

# Diagnosability and fault detection in PNs - Part I: graph-based approaches

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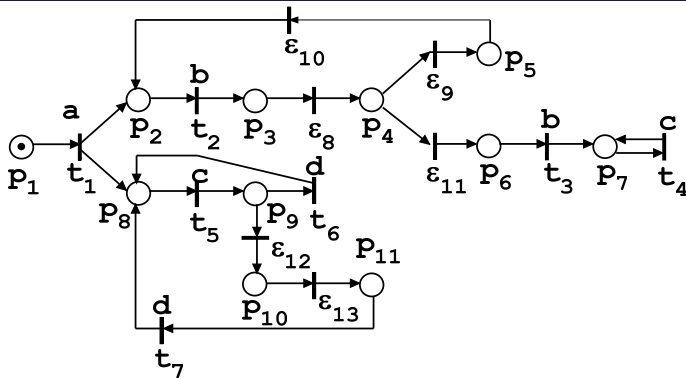
- 1 Discrete Event Systems (DES), Languages and Automata
- 2 Petri nets (PNs) and their twofold representation to model DES
- 3 MILP and ILP formulations: logical conditions, binary variables “do everything”, and variable connecting
- 4 Adding uncertainty: unobservable events and observers for finite state automata and PNs
- 5 Augmenting the observers: diagnosability of prefix-closed languages, diagnosers and the fault detection for finite state automata
- 6 **Diagnosability and fault detection in PNs - Part I: graph-based approaches**
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- 1 Introducing fault diagnosis in PN context
  - Main PN approaches classification
- 2 BRG
  - Minimal explanations
  - Minimal justifications
  - Basis marking
  - Fault diagnosis
- 3 Verifier Net
- 4 Net unfolding

- Basis Reachability Graph [CabasinoCEP2011];
- Verifier net [CabasinoGiua:CDC2009b];
- Net unfolding approach [Benveniste03];
- Integer linear programming approaches [DotoliAutomatica09,BasileAutomatica2012] (treated in detail in Lesson 7).
- Fault free model approach (not treated)

Given an observed word of event  $w$ :

- $\mathcal{S}(w)$  is the set containing all sequences of labeled transitions that are consistent with  $w$ , i.e., the set of all possible occurring sequences that produce observation  $w$  from the initial marking;
- $\mathcal{C}(w)$  is the set of reachable markings that are consistent with  $w$ , i.e., the set of all possible markings in which the system can be after the firing of  $w$  from the initial marking.



Consider the observation  $w = ab$ ,

$T_o = \{t_1, t_2, t_3, t_4, t_5, t_6, t_7\}$ ,  $T_u = \{\epsilon_8, \epsilon_9, \epsilon_{10}, \epsilon_{11}, \epsilon_{12}, \epsilon_{13}\}$

$S(w) = t_1 t_2, t_1 t_2 \epsilon_8, t_1 t_2 \epsilon_8 \epsilon_9, t_1 t_2 \epsilon_8 \epsilon_9 \epsilon_{10}, t_1 t_2 \epsilon_8 \epsilon_{11}$

$C(w) = \{[00100001000], [00010001000], [0\ 0\ 0\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 0], [01000001000], [00000101000]\}$

Given a marking  $\mathbf{m}$  and an observable transitions  $t \in T_o$ , let

$$\Sigma(\mathbf{m}, t) = \{\sigma \in T_u^* \mid \mathbf{m}[\sigma] \mathbf{m}' \text{ s.t. } \mathbf{m}' \geq \mathbf{Pre}(\cdot, t)\}$$

is the set of all the *explanations* of  $t$  at  $\mathbf{m}$ , and let

$$Y(\mathbf{m}, t) = \pi(\Sigma(\mathbf{m}, t))$$

be the corresponding set of firing count vectors, called e-vectors (explanation vectors).

Notice that  $\sigma = \pi(\sigma)$ .

Given a marking  $\mathbf{m}$  and an observable transitions  $t \in T_o$ , let

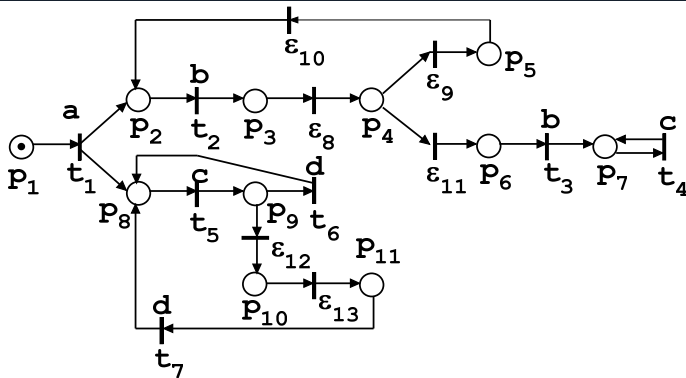
$$\Sigma_{min}(\mathbf{m}, t) = \{\sigma \in \Sigma(\mathbf{m}, t) \mid \nexists \sigma' \in \Sigma(\mathbf{m}, t) : \pi(\sigma') \preceq \pi(\sigma)\}$$

is the set of all the *minimal explanations* of  $t$  at  $\mathbf{m}$ , and let

$$Y(\mathbf{m}, t) = \pi(\Sigma_{min}(\mathbf{m}, t))$$

be the corresponding set of firing count vectors, called minimal e-vectors.





