Formal languages

Discrete Event Systems and Supervisory Control

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UNIVERSITA' DEGLI STUDI DI NAPOLI FEDERICO II
DIPARTIMENTO DI INGEGNERIA ELETTRICA E DELLE TECNOLOGIE DELL'INFORMAZIONE

Outline

Alphabet and words

- Given a DES, the events that may occur can be seen as elements (*symbols*) of an **alphabet** E set

$$E = \{a, b, c, \ldots\},$$

where the symbols $a, b, c, \ldots$ are used to denotes events

- A **word** (**string, trace**) w is a sequence of event of **finite length**
Example: $w = e^1 e^2 e^3 = aab$
- $|w|$ denotes the length of a word is denoted (some authors use $\|w\|$)
- ε denotes the empty word or silent event, i.e. $|\varepsilon| = 0$

- A language L defined over an alphabet E is a set of words defined on the symbols of E
- **The cardinality of a language L can be either finite or infinite**
- Being $E = \{a, b, c\}$
  - $L_1 = \{\varepsilon\,, a\,, abb\}$
  - $L_2 = \{\text{all words that starts with the event } a\}$
  - $L_3 = \{\varepsilon\,, b\,, b\,, bab\}$

Concatenation of strings

- The key operation among words is the **concatenation**
- The concatenation of two words w_1 and w_2 is the new string w consisting of the events in w_1 immediately followed by the events in w_2, and it is denoted $w = w_1 w_2$
- In general, if $u = w_1 w_2$ and $v = w_2 w_1$ it does not necessarily follows that $u = v \rightarrow$ **concatenation is not commutative**
- The empty word ε is the identity element of concatenation, i.e. $w\varepsilon = \varepsilon w = w$
- Given a word $w = tuv$ the following terminology is adopted
  - t is called **prefix** of w
  - u is called **substring** of w
  - v is called **suffix** of w

Kleene closure of E

- The **Kleene closure** of an alphabet E is the set of all the finite-length words defined on the elements of E and is denoted with E^*
- The empty word ε is always contained in E^*
- Example:
  - $E = \{\alpha, \beta\}$
  - $E^* = \{\varepsilon\,, \alpha\,, \beta\,, \alpha\alpha\,, \alpha\beta\,, \beta\alpha\,, \beta\beta\,, \alpha\alpha\alpha\,, \ldots\}$
- E^* contains every possible language L defined on the symbols of E

Set operations

- Being sets, all the set operations are defined also on languages:
  - **union** $L_1 \cup L_2$
  - **intersection** $L_1 \cap L_2$
  - **difference** $L_1 \setminus L_2$
  - **complement with respect to** E^* $E^* \setminus L$
- Language specific operations are
  - Concatenation (of languages)
  - Prefix-closure
  - Kleen-closure

Concatenation of languages

Given two languages $L_a\,, L_b \subseteq E^*$, the concatenation $L_a L_b$ is

$$L_a L_b := \{w \in E^* \;:\; w = w_a w_b \text{ with } w_a \in L_a\,, w_b \in L_b\}$$

- Let $L \subseteq E^*$, its prefix-closure is

$$\bar{L} := \{w \in E^* \ : \ \exists \ t \in E^* \text{ such that } wt \in L\}$$

- It is always $L \subseteq \bar{L}$
- L is said to be **prefix-closed** if $L = \bar{L}$

Kleene-closure

- Let $L \subseteq E^*$, its Kleene-closure is

$$L^* := \{\varepsilon\} \cup L \cup LL \cup LLL \cup \ldots$$

- The Kleene-closure is idempotent, i.e. $(L^*)^* = L^*$

Operator precedence

- Closures comes first. . .
- . . .then concatenations. . .
- . . .finally set operators

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- Closures comes first. . .
- . . .then concatenations. . .
- . . .finally set operators
- **unless there are brackets**

Special cases

- Always remember that $\varepsilon \neq \emptyset$
- If $L = \emptyset \Rightarrow \bar{L} = \emptyset$
- If $L \neq \emptyset \Rightarrow \varepsilon \in \bar{L}$
- $\emptyset^* = \{\varepsilon\}$
- $\{\varepsilon\}^* = \{\varepsilon\}$

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