Nondeterministic automata and observers

Discrete Event Systems and Supervisory Control

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UNIVERSITA' DEGLI STUDI DI NAPOLI FEDERICO II
DIPARTIMENTO DI INGEGNERIA ELETTRICA E DELLE TECNOLOGIE DELL'INFORMAZIONE

Lack of sensors

- The primary source of nondeterminism is the limitations of the sensors attached to the system
- This results in unobservable events that causes a change in the state that cannot be directly *measured*
- From the point of view of an *external observer*, the occurrence of an unobservable event is equivalent to the occurrence of the silent event ε

Unknown initial state

- Another way to model uncertainty about the system behaviour can be the lack of knowledge about the initial state
- Sometime it is assumed that the initial state of a DES is one among a set of states

Partial knowledge of the system dynamic

- There can be also uncertainty on the effects due to the occurrence of an event...
- ... or uncertainty due to undistinguishable events
- Both sources of uncertainty can be modelled as an event that, from a given state x, can cause transitions to more than one state
- In this case the state transition function becomes nondeterministic

$$f : X \times E \mapsto 2^X$$

Unobservable events

- When unobservable events are used to model the uncertain system, we can assume that $E = E_o \cup E_{uo}$ with
  - E_o the set of observable events
  - E_{uo} the set of unobservable events
  - $E_o \cap E_{uo} = \emptyset$
- For an *external* observer the occurrence of and event $e \in E_{uo}$ is equivalent to the occurrence of ε
- The projection function can be used to *filter out* the unobservable events from the words generated by the system

Projection

$$Pr : E^* \mapsto E_o^*$$

$$\begin{cases} Pr(\varepsilon) := \varepsilon & \\ Pr(e) := e & \text{if } e \in E_o \\ Pr(e) := \varepsilon & \text{if } e \in E_{uo} \\ Pr(we) := Pr(w)Pr(e) & w \in E^*, e \in E \end{cases}$$

- Given a word $w \in E^*$ generated by the uncertain model, its protection $Pr(w) \in E_o^*$ represents what an *external* observer can measure

Nondeterministic automata

- **Nondeterministic automata** permit to take into account all the sources of uncertainty that have been introduced so far
- A nondeterministic automata (**NDA**) is defined a 6-ple

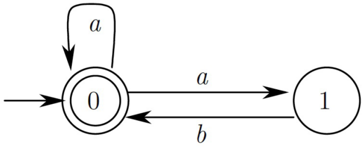
$$G_{nd} = (X\,, E \cup \{\varepsilon\}\,, f_{nd}\,, \Gamma\,, x_0\,, X_m)$$

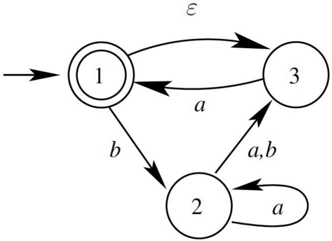
- The silent event ε is included in the set of events that drive the systems dynamic
- The transition function is defined as

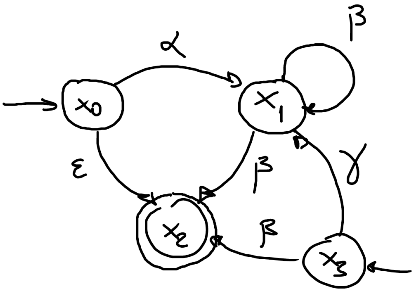
$$f_{nd} : X \times E \cup \{\varepsilon\} \mapsto 2^X$$

  that is $f_{nd}(x\,, e) \subseteq X$, when defined (uncertainty on the *conseguences* of a given event)
- The initial state may be itself a set of states, that is $x_0 \subseteq X$

Examples







Extending the transition function to the nondeterministic case

- For (logic) deterministic automata it is $f(x, \varepsilon) = x$ (in the deterministic case ε is used as *empty string*, rather than silent event)
- ε-reach of a state x

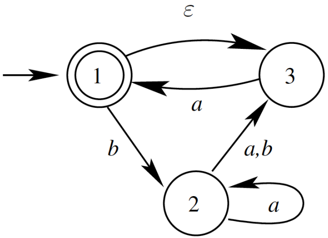
$\varepsilon R(x) = \{$all the states that can be reached from x following a silent transition$\}$

- **By definition it is** $x \in \varepsilon R(x)$
- If $B \in X$, then

$$\varepsilon R(B) = \bigcup_{x \in B} \varepsilon R(x)$$

- It is then possible to extend f_{nd} as
  - $f_{nd}^{ext}(x, \varepsilon) := \varepsilon R(x)$
  - $f_{nd}^{ext}(x, we) := \varepsilon R\left(\{z \in X \mid z \in f_{nd}(y, e) \text{ for some } y \in f_{nd}^{ext}(x, w)\}\right)$ with $w \in E^*$ and $e \in E$
- In general it is $f_{nd}(x, e) \subseteq f_{nd}^{ext}(x, e)$ with $e \in E \cup \{\varepsilon\}$