$$\Sigma(\mathbf{m}_0, t_1) = \{\epsilon\}, \Sigma(\mathbf{m}_0, t_2) = \{\emptyset\}$$

$$\text{Let } \mathbf{m} = [00100001000],$$

$$\Sigma(\mathbf{m}, t_5) = \{\epsilon, \epsilon_8, \epsilon_8\epsilon_9, \epsilon_8\epsilon_{11}, \epsilon_8\epsilon_9\epsilon_{10}\}, \Sigma_{\min}(\mathbf{m}, t_5) = \{\epsilon\},$$

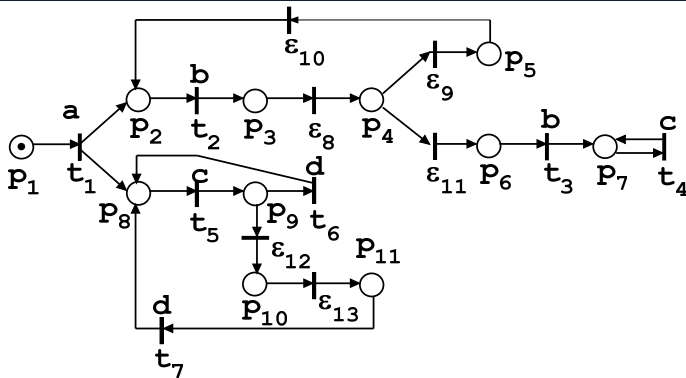
$$Y(\mathbf{m}, t_5) = \{[000000]^T, [100000]^T, [110000]^T,$$

$$[100100]^T, [111000]^T,$$

$$Y_{\min}(\mathbf{m}, t_5) = [000000]^T.$$

Given an observed word of event  $w$ :

- $\mathcal{J}(w)$  the set of justifications, i.e., the set of all minimal sequences of unobservable transitions interleaved with  $w$  and whose occurrence enables  $w$ ;
- $\hat{\mathcal{J}}(w)$  is the set of pairs whose first element is the sequence  $\sigma_o \in T_o^*$  labeled  $w$  and whose second element is the corresponding justification (sequence of unobservable transitions interleaved with  $\sigma_o$  whose firing enables  $\sigma_o$  and whose firing vector is minimal);  $\mathbf{y}$  are the firing vectors of these justification sequences, they are called j-vectors.
- $\hat{Y}_{min}(\mathbf{m}_0, w)$  is the set of pairs whose first element is the sequence  $\sigma_o \in T_o^*$  labeled  $w$  and the second is the corresponding j-vector.



Consider the observation  $w = ab$ ,

$$\hat{\mathcal{J}}(w) = \{(t_1 t_2, \epsilon)\}, \hat{Y}_{min}(\mathbf{m}_0, w) = \{(t_1 t_2, \mathbf{0})\}$$

Consider the observation  $w = acd$ ,

$$\hat{\mathcal{J}}(w) = \{(t_1 t_5 t_6, \epsilon), (t_1 t_5 t_7, \epsilon_{12} \epsilon_{13})\},$$

$$\hat{Y}_{min}(\mathbf{m}_0, w) = \{(t_1 t_5 t_6, \mathbf{0}), (t_1 t_5 t_7, [000011]^T)\}$$

- Any marking reached from  $\mathbf{m}_0$  firing  $\sigma_o$  labeled  $w$  and interleaved with its justification  $\sigma_u$  is called *basis marking* and the  $j$ -vector corresponding to  $\sigma_o$  is called  $j$ -vector of the basis marking and denoted by  $\mathbf{m}_b$ .  
More than one justification exists for a word  $w$  - the set  $\hat{\mathcal{J}}(w)$  is not a singleton - the basis marking is not unique.
- The set of couples  $(\mathbf{m}_b, \mathbf{y})$ , i.e. (basis marking,  $j$ -vector), consistent with  $w$  is denoted  $\mathcal{M}(w)$ .

$\mathcal{M}(w)$  only keep track of the basis markings that are reached and of the firing vector (not the sequences) relative to sequences of unobservable transitions that have fired to reach them.

Minimal explanations are function of a generic marking  $m$ ,  
justifications are function of the initial marking  $m_0$ .

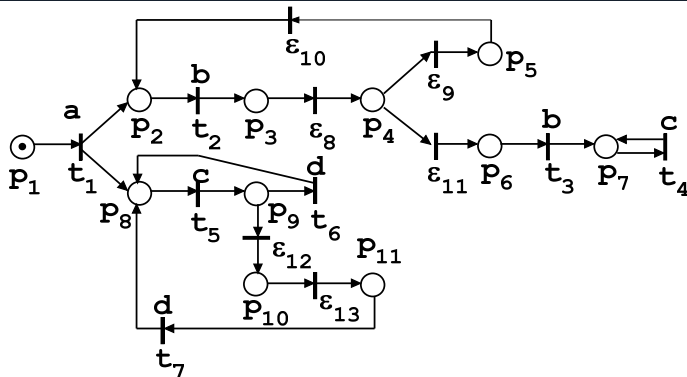
Justifications are function of the initial marking, but in case of acyclic unobservable subnets they can be recursively computed summing up minimal explanations.

