Simultaneous control of modes with multiple toroidal periodicity in tokamak plasmas

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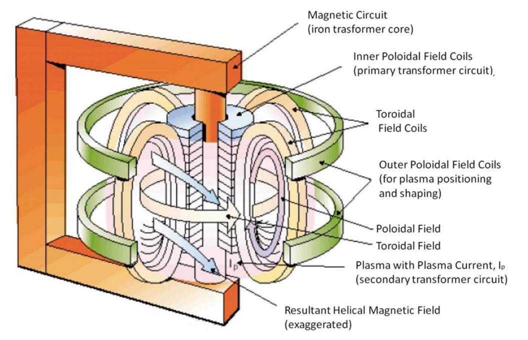
Outline

1. **Introduction**
   - Contribution of the paper
2. **Controller Architecture**
3. **Plant Model**
4. **Controller Design**
5. **Simulation Results**

Tokamak

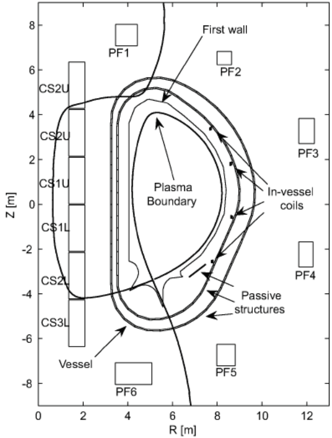


A tokamak is an electromagnetic machine containing a fully ionised gas (plasma) at about 100 million degrees within a torus shaped vacuum vessel. Poloidal and toroidal field coils, together with the plasma current, generate a spiralling magnetic field that confines the plasma.

Motivations

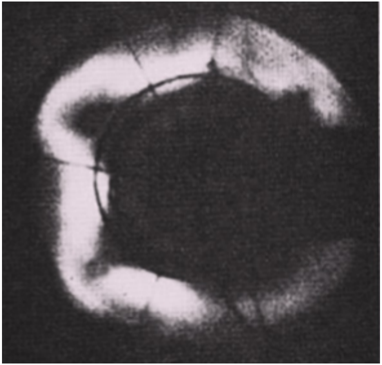
- Tokamaks cannot be operated without reliable and robust plasma control systems
- Tokamak control systems have to deal with different kinds of instabilities related to the presence of a resistive wall that surrounds the plasma
- These instabilities are known as **Resistive Wall Modes** (RWMs) and are both axisymmetric and non-axisymmetric

Resistive Wall Modes - 1



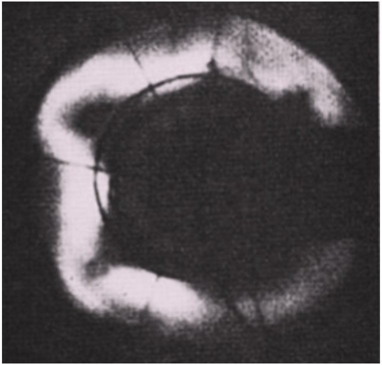
- The main instability is due to an axisymmetric ($n = 0$) mode, the so-called axisymmetric Vertical Displacement Event, which occurs whenever a plasma with a vertical elongated poloidal cross-section is operated

Resistive Wall Modes - 2



- Another important plasma instability is the one called kink instability, which is the main non-axisymmetric ($n = 1$) mode

Resistive Wall Modes - 2



- Another important plasma instability is the one called kink instability, which is the main non-axisymmetric ($n = 1$) mode
- The kink instability arises when the *plasma pressure* exceeds a certain threshold $\rightarrow$ it is similar to a garden hose kinking when it is suddenly pressurized

Control RWMs

- Elongated plasmas enable to increase the energy confinement time, which is an essential criterion for realizing sustained fusion, but they are vertically unstable
  - The use of an active feedback system, usually called **vertical stabilization system** is required

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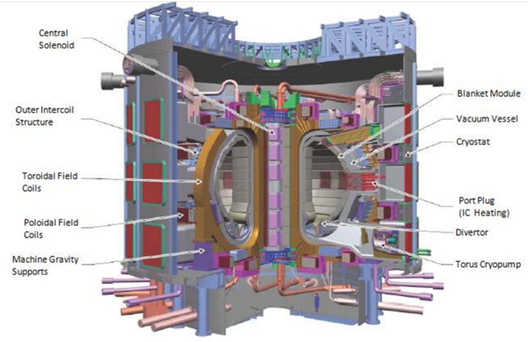
- Elongated plasmas enable to increase the energy confinement time, which is an essential criterion for realizing sustained fusion, but they are vertically unstable
  - The use of an active feedback system, usually called **vertical stabilization system** is required
- Modern tokamak devices operate at high plasma pressure, hence a kink instability is most likely to occur
  - a **control system to stabilize also the $n = 1$ mode** becomes necessary

