Simulazione di sistemi non lineari
Nonlinear Current Allocator for Current Limit Avoidance at the JET Tokamak

Gianmaria De Tommasi[1]

[1]Università degli Studi di Napoli Federico II
detommas@unina.it

Ottobre 2012

Corsi AnsaldoBreda

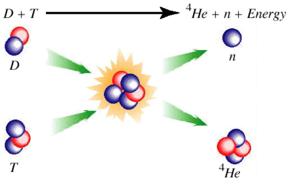
Outline

Nuclear Fusion for Dummies

Main Aim

Production of energy by means of a fusion reaction

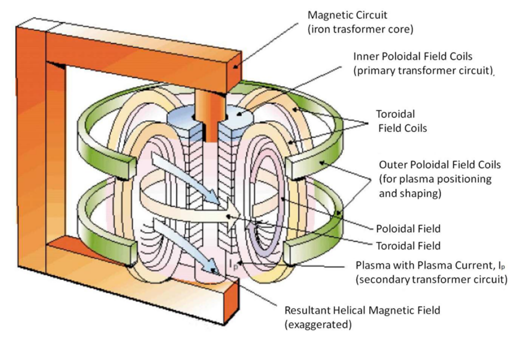
$$D + T \rightarrow {}^{4}\mathrm{He} + n$$



Plasma

- High temperature and pressure are needed
- Fully ionised gas $\rightarrow$ Plasma
- Magnetic field is needed to confine the plasma

What is a Tokamak ?

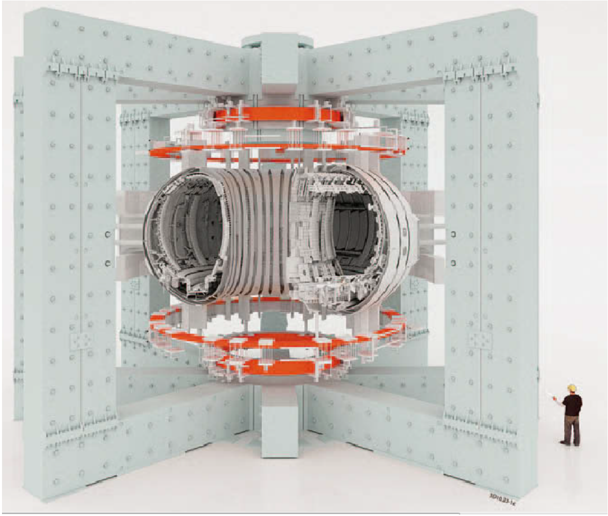


A tokamak is an electromagnetic machine containing a fully ionised gas (plasma) at about 100 million degrees within a torus shaped vacuum vessel. Poloidal and toroidal field coils, together with the plasma current, generate a spiralling magnetic field that confines the plasma.

The JET tokamak - 1

- The Joint European Torus (JET) is an example of successful European collaboration.
- JET is still the world's largest tokamak
- JET has been built in the early eighties, and it was designed to allow the exploration of the plasma regimes in proximity of break-even, the condition at which the ratio between produced fusion power and input heating power is unity
- At the time of its construction, JET was a large step in scale from existing experiments

The JET tokamak - 2



Plasma axisymmetric model - 1

Model Inputs

The *input variables* are:

- The voltage applied to the active coils v
- The plasma current I_p
- The poloidal beta β_p
- The internal inductance l_i

I_p, β_p and l_i

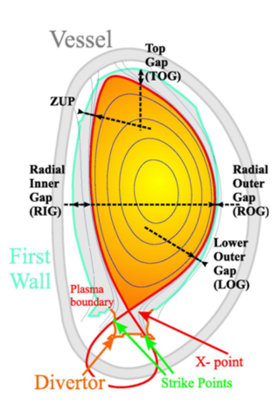
I_p, β_p and l_i are used to specify the current density distribution inside the plasma region.