*$\mathcal{M}(w)$  computation*

*After a certain word  $w'$  has been observed, a new observable  $t$  fires and its label  $l = \mathcal{L}(t)$  is observed.*

*Consider all basis markings at the observation  $w't$  and select among them those that may have allowed at least the firing of one transition labelled by  $t$ , also taking into account that this may have required the firing of the minimal explanations and thus the corresponding minimal  $e$ -vectors.*

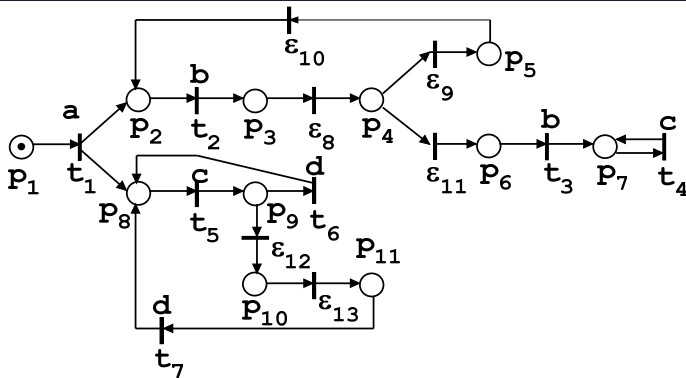
*Update  $\mathcal{M}(w't)$  including all pairs of new basis markings and  $j$ -vectors, taking into account that for each basis marking at  $w't$  it may correspond more than one  $j$ -vector.*



Consider the observation  $w = ab$ ,

$$\hat{\mathcal{J}}(w) = \{(t_1 t_2, \epsilon)\}$$

$$\mathbf{m}_b = [00100001000]^T, \text{ and } \mathcal{M}(w) = \{(\mathbf{m}_b, \mathbf{0})\}.$$



Consider the observation  $w = acd$ ,

$$\hat{\mathcal{J}}(w) = \{(t_1 t_5 t_6, \epsilon), (t_1 t_5 t_7, \epsilon_{12} \epsilon_{13})\}$$

$$\mathbf{m}'_b = [0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0]^T$$

$\mathcal{M}(w) = \{(\mathbf{m}'_b, \mathbf{0}), (\mathbf{m}'_b, [0 \ 0 \ 0 \ 0 \ 1 \ 1]^T)\}$  since all the above  $j$ -vectors lead to the same basis marking.

If the unobservable net is acyclic,

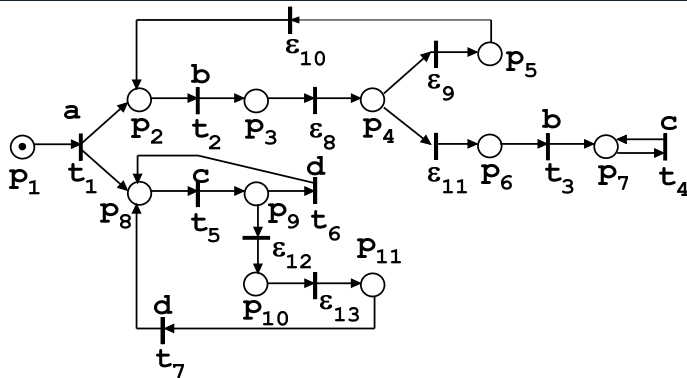
- $\mathcal{M}_{basis}(w) = \{\mathbf{m} \mid \exists \mathbf{y} \text{ and } (\mathbf{m}, \mathbf{y}) \in \mathcal{M}(w)\}$
- $\mathcal{C}(w) = \{\mathbf{m} \in \mathbb{N}^m \mid \mathbf{m} = \mathbf{m}_b + \mathbf{C}_u \cdot \mathbf{y} : \mathbf{y} \geq \mathbf{0} \text{ and } \mathbf{m}_b \in \mathcal{M}_{basis}(w)\}$

The set  $\mathcal{C}(w)$  can be characterized in linear algebraic terms given the set  $\mathcal{M}_{basis}(w)$ , thus not requiring exhaustive enumeration. This is the main advantage of the approach.



Assume that the set of fault transitions is partitioned into  $r$  subsets,  $T_f^i$ ,  $i = 1 \dots r$ , each one corresponding to a fault class. The following four cases can be distinguished, each one corresponding to an increasing level of alarm (the diagnosis state varies from 0 to 3).

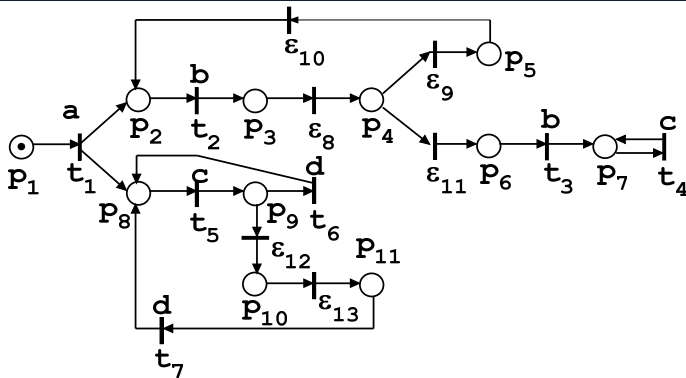
- $\Delta(w, T_f^i) = 0$  - No sequence in  $\mathcal{J}(w)$  contains a transition in  $T_f^i$ , thus no fault in the  $i$ -th class has occurred.
- $\Delta(w, T_f^i) = 1$  - Some transitions in  $T_f^i$  may have occurred but none of them was contained in a justification of  $w$ .
- $\Delta(w, T_f^i) = 2$  - Some transitions in  $T_f^i$  may have occurred and are contained in some of the justifications of  $w$ . However, not all justifications of  $w$  contain transitions in  $T_f^i$ .
- $\Delta(w, T_f^i) = 3$  - All justifications of  $w$  contain transitions in  $T_f^i$ , thus a fault must have occurred.