Example



- $f_{nd}(1, \varepsilon) = \{3\}$; $f_{nd}^{ext}(1, \varepsilon) = \{1, 3\}$
- $f_{nd}(3, a) = \{1\}$; $f_{nd}^{ext}(3, a) = \{1, 3\}$
- $f_{nd}(3, a) = \{1\}$; $f_{nd}^{ext}(3, a) = \{1, 3\}$
- $f_{nd}(2, a) = f_{nd}^{ext}(2, a) = \{2, 3\}$
- $f_{nd}(2, b) = f_{nd}^{ext}(2, b) = \{3\}$
- $f_{nd}(1, bba) = \{1\}$; $f_{nd}^{ext}(1, bba) = \{1, 3\}$

Given the notion of extended transition function f_{nd}^{ext}, it is possible to define the languages generated and marked by a NDA

Language generated by G_{nd} – $\mathcal{L}(G_{nd})$

$$\mathcal{L}(G_{nd}) = \left\{w \in E^* \mid \exists\, x \in x_0 \text{ s.t. } f_{nd}^{ext}(x\,, w) \text{ is defined}\right\}$$

Language marked by G_nd – $\mathcal{L}_m(G_{nd})$

$$\mathcal{L}_m(G_{nd}) = \left\{w \in \mathcal{L}(G_{nd}) \mid \exists\, x \in x_0 \text{ s.t. } f_{nd}^{ext}(x\,, w) \cap X_m \neq \emptyset\right\}$$

Logic nondeterminism vs stochastic nondeterminism

- Here we are dealing with nondeterminism in the context of logic automata
- Nondeterminism can be associated also to the timing of event occurrences
- The inclusion of this further source of nondeterminism calls for the use of stochastic models. . .
  - Stochastic automata
  - Generalized Semi-Markov Process
  - Markov chains
  - . . .
- . . .which are out of the scope of these lectures :)

- The **observer** is a deterministic automaton that is equivalent to a given NDA
  - Equivalence in terms of languages
- If the NDA has finite state space, then also the observer will be a FSM
- The observer allows us to estimate the state of a NDA

Building the observer

Let $G_{nd} = (X, E \cup \{\varepsilon\}, f_{nd}, x_0, X_m)$ be a NDA. Its observer is the deterministic automaton

$$Obs(G_{nd}) = (X_{obs}, E, f_{obs}, x_{0,obs}, X_{m,obs})$$

where

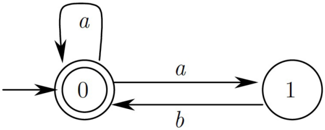
- $x_{0,obs} := \varepsilon R(x_0)$
- For each $B \in X_{obs}$ and $e \in E$, the transition function of the observer is defined as

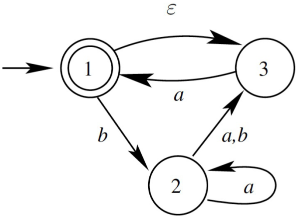
$$f_{obs}(B, e) := \varepsilon R(\{x \in X \mid \exists\, x_e \in B \text{ s.t. } x \in f(x_e, e)\})$$

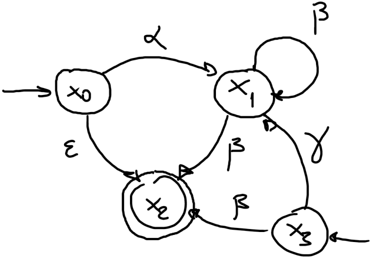
therefore the state $f_{obs}(B, e)$ is included in X_{obs}

- $X_{m,obs} := \{B \in X_{obs} \mid B \cap X_m \neq \emptyset\}$

Let's try to build the observer for these NDA!







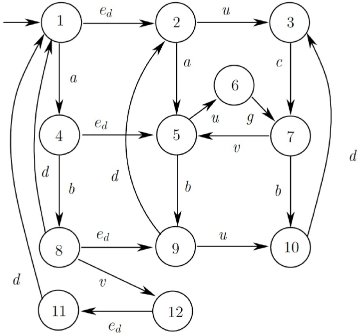
NDA and regular languages

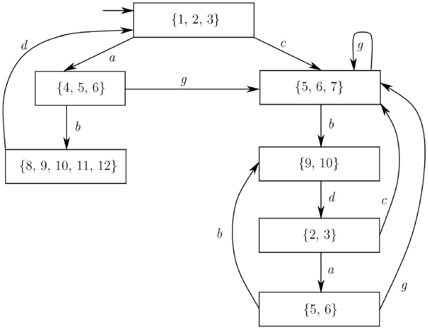
- Given a NDA G_{nd} with finite state space, its observer $Obs(G_{nd})$
  - has finite state space as well
  - is equivalent to G_{nd}
- Indeed, by definition it is
  - $\mathcal{L}(G_{nd}) = \mathcal{L}\left(Obs(G_{nd})\right)$
  - $\mathcal{L}_m(G_{nd}) = \mathcal{L}_m\left(Obs(G_{nd})\right)$
- **Finite-state NDA *speaks* regular languages as deterministic FSM**

Observer of deterministic automata with unobservable events

- The observer can be used to estimate the state of a partially observed **deterministic** automaton, i.e. a deterministic automaton with $E = E_o \cup E_{uo}$
- It is sufficient to *replace* the unobservable events with the silent transition $\rightarrow$ a NDA is derived from the deterministic automaton with unobservable events

Set of unobservable events $E_{uo} = e_d \, , u \, , v$





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