

Non-interference and opacity enforcement

From observability to privacy and security in discrete event systems

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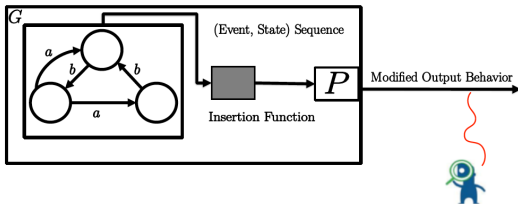
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- 1 Discrete Event Systems (DES), Languages and Automata
- 2 Petri nets (PNs) and their twofold representation to model DES
- 3 MILP and ILP formulations: logical conditions, binary variables “do everything”, and variable connecting
- 4 Adding uncertainty: unobservable events and observers for finite state automata and PNs
- 5 Augmenting the observers: diagnosability of prefix-closed languages, diagnosers and the fault detection for finite state automata
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- 7 Diagnosability and fault detection in PNs - Part II: algebraic approaches for bounded systems
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- 9 **Non-interference and opacity enforcement**
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- 1 Non-interference enforcement via supervisory control
- 2 Open issues
- 3 The assessment

- **Enforcement by Supervisory Control** → to restrict the system's behaviour in order to preserve the security/privacy property

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- **Enforcement by insertion/obfuscation** → to input or mask observable events of the systems so to output (possibly) modified information to the malicious observers



Taken from



C. Keroglou and S. Lafortune

Embedded Insertion Functions for Opacity Enforcement,
IEEE Transactions on Automatic Control, 2020

Main assumptions

- The net system $\mathcal{S} = \langle N, m_0 \rangle$ is **bounded**

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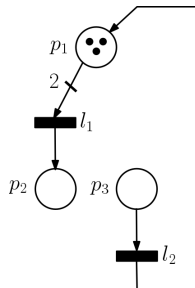
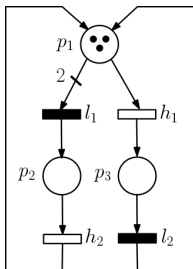
- The net system $\mathcal{S} = \langle N, m_0 \rangle$ is **bounded**
- The net system is assumed to be unlabeled
- The P/T net: $N = (P, L, H, \text{Pre}, \text{Post})$, with $L \cap H = \emptyset$
 - L low-level transitions
 - H high-level transitions
 - $T = L \cup H$

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Objective: to exploit the twofold representation of PN systems to find algebraic conditions to assess SNNI

- The *low-level system* is the system *induced* by the low-level transitions
- $L = \{l_1, l_2\}$ and $H = \{h_1, h_2\}$



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- \mathcal{S} is SNNI *if and only if*

$$Pr_L(\mathcal{L}(N, \mathbf{m}_0)) = \mathcal{L}(N_L, \mathbf{m}_0)$$



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- There exists a set of ρ integer vectors $\mathbf{s}_1, \dots, \mathbf{s}_\rho$ with $\rho \leq |\sigma|$ such that the following linear constraints are fulfilled

$$\left\{ \begin{array}{l} \mathbf{m} \geq \mathbf{Pre} \cdot \mathbf{s}_1 \\ \mathbf{m} + \mathbf{C} \cdot \mathbf{s}_1 \geq \mathbf{Pre} \cdot \mathbf{s}_2 \\ \dots \\ \mathbf{m} + \mathbf{C} \cdot \sum_{i=1}^{\rho-1} \mathbf{s}_i \geq \mathbf{Pre} \cdot \mathbf{s}_\rho \\ \sum_{i=1}^{\rho} \mathbf{s}_i = \pi(\sigma) \end{array} \right. \quad (1)$$

iff there exists at least one sequence σ , which is enabled under the marking \mathbf{m} and such that $\pi(\sigma) = \sigma$

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- **For a bounded net, given a sufficiently large number of inequality constraints (1), it is possible to describe the $R(N, m_0)$ set \rightarrow let assume that J inequalities are sufficient to this purpose**

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- If it is possible to have at least one additional firing of t in the original net system, this implies interference
- The other source of interference is the possibility of using high-level transitions to enable the firing of t