Plasma axisymmetric model - 2

Model outputs

Different model outputs can be chosen:

- fluxes and fields where the magnetic sensors are located
- currents in the active and passive circuits
- plasma radial and vertical position (1st and 2nd moment of the plasma current density)
- geometrical descriptors describing the plasma shape (gaps, x-point and strike points positions)



Lumped parameters approximation

By using finite-elements methods, **nonlinear** lumped parameters approximation of the PDEs model is obtained

$$\frac{\mathrm{d}}{\mathrm{dt}}\Big[\mathcal{M}\big(\mathbf{y}(t), \beta_p(t), l_i(t)\big)\mathbf{I}(t)\Big] + \mathbf{R}\mathbf{I}(t) = \mathbf{U}(t)\,,$$

$$\mathbf{y}(t) = \mathcal{Y}\big(\mathbf{I}(t), \beta_p(t), l_i(t)\big)\,.$$

where:

- $\mathbf{y}(t)$ are the output to be controlled
- $\mathbf{I}(t) = \left[\mathbf{I}_{PF}^T(t)\ \mathbf{I}_e^T(t)\ I_p(t)\right]^T$ is the currents vector, which includes the currents in the active coils $\mathbf{I}_{PF}(t)$, the eddy currents in the passive structures $\mathbf{I}_e(t)$, and the plasma current $I_p(t)$
- $\mathbf{U}(t) = \left[\mathbf{U}_{PF}^T(t)\ \mathbf{0}^T\ 0\right]^T$ is the input voltages vector
- $\mathcal{M}(\cdot)$ is the mutual inductance nonlinear function
- $\mathbf{R}$ is the resistance matrix
- $\mathcal{Y}(\cdot)$ is the output nonlinear function

Plasma linearized model

Starting from the nonlinear lumped parameters model, the following plasma linearized state space model can be easily obtained:

$$\delta\dot{\mathbf{x}}(t) = \mathbf{A}\delta\mathbf{x}(t) + \mathbf{B}\delta\mathbf{u}(t) + \mathbf{E}\delta\dot{\mathbf{w}}(t), \quad (1)$$

$$\delta\mathbf{y}(t) = \mathbf{C}\,\delta\mathbf{I}_{PF}(t) + \mathbf{F}\delta\mathbf{w}(t), \quad (2)$$

where:

- **A**, **B**, **E**, **C** and **F** are the model matrices
- $\delta\mathbf{x}(t) = \left[\delta\mathbf{I}_{PF}^T(t)\ \delta\mathbf{I}_e^T(t)\ \delta I_p(t)\right]^T$ is the state space vector
- $\delta\mathbf{u}(t) = \left[\delta\mathbf{U}_{PF}^T(t)\ \mathbf{0}^T\ 0\right]^T$ are the input voltages variations
- $\delta\mathbf{w}(t) = \left[\delta\beta_p(t)\ \delta l_i(t)\right]^T$ are the β_p and l_i variations
- $\delta\mathbf{y}(t)$ are the output variations

The model (1)–(2) relates the variations of the PF currents to the variations of the outputs around a given equilibrium

Plasma Shape Control

- The problem of controlling the plasma shape is probably the most understood and mature of all the control problems in a tokamak
- The actuators are the Poloidal Field coils, that produce the magnetic field acting on the plasma
- The controlled variables are a finite number of geometrical descriptors chosen to describe the plasma shape

Plasma Shape Control

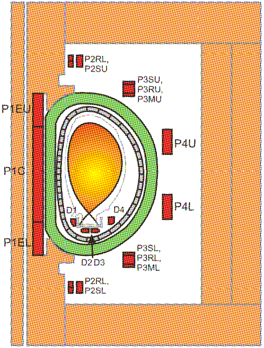
- The problem of controlling the plasma shape is probably the most understood and mature of all the control problems in a tokamak
- The actuators are the Poloidal Field coils, that produce the magnetic field acting on the plasma
- The controlled variables are a finite number of geometrical descriptors chosen to describe the plasma shape

Objectives

- Precise control of plasma boundary
- Counteract the effect of disturbances (β_p and l_i variations)