Let  $T_f^1 = \{\epsilon_{11}\}$ ,  $T_f^2 = \{\epsilon_{12}\}$ .

Consider the observation  $w = a$ ,

$\Delta(w, T_f^1) = \Delta(w, T_f^2) = 0$  since  $\hat{J}(w) = \{(t_1, \epsilon)\}$  and  $S(w) = \{t_1\}$

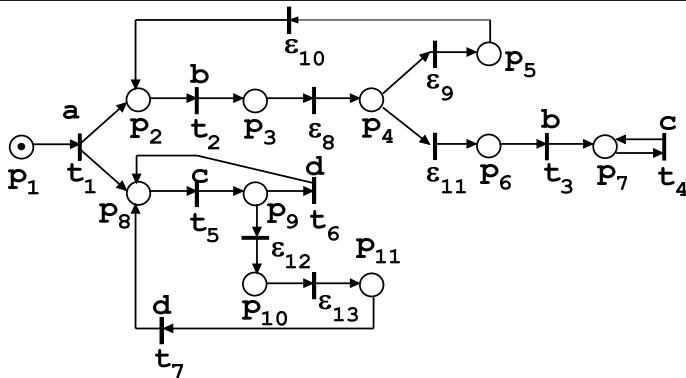


Let  $T_f^1 = \{\epsilon_{11}\}$ ,  $T_f^2 = \{\epsilon_{12}\}$ .

Consider the observation  $w = ab$ ,

$\Delta(w, T_f^1) = 1$  and  $\Delta(w, T_f^2) = 0$  since  $\hat{\mathcal{J}}(w) = \{(t_1 t_2, \epsilon)\}$  and

$\mathcal{S}(w) = \{t_1 t_2, t_1 t_2 \epsilon_8, t_1 t_2 \epsilon_8 \epsilon_9, t_1 t_2 \epsilon_8 \epsilon_9 \epsilon_{10}, t_1 t_2 \epsilon_8 \epsilon_{11}\}$



Let  $T_f^1 = \{\epsilon_{11}\}$ ,  $T_f^2 = \{\epsilon_{12}\}$ .

Consider the observation  $w = abb$ ,

$\Delta(w, T_f^1) = 2$  and  $\Delta(w, T_f^2) = 0$  since

$\hat{J}(w) = \{(t_1 t_2 t_2, \epsilon_8 \epsilon_9 \epsilon_{10})(t_1 t_2 t_3, \epsilon_8 \epsilon_{11})\}$  and

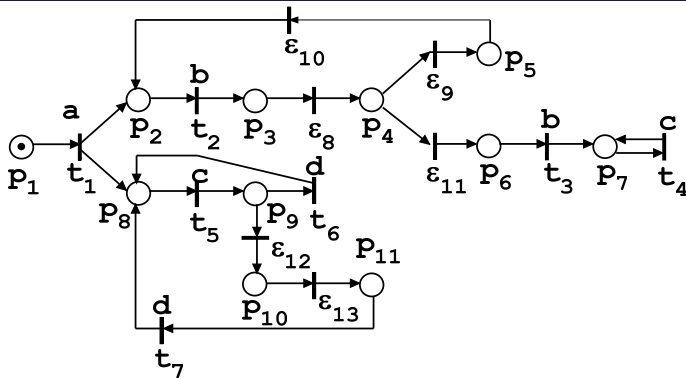
$S(w) = \{t_1 t_2 \epsilon_8 \epsilon_9 \epsilon_{10} t_2, t_1 t_2 \epsilon_8 \epsilon_9 \epsilon_{10} t_2 \epsilon_8, t_1 t_2 \epsilon_8 \epsilon_9 \epsilon_{10} t_2 \epsilon_8 \epsilon_9, t_1 t_2 \epsilon_8 \epsilon_9 \epsilon_{10} t_2 \epsilon_8 \epsilon_9 \epsilon_{10}, t_1 t_2 \epsilon_8 \epsilon_9 \epsilon_{10} t_2 \epsilon_8 \epsilon_{11}, t_1 t_2 \epsilon_8 \epsilon_{11} t_3\}$

From the analysis of  $\mathcal{M}(w)$  it is possible to determine the state 2 and 3, while to distinguish between state 0 and 1, for a PN whose unobservable subnet is acyclic, an integer linear programming problem can be used.