Maximum number of firings of a low-level transition in \mathcal{S}_L



Given J constraints in (1), the maximum number of firings for $t \in L$ in \mathcal{S}_L can be computed as the solution of the ILP

$$\varphi_t = \max \sum_{i=1}^J \sigma_i(t)$$

subject to

$$\left\{ \begin{array}{l} \mathbf{m}_0 \geq \mathbf{Pre}_L \cdot \sigma_1 \\ \mathbf{m}_0 + \mathbf{C}_L \cdot \sigma_1 \geq \mathbf{Pre}_L \cdot \sigma_2 \\ \dots \\ \mathbf{m}_0 + \mathbf{C}_L \cdot \sum_{i=1}^{J-1} \sigma_i \geq \mathbf{Pre}_L \cdot \sigma_J \\ \mathbf{m}_0 + \mathbf{C}_L \cdot \sum_{i=1}^J \sigma_i \geq \mathbf{0} \\ \sigma_i \in \mathbb{N}^n, \quad i = 1, 2, \dots, J \end{array} \right.$$

SNNI assessment in DES modeled as Petri nets (I)

Given a K -bounded system \mathcal{S} , let consider the two ILP problems

$$\min \sum_{i=1}^J \sum_{t_h \in H} x_i(t_h) \quad (2)$$

subject to

$$\mathcal{X}(m_0, \varphi_t) : \begin{cases} m_0 \geq \text{Pre} \cdot x_1 \\ m_0 + C \cdot x_1 \geq \text{Pre} \cdot x_2 \\ \dots \\ m_0 + C \cdot \sum_{i=1}^{J-1} x_i \geq \text{Pre} \cdot x_J \\ m_0 + C \cdot \sum_{i=1}^J x_i \geq 0 \\ \sum_{i=1}^J x_i(t) \geq \varphi_t + 1 \\ x_i \in \mathbb{N}^n, \quad i = 1, 2, \dots, J \end{cases} \quad (3a)$$

(3b)

(3c)

$$\min \left[\sum_{i=1}^J \sum_{t_l \in L} y_i(t_l) + \epsilon \sum_{i=1}^J \sum_{t_h \in H} y_i(t_h) \right] \quad (4)$$

subject to

$$\mathcal{Y}(m_0, \varphi_t) : \begin{cases} m_0 \geq \text{Pre} \cdot y_1 \\ m_0 + C \cdot y_1 \geq \text{Pre} \cdot y_2 \\ \dots \\ m_0 + C \cdot \sum_{i=1}^{J-1} y_i \geq \text{Pre} \cdot y_J \\ m_0 + C \cdot \sum_{i=1}^J y_i \geq 0 \\ \sum_{i=1}^J y_i(t) = \varphi_t \\ y_i \in \mathbb{N}^n, \quad i = 1, 2, \dots, J \end{cases} \quad (5a)$$

(5b)

(5c)

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with $\epsilon < (K \cdot \text{card}(H) \cdot J)^{-1}$

System S is SNNI **iff** the following two conditions hold for each $t \in L$

- 1) the ILP problem (2)-(3) does not admit a solution
- 2) the solution of the ILP problem (4)-(5) $\tilde{\mathbf{y}}_1, \dots, \tilde{\mathbf{y}}_J \in \mathbb{N}^n$ is such that $\sum_{i=1}^J \sum_{t_h \in H} \tilde{\mathbf{y}}_j(t_h) = 0$

The static SNNI enforcement problem

Given a bounded and not SNNI net system $\mathcal{S} = \langle N, \mathbf{m}_0 \rangle$, and given a set of *potentially selectable* transitions $\mathcal{PS} \subseteq H$, find a subset $\mathcal{D} \subseteq \mathcal{PS}$ such that if $\mathcal{E} = (L \cup H) \setminus \mathcal{D}$, then

- i) the system $\mathcal{S}_{\mathcal{E}} = \langle N_{\mathcal{E}}, \mathbf{m}_0 \rangle$ is SNNI
- ii) for all $\mathcal{D}' \subseteq \mathcal{PS}$ such that $\mathcal{D}' \subset \mathcal{D}$, if $\mathcal{E}' = (L \cup H) \setminus \mathcal{D}'$, then the system $\mathcal{S}_{\mathcal{E}'} = \langle N_{\mathcal{E}'}, \mathbf{m}_0 \rangle$ is not SNNI

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- The solution to the static enforcement problem **is not necessarily unique**

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- The solution to the static enforcement problem **is not necessarily unique**
- There may exist several subsets $\mathcal{D}_1, \dots, \mathcal{D}_k \subset \mathcal{PS}$ that satisfy conditions **i)** and **ii)**

Does the static SNNI problem admit a solution?