Plasma Shape Control

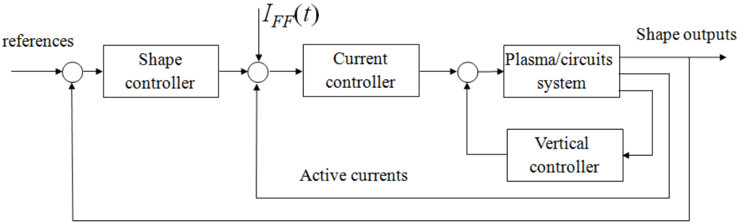
- The problem of controlling the plasma shape is probably the most understood and mature of all the control problems in a tokamak
- The actuators are the Poloidal Field coils, that produce the magnetic field acting on the plasma
- The controlled variables are a finite number of geometrical descriptors chosen to describe the plasma shape



Objectives

- Precise control of plasma boundary
- Counteract the effect of disturbances (β_p and l_i variations)
- Manage saturation of the actuators (currents in the PF coils)

Control scheme



- The scenario is usually specified in terms of feed-forward currents $I_{FF}(t)$.
- It is convenient that the SC generates current references
- A PF currents controller must be designed

Plasma shape control at the JET tokamak

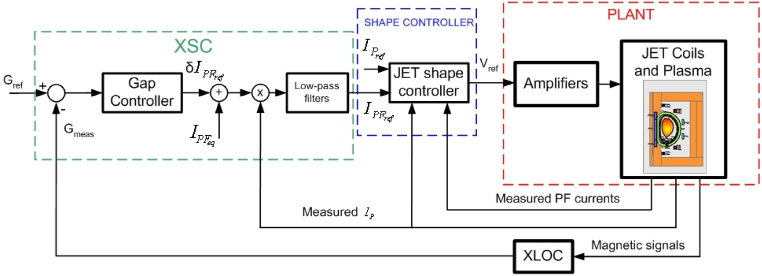
Two different shape controllers are available at the JET tokamak

- the *standard* Shape Controller (SC). This controller can be set in *full current control mode* (acting as a PF currents controler)
- the eXtreme Shape Controller (XSC)

XSC "philosophy"

- To control the plasma shape in JET, in principle 8 *knobs* are available, namely the currents in the PF circuits except $P1$ which is used only to control the plasma current
- As a matter of fact, these 8 knobs do not practically guarantee 8 degrees of freedom to change the plasma shape
- Indeed there are 2 or 3 current combinations that cause small effects on the shape (depending on the considered equilibrium).
- **The design of the XSC is model-based. Different controller gains must be designed for each different plasma equilibrium, in order to achieve the desired performances**

XSC - Controller scheme



eXtreme Shape Controller (XSC)

- The *eXtreme Shape Controller* (XSC) controls the whole plasma shape, specified as a set of 32 geometrical descriptors, calculating the PF coil current references.

eXtreme Shape Controller (XSC)

- The *eXtreme Shape Controller* (XSC) controls the whole plasma shape, specified as a set of 32 geometrical descriptors, calculating the PF coil current references.
- Let $\mathbf{I}_{PF_N}(t)$ be the PF currents normalized to the equilibrium plasma current, it is

$$\delta\mathbf{g}(t) = \mathbf{C}\ \delta\mathbf{I}_{PF_N}(t).$$

It follows that the plasma boundary descriptors have the same dynamic response of the PF currents.

eXtreme Shape Controller (XSC)

- The *eXtreme Shape Controller* (XSC) controls the whole plasma shape, specified as a set of 32 geometrical descriptors, calculating the PF coil current references.
- Let $\mathbf{I}_{PF_N}(t)$ be the PF currents normalized to the equilibrium plasma current, it is

$$\delta\mathbf{g}(t) = \mathbf{C}\ \delta\mathbf{I}_{PF_N}(t).$$

It follows that the plasma boundary descriptors have the same dynamic response of the PF currents.

