} C_{L_\alpha} & C_{L_{\delta e}} \\ C_{M_\alpha} & C_{M_{\delta e}} \end{bmatrix} \begin{bmatrix} \alpha_B \\ \delta_e \end{bmatrix} = \begin{bmatrix} C_L - C_{L0} - C_{L_{iH}} i_H \\ -C_{M0} - C_{M_{mot}} - C_{M_{iH}} i_H - C_{M_q} \hat{q} \end{bmatrix} \tag{1.18}$$

αB = 0.084 rad = 4.8 deg

δe = -0.029 rad = -1.7 deg

Now it is possible to represent the evolution of the pitching moment coefficient vs the body angle of attack.

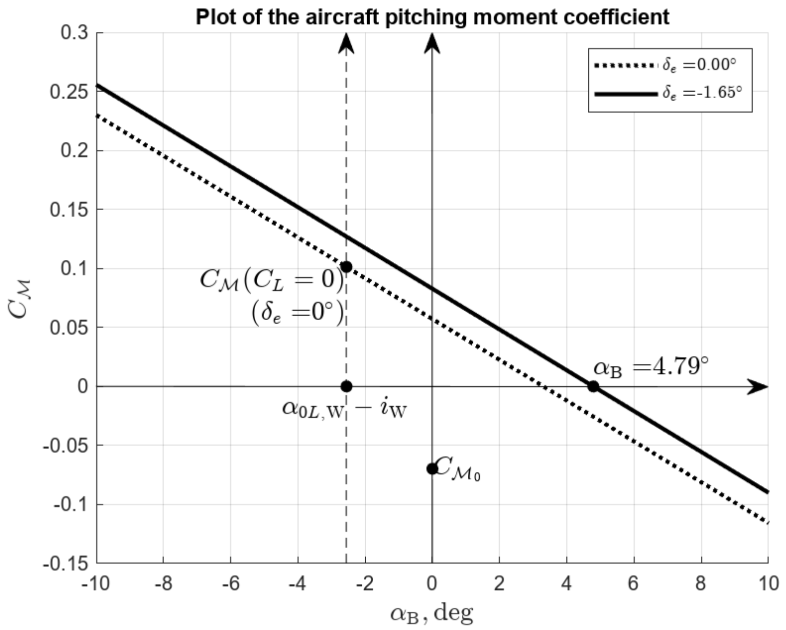

Figure 5- Plot of the aircraft pitching moment coefficient

LH = -153 N = -15.6 kgf

Hinge Moment

The estimation of the hinge moment

$$C_{h_\alpha} = -0.55 k_1\left(\frac{c_e}{c}\right) k_1\left(\frac{t}{c}\right) k_1(\mathrm{BR}) k_1\left(\frac{1}{A}\right) \tag{1.23}$$

Cha = -0.146 rad^-1

$$C_{h_\delta} = -0.89 k_2\left(\frac{c_e}{c}\right) k_2\left(\frac{t}{c}\right) k_2(\mathrm{BR}) k_2\left(\frac{1}{A}\right) \tag{1.24}$$

Chd = -0.489 rad^-1

Neutral points and SMs

Stick-Fixed

The neutral point for stick-fixed condition is the solution of the following equation:

$$C_{M_\alpha} = C_{L_{\alpha,W}}(\bar{x}_G - \bar{x}_{\text{ac,WB}}) - \eta_H \frac{S_H}{S}(\bar{x}_{\text{ac},H} - \bar{x}_G) C_{L_{\alpha,H}} \left[1 - \left(\frac{d\varepsilon}{d\alpha}\right)_H\right] = 0 \quad (1.25)$$

By the position $\bar{x}_G = \bar{x}_N$:

```
xN = 0.44
```

Now it's possible to study the $\text{SM} = \bar{x}_G - \bar{x}_N$:

```
SM = -0.19

The aircraft is stable.
```

If the approximated formula is used:

```
xN approx = 0.46

SM approx = -0.21
```

Stick-Free

```
F = 0.871
```

The neutral point for stick-free condition is the solution of the following equation:

$$C_{M_\alpha}{}' = C_{L_{\alpha,W}}(\bar{x}_G - \bar{x}_{\text{ac,WB}}) - \eta_H \frac{S_H}{S}(\bar{x}_{\text{ac},H} - \bar{x}_G) C_{L_{\alpha,H}} \left[1 - \left(\frac{d\varepsilon}{d\alpha}\right)_H\right] F = 0 \quad (1.26)$$

By the position $\bar{x}_G = \bar{x}_N{}'$:

```
xN' = 0.41
```

Now it's possible to study the $\text{SM} = \bar{x}_G - \bar{x}_N$:

```
SM' = -0.16

The aircraft is stable.
```

If the approximated formula is used:

```
xN' approx = 0.42

SM' approx = -0.17
```

2.2.4 Lateral-Directional stability

```
Selected input: the sideslip angle is β = -3 deg with a roll rate p = 0 deg/s. The yaw rate is r
= 0 deg/s.
```

To study lateral-directional stability of the aircraft, two values are calculated by a system of two equations. The values are aileron deflection δ_a and rudder deflection δ_r by considering the assigned side-slip angle β. The equations are:

1. Lateral Stability;
2. Directional Stability.

Lateral stability

	Derivative	Value	Unit
1	'CL0'	0.000	' '
2	'CLβ'	-0.395	'rad-1'
3	'CLδa'	-0.153	'rad-1'
4	'Cℒδr'	0.056	'rad-1'
5	'Cℒp'	-0.848	'rad-1'
6	'Cℒr'	0.183	'rad-1'
7	'Cℒmot'	-0.0003	' '

Directional stability

	Derivative	Value	Unit
8	'C𝒩0'	0	' '
9	'C𝒩β'	0.462	'rad-1'
10	'C𝒩δa'	0	'rad-1'
11	'C𝒩δr'	-0.298	'rad-1'
12	'C𝒩p'	0.239	'rad-1'
13	'C𝒩r'	-0.974	'rad-1'
14	'C𝒩mot'	0	' '

Solving the system

The deflection values of the aileron and the rudder are the unknown values of the following system:

$$\begin{bmatrix} C_{\mathscr{L}_{\delta a}} & C_{\mathscr{L}_{\delta r}} \\ C_{\mathscr{N}_{\delta a}} & C_{\mathscr{N}_{\delta r}} \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} = \begin{bmatrix} -\left(C_{\mathscr{L}_0} + C_{\mathscr{L}_\beta}\beta + C_{\mathscr{L}_p}\hat{p} + C_{\mathscr{L}_r}\hat{r} + C_{\mathscr{L}_{\text{mot}}}\right) \\ -\left(C_{\mathscr{N}_0} + C_{\mathscr{N}_\beta}\beta + C_{\mathscr{N}_p}\hat{p} + C_{\mathscr{N}_r}\hat{r} + C_{\mathscr{N}_{\text{mot}}}\right) \end{bmatrix} \tag{1.53}$$

```
δa = 0.106 rad = 6.1 deg

δr = -0.081 rad = -4.7 deg
```

One-Engine-Inoperative condition

```
The aircraft is a single-engine type. The engine does not fail. The lateral-directional
equilibrium will NOT be updated.
```

2.3 Tecnam P-2012

2.3.1 Input Data

Figure 6- Tecnam P-2012

Select the following parameters:

1. Speed.
2. Load
3. Pitch rate
4. Beta
5. Roll rate
6. Yaw rate
7. OEI
8. Propeller rotation direction

Input data are here below summarized:

```
AIRCRAFT NAME: Tecnam P2012
```

Geometry

```
WING: b = 14.00 m | cr = 2.30 m | ct = 1.36 m | iW = 0.035 rad | Λ = 0.500 rad | Γ = 0.050 rad
HORIZONTAL TAILPLANE: bH = 5.30 m | SH = 7.30 m2 | lH = 5.35 m | iH = 0.000 rad | ce_c = 0.25 |
BR = 0.02 | t_c = 0.10
VERTICAL TAILPLANE: SV = 3.50 m2 | lV = 7.00 m | hV = 1.10 m
ENGINE: d_mot = 1.500 m | Xt = 1.500 m | Zt = 1.500 m | D = 2.500 m
```

Aerodynamics

```
GLOBAL: CD0 = 0.03 | e = 0.80 | xcg = 0.30 | CRoll_0 = 0.000 | CN_0 = 0.000
WING: xac_W = 0.25 | Cl_alphaW = 6.30 rad-1 | alpha_0l_W = -0.01 rad | eW = 0.90 | CM_ac_W = -
0.070 rad-1 | CRoll_p_W = -0.693 | CRoll_delta_a = -0.200 rad-1 | CRoll_beta_wpos = -0.001 rad-1
| CN_delta_a = 0.000 rad-1  CN_r_W = 0.000 | τa = 0.40 | CN_beta_W = 0.000 rad-1 | CN_p_Wtilt =
0.000 rad-1 | CN_p_W_tipsuc = 0.000 rad-1 | CN_r_W = 0.000 rad-1
HORIZONTAL TAILPLANE: Cl_alphaH = 6.300 rad-1 | eH = 0.90 | ηH = 1.00 | τe = 0.45 | CH_alpha_e =
-0.040 rad-1 | CH_deltaE_e = -0.750 rad-1
VERTICAL TAILPLANE: CL_alphaV = 3.000 rad-1 | ηV = 1.00 | dσ/dβ = 0.150 | τr = 0.45 | CN_p_V =
0.000 | CN_r_V = 0.000 | dσ/dp = 0.000 | dσ/dr = 0.000
FUSELAGE: CM_0_B = -0.060 | CM_alpha_B(W) = 0.350 rad-1 | CN_beta_B = -0.080 rad-1
```

Powerplant

```
POWERPLANT: Engines number = 2 | Pa = 280 kW | Np = 15.0 kgf | J = 1.00
```

2.3.2 Preliminary calculations

Preliminary calculations used in the entire code are shown below.