The static SNNI enforcement problem admits a solution **iff** the algorithm returns **true**

```
Input:  $\mathcal{S} = \langle N, m_0 \rangle, \mathcal{PS} \subseteq H,$   
        $J, \mathcal{T},$  and  $\varphi_{\bar{t}}$  for all  $\bar{t} \in \mathcal{T}$   
Output: return true if Problem 1 admits a solution,  
        false otherwise  
1  $\mathcal{E} := (L \cup H) \setminus \mathcal{PS};$  /* assume that all the  
   transitions in  $\mathcal{PS}$  are disabled */  
2 foreach  $\bar{t}_i \in \mathcal{T}$  do  
3   solve the ILP problem  
    $\min \left[ \sum_{i=1}^J \sum_{t \in L} \mathbf{y}_i(t) + \varepsilon \cdot \sum_{i=1}^J \sum_{t \in H \setminus \mathcal{PS}} \mathbf{y}_i(t) \right]$   
   subject to the set of constraints  $\mathcal{Y}_{\mathcal{E}}(\mathbf{m}_0, \varphi_{\bar{t}});$   
4   let  $\tilde{\mathbf{y}}_i,$  with  $i = 1, \dots, J,$  be the solution of the ILP  
   problem solved at Step 3;  
5   if  $\sum_{i=1}^J \sum_{t \in H \setminus \mathcal{PS}} \tilde{\mathbf{y}}_i(t) > 0;$  /* interference  
   is due to non selectable high-level  
   transitions */  
6   then  
7   | return false  
8   end  
9   solve the ILP problem  $\min \sum_{i=1}^J \sum_{t \in H \setminus \mathcal{PS}} \mathbf{x}_i(t)$   
   subject to the set of constraints  $\mathcal{X}_{\mathcal{E}}(\mathbf{m}_0, \varphi_{\bar{t}});$   
10  let  $\tilde{\mathbf{x}}_i,$  with  $i = 1, \dots, J,$  be the solution of the ILP  
   problem solved at Step 9;  
11  if  $\sum_{i=1}^J \sum_{t \in H \setminus \mathcal{PS}} \tilde{\mathbf{x}}_i(t) > 0;$  /* interference  
   is due to non selectable high-level  
   transitions */  
12  then  
13  | return false  
14  end  
15 end  
16 return true
```