- The XSC design has been based on the **C** matrix. Since the number of independent control variables is less than the number of outputs to regulate, it is not possible to track a generic set of references with zero steady-state error.

$$\delta\mathbf{I}_{PF_{Nreq}} = \mathbf{C}^{\dagger}\delta\mathbf{g}_{error}$$

eXtreme Shape Controller (XSC)

- The XSC has then been implemented introducing weight matrices both for the geometrical descriptors and for the PF coil currents.

eXtreme Shape Controller (XSC)

- The XSC has then been implemented introducing weight matrices both for the geometrical descriptors and for the PF coil currents.
- The determination of the controller gains is based on the Singular Value Decomposition (SVD) of the following weighted output matrix:

$$\widetilde{\mathbf{C}} = \widetilde{\mathbf{Q}}\,\mathbf{C}\,\widetilde{\mathbf{R}}^{-1} = \widetilde{\mathbf{U}}\,\widetilde{\mathbf{S}}\,\widetilde{\mathbf{V}}^{T},$$

where $\widetilde{\mathbf{Q}}$ and $\widetilde{\mathbf{R}}$ are two diagonal matrices.

eXtreme Shape Controller (XSC)

- The XSC has then been implemented introducing weight matrices both for the geometrical descriptors and for the PF coil currents.
- The determination of the controller gains is based on the Singular Value Decomposition (SVD) of the following weighted output matrix:

$$\widetilde{\mathbf{C}} = \widetilde{\mathbf{Q}}\,\mathbf{C}\,\widetilde{\mathbf{R}}^{-1} = \widetilde{\mathbf{U}}\,\widetilde{\mathbf{S}}\,\widetilde{\mathbf{V}}^{T},$$

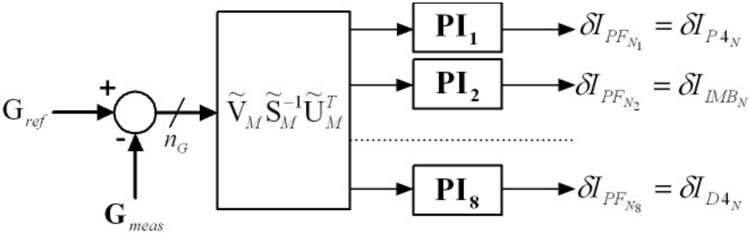
  where $\widetilde{\mathbf{Q}}$ and $\widetilde{\mathbf{R}}$ are two diagonal matrices.
- The XSC minimizes the cost function

$$\widetilde{J}_1 = \lim_{t\to+\infty} (\delta\mathbf{g}_{ref} - \delta\mathbf{g}(t))^T \widetilde{\mathbf{Q}}^T \widetilde{\mathbf{Q}} (\delta\mathbf{g}_{ref} - \delta\mathbf{g}(t)),$$

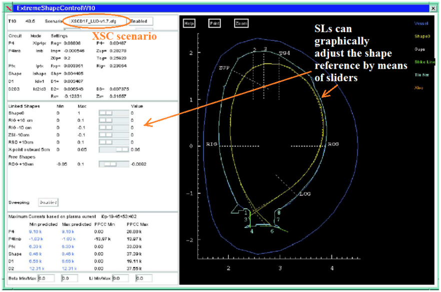
  using $\bar{n} < 8$ degrees of freedom, while the remaining $8 - \bar{n}$ degrees of freedom are exploited to minimize

$$\widetilde{J}_2 = \lim_{t\to+\infty} \delta\mathbf{I}_{PF_N}(t)^T \widetilde{\mathbf{R}}^T \widetilde{\mathbf{R}} \delta\mathbf{I}_{PF_N}(t).$$

XSC - Gap controller



XSC and CLA

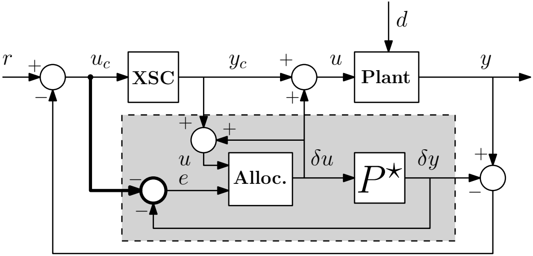


- The XSC allows the SLs to directly specify the target shape, without specifying the PF current waveforms
- The PF current waveforms are *automatically* computed by the model-based control algorithm
- The PF currents may saturate during the experiment
- The Current Limit Avoidance System (CLA) has been recently designed and implemented **to avoid current saturations in the PF coils when the XSC is used to control the plasma shape**