In particular, let  $w$  be an observed word such that for all  $(\mathbf{m}, \mathbf{y}) \in \mathcal{M}(w)$  it holds  $\mathbf{y}(t_f) = \mathbf{0}$ ,  $\forall t_f \in T_f^i$ . Consider the constraint set

$$\mathcal{T}(\mathbf{m}, T_f^i) = \begin{cases} \mathbf{m} + \mathbf{C}_u \cdot \mathbf{z} \geq \mathbf{0} \\ \sum_{t_f \in T_f^i} \mathbf{z}(t_f) > \mathbf{0} \\ \mathbf{z} \in \mathbb{N}^{n_u} \end{cases}$$

If  $\mathcal{T}(\mathbf{m}, T_f^i)$  is not feasible  $\forall (\mathbf{m}, \mathbf{y}) \in \mathcal{M}(w)$ , a fault in the class  $T_f^i$  cannot have occurred ( $\Delta(w, T_f^i) = 0$ ), otherwise if  $\exists (\mathbf{m}, \mathbf{y}) \in \mathcal{M}(w)$  such that  $\mathcal{T}(\mathbf{m}, T_f^i)$  is feasible, a fault in the class  $T_f^i$  may have occurred ( $\Delta(w, T_f^i) = 1$ ).

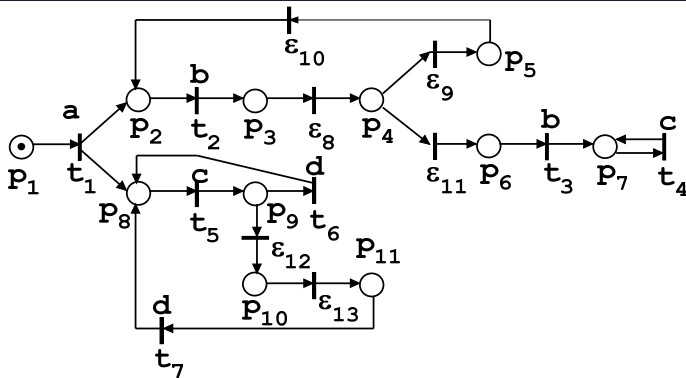


Let  $T_f^1 = \{\epsilon_{11}\}$ ,  $T_f^2 = \{\epsilon_{12}\}$ .

Consider the observation  $w = a$ ,

$\mathcal{M}(w) = \{(\mathbf{m}_1, \mathbf{0})\}$ , where  $\mathbf{m}_1 = [0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0]^T$

$\mathcal{T}(\mathbf{m}_1, T_f^i)$  is not feasible for both fault classes,  $\Delta(w, T_f^1) = 0$   
and  $\Delta(w, T_f^2) = 0$ .



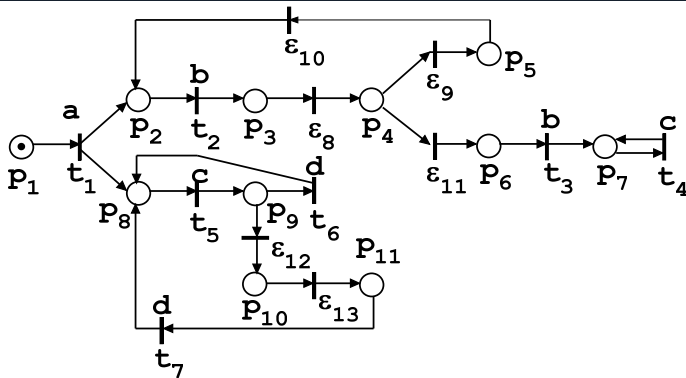
Let  $T_f^1 = \{\epsilon_{11}\}$ ,  $T_f^2 = \{\epsilon_{12}\}$ .

Consider the observation  $w = ab$ ,

$\mathcal{M}(w) = \{(\mathbf{m}_2, \mathbf{0})\}$ , where  $\mathbf{m}_2 = [00100001000]^T$

$\mathcal{T}(\mathbf{m}_2, T_f^i)$  is feasible only for  $i = 1$ ,  $\Delta(w, T_f^1) = 1$  and

$\Delta(w, T_f^2) = 0$ .



Let  $T_f^1 = \{\epsilon_{11}\}$ ,  $T_f^2 = \{\epsilon_{12}\}$ .

Consider the observation  $w = abb$ ,

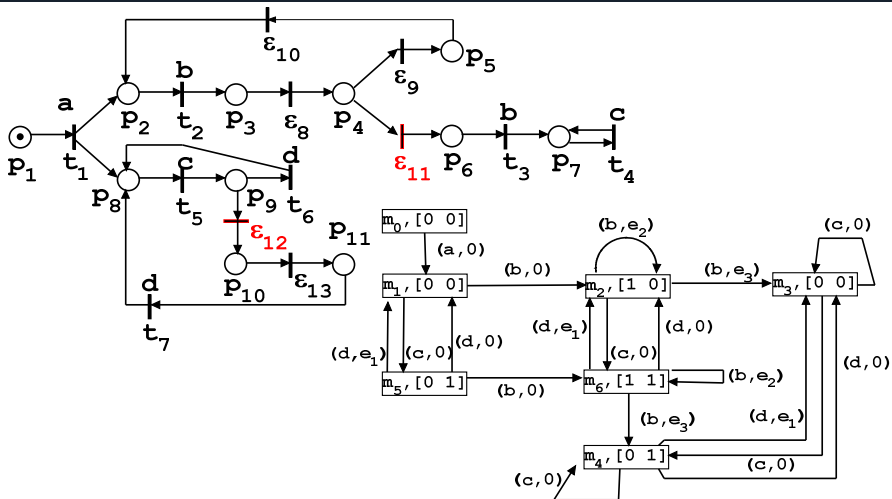
$\mathcal{M}(w) = \{(\mathbf{m}_2, [1\ 1\ 1\ 0\ 0\ 0]), (\mathbf{m}_3, [1\ 0\ 0\ 1\ 0\ 0])\}$ , where