Compute a solution to the static enforcement problem



```
Input:  $\mathcal{S} = \langle N, m_0 \rangle, \mathcal{PS} \subseteq H,$   
        $J, \mathcal{T},$  and  $\varphi_{\tilde{t}}$  for all  $\tilde{t} \in \mathcal{T}$   
Output: a set  $\mathcal{D} \subseteq \mathcal{PS}$  that solves Problem 1  
Precondition: Algorithm 1 returns true  
1  $\mathcal{NS} := (L \cup H) \setminus \mathcal{PS};$  /* set of the  
   transitions that are always enabled */  
2  $\mathcal{CS} := \mathcal{PS};$  /* initialize the set of  
   selectable transitions */  
3  $\mathcal{D} := \emptyset;$  /* initialize  $\mathcal{D}$  */  
4  $\mathcal{E} := \mathcal{NS} \cup \mathcal{CS};$  /* initialize the set of  
   enabled transitions */  
5 foreach  $\tilde{t}_i \in \mathcal{T}$  do  
6   solve the ILP problem  
    $\min \left[ \sum_{i=1}^J \sum_{t \in L} y_i(t) + \epsilon \sum_{i=1}^J \sum_{t \in \mathcal{CS}} y_i(t) \right]$   
   subject to the set of constraints  $\mathcal{Y}_{\mathcal{E}}(m_0, \varphi_{\tilde{t}});$   
7   let  $\tilde{y}_i,$  with  $i = 1, \dots, J,$  be the solution of the ILP  
   problem solved at Step 6;  
8   if  $\sum_{i=1}^J \sum_{t \in \mathcal{CS}} \tilde{y}_j(t) > 0$  then  
9     let  $\tilde{t} \in \mathcal{CS}$  be such that  $\sum_{i=1}^J \tilde{y}_i(\tilde{t}) > 0;$   
     /* choose one  $\tilde{t} \in \mathcal{CS}$  that causes  
     interference and update the  
     various sets */  
10     $\mathcal{D} := \mathcal{D} \cup \{\tilde{t}\};$   
11     $\mathcal{CS} := \mathcal{CS} \setminus \{\tilde{t}\};$   
12     $\mathcal{E} := \mathcal{NS} \cup \mathcal{CS};$   
13    if  $\mathcal{CS} \neq \emptyset$  then  
14      go to Step 6  
15    else  
16      Problem 1 admits the solution  $\mathcal{D} = \mathcal{PS}$  and  
      the algorithm terminates  
17    end  
18 end
```

```
19 solve the ILP problem  $\min \sum_{i=1}^J \sum_{t \in \mathcal{CS}} x_i(t)$   
   subject to the set of constraints  $\mathcal{X}_{\mathcal{E}}(m_0, \varphi_{\tilde{t}});$   
20 let  $\tilde{x}_i,$  with  $i = 1, \dots, J,$  be the solution of the ILP  
   problem solved at Step 19;  
21 if  $\sum_{i=1}^J \sum_{t \in \mathcal{CS}} \tilde{x}_i(t) > 0$  then  
22   let  $\tilde{t} \in \mathcal{CS}$  be such that  $\sum_{i=1}^J \tilde{x}_i(\tilde{t}) > 0;$   
   /* choose one  $\tilde{t} \in \mathcal{CS}$  that causes  
   interference and update the  
   various sets */  
23    $\mathcal{D} := \mathcal{D} \cup \{\tilde{t}\};$   
24    $\mathcal{CS} := \mathcal{CS} \setminus \{\tilde{t}\};$   
25    $\mathcal{E} := \mathcal{NS} \cup \mathcal{CS};$   
26   if  $\mathcal{CS} \neq \emptyset$  then  
27     go to Step 19  
28   else  
29     Problem 1 admits the solution  $\mathcal{D} = \mathcal{PS}$  and  
     the algorithm terminates  
30   end  
31 end  
32 end
```

Compute all the solutions to the static enforcement problem



Algorithm to compute all the solutions to the static SNNI enforcement problem

```
Input:  $\mathcal{S} = \langle N, m_0 \rangle, \mathcal{PS} \subseteq H,$   
           $J, \mathcal{T},$  and  $\varphi_{\bar{t}}$  for all  $\bar{t} \in \mathcal{T}$   
Output: the set  $SOL$  of all the solutions to Problem 1  
Precondition: Algorithm 1 returns true  
1  $SOL := \emptyset;$        /* initialize the output */  
2 run Algorithm 2 and let  $\bar{\mathcal{D}}$  be the output;  
3 add  $\bar{\mathcal{D}}$  to  $SOL$  and flag it as “new”;  
4 foreach  $\mathcal{D} \in SOL$  flagged as “new” do  
5     flag  $\mathcal{D}$  as “old”;  
6     foreach  $t \in \mathcal{D}$  do  
7         if Algorithm 1 returns true when executed using  
            $\mathcal{PS} \setminus \{t\}$  as set of potentially selectable  
           transitions then  
8             run Algorithm 2 using  $\mathcal{PS} \setminus \{t\}$  as set of  
               potentially selectable transitions and let  $\mathcal{D}'$   
               the solution;  
9             add  $\mathcal{D}'$  to  $SOL$  and flag it as “new”  
10        end  
11     end  
12 end
```

Set SOL of the solutions to the static enforcement problem

Given a bounded and not SNNI net system $\mathcal{S} = \langle N, \mathbf{m}_0 \rangle$ and a set of potentially selectable transitions $\mathcal{PS} \subseteq H$, let SOL be the family of solutions to the static SNNI enforcement problem, i.e.