The Current Limit Avoidance System - 1

- The CLA uses the redundancy of the PF coils system to automatically obtain almost the same plasma shape with a different combination of currents in the PF coils
- In the presence of disturbances (e.g., variations of the internal inductance l_i and of the poloidal beta β_p), it tries to avoid the current saturations by “relaxing” the plasma shape constraints
- **Thanks to the CLA safe operations can be guaranteed**

The Current Limit Avoidance System - 2



- The proposed current allocation scheme aims keeping the value of the plant inputs (PF currents) inside a desirable region, meanwhile ensuring a small tracking error on the plasma shape *at steady state*
- $P^\star$ is the plant steady-state gain

The Current Limit Avoidance System - 3

The allocator equations are given by

$$\dot{x}_a = -KB_0^T \begin{bmatrix} I \\ P^\star \end{bmatrix}^T (\nabla J)^T \Big|_{(u\,,e)}, \tag{3a}$$

$$\delta u = B_0 x_a, \tag{3b}$$

$$\delta y = P^\star B_0 x_a \tag{3c}$$

- $J(u^\star\,,e^\star)$ is a continuously differentiable cost function that penalizes (at steady-state)
  - large PF currents
  - large plasma shape error
- The key property of the current allocator algorithm (3) is that, for each constant current request of the XSC, it has a unique globally asymptotically stable equilibrium $x_a^\star$ coinciding with the unique global minimizer $J(\cdot\,,\cdot)$

The CLA scenario

When designing the current allocator, **a large number of parameters must be specified** by the user once the reference plasma equilibrium has been chosen:

- the two matrices $P^\star$ and B_0, which are strictly related to the linearized plasma model
- the K matrix
- the gradient of the cost function J must be specified by the user. In particular, the gradient of J on each *channel* is assumed to be piecewise linear

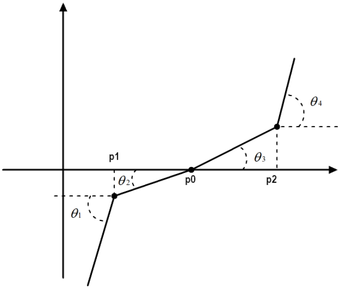
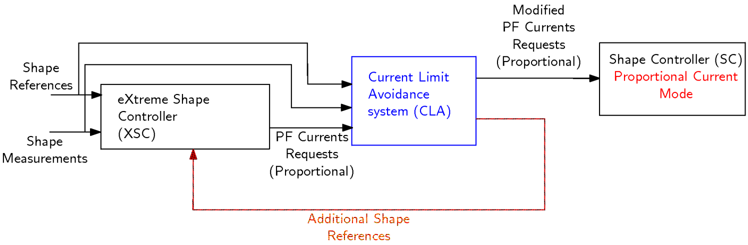


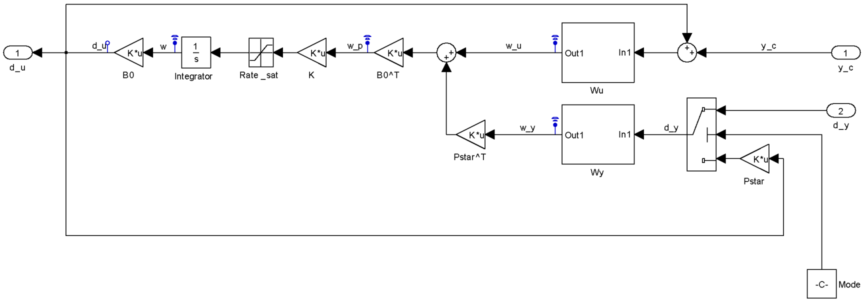
Figure: Piecewise linear function used to specify the gradient of the cost function J for each *allocated* channel. For each channel 7 parameters must be specified.