- **Taper ratio**

```
λ = 0.59
```

- **Wing area**

```
S = 25.62 m²
```

- **Dynamic pressure**

```
q∞ = 6125.0 Pa
```

- **Mean aerodynamic chord**

```
m.a.c. = 1.87 m
```

- **Lift coefficient**

```
CL = 0.36
```

- **Aspect ratio** (wing and horizontal tail, respectively)

```
AR = 7.65
ARH = 3.85
```

- **Wing lift curve slope**

```
CLα,W = 4.879 rad-1 = 0.085 deg-1
```

- **Wing-Body's lift coefficient at zero lift**

```
CL0,W = 0.22
```

- **Horizontal tail lift curve slope**

```
CLα,H = 3.990 rad-1 = 0.070 deg-1
```

- **Mean value of downwash at** $\alpha_{\mathrm{B}} = 0$

```
ε0 = 0.020 rad = 1.2 deg
```

- **Downwash gradient**

```
dε/dα = 0.451
```

2.3.3 Longitudinal equilibrium, stability and control

```
Selected input: the aircraft speed is V = 80 m/s with a load factor n = 1. The pitch rate is q = 0 deg/s.
```

To study longitudinal stability and control of the aircraft, two values are calculated by a system of two equations. The values are the angle of attack and the equilibrium elevator angle by considering the assigned horizontal tail incidence angle i_{H}. The equations are:

1. Vertical Translational Equilibrium;
2. Rotational Equilibrium.

Vertical translation equilibrium

	Derivative	Value	Unit
1	'CL0'	0.1965	' '
2	'CLα'	5.5030	'rad-1'
3	'CLδe'	0.5116	'rad-1'
4	'CLiH'	1.1368	'rad-1'

Rotational equilibrium

	Derivative	Value	Unit
5	'CM0'	-0.0373	' '
6	'CMα'	-1.1910	'rad-1'

	Derivative	Value	Unit
7	'CMδe'	-1.4634	'rad-1'
8	'CMiH'	-3.2520	'rad-1'
9	'CMq'	-18.6051	'rad-1'
10	'CMmot'	0.0263	' '

Solving the system

The values of the angle of attack and the deflection of the elevator are the solution of the following system:

$$\begin{bmatrix} C_{L_\alpha} & C_{L_{\delta e}} \\ C_{M_\alpha} & C_{M_{\delta e}} \end{bmatrix} \begin{bmatrix} \alpha_B \\ \delta_e \end{bmatrix} = \begin{bmatrix} C_L - C_{L0} - C_{L_{iH}} i_H \\ -C_{M0} - C_{M_{mot}} - C_{M_{iH}} i_H - C_{M_q} \hat{q} \end{bmatrix} \quad (1.18)$$

```
αB = 0.033 rad = 1.9 deg

δe = -0.032rad = -1.8 deg
```

Now it is possible to represent the evolution of the pitching moment coefficient vs the body angle of attack.

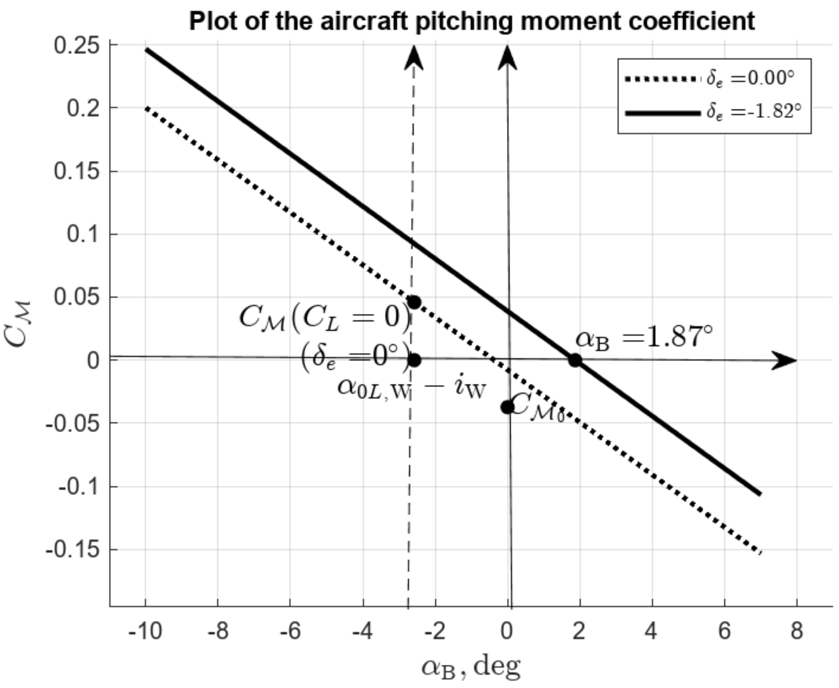

Figure 7 -Plot of the aircraft pitching moment coefficient

```
LH = -1908 N = -194.6 kgf
```

Hinge Moment

The estimation of the hinge moment

$$C_{h_\alpha} = -0.55 k_1 \left(\frac{c_e}{c}\right) k_1 \left(\frac{t}{c}\right) k_1 (\mathrm{BR}) k_1 \left(\frac{1}{A}\right) \tag{1.23}$$

```
Cha = -0.252 rad^-1
```

$$C_{h_\delta} = -0.89 k_2 \left(\frac{c_e}{c}\right) k_2 \left(\frac{t}{c}\right) k_2 (\mathrm{BR}) k_2 \left(\frac{1}{A}\right) \tag{1.24}$$

```
Chd = -0.624 rad^-1
```

Neutral points and SMs

Stick-Fixed

The neutral point for stick-fixed condition is the solution of the following equation:

$$C_{M_\alpha} = C_{L_{\alpha,W}}(\bar{x_G} - \bar{x_{\mathrm{ac,WB}}}) - \eta_H \frac{S_H}{S} (\bar{x_{\mathrm{ac},H}} - \bar{x_G}) C_{L_{\alpha,H}} \left[1 - \left(\frac{d\varepsilon}{d\alpha}\right)_H\right] = 0 \tag{1.25}$$

By the position $\bar{x_G} = \bar{x_N}$:

```
xN = 0.52
```

Now it's possible to study the $\mathrm{SM} = \bar{x_G} - \bar{x_N}$:

```
SM = -0.22

The aircraft is stable.
```

If the approximated formula is used:

```
xN approx = 0.54

SM approx = -0.24
```

Stick-Free

```
F = 0.781
```

The neutral point for stick-free condition is the solution of the following equation:

$$C_{M_\alpha}' = C_{L_{\alpha,W}}(\bar{x_G} - \bar{x_{\mathrm{ac,WB}}}) - \eta_H \frac{S_H}{S} (\bar{x_{\mathrm{ac},H}} - \bar{x_G}) C_{L_{\alpha,H}} \left[1 - \left(\frac{d\varepsilon}{d\alpha}\right)_H\right] F = 0 \tag{1.26}$$

By the position $\bar{x_G} = \bar{x_N}'$:

```
xN' = 0.45
```

Now it's possible to study the $\mathrm{SM} = \bar{x_G} - \bar{x_N}$:

```
SM' = -0.15

The aircraft is stable.
```

If the approximated formula is used:

```
xN' approx = 0.46

SM' approx = -0.16
```

2.3.4 Lateral-Directional stability

```
Selected input: the sideslip angle is β = -3 deg with a roll rate p = 0 deg/s. The yaw rate is r
= 0 deg/s.
```

To study lateral-directional stability of the aircraft, two values are calculated by a system of two equations. The values are aileron deflection δ_a and rudder deflection δ_r by considering the assigned side-slip angle β. The equations are:

1. Lateral Stability;
2. Directional Stability.

Lateral stability

	Derivative	Value	Unit
1	'Cℒ0'	0	''
2	'Cℒβ'	-0.0863	'rad-1'
3	'Cℒδa'	-0.1503	'rad-1'
4	'Cℒδr'	0.0145	'rad-1'
5	'Cℒp'	-0.8361	'rad-1'
6	'Cℒr'	0.0322	'rad-1'
7	'Cℒmot'	-0.0010	''

Directional stability

	Derivative	Value	Unit

	Derivative	Value	Unit
8	'C𝒩0'	0	' '
9	'C𝒩β'	0.0942	'rad-1'
10	'C𝒩δa'	0	'rad-1'
11	'C𝒩δr'	-0.0922	'rad-1'
12	'C𝒩p'	0.0672	'rad-1'
13	'C𝒩r'	-0.2049	'rad-1'
14	'C𝒩mot'	0.000	' '

Solving the system

The deflection values of the aileron and the rudder are the unknown values of the following system:

$$\begin{bmatrix} C_{\mathscr{L}_{\delta a}} & C_{\mathscr{L}_{\delta r}} \\ C_{\mathcal{N}_{\delta a}} & C_{\mathcal{N}_{\delta r}} \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} = \begin{bmatrix} -\left(C_{\mathscr{L}_0} + C_{\mathscr{L}_\beta}\beta + C_{\mathscr{L}_p}\widehat{p} + C_{\mathscr{L}_r}\widehat{r} + C_{\mathscr{L}_{\text{mot}}}\right) \\ -\left(C_{\mathcal{N}_0} + C_{\mathcal{N}_\beta}\beta + C_{\mathcal{N}_p}\widehat{p} + C_{\mathcal{N}_r}\widehat{r} + C_{\mathcal{N}_{\text{mot}}}\right) \end{bmatrix} \quad (1.53)$$

```
δa = 0.019 rad = 1.1 deg

δr = -0.053 rad = -3.1 deg
```

One-Engine-Inoperative condition

```
The engine that you have chosen fails. The lateral-directional equilibrium will be updated
considering the asymmetrical thrust generated by the operating engine.

T = 3688 N = 376 kgf

C𝒩mot = -0.0039

δa = 0.015 rad = 0.8 deg

δr = -0.096 rad = -5.5 deg
```

2.4 Boeing 737

2.4.1 Input Data

Figure 8-Boeing 737

Select the following parameters:

1. Speed.
2. Load
3. Pitch rate
4. Beta
5. Roll rate
6. Yaw rate
7. OEI
8. Propeller rotation direction

Input data are here below summarized:

```
AIRCRAFT NAME: Boeing737
```

Geometry

```
WING: b = 34.30 m | cr = 4.47 m | ct = 1.25 m | iW = 0.035 rad | Λ = 0.440 rad | Γ = 0.110 rad
HORIZONTAL TAILPLANE: bH = 8.60 m | SH = 32.80 m2 | lH = 13.36 m | iH = -0.035 rad | ce_c = 0.25
| BR = 0.02 | t_c = 0.10
VERTICAL TAILPLANE: SV = 26.40 m2 | lV = 7.60 m | hV = 4.00 m
ENGINE: d_mot= Yt = 5.000 m | Xt = 0.000 m | Zt = 0.000 m | D = 2.000 m
```

Aerodynamics

```
GLOBAL: CD0 = 0.025 | e = 0.82 | xcg = 0.25 | CRoll_0 = 0.000 | CN_0 = 0.000
WING: xac_W = 0.29 | Cl_alphaW = 6.30 rad-1 | alpha_0l_W = -0.02 rad | eW = 0.90 | CM_ac_W = -
0.068 rad-1 | CRoll_p_W = -0.013 | CRoll_delta_a = -0.005 rad-1 | CRoll_beta_wpos = -0.001 rad-1
```

| CN_delta_a = 0.000 rad-1 | CN_r_W = 0.000 | τa = 0.40 | CN_beta_W = 0.000| CN_p_Wtilt = 0.000 | CN_p_W_tipsuc = 0.000 | CN_r_W = 0.000
HORIZONTAL TAILPLANE: Cl_alphaH = 6.300 rad-1 | eH = 0.90 | ηH = 1.00 | τe = 0.38 | CH_alpha_e = -0.400 rad-1 | CH_deltaE_e = -0.800 rad-1
VERTICAL TAILPLANE: CL_alphaV = 2.800 rad-1 | ηV = 1.00 | dσ/dβ = 0.140 | τr = 0.45 | CN_p_V = 0.000 | CN_r_V = 0.000 | dσ/dp = 0.000 | dσ/dr = 0.000
FUSELAGE: CM_0_B = -0.060 | CM_alpha_B(W) = 0.144 rad-1 | CN_beta_B = -0.070 rad-1

Powerplant

POWERPLANT: Engines number = 2 | Pa = 16140 kW

2.4.2 Preliminary calculations

Preliminary calculations used in the entire code are shown below.

- **Taper ratio**

λ = 0.28

- **Wing area**

S = 98.10 m²

- **Dynamic pressure**

q∞ = 6125.0 Pa

- **Mean aerodynamic chord**

m.a.c. = 3.16 m

- **Lift coefficient**

CL = 0.98

- **Aspect ratio** (wing and horizontal tail, respectively)

AR = 11.99
ARH = 2.25

- **Wing lift curve slope**

CLα,W = 5.313 rad-1 = 0.093 deg-1

- **Wing-Body's lift coefficient at zero lift**

CL0,W = 0.29

- **Horizontal tail lift curve slope**

CLα,H = 3.169 rad-1 = 0.055 deg-1

- **Mean value of downwash at** $\alpha_{\mathrm{B}} = 0$

ε0 = 0.017 rad = 1.0 deg

- **Downwash gradient**

dε/dα = 0.313

2.4.3 Longitudinal equilibrium, stability and control

```
Selected input: the aircraft speed is V = 250 m/s with a load factor n = 1. The pitch rate is q = 0 deg/s.
```

To study longitudinal stability and control of the aircraft, two values are calculated by a system of two equations. The values are the angle of attack and the equilibrium elevator angle by considering the assigned horizontal tail incidence angle i_{H}. The equations are:

1. Vertical Translational Equilibrium;
2. Rotational Equilibrium.

Vertical translation equilibrium

	Derivative	Value	Unit
1	'CL0'	0.2740	' '
2	'CLα'	6.0404	'rad-1'
3	'CLδe'	0.4026	'rad-1'
4	'CLiH'	1.0595	'rad-1'

Rotational equilibrium

	Derivative	Value	Unit
5	'CM0'	-0.0546	' '
6	'CMα'	-3.1422	'rad-1'
7	'CMδe'	-1.7011	'rad-1'
8	'CMiH'	-4.4764	'rad-1'
9	'CMq'	-37.8262	'rad-1'
10	'CMmot'	0	' '

Solving the system

The values of the angle of attack and the deflection of the elevator are the solution of the following system:

$$\begin{bmatrix} C_{L_\alpha} & C_{L_{\delta e}} \\ C_{M_\alpha} & C_{M_{\delta e}} \end{bmatrix} \begin{bmatrix} \alpha_B \\ \delta_e \end{bmatrix} = \begin{bmatrix} C_L - C_{L0} - C_{L_{iH}} i_H \\ -C_{M0} - C_{M_{mot}} - C_{M_{iH}} i_H - C_{M_q} \hat{q} \end{bmatrix} \tag{1.18}$$

αB = 0.045 rad = 2.6 deg

δe = -0.023 rad = -1.3 deg

Now it is possible to represent the evolution of the pitching moment coefficient vs the body angle of attack.