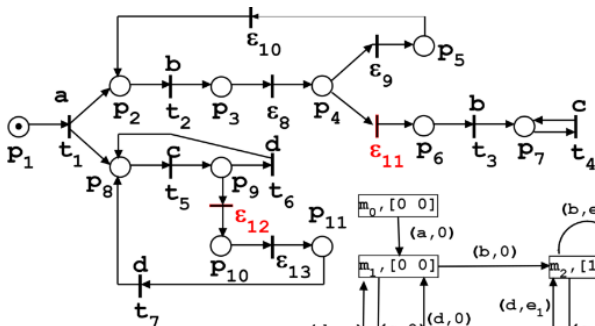
$\mathbf{m}_3 = [0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0]^T$ ,  $\Delta(w, T_f^1) = 2$  and, being  $\mathcal{T}(\mathbf{m}_2, T_f^2)$  and  $\mathcal{T}(\mathbf{m}_3, T_f^2)$  both not feasible  $\Delta(w, T_f^2) = 0$ .



- Using the concept of basis marking and e-vector, a graph that collects all the information needed to perform fault diagnosis as well as to study the diagnosability for bounded net systems can be built, and so a mixed (quite compiled) diagnoser is obtained.
- This graph is called Basis Reachability Graph (BRG) and its nodes form a strict subset of the reachability space of the system.
- The net marking is assumed not observable.
- The net system is assumed to be **bounded**.
- Graphs derived from BRG have been proposed to study diagnosability of bounded PN systems.

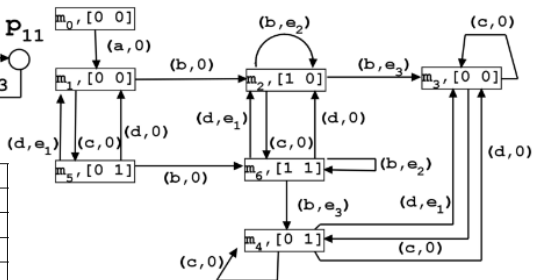
- The BRG is a deterministic graph that has as many nodes as the number of possible basis markings.
- To each node is associated a different basis marking  $\mathbf{m}$  and a row vector with as many entries as the number of fault classes. The entries of this vector may only take binary values: 1 if  $\mathcal{T}(\mathbf{m}, T_f^i)$  is feasible, 0 if  $\mathcal{T}(\mathbf{m}, T_f^i)$  otherwise.
- Arcs are labeled with observable events in  $L$  and e-vectors: an arc exists from a node containing the basis marking  $\mathbf{m}$  to a node containing the basis marking  $\mathbf{m}'$  if and only if there exists a transition  $t$  for which an explanation exists at  $\mathbf{m}$  and the firing of  $t$  and one of its minimal explanations leads to  $\mathbf{m}'$ . The arc going from  $\mathbf{m}$  to  $\mathbf{m}'$  is labeled  $(\mathcal{L}(t), \mathbf{e})$ , where  $\mathbf{e} \in Y_{min}(\mathbf{m}, t)$  and  $\mathbf{m}' = \mathbf{m} + \mathbf{C}_u \mathbf{e} + \mathbf{C}(\cdot, t)$ .

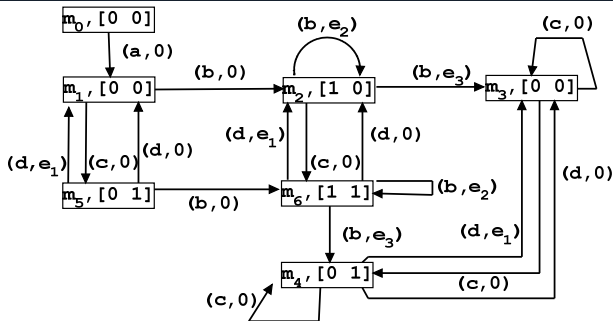




	$\epsilon_8$	$\epsilon_9$	$\epsilon_{10}$	$\epsilon_{11}$	$\epsilon_{12}$	$\epsilon_{13}$
$e_1$	0	0	0	0	1	1
$e_2$	1	1	1	0	0	0
$e_3$	1	0	0	1	0	0

$m_0$	[ 1 0 0 0 0 0 0 0 0 0 0 ] <sup>T</sup>
$m_1$	[ 0 1 0 0 0 0 0 0 1 0 0 ] <sup>T</sup>
$m_2$	[ 0 0 1 0 0 0 0 0 1 0 0 ] <sup>T</sup>
$m_3$	[ 0 0 0 0 0 0 0 1 1 0 0 ] <sup>T</sup>
$m_4$	[ 0 0 0 0 0 0 0 1 0 1 0 ] <sup>T</sup>
$m_5$	[ 0 1 0 0 0 0 0 0 1 0 0 ] <sup>T</sup>
$m_6$	[ 0 0 1 0 0 0 0 0 1 0 0 ] <sup>T</sup>