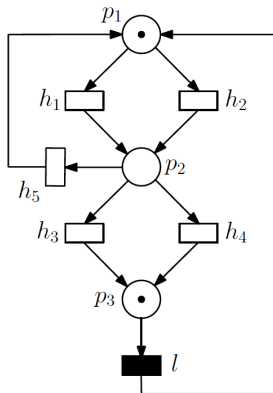
$$SOL = \{ \mathcal{D} \mid \mathcal{D} \subseteq \mathcal{PS} \text{ and } \mathcal{D} \text{ solves the static SNNI enforcement problem} \}$$

and let

$$\overline{SOL} = \{ \mathcal{E} \subseteq L \cup H \mid \mathcal{E} = (L \cup H) \setminus \mathcal{D}, \text{ with } \mathcal{D} \in SOL \}$$

be the family of transitions in \mathcal{PS} that can be enabled without violating SNNI. If SOL is not empty, then the following properties hold

- a) \overline{SOL} is not empty
- b) \overline{SOL} is not closed under union



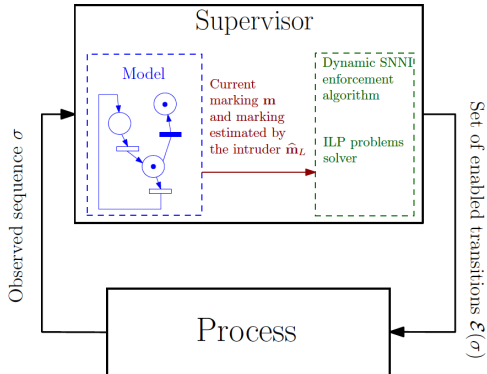
- The static SNNI enforcement problem admits multiple solutions when $\mathcal{PS} = H = \{h_1, h_2, h_3, h_4, h_5\}$
- $SOL = \{\{h_1, h_2\}, \{h_3, h_4\}\}$
- $\overline{SOL} = \{\{l, h_3, h_4, h_5\}, \{l, h_1, h_2, h_5\}\}$
- The union of sets in \overline{SOL} is equal to $L \cup H$ that does not belong to \overline{SOL}

the dynamic SNNI enforcement problem

Given a bounded and not SNNI net system $\mathcal{S} = \langle N, \mathbf{m}_0 \rangle$, a set of potentially selectable transitions $\mathcal{PS} \subseteq H$, and a reachable marking $\mathbf{m} \in R(N, \mathbf{m}_0)$, find a subset $\mathcal{D}(\mathbf{m}) \subseteq \mathcal{PS}$ such that if $\mathcal{E}(\mathbf{m}) = (L \cup H) \setminus \mathcal{D}(\mathbf{m})$, then

- i) The system $\mathcal{S}_{\mathcal{E}(\mathbf{m})} = \langle N_{\mathcal{E}(\mathbf{m})}, \mathbf{m} \rangle$ is SNNI
- ii) for all $\mathcal{D}' \subseteq \mathcal{PS}$ such that $\mathcal{D}' \subset \mathcal{D}(\mathbf{m})$, if $\mathcal{E}' = (L \cup H) \setminus \mathcal{D}'$, then the system $\mathcal{S}_{\mathcal{E}'} = \langle N_{\mathcal{E}'}, \mathbf{m} \rangle$ is not SNNI

Control architecture for dynamic SNNI enforcement



When the firing of the k -th transition t is observed, then the supervisor updates the two state vectors estimates

$$\begin{aligned} \mathbf{m}(k) &= \mathbf{m}(k-1) + \mathbf{C}(\cdot, t), \\ \begin{cases} \hat{\mathbf{m}}_L(k) = \hat{\mathbf{m}}_L(k-1) + \mathbf{C}(\cdot, t), & \text{if } t \in L, \\ \hat{\mathbf{m}}_L(k) = \hat{\mathbf{m}}_L(k-1), & \text{otherwise} \end{cases} \end{aligned}$$

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- When the observed sequence is equal to ε , the supervisor disables all the high-level transitions that belong to at least one of the solutions to the static enforcement problem, i.e. the initial guess for the set of high-level transitions to be disabled is

$$\mathcal{D}(\mathbf{m}_{-1}) = \bigcup_{\bar{\mathcal{D}} \in SOL} \bar{\mathcal{D}},$$

hence, the initial guess for the enabled transitions is $\mathcal{E}(\mathbf{m}_{-1}) = (L \cup H) \setminus \mathcal{D}(\mathbf{m}_{-1})$

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- $\mathcal{D}(\mathbf{m}_{-1})$ and $\mathcal{E}(\mathbf{m}_{-1})$ are used to compute $\mathcal{D}(\mathbf{m}_0)$ and $\mathcal{E}(\mathbf{m}_0) = (L \cup H) \setminus \mathcal{D}(\mathbf{m}_0)$