The CLA Architecture



The CLA block is inserted between the XSC and the Shape Controller set in *Current Control Mode*

The CLA block diagram

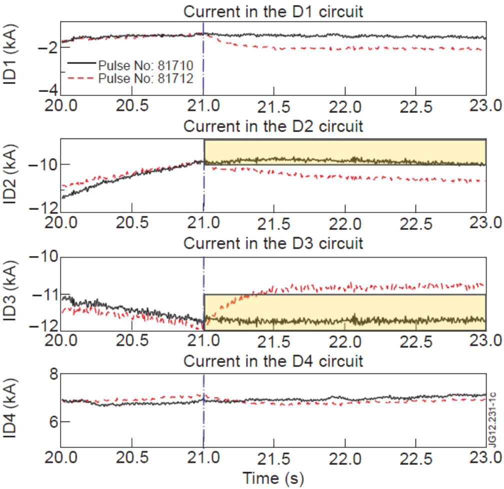


Experimental results

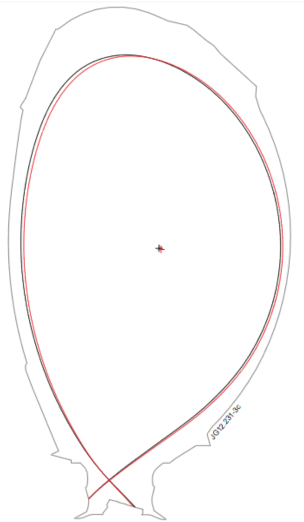
The following strategy has been adopted to carry out the experiment

- first the reference pulse was run (pulse 81710), where the XSC without CLA has successfully controlled the plasma shape between 20 s and 23 s.
- The CLA has been then enabled starting from 21 s, in order to limit the currents in the four divertor coils within a range smaller than the available one
  - pulse 81712 both the currents in $D2$ and $D3$ have been limited between $[-31.5\,,-10]$ kA and $[-11\,,-2]$ kA
  - in pulse 81715 two further limits have been added, one on $D1$ ($[-16.5\,,-4]$ kA) and one on $D4$ ($[0\,,6]$ kA)

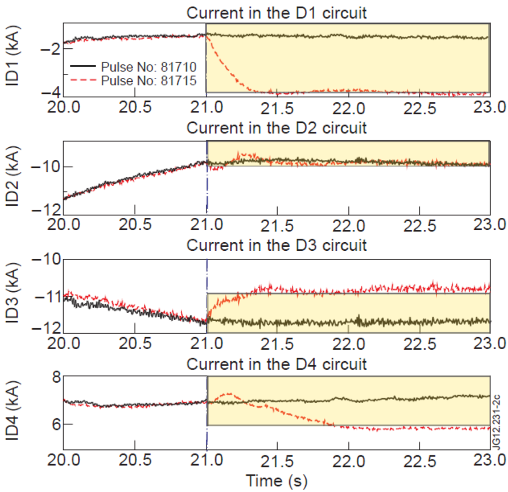
Pulse 81712 - 1



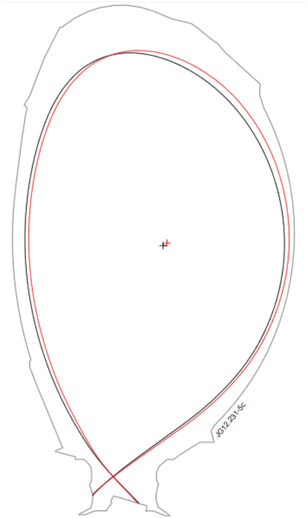
Pulse 81712 - 2



Pulse 81715 - 1



Pulse 81715 - 2



References

The current allocation at JET

L. Zaccarian
Dynamic allocation for input redundant control systems
Automatica, vol. 45, no. 6, pp. 1431–1438, Jun. 2009.

G. De Tommasi et al.
Nonlinear dynamic allocator for optimal input/output performance trade-off: Application to the JET tokamak shape controller
Automatica, vol. 47, no. 5, pp. 981–987, May 2011.

G. Varano et al.
Performance assessment of a dynamic current allocator for the JET eXtreme Shape Controller
Fusion Engineering and Design, vol. 86, no. 6-8, pp. 1057–1060, Oct. 2011.