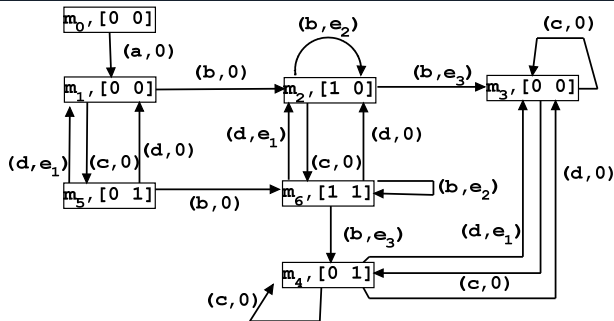
$$m_0 = [100000000000]^T, m_1 = [01000001000]^T,$$

$$m_2 = [00100001000]^T, m_3 = [00000011000]^T,$$

$$m_4 = [00000010100]^T, m_5 = [01000000100]^T,$$

$$m_6 = [00100000100]^T,$$

$e_1 = [000011], e_2 = [111000], e_3 = [100100]$  where the  $e_i, i = 1..3$  vector's components refers respectively to  $\epsilon_8, \epsilon_9, \epsilon_{10}, \epsilon_{11}, \epsilon_{12}$  and  $\epsilon_{13}$ .



Let  $T_f^1 = \{\epsilon_{11}\}$ ,  $T_f^2 = \{\epsilon_{12}\}$ .

Consider the observation  $w = abbc$ ,  $\Delta(w, T_f^1) = 2$  and  $\Delta(w, T_f^2) = 1$  since

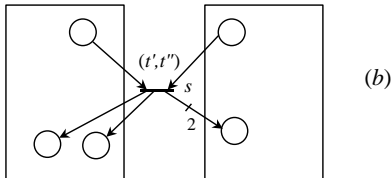
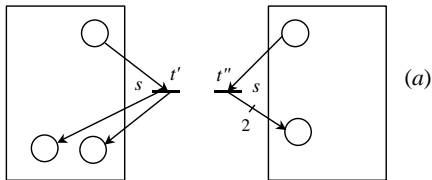
$\mathcal{M}(w) = \{(\mathbf{m}_6, \mathbf{y}_1), (\mathbf{m}_3, \mathbf{y}_2), (\mathbf{m}_4, \mathbf{y}_2)\}$ , with

$\mathbf{y}_1 = \mathbf{e}_2 = [1 \ 1 \ 1 \ 0 \ 0 \ 0]$ ,  $\mathbf{y}_2 = \mathbf{e}_3 = [1 \ 0 \ 0 \ 1 \ 0 \ 0]$  and the row vector associated to  $\mathbf{m}_6$ ,  $\mathbf{m}_3$  and  $\mathbf{m}_4$  are  $[1, 1]$ ,  $[0, 0]$  and  $[0, 1]$ .

The term *verifier* is motivated since it has similarity with the verifier automaton used in the study of the diagnosability of discrete event systems modeled by finite-state automata.

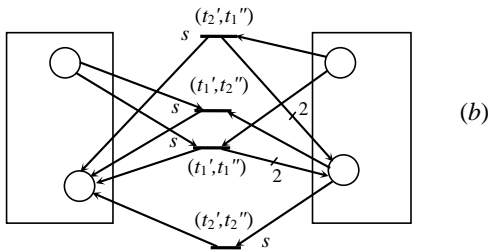
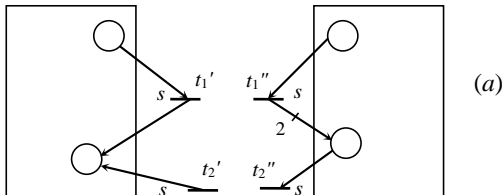
The verifier net is built from a PN system, by considering the parallel composition of the PN system with the PN system induced by the set of faulty transitions  $T_f$ , i.e. obtained from the restriction of pre- and post- incidence matrix to  $T \setminus T_f$ . The synchronization is performed only wrt the observable transitions.

The approach works for **unbounded nets** requires to search for the existence of cycles in the coverability graph of the verifier net involving a faulty node, which are nodes reachable by firing a fault transition.

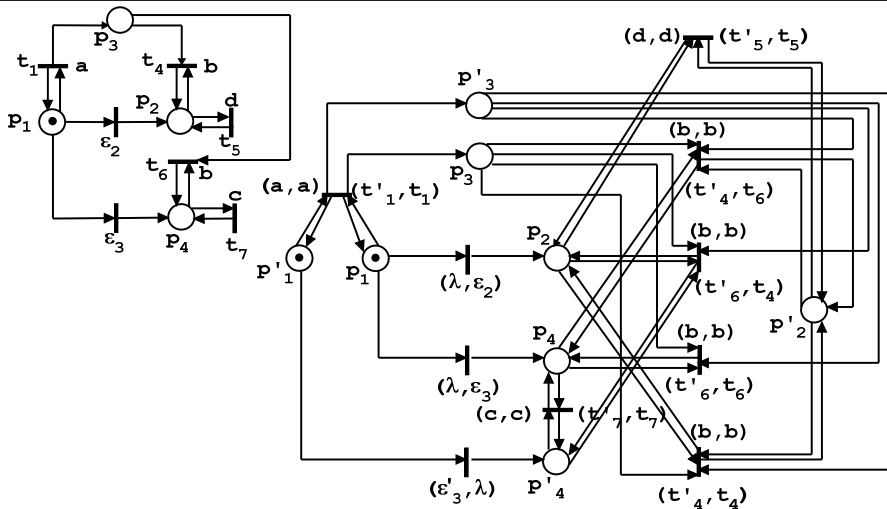


Parallel composition in absence of transitions sharing the same label.