Input: $N, \mathbf{m}(k), \widehat{\mathbf{m}}_L(k), J, \mathcal{T}$
Input/Output: $\mathcal{D}(\cdot), \mathcal{E}(\cdot)$
Precondition: Execute Algorithm 3

```

1 foreach  $t_l \in \mathcal{T}$  do
2   let  $\bar{s}_{t_l} = \max \sum_{i=1}^J \mathbf{s}_i(t_l)$ 
   subject to
   
$$\begin{cases} \widehat{\mathbf{m}}_L(k) \geq \text{Pre}_{\mathcal{E}(\mathbf{m}(k-1))} \cdot \mathbf{s}_1 \\ \widehat{\mathbf{m}}_L(k) + \mathbf{C}_{\mathcal{E}(\mathbf{m}(k-1))} \cdot \mathbf{s}_1 \geq \text{Pre}_{\mathcal{E}(\mathbf{m}(k-1))} \cdot \mathbf{s}_2 \\ \dots \\ \widehat{\mathbf{m}}_L(k) + \mathbf{C}_{\mathcal{E}(\mathbf{m}(k-1))} \cdot \sum_{i=1}^{J-1} \mathbf{s}_i \geq \text{Pre}_{\mathcal{E}(\mathbf{m}(k-1))} \\ \widehat{\mathbf{m}}_L(k) + \mathbf{C}_{\mathcal{E}(\mathbf{m}(k-1))} \cdot \sum_{i=1}^J \mathbf{s}_i \geq \mathbf{0} \\ \mathbf{s}_i \in \mathbb{N}^{\text{card}(\mathcal{E})}, \quad i = 1, 2, \dots, J \end{cases}$$

3 end
4  $\mathcal{DD} := \mathcal{D}(\mathbf{m}(k-1));$ 
5  $\mathcal{EE} := \mathcal{E}(\mathbf{m}(k-1));$ 

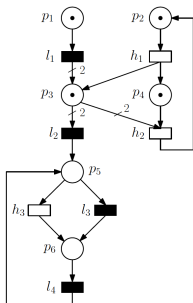
```

```

6 foreach  $t_h \in \mathcal{D}(\mathbf{m}(k-1))$  such that  $\mathbf{m}(k) \geq \text{Pre}(\cdot, t_h)$ 
  do
7    $\mathcal{E}' := \mathcal{EE} \cup \{t_h\};$ 
8   keep_disabled:=false;
9   foreach  $t_l \in \mathcal{T}$  do
10    if the ILP problem  $\min \sum_{i=1}^J x_i(t_h)$  subject to
      the set of constraints  $\mathcal{X}_{\mathcal{E}'}(\mathbf{m}(k), \bar{s}_{t_l})$  admits a
      solution then
11     keep_disabled:=true;
12     break
13    end
14    if the ILP problem
       $\min \left[ \sum_{i=1}^J \sum_{t \in L} \mathbf{y}_i(t) + \epsilon \sum_{i=1}^J \mathbf{y}_i(t_h) \right]$ 
      subject to the set of
      constraints  $\mathcal{Y}_{\mathcal{E}'}(\widehat{\mathbf{m}}_L(k), \bar{s}_{t_l})$  admits a
      solution  $\mathbf{y}'_i, i = 1, \dots, J$  such that
       $\sum_{i=1}^J \mathbf{y}'_i(t_h) > 0$ 
15    and the ILP problem
       $\min \left[ \sum_{i=1}^J \sum_{t \in L} \mathbf{y}_i(t) + \epsilon \sum_{i=1}^J \mathbf{y}_i(t_h) \right]$ 
      subject to the set of constraints  $\mathcal{Y}_{\mathcal{E}'}(\mathbf{m}(k), \bar{s}_{t_l})$ 
      admits a solution  $\mathbf{y}''_i, i = 1, \dots, J$  such that
       $\sum_{i=1}^J \mathbf{y}''_i(t_h) > 0$  then
16     keep_disabled:=true;
17     break
18    end
19    end
20    if keep_disabled=false then
21      $\mathcal{DD} := \mathcal{DD} \setminus \{t_h\};$ 
22      $\mathcal{EE} := \mathcal{EE} \cup \{t_h\}$ 
23    end
24  end
25   $\mathcal{D}(\mathbf{m}(k)) = \mathcal{DD};$ 
26   $\mathcal{E}(\mathbf{m}(k)) = \mathcal{EE};$ 