Control of RWMs in ITER

- In this paper we propose a control architecture that enables to control $n = 0$ and $n = 1$ instabilities in ITER
- The proposed solution allows us to minimize the control effort in terms of amplitude of the currents in the coils

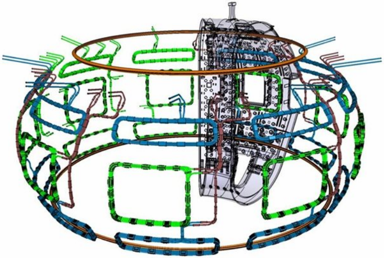
ITER



ITER is a joint venture of 7 participant teams (EU plus Switzerland, Japan, the People's Republic of China, India, the Republis of Korea, Russia and USA). It has been designed to demonstrate the feasibility of fusion energy for peaceful purposes.

Control coils

- The stabilization of the $n = 0$ mode is achieved by using the axisymmetric in-vessel coils, which is referred to as VS3 circuit
- The 27 non-axisymmetric coils so-called ELM coils, are used to stabilize the $n = 1$ mode



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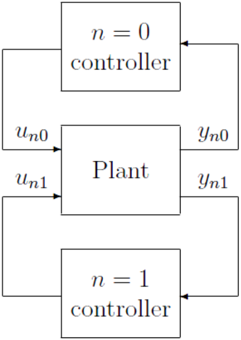
- When designing the controller there are thermal constraints that must be taken into account, and that limit the *rms* value of the current in these internal copper coils
- Two separate controllers have been designed
  - the $n = 0$ controller stabilizes the $n = 0$ mode trying not to induce any $n = 1$ mode and keeping the currents in the ELM coils as low as possible
  - the $n = 1$ controller is an LQ optimal controller that stabilizes the $n = 1$ instability without inducing any $n = 0$ mode



Voltages applied to the ELM coils

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$$u_i = \Theta \cdot \begin{pmatrix} u_{Ai} \\ u_{Bi} \end{pmatrix} + hu_{i0}\,, \quad i = 1, 2, 3\,, \tag{1}$$

where $h \in \mathbb{R}^{9\times 1}$ is a vector whose elements are all equal to 1, and

$$\Theta = \begin{pmatrix} \cos\eta_1 & \sin\eta_1 \\ \cos\eta_2 & \sin\eta_2 \\ \dots & \dots \\ \cos\eta_9 & \sin\eta_9 \end{pmatrix} \in \mathbb{R}^{9\times 2}\,.$$

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- The u_{i0} terms are used by the $n = 0$ mode stabilization controller in order to minimize the amplitude of the $i_{ELM_{0,i}}$ currents
- The u_{Ai} and u_{Bi} terms counteract the $n = 1$ perturbation without stimulating the $n = 0$ mode

Plasma vertical position

- Analogously to (1), the estimated plasma vertical position z_i in the generic poloidal section i, with $i \in 1, 2, 3$ has been approximated as

$$z_i = z_0 + z_A \cos\varphi_i + z_B \sin\varphi_i \,, \qquad (2)$$

- z_0 is the average vertical position along the toroidal angle, calculated as $z_0 = (z_1 + z_2 + z_3)/3$
- z_A and z_B in (2) are given by

$$\begin{pmatrix} z_A \\ z_B \end{pmatrix} = M^\dagger \begin{pmatrix} z_1 - z_0 \\ z_2 - z_0 \\ z_3 - z_0 \end{pmatrix} \,, \qquad (3)$$

where

$$M = \begin{pmatrix} \cos\varphi_1 & \sin\varphi_1 \\ \cos\varphi_2 & \sin\varphi_2 \\ \cos\varphi_3 & \sin\varphi_3 \end{pmatrix}$$

Plant model - 1

- The ITER tokamak has been discretized with a 3D finite elements mesh, made of 4970 hexahedral elements, giving rise to $N = 4135$ discrete degrees of freedom
- The considered plasma equilibrium is a $I_p = 9\ MA$ configuration, with a normalized $\beta_N = 2.94$ (this parameter quantifies the plasma pressure)