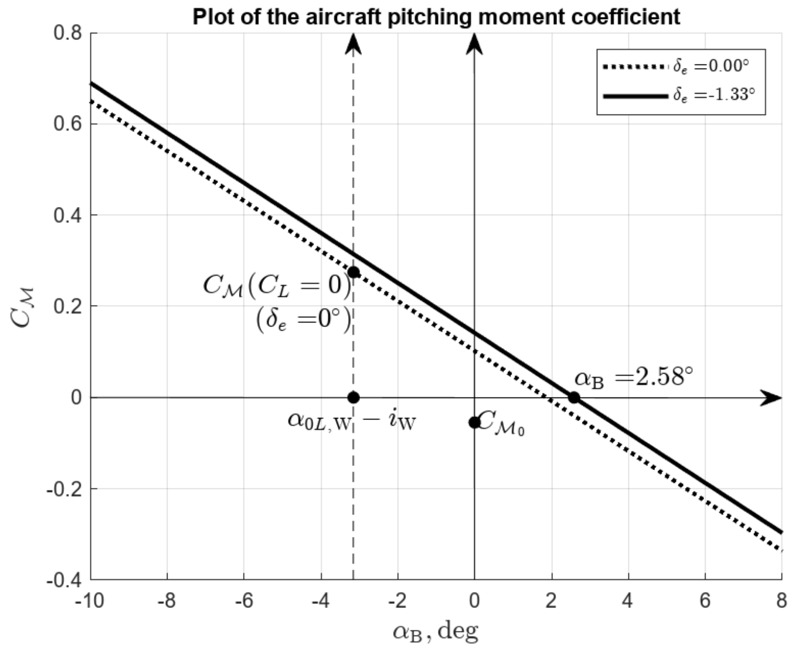

Figure 9- Plot of the aircraft pitching moment coefficient

LH = -37590 N = -3833.1 kgf

Hinge Moment

The estimation of the hinge moment

$$C_{h_\alpha} = -0.55 k_1\left(\frac{c_e}{c}\right) k_1\left(\frac{t}{c}\right) k_1(\mathrm{BR}) k_1\left(\frac{1}{A}\right) \tag{1.23}$$

Cha = -0.160 rad^-1

$$C_{h_\delta} = -0.89 k_2\left(\frac{c_e}{c}\right) k_2\left(\frac{t}{c}\right) k_2(\mathrm{BR}) k_2\left(\frac{1}{A}\right) \tag{1.24}$$

Chd = -0.514 rad^-1

Neutral points and SMs

Stick-Fixed

The neutral point for stick-fixed condition is the solution of the following equation:

$$C_{M_\alpha} = C_{L_{\alpha,W}}(\bar{x}_G - \bar{x}_{\text{ac,WB}}) - \eta_H \frac{S_H}{S}(\bar{x}_{\text{ac},H} - \bar{x}_G)C_{L_{\alpha,H}}\left[1 - \left(\frac{d\varepsilon}{d\alpha}\right)_H\right] = 0 \quad (1.25)$$

By the position $\bar{x}_G = \bar{x}_N$:

```
xN = 0.77
```

Now it's possible to study the $\text{SM} = \bar{x}_G - \bar{x}_N$:

```
SM = -0.52

The aircraft is stable.
```

If the approximated formula is used:

```
xN approx = 0.86

SM approx = -0.59
```

Stick-Free

```
F = 0.810
```

The neutral point for stick-free condition is the solution of the following equation:

$$C_{M_\alpha}{}' = C_{L_{\alpha,W}}(\bar{x}_G - \bar{x}_{\text{ac,WB}}) - \eta_H \frac{S_H}{S}(\bar{x}_{\text{ac},H} - \bar{x}_G)C_{L_{\alpha,H}}\left[1 - \left(\frac{d\varepsilon}{d\alpha}\right)_H\right]F = 0 \quad (1.26)$$

By the position $\bar{x}_G = \bar{x}_N{}'$:

```
xN' = 0.68
```

Now it's possible to study the $\text{SM} = \bar{x}_G - \bar{x}_N$:

```
SM' = -0.43

The aircraft is stable.
```

If the approximated formula is used:

```
xN' approx = 0.73

SM' approx = -0.48
```

2.4.4 Lateral-Directional stability

```
Selected input: the sideslip angle is β = -3 deg with a roll rate p = 0 deg/s. The yaw rate is r
= 0 deg/s.
```

To study lateral-directional stability of the aircraft, two values are calculated by a system of two equations. The values are aileron deflection δ_a and rudder deflection δ_r by considering the assigned side-slip angle β. The equations are:

1. Lateral Stability;
2. Directional Stability.

Lateral stability

	Derivative	Value	Unit
1	'Cℒ0'	0	' '
2	'Cℒβ'	-0.2017	'rad-1'
3	'Cℒδa'	-0.1237	'rad-1'
4	'Cℒδr'	0.0395	'rad-1'
5	'Cℒp'	-0.9995	'rad-1'
6	'Cℒr'	0.0389	'rad-1'
7	'Cℒmot'	-0.0025	' '

Directional stability

	Derivative	Value	Unit
8	'C𝒩0'	0	' '
9	'C𝒩β'	0.0736	'rad-1'
10	'C𝒩δa'	0	'rad-1'
11	'C𝒩δr'	-0.0751	'rad-1'
12	'C𝒩p'	0.1095	'rad-1'
13	'C𝒩r'	-0.0740	'rad-1'
14	'C𝒩mot'	0.000	' '

Solving the system

The deflection values of the aileron and the rudder are the unknown values of the following system:

$$\begin{bmatrix} C_{\mathscr{L}_{\delta a}} & C_{\mathscr{L}_{\delta r}} \\ C_{\mathscr{N}_{\delta a}} & C_{\mathscr{N}_{\delta r}} \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} = \begin{bmatrix} -\left(C_{\mathscr{L}_0} + C_{\mathscr{L}_\beta}\beta + C_{\mathscr{L}_p}\hat{p} + C_{\mathscr{L}_r}\hat{r} + C_{\mathscr{L}_{\text{mot}}}\right) \\ -\left(C_{\mathscr{N}_0} + C_{\mathscr{N}_\beta}\beta + C_{\mathscr{N}_p}\hat{p} + C_{\mathscr{N}_r}\hat{r} + C_{\mathscr{N}_{\text{mot}}}\right) \end{bmatrix} \tag{1.53}$$

```
δa = 0.049 rad = 2.8 deg

δr = -0.051 rad = -2.9 deg
```

One-Engine-Inoperative condition

```
The engine that you have chosen fails. The lateral-directional equilibrium will be updated
considering the asymmetrical thrust generated by the operating engine.
T = 33684 N = 3434 kgf
C𝒩mot = -0.0082
δa = 0.014 rad = 0.8 deg
δr = -0.160 rad = -9.2 deg
```

2.5 Boeing 777

2.5.1 Input Data

Figure 10-Boeing 777

Select the following parameters:

1. Speed.
2. Load

3. Pitch rate
4. Beta
5. Roll rate
6. Yaw rate
7. OEI
8. Propeller rotation direction

Input data are here below summarized:

```
AIRCRAFT NAME: Boeing777
```

Geometry

```
WING: b = 60.00 m | cr = 12.98 m | ct = 3.11 m | iW = 0.035 rad | Λ = 0.440 rad | Γ = 0.110 rad
HORIZONTAL TAILPLANE: bH = 14.80 m | SH = 144.00 m2 | lH = 63.70 m | iH = -0.035 rad | ce_c =
0.25 | BR = 0.02 | t_c = 0.10
VERTICAL TAILPLANE: SV = 44.80 m2 | lV = 5.90 m | hV = 7.60 m
ENGINE: d_mot= Yt = 5.000 m | Xt = 0.000 m | Zt = 0.000 m | D = 2.000 m
```

Aerodynamics

```
GLOBAL: CD0 = 0.019 | e = 0.80 | xcg = 0.25 | CRoll_0 = 0.000 | CN_0 = 0.000
WING: xac_W = 0.29 | Cl_alphaW = 6.30 rad-1 | alpha_0l_W = 0.02 rad | eW = 0.90 | CM_ac_W = -
0.068 rad-1 | CRoll_p_W = -0.013 | CRoll_delta_a = -0.005 rad-1 | CRoll_beta_wpos = -0.001 rad-1
| CN_delta_a = 0.000 rad-1 | CN_r_W = 0.000 | τa = 0.40 | CN_beta_W = 0.000| CN_p_Wtilt = 0.000 |
CN_p_W_tipsuc = 0.000 | CN_r_W = 0.000
HORIZONTAL TAILPLANE: Cl_alphaH = 6.300 rad-1 | eH = 0.90 | ηH = 1.00 | τe = 0.40 | CH_alpha_e =
-0.043 rad-1 | CH_deltaE_e = -0.750 rad-1
VERTICAL TAILPLANE: CL_alphaV = 3.000 rad-1 | ηV = 1.00 | dσ/dβ = 0.150 | τr = 0.45 | CN_p_V =
0.000 | CN_r_V = 0.000 | dσ/dp = 0.000 | dσ/dr = 0.000
FUSELAGE: CM_0_B = -0.070 | CM_alpha_B(W) = 0.200 rad-1 | CN_beta_B = -0.070 rad-1
```

Powerplant

```
POWERPLANT: Engines number = 2 | Pa = 489514 kW
```

2.5.2 Preliminary calculations

Preliminary calculations used in the entire code are shown below.