```


Example



Observed sequence	\mathbf{m}	$\bar{\mathbf{m}}_L$	\bar{s}_{12}	\bar{s}_{13}	\bar{s}_{14}	Comment	$\mathcal{D}(\cdot)$
ε	$(1 \ 1 \ 1 \ 1 \ 0 \ 0)^T$	$(1 \ 1 \ 1 \ 1 \ 0 \ 0)^T$	1	3	3	Under the initial marking h_1 is the only transition in $\mathcal{D}(\cdot)$ that could be potentially enabled. However, for h_1 the ILP problem (14) admits a solution when l_2 is considered. Hence, it must be kept in $\mathcal{D}(\cdot)$.	$\{h_1, h_3\}$
l_1	$(0 \ 1 \ 3 \ 1 \ 0 \ 0)^T$	$(0 \ 1 \ 3 \ 1 \ 0 \ 0)^T$	1	4	3	h_1 is still the only transition in $\mathcal{D}(\cdot)$ that could be enabled, but also in this case the ILP problem (14) admits a solution when l_2 is considered.	$\{h_1, h_3\}$
$l_1 h_2$	$(0 \ 2 \ 1 \ 0 \ 0 \ 0)^T$	$(0 \ 1 \ 3 \ 1 \ 0 \ 0)^T$	1	4	3	In this case h_1 can be moved to $\mathcal{E}(\cdot)$.	$\{h_3\}$
$l_1 h_2 h_1$	$(0 \ 1 \ 2 \ 1 \ 0 \ 0)^T$	$(0 \ 1 \ 3 \ 1 \ 0 \ 0)^T$	1	4	3	h_3 is the only transition in $\mathcal{D}(\cdot)$, but is not enabled under \mathbf{m} , therefore Algorithm 4 does not need to be executed.	$\{h_3\}$
$l_1 h_2 h_1 h_1$	$(0 \ 0 \ 3 \ 2 \ 0 \ 0)^T$	$(0 \ 1 \ 3 \ 1 \ 0 \ 0)^T$	1	4	3	h_3 is still not enabled under \mathbf{m} .	$\{h_3\}$
$l_1 h_2 h_1 h_1 l_2$	$(0 \ 0 \ 1 \ 2 \ 1 \ 0)^T$	$(0 \ 1 \ 1 \ 1 \ 1 \ 0)^T$	0	4	4	h_3 could now be potentially enabled, but when the low-level transition l_4 is considered, both ILP problems (15) and (16) admit a solution $\bar{\mathbf{y}}_i$, $i = 1, \dots, J$, such that $\sum_{i=1}^J \bar{\mathbf{y}}_i(h_3) > 0$. Therefore, h_3 must be kept disabled.	$\{h_3\}$

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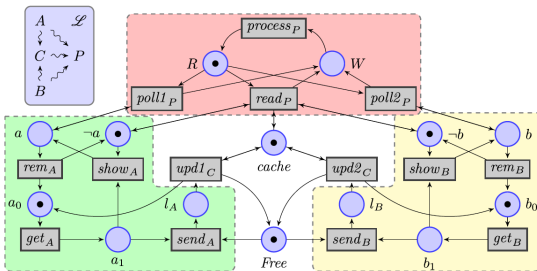
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- See also



X. Yin

Verification of Prognosability for Labeled Petri Nets,
IEEE Transactions on Automatic Control, 2018

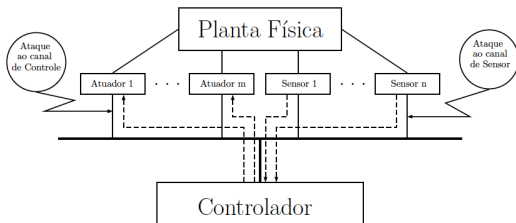


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Theoretical Computer Science, 2018



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Security Against Network Attacks in Supervisory Control Systems,
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- Non-interference enforcement by means of supervisory control



F. Basile, G. De Tommasi and C. Sterle

Non-interference enforcement via supervisory control in bounded Petri nets,

IEEE Transactions on Automatic Control, to appear 2021

Subjects you should focus on

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The assessment

- When you're ready I will send you 2/3 exercises
- You will have one week to send me back what you have done

Non-interference and opacity enforcement

From observability to privacy and security in discrete event systems

Prof. Gianmaria DE TOMMASI
Email: detommas@unina.it

December 2020