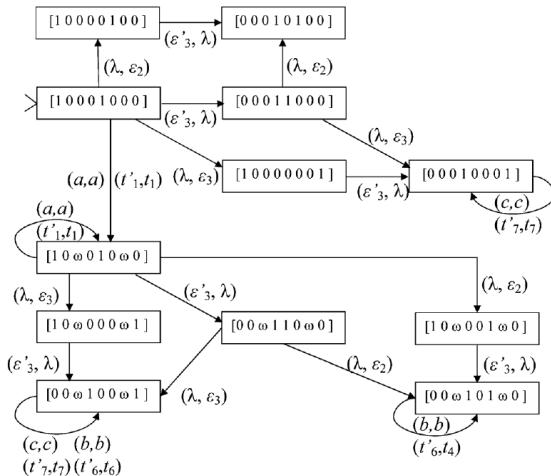




Parallel composition in presence of transitions sharing the same label.



$$T_f = \{\epsilon_2\}$$

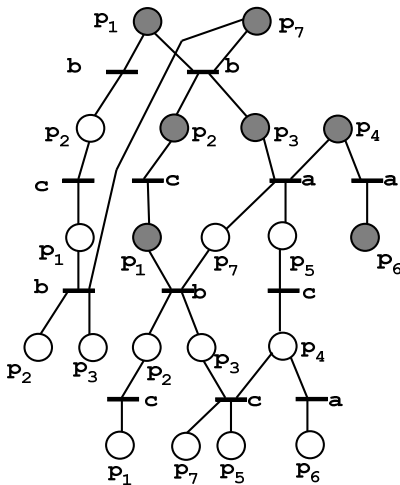
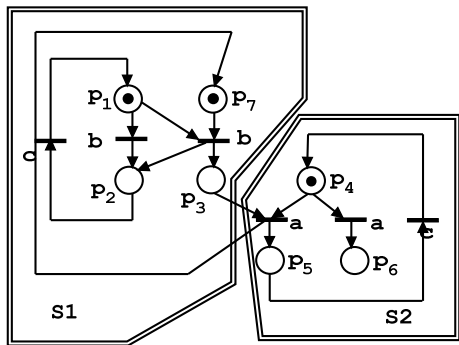


The self loop at  $[00\omega 101\omega 0]^T$ , that is a faulty state, labeled by  $(t'_6, t_4)$  must be checked and it can be verified that it is not a repetitive sequence. The net is diagnosable.

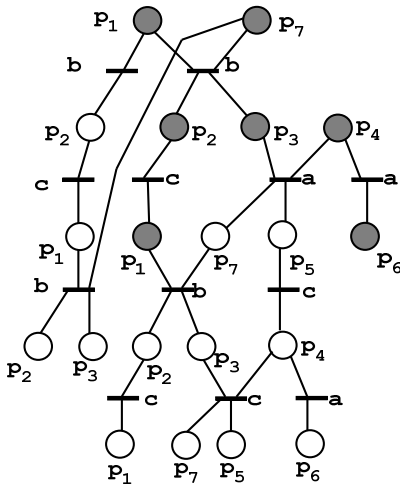
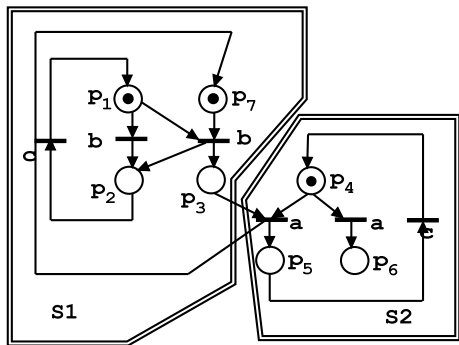
Net unfolding approach has been conceived in control architecture when the supervisor collects, not a sequence of events, but rather a partially ordered set of events.

Net unfolding approach allows to device fault diagnosis algorithm robust against a wrong event interleaving.

Net unfoldings are branching structures suitable to represent the set of executions of a PN using an asynchronous semantic with local states and partially ordered time. In this structure, common prefixes of executions are shared, and executions differing only in the interleaving of their transition firings are represented only once.



A configuration is a sub-net of the occurrence net, which is conflict-free (no two nodes are in conflict), and causally closed.



The configuration shown in grey is a diagnosis for the observation  $(b, S_1) (a, S_2) (c, S_1)$  as well as for  $(b, S_1) (c, S_1) (a, S_2)$  but not for  $(c, S_1) (b, S_1) (a, S_2)$ .

## ■ BRG



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## ■ Verifier net



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## ■ Net unfolding



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# Diagnosability and fault detection in PNs - Part I: graph-based approaches

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