Plant model - 2

- For controller design purposes the following linearized model can be considered

$$\dot{x} = Ax + B_{n0}u_{n0} + B_{n1}u_{n1} \tag{4a}$$

$$\begin{pmatrix} y_{n0} \\ y_{n1} \end{pmatrix} = \begin{pmatrix} C_{n0} \\ C_{n1} \end{pmatrix} x + \begin{pmatrix} D_{n0} \\ 0 \end{pmatrix} u_{n0} \tag{4b}$$

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- $y_{n1} = (z_A\ z_B)^T$ indicates the outputs controlled by the $n = 1$ controller

Unstable modes

- The dynamic matrix A has three unstable eigenvalues
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- Each of the related eigenvectors corresponds to a specific current pattern inside the three-dimensional structure
- The first unstable eigenvalue (around $5.6s^{-1}$) shows an almost axisymmetric current density pattern, and hence corresponds to the $n = 0$ RWM (VDE)
- The other two unstable modes have coinciding values (around $17s^{-1}$) and correspond to two $n = 1$ current density patterns (external kink), which are identical apart from a shift of $\pi/2$ in the toroidal direction

Observability and controllability of the unstable modes

- The two $n = 1$ unstable modes are structurally neither controllable from u_{n0} nor observable from y_{n0}
- Similarly the $n = 0$ unstable mode is neither controllable from u_{n1} nor observable from y_{n1}

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- The two $n = 1$ unstable modes are structurally neither controllable from u_{n0} nor observable from y_{n0}
- Similarly the $n = 0$ unstable mode is neither controllable from u_{n1} nor observable from y_{n1}
- The controller design is split in the design of two separate stabilizing controller, for the $n = 0$ and $n = 1$ modes, respectively

Minimal realizations of the plant model

- The $n = 0$ controller is designed considering the following state space model

$$\dot{\xi} = \widehat{A}_{n0}\xi + \widehat{B}_{n0}u_{n0} \tag{5a}$$

$$y_{n0} = \widehat{C}_{n0}\xi + \widehat{D}_{n0}u_{n0} \tag{5b}$$

where $\widehat{A}_{n0}$, $\widehat{B}_{n0}$, $\widehat{C}_{n0}$, and $\widehat{D}_{n0}$ correspond to a minimal realization of (4) as seen from input u_{n0} to output y_{n0}

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- Similarly, the $n = 1$ controller is designed on the basis of the plant

$$\dot{\zeta} = \widehat{A}_{n1}\zeta + \widehat{B}_{n1}u_{n1} \tag{6a}$$

$$y_{n1} = \widehat{C}_{n1}\zeta \tag{6b}$$

The $n = 0$ controller

- Given the high order of model (5), the design of the $n = 0$ controller has been carried out the reduced order model

$$\dot{\tilde{\xi}} = \widetilde{A}_{n0}\tilde{\xi} + \widetilde{B}_{n0}u_{n0}\,, \quad \tilde{\xi}(t_0) = \tilde{\xi}_0 \tag{7a}$$

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- The controller has been designed as a state feedback controller

$$u_{n0} = K_{n0}\tilde{\xi}\,, \tag{8}$$

  where the control gain matrix $K_{n0} \in \mathbb{R}^{4\times n_r}$ is chosen in order to satisfy the following condition

$$\sup_{t\in[0,+\infty[} \frac{y_{n0}^T(t)Qy_{n0}(t)}{\tilde{\xi}_0^T R\tilde{\xi}_0} < \gamma_{n0}\,, \tag{9}$$

  with $Q \in \mathbb{R}^{5\times 5}$ and $R \in \mathbb{R}^{n_r\times n_r}$ being two positive definite matrices and $\gamma_{n0} > 0$.

Design theorem

Theorem

Let us consider closed loop system

$$\dot{\tilde{\xi}} = (\widetilde{A}_{n0} + \widetilde{B}_{n0} K_{n0})\tilde{\xi}\,, \quad \tilde{\xi}(t_0) = \tilde{\xi}_0 \tag{10a}$$

$$y_{n0} = (\widetilde{C}_{n0} + \widetilde{D}_{n0} K_{n0})\tilde{\xi}\,. \tag{10b}$$

and condition (9)*. If there exist a positive definite matrix* $Y \in \mathbb{R}^{n_r \times n_r}$ *and a matrix* $W \in \mathbb{R}^{4 \times n_r}$ *such that*

$$\widetilde{A}_{n0} Y + Y \widetilde{A}_{n0}^T + \widetilde{B}_{n0} W + W^T \widetilde{B}_{n0}^T < 0\,, \tag{11