- Taper ratio

```
λ = 0.24
```

- Wing area

```
S = 482.70 m²
```

- Dynamic pressure

```
q∞ = 6125.0 Pa
```

- Mean aerodynamic chord

```
m.a.c. = 9.05 m
```

- Lift coefficient

```
CL = 0.76
```

- **Aspect ratio** (wing and horizontal tail, respectively)

```
AR = 7.46
ARH = 1.52
```

- **Wing lift curve slope**

```
CLα,W = 4.851 rad-1 = 0.085 deg-1
```

- **Wing-Body's lift coefficient at zero lift**

```
CL0,W = 0.07
```

- **Horizontal tail lift curve slope**

```
CLα,H = 2.556 rad-1 = 0.045 deg-1
```

- **Mean value of downwash at** $\alpha_{\mathrm{B}} = 0$

```
ε0 = 0.007 rad = 0.4  deg
```

- **Downwash gradient**

```
dε/dα = 0.460
```

2.5.3 Longitudinal equilibrium, stability and control

```
Selected input: the aircraft speed is V = 250 m/s with a load factor n = 1. The pitch rate is q = 0 deg/s.
```

To study longitudinal stability and control of the aircraft, two values are calculated by a system of two equations. The values are the angle of attack and the equilibrium elevator angle by considering the assigned horizontal tail incidence angle i_{H}. The equations are:

3. Vertical Translational Equilibrium;
4. Rotational Equilibrium.

Vertical translation equilibrium

	Derivative	Value	Unit
1	'CL0'	0.0675	''
2	'CLα'	5.2625	'rad-1'
3	'CLδe'	0.3050	'rad-1'
4	'CLiH'	0.7625	'rad-1'

Rotational equilibrium

	Derivative	Value	Unit
5	'CM0'	-0.1009	''

	Derivative	Value	Unit
6	'CMα'	-2.8905	'rad-1'
7	'CMδe'	-2.1458	'rad-1'
8	'CMiH'	-5.3646	'rad-1'
9	'CMq'	-75.4847	'rad-1'
10	'CMmot'	0	' '

Solving the system

The values of the angle of attack and the deflection of the elevator are the solution of the following system:

$$\begin{bmatrix} C_{L_\alpha} & C_{L_{\delta e}} \\ C_{M_\alpha} & C_{M_{\delta e}} \end{bmatrix} \begin{bmatrix} \alpha_B \\ \delta_e \end{bmatrix} = \begin{bmatrix} C_L - C_{\mathrm{L0}} - C_{L_{\mathrm{iH}}} i_H \\ -C_{\mathrm{M0}} - C_{M_{\mathrm{mot}}} - C_{M_{\mathrm{iH}}} i_H - C_{M_q} \hat{q} \end{bmatrix} \tag{1.18}$$

```
αB = 0.069 rad = 4.0 deg

δe = -0.053 rad = -3.0 deg
```

Now it is possible to represent the evolution of the pitching moment coefficient vs the body angle of attack.

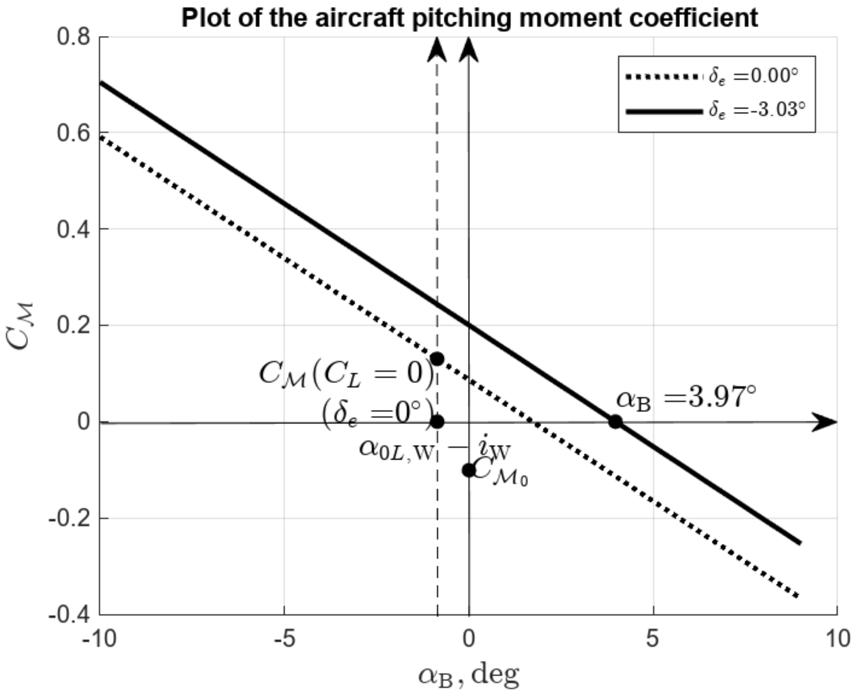

Figure 11- Plot of the aircraft pitching moment coefficient

```
LH = -57587 N = -5872.3 kgf
```

Hinge Moment

The estimation of the hinge moment

$$C_{h_\alpha} = -0.55 k_1 \left(\frac{c_e}{c}\right) k_1 \left(\frac{t}{c}\right) k_1 (\mathrm{BR}) k_1 \left(\frac{1}{A}\right) \tag{1.23}$$

```
Cha = -0.146 rad^-1
```

$$C_{h_\delta} = -0.89 k_2 \left(\frac{c_e}{c}\right) k_2 \left(\frac{t}{c}\right) k_2 (\mathrm{BR}) k_2 \left(\frac{1}{A}\right) \tag{1.24}$$

```
Chd = -0.489 rad^-1
```

Neutral points and SMs

Stick-Fixed

The neutral point for stick-fixed condition is the solution of the following equation:

$$C_{M_\alpha} = C_{L_{\alpha,W}}(\bar{x}_G - \bar{x}_{\mathrm{ac,WB}}) - \eta_H \frac{S_H}{S}(\bar{x}_{\mathrm{ac},H} - \bar{x}_G) C_{L_{\alpha,H}} \left[1 - \left(\frac{d\varepsilon}{d\alpha}\right)_H\right] = 0 \tag{1.25}$$

By the position $\bar{x}_G = \bar{x}_N$:

```
xN = 0.80
```

Now it's possible to study the $\mathrm{SM} = \bar{x}_G - \bar{x}_N$:

```
SM = -0.55

The aircraft is stable.
```

If the approximated formula is used:

```
xN approx = 0.85

SM approx = -0.60
```

Stick-Free

```
F = 0.977
```

The neutral point for stick-free condition is the solution of the following equation:

$$C_{M_\alpha}' = C_{L_{\alpha,W}}(\bar{x}_G - \bar{x}_{\mathrm{ac,WB}}) - \eta_H \frac{S_H}{S}(\bar{x}_{\mathrm{ac},H} - \bar{x}_G) C_{L_{\alpha,H}} \left[1 - \left(\frac{d\varepsilon}{d\alpha}\right)_H\right] F = 0 \tag{1.26}$$

By the position $\bar{x}_G = \bar{x}_N'$:

```
xN' = 0.79
```

Now it's possible to study the $\mathrm{SM} = \bar{x}_G - \bar{x}_N$:

```
SM' = -0.54

The aircraft is stable.
```

If the approximated formula is used:

```
xN' approx = 0.83

SM' approx = -0.58
```

2.5.4 Lateral-Directional stability

```
Selected input: the sideslip angle is β = -3 deg with a roll rate p = 0 deg/s. The yaw rate is r
= 0 deg/s.
```

To study lateral-directional stability of the aircraft, two values are calculated by a system of two equations. The values are aileron deflection δ_a and rudder deflection δ_r by considering the assigned side-slip angle β. The equations are:

3. Lateral Stability;
4. Directional Stability.

Lateral stability

	Derivative	Value	Unit
1	'Cℒ0'	0	''
2	'Cℒβ'	-0.1421	'rad-1'
3	'Cℒδa'	-0.1069	'rad-1'
4	'Cℒδr'	0.0159	'rad-1'
5	'Cℒp'	-0.9188	'rad-1'
6	'Cℒr'	0.0069	'rad-1'
7	'Cℒmot'	-0.0088	''

Directional stability

	Derivative	Value	Unit

	Derivative	Value	Unit
8	'C𝒩0'	0	' '
9	'C𝒩β'	-0.0467	'rad-1'
10	'C𝒩δa'	0	'rad-1'
11	'C𝒩δr'	-0.0123	'rad-1'
12	'C𝒩p'	0.0873	'rad-1'
13	'C𝒩r'	-0.0054	'rad-1'
14	'C𝒩mot'	0.000	' '

Solving the system

The deflection values of the aileron and the rudder are the unknown values of the following system:

$$\begin{bmatrix} C_{\mathscr{L}_{\delta a}} & C_{\mathscr{L}_{\delta r}} \\ C_{\mathscr{N}_{\delta a}} & C_{\mathscr{N}_{\delta r}} \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} = \begin{bmatrix} -\left(C_{\mathscr{L}_0} + C_{\mathscr{L}_\beta}\beta + C_{\mathscr{L}_p}\widehat{p} + C_{\mathscr{L}_r}\widehat{r} + C_{\mathscr{L}_{\mathrm{mot}}}\right) \\ -\left(C_{\mathscr{N}_0} + C_{\mathscr{N}_\beta}\beta + C_{\mathscr{N}_p}\widehat{p} + C_{\mathscr{N}_r}\widehat{r} + C_{\mathscr{N}_{\mathrm{mot}}}\right) \end{bmatrix} \quad (1.53)$$

```
δa = 0.017 rad = 1.0 deg

δr = 0.199 rad = 11.4 deg
```

One-Engine-Inoperative condition

```
The engine that you have chosen fails. The lateral-directional equilibrium will be updated
considering the asymmetrical thrust generated by the operating engine.
T = 148038 N = 15091 kgf
C𝒩mot = -0.0042
δa = -0.033 rad = -1.9 deg
δr = -0.140 rad = -8.0 deg
```

2.6 Beechcraft KingAir

2.6.1 Input Data

Figure 12-Beechcraft KingAir

Select the following parameters:

1. Speed.
2. Load
3. Pitch rate
4. Beta
5. Roll rate
6. Yaw rate
7. OEI
8. Propeller rotation direction

Input data are here below summarized:

```
AIRCRAFT NAME: Beechcraft KingAir
```

Geometry

```
WING: b = 17.65 m | cr = 2.15 m | ct = 1.07 m | iW = 0.035 rad | Λ = 0.200 rad | Γ = 0.120 rad
HORIZONTAL TAILPLANE: bH = 4.60 m | SH = 4.60 m2 | lH = 4.90 m | iH = -0.035 rad | ce_c = 0.25 |
BR = 0.02 | t_c = 0.10
VERTICAL TAILPLANE: SV = 3.70 m2 | lV = 5.75 m | hV = 1.35 m
ENGINE: d_mot = 3.200 m | Xt = 1.300 m | Zt = 0.030 m | D = 2.200 m
```

Aerodynamics

```
GLOBAL: CD0 = 0.03 | e = 0.80 | xcg = 0.26 | CRoll_0 = 0.000 | CN_0 = 0.000
WING: xac_W = 0.28 | Cl_alphaW = 6.13 rad-1 | alpha_0l_W = -0.02 rad | eW = 0.90 | CM_ac_W = -
0.074 rad-1 | CRoll_p_W = 0.000 | CRoll_delta_a = 0.000 rad-1 | CRoll_beta_wpos = 0.000 rad-1 |
CN_delta_a = 0.000 rad-1  CN_r_W = 0.000 | τa = 0.40 | CN_beta_W = 0.000 rad-1 | CN_p_Wtilt =
0.000 rad-1 | CN_p_W_tipsuc = 0.000 rad-1 | CN_r_W = 0.000 rad-1
HORIZONTAL TAILPLANE: Cl_alphaH = 6.400 rad-1 | eH = 0.90 | ηH = 0.95 | τe = 0.40 | CH_alpha_e =
-0.440 rad-1 | CH_deltaE_e = -0.810 rad-1
VERTICAL TAILPLANE: CL_alphaV = 3.040 rad-1 | ηV = 1.00 | dσ/dβ = 0.120 | τr = 0.48 | CN_p_V =
0.000 | CN_r_V = 0.000 | dσ/dp = 0.000 | dσ/dr = 0.000
FUSELAGE: CM_0_B = -0.060 | CM_alpha_B(W) = 0.573 rad-1 | CN_beta_B = -0.132 rad-1
```

Powerplant

```
POWERPLANT: Engines number = 2 | Pa = 410 kW | Np = 11.0 kgf | J = 1.00
```

2.6.2 Preliminary calculations

Preliminary calculations used in the entire code are shown below.

- Taper ratio

```
λ = 0.50
```

- Wing area

```
S = 28.420 m²
```

- Dynamic pressure

```
q∞ = 6125.0 Pa
```

- Mean aerodynamic chord

```
m.a.c. = 1.67 m
```

- Lift coefficient

```
CL = 0.47
```

- Aspect ratio (wing and horizontal tail, respectively)

```
AR = 10.96
ARH = 4.60
```

- Wing lift curve slope

```
CLα,W = 5.118 rad-1 = 0.089 deg-1
```

- Wing-Body's lift coefficient at zero lift

```
CL0,W = 0.29
```

- **Horizontal tail lift curve slope**

```
CLα,H = 4.289 rad-1 = 0.075 deg-1
```

- **Mean value of downwash at** $\alpha_{\mathrm{B}} = 0$

```
ε0 = 0.019 rad = 1.1 deg
```

- **Downwash gradient**

```
dε/dα = 0.330
```

2.6.3 Longitudinal equilibrium, stability and control

```
Selected input: the aircraft speed is V = 100 m/s with a load factor n = 1. The pitch rate is q = 0 deg/s.
```

To study longitudinal stability and control of the aircraft, two values are calculated by a system of two equations. The values are the angle of attack and the equilibrium elevator angle by considering the assigned horizontal tail incidence angle i_{H}. The equations are:

5. Vertical Translational Equilibrium;
6. Rotational Equilibrium.

Vertical translation equilibrium

	Derivative	Value	Unit
1	'CL0'	0.2793	' '
2	'CLα'	5.5597	'rad-1'
3	'CLδe'	0.2639	'rad-1'
4	'CLiH'	0.6596	'rad-1'

Rotational equilibrium

	Derivative	Value	Unit
5	'CM0'	-0.0708	' '
6	'CMα'	-0.8254	'rad-1'
7	'CMδe'	-0.7740	'rad-1'
8	'CMiH'	-1.9350	'rad-1'
9	'CMq'	-11.3526	'rad-1'

	Derivative	Value	Unit
10	'CMmot'	0.0007	' '

Solving the system

The values of the angle of attack and the deflection of the elevator are the solution of the following system:

$$\begin{bmatrix} C_{L_\alpha} & C_{L_{\delta e}} \\ C_{M_\alpha} & C_{M_{\delta e}} \end{bmatrix} \begin{bmatrix} \alpha_B \\ \delta_e \end{bmatrix} = \begin{bmatrix} C_L - C_{\mathrm{L0}} - C_{L_{\mathrm{iH}}} i_H \\ -C_{\mathrm{M0}} - C_{M_{\mathrm{mot}}} - C_{M_{\mathrm{iH}}} i_H - C_{M_q} \widehat{q} \end{bmatrix} \tag{1.18}$$

```
αB = 0.041 rad = 2.4 deg
```

```
δe = -0.047 rad = -2.7 deg
```

Now it is possible to represent the evolution of the pitching moment coefficient vs the body angle of attack.

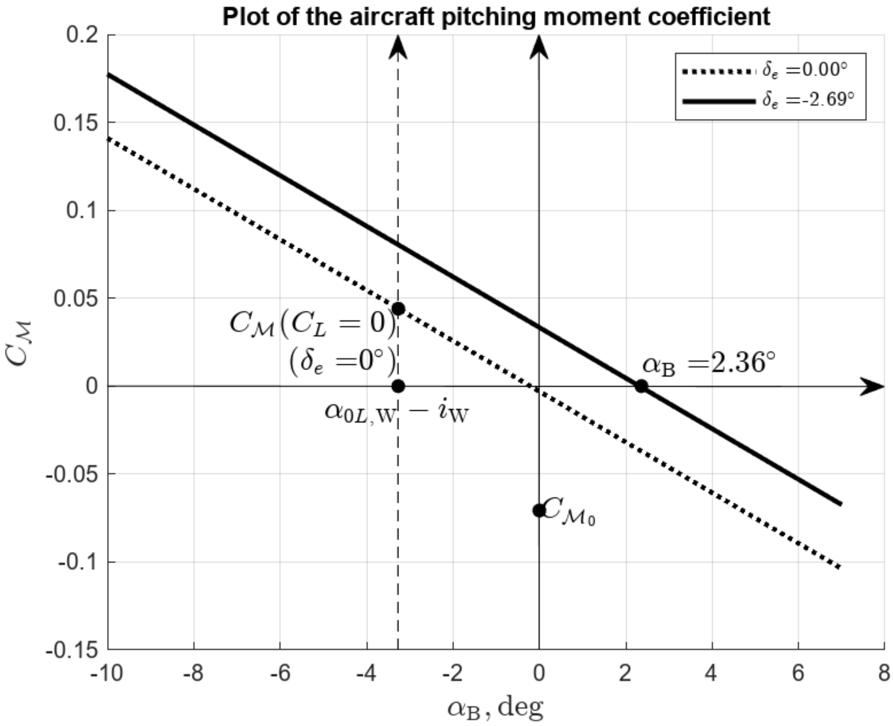

Figure 13- Plot of the aircraft pitching moment coefficient

```
LH = -5645 N = -575.6 kg
```

Hinge Moment

The estimation of the hinge moment

$$C_{h_\alpha} = -0.55 k_1 \left(\frac{c_e}{c}\right) k_1 \left(\frac{t}{c}\right) k_1 (\mathrm{BR}) k_1 \left(\frac{1}{A}\right) \tag{1.23}$$

```
Cha = -0.285 rad^-1
```

$$C_{h_\delta} = -0.89 k_2 \left(\frac{c_e}{c}\right) k_2 \left(\frac{t}{c}\right) k_2 (\mathrm{BR}) k_2 \left(\frac{1}{A}\right) \tag{1.24}$$

```
Chd = -0.655 rad^-1
```

Neutral points and SMs

Stick-Fixed

The neutral point for stick-fixed condition is the solution of the following equation:

$$C_{M_\alpha} = C_{L_{\alpha,W}}(\bar{x}_G - \bar{x}_{\mathrm{ac,WB}}) - \eta_H \frac{S_H}{S} (\bar{x}_{\mathrm{ac},H} - \bar{x}_G) C_{L_{\alpha,H}} \left[1 - \left(\frac{d\varepsilon}{d\alpha}\right)_H\right] = 0 \tag{1.25}$$

By the position $\bar{x}_G = \bar{x}_N$:

```
xN = 0.41
```

Now it's possible to study the $\mathrm{SM} = \bar{x}_G - \bar{x}_N$:

```
SM = -0.15
```

```
The aircraft is stable.
```

If the approximated formula is used:

```
xN approx = 0.42
```

```
SM approx = -0.16
```

Stick-Free

```
F = 0.783
```

The neutral point for stick-free condition is the solution of the following equation:

$$C_{M_\alpha}' = C_{L_{\alpha,W}}(\bar{x}_G - \bar{x}_{\mathrm{ac,WB}}) - \eta_H \frac{S_H}{S} (\bar{x}_{\mathrm{ac},H} - \bar{x}_G) C_{L_{\alpha,H}} \left[1 - \left(\frac{d\varepsilon}{d\alpha}\right)_H\right] F = 0 \tag{1.26}$$

By the position $\bar{x}_G = \bar{x}_N'$:

```
xN' = 0.36
```

Now it's possible to study the $\mathrm{SM} = \bar{x}_G - \bar{x}_N$:

```
SM' = -0.10
```

```
The aircraft is stable.
```

If the approximated formula is used:

```
xN' approx = 0.37

SM' approx = -0.11
```

2.6.4 Lateral-Directional stability

```
Selected input: the sideslip angle is β = -3 deg with a roll rate p = 0 deg/s. The yaw rate is r
= 0 deg/s.
```

To study lateral-directional stability of the aircraft, two values are calculated by a system of two equations. The values are aileron deflection δ_a and rudder deflection δ_r by considering the assigned side-slip angle β. The equations are:

5. Lateral Stability;
6. Directional Stability.

Lateral stability

	Derivative	Value	Unit
1	'Cℒ0'	0	' '
2	'Cℒβ'	-0.1642	'rad-1'
3	'Cℒδa'	-0.1478	'rad-1'
4	'Cℒδr'	0.0145	'rad-1'
5	'Cℒp'	-0.8896	'rad-1'
6	'Cℒr'	0.0197	'rad-1'
7	'Cℒmot'	-0.0005	' '

Directional stability

	Derivative	Value	Unit
8	'C𝒩0'	0	' '
9	'C𝒩β'	-0.0185	'rad-1'
10	'C𝒩δa'	0	'rad-1'

	Derivative	Value	Unit
11	'C𝒩δr'	-0.0619	'rad-1'
12	'C𝒩p'	0.0525	'rad-1'
13	'C𝒩r'	-0.0840	'rad-1'
14	'C𝒩mot'	0.000	' '

Solving the system

The deflection values of the aileron and the rudder are the unknown values of the following system:

$$\begin{bmatrix} C_{\mathscr{L}_{\delta a}} & C_{\mathscr{L}_{\delta r}} \\ C_{\mathcal{N}_{\delta a}} & C_{\mathcal{N}_{\delta r}} \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} = \begin{bmatrix} -\left(C_{\mathscr{L}_0} + C_{\mathscr{L}_\beta}\beta + C_{\mathscr{L}_p}\hat{p} + C_{\mathscr{L}_r}\hat{r} + C_{\mathscr{L}_{\text{mot}}}\right) \\ -\left(C_{\mathcal{N}_0} + C_{\mathcal{N}_\beta}\beta + C_{\mathcal{N}_p}\hat{p} + C_{\mathcal{N}_r}\hat{r} + C_{\mathcal{N}_{\text{mot}}}\right) \end{bmatrix} \tag{1.53}$$

```
δa = 0.057 rad = 3.2 deg

δr = 0.016 rad = 0.9 deg
```

One-Engine-Inoperative condition

```
The engine that you have chosen fails. The lateral-directional equilibrium will be updated
considering the asymmetrical thrust generated by the operating engine.
T = 5627 N = 574 kgf
C𝒩mot = -0.0064
δa = 0.047 rad = 2.7 deg
δr = -0.079 rad = -4.5 deg
```

2.7 Dornier Do 328

2.7.1 Input Data

Figure 14-Dornier Do 328

Select the following parameters:

1. Speed.
2. Load
3. Pitch rate
4. Beta
5. Roll rate
6. Yaw rate
7. OEI
8. Propeller rotation direction

Input data are here below summarized:

```
AIRCRAFT NAME: Dornier_328
```

Geometry

```
WING: b = 21.00 m | cr = 2.15 m | ct = 1.60 m | iW = 0.035 rad | Λ = 0.170 rad | Γ = 0.110 rad
HORIZONTAL TAILPLANE: bH = 5.18 m | SH = 25.20 m2 | lH = 4.30 m | iH = -0.035 rad | ce_c = 0.25 |
BR = 0.02 | t_c = 0.10
```

```
VERTICAL TAILPLANE: SV = 5.40 m2 | lV = 3.46 m | hV = 1.48 m
ENGINE: d_mot = 0.530 m | Xt = 0.000 m | Zt = 0.000 m | D = 0.510 m
```

Aerodynamics

```
GLOBAL: CD0 = 0.03 | e = 0.80 | xcg = 0.30 | CRoll_0 = 0.000 | CN_0 = 0.000
WING: xac_W = 0.25 | Cl_alphaW = 6.02 rad-1 | alpha_0l_W = -0.02 rad | eW = 0.90 | CM_ac_W = -
0.074 rad-1 | CRoll_p_W = 0.000 | CRoll_delta_a = 0.000 rad-1 | CRoll_beta_wpos = 0.000 rad-1 |
CN_delta_a = 0.000 rad-1  CN_r_W = 0.000 | τa = 0.40 | CN_beta_W = 0.000 rad-1 | CN_p_Wtilt =
0.000 rad-1 | CN_p_W_tipsuc = 0.000 rad-1 | CN_r_W = 0.000 rad-1
HORIZONTAL TAILPLANE: Cl_alphaH = 6.300 rad-1 | eH = 0.90 | ηH = 0.95 | τe = 0.40 | CH_alpha_e =
-0.440 rad-1 | CH_deltaE_e = -0.810 rad-1
VERTICAL TAILPLANE: CL_alphaV = 3.400 rad-1 | ηV = 1.00 | dσ/dβ = 0.120 | τr = 0.48 | CN_p_V =
0.000 | CN_r_V = 0.000 | dσ/dp = 0.000 | dσ/dr = 0.000
FUSELAGE: CM_0_B = -0.061 | CM_alpha_B(W) = 0.370 rad-1 | CN_beta_B = -0.090 rad-1
```

Powerplant

```
POWERPLANT: Engines number = 2 | Pa = 2 kW | Np = 11.0 kgf | J = 1.00
```

2.7.2 Preliminary calculations

Preliminary calculations used in the entire code are shown below.

- Taper ratio

```
λ = 0.74
```

- Wing area

```
S = 39.38 m²
```

- Dynamic pressure

```
q∞ = 6125.0 Pa
```

- Mean aerodynamic chord

```
m.a.c. = 1.89 m
```

- Lift coefficient

```
CL = 0.57
```

- Aspect ratio (wing and horizontal tail, respectively)

```
AR = 11.20
ARH = 1.06
```

- Wing lift curve slope

```
CLα,W = 5.058 rad-1 = 0.088 deg-1
```

- Wing-Body's lift coefficient at zero lift

```
CL0,W = 0.28
```

- Horizontal tail lift curve slope

```
CLα,H = 2.037 rad-1 = 0.036 deg-1
```

- **Mean value of downwash at $\alpha_{\mathrm{B}} = 0$**

```
ε0 = 0.018 rad = 1.0 deg
```

- **Downwash gradient**

```
dε/dα = 0.319
```

2.7.3 Longitudinal equilibrium, stability and control

```
Selected input: the aircraft speed is V = 100 m/s with a load factor n = 1. The pitch rate is q = 0 deg/s.
```

To study longitudinal stability and control of the aircraft, two values are calculated by a system of two equations. The values are the angle of attack and the equilibrium elevator angle by considering the assigned horizontal tail incidence angle i_{H}. The equations are:

7. Vertical Translational Equilibrium;
8. Rotational Equilibrium.

Vertical translation equilibrium

	Derivative	Value	Unit
1	'CL0'	0.2564	' '
2	'CLα'	5.9013	'rad-1'
3	'CLδe'	0.4954	'rad-1'
4	'CLiH'	1.2386	'rad-1'

Rotational equilibrium

	Derivative	Value	Unit
5	'CM0'	-0.0512	' '
6	'CMα'	-1.2963	'rad-1'
7	'CMδe'	-1.1281	'rad-1'
8	'CMiH'	-2.8202	'rad-1'
9	'CMq'	-12.8433	'rad-1'
10	'CMmot'	0	' '

Solving the system

The values of the angle of attack and the deflection of the elevator are the solution of the following system:

$$\begin{bmatrix} C_{L_\alpha} & C_{L_{\delta e}} \\ C_{M_\alpha} & C_{M_{\delta e}} \end{bmatrix} \begin{bmatrix} \alpha_B \\ \delta_e \end{bmatrix} = \begin{bmatrix} C_L - C_{L0} - C_{L_{iH}} i_H \\ -C_{M0} - C_{M_{mot}} - C_{M_{iH}} i_H - C_{M_q} \hat{q} \end{bmatrix} \quad (1.18)$$

```
αB = 0.063 rad = 3.6 deg
```

```
δe = -0.030 rad = -1.7 deg
```

Now it is possible to represent the evolution of the pitching moment coefficient vs the body angle of attack.

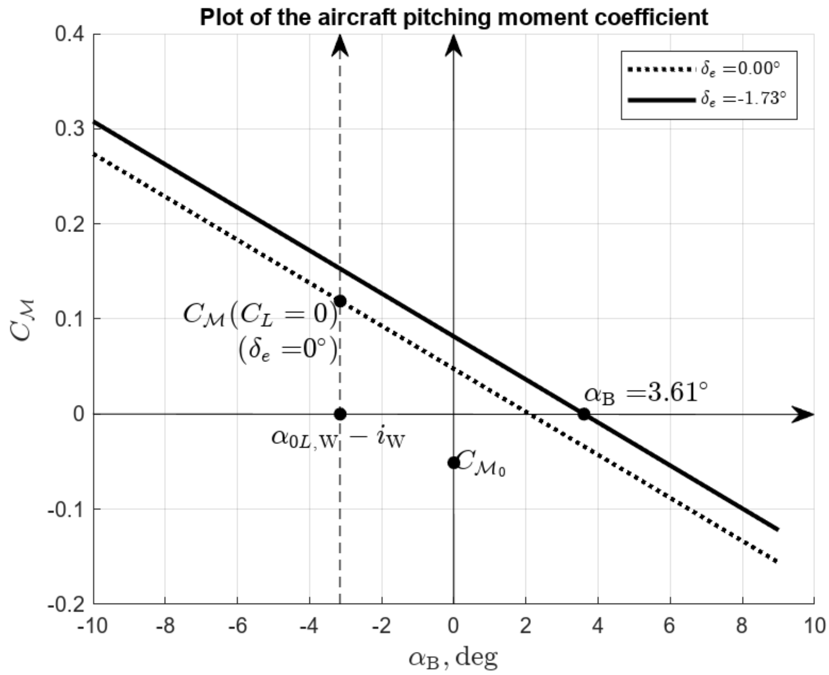

Figure 15- Plot of the aircraft pitching moment coefficient

```
LH = -6518 N = -664.7 kgf
```

Hinge Moment

The estimation of the hinge moment

$$C_{h_\alpha} = -0.55 k_1 \left(\frac{c_e}{c}\right) k_1 \left(\frac{t}{c}\right) k_1 (\mathrm{BR}) k_1 \left(\frac{1}{A}\right) \quad (1.23)$$

```
Cha = -0.146 rad^-1
```

$$C_{h_\delta} = -0.89 k_2\left(\frac{c_e}{c}\right) k_2\left(\frac{t}{c}\right) k_2(\mathrm{BR}) k_2\left(\frac{1}{A}\right) \tag{1.24}$$

```
Chd = -0.489 rad^-1
```

Neutral points and SMs

Stick-Fixed

The neutral point for stick-fixed condition is the solution of the following equation:

$$C_{M_\alpha} = C_{L_{\alpha,W}}(\bar{x_G} - \bar{x}_{\mathrm{ac,WB}}) - \eta_H \frac{S_H}{S}(\bar{x}_{\mathrm{ac},H} - \bar{x_G}) C_{L_{\alpha,H}}\left[1 - \left(\frac{d\varepsilon}{d\alpha}\right)_H\right] = 0 \tag{1.25}$$

By the position $\bar{x_G} = \bar{x_N}$:

```
xN = 0.52
```

Now it's possible to study the $\mathrm{SM} = \bar{x_G} - \bar{x_N}$:

```
SM = -0.22
```

```
The aircraft is stable.
```

If the approximated formula is used:

```
xN approx = 0.56
```

```
SM approx = -0.26
```

Stick-Free

```
F = 0.783
```

The neutral point for stick-free condition is the solution of the following equation:

$$C_{M_\alpha}' = C_{L_{\alpha,W}}(\bar{x_G} - \bar{x}_{\mathrm{ac,WB}}) - \eta_H \frac{S_H}{S}(\bar{x}_{\mathrm{ac},H} - \bar{x_G}) C_{L_{\alpha,H}}\left[1 - \left(\frac{d\varepsilon}{d\alpha}\right)_H\right] F = 0 \tag{1.26}$$

By the position $\bar{x_G} = \bar{x_N}'$:

```
xN' = 0.45
```

Now it's possible to study the $\mathrm{SM} = \bar{x_G} - \bar{x_N}$:

```
SM' = -0.15
```

```
The aircraft is stable.
```

If the approximated formula is used:

```
xN' approx = 0.47
```

```
SM' approx = -0.17
```

2.7.4 Lateral-Directional stability

```
Selected input: the sideslip angle is β = -3 deg with a roll rate p = 0 deg/s. The yaw rate is r
= 0 deg/s.
```

To study lateral-directional stability of the aircraft, two values are calculated by a system of two equations. The values are aileron deflection δ_a and rudder deflection δ_r by considering the assigned side-slip angle β. The equations are:

7. Lateral Stability;
8. Directional Stability.

Lateral stability

	Derivative	Value	Unit
1	'Cℒ0'	0	' '
2	'Cℒβ'	-0.1628	'rad-1'
3	'Cℒδa'	-0.1695	'rad-1'
4	'Cℒδr'	0.0158	'rad-1'
5	'Cℒp'	-0.8537	'rad-1'
6	'Cℒr'	0.0108	'rad-1'
7	'Cℒmot'	-0	' '

Directional stability

	Derivative	Value	Unit
8	'C𝒩0'	0	' '
9	'C𝒩β'	-0.0224	'rad-1'
10	'C𝒩δa'	0	'rad-1'
11	'C𝒩δr'	-0.0369	'rad-1'
12	'C𝒩p'	0.0509	'rad-1'

	Derivative	Value	Unit
13	'C𝒩r'	-0.0253	'rad-1'
14	'C𝒩mot'	0.000	' '

Solving the system

The deflection values of the aileron and the rudder are the unknown values of the following system:

$$\begin{bmatrix} C_{\mathscr{L}_{\delta a}} & C_{\mathscr{L}_{\delta r}} \\ C_{\mathcal{N}_{\delta a}} & C_{\mathcal{N}_{\delta r}} \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} = \begin{bmatrix} -\left(C_{\mathscr{L}_0} + C_{\mathscr{L}_\beta}\beta + C_{\mathscr{L}_p}\hat{p} + C_{\mathscr{L}_r}\hat{r} + C_{\mathscr{L}_{\text{mot}}}\right) \\ -\left(C_{\mathcal{N}_0} + C_{\mathcal{N}_\beta}\beta + C_{\mathcal{N}_p}\hat{p} + C_{\mathcal{N}_r}\hat{r} + C_{\mathcal{N}_{\text{mot}}}\right) \end{bmatrix} \tag{1.53}$$

```
δa = 0.053 rad = 3.1 deg
```

```
δr = 0.032 rad = 1.8 deg
```

One-Engine-Inoperative condition

```
The engine that you have chosen fails. The lateral-directional equilibrium will be updated
considering the asymmetrical thrust generated by the operating engine.
T = 9531 N = 972 kgf
C𝒩mot = -0.0010
δa = 0.051 rad = 2.9 deg
δr = 0.005 rad = 0.3 deg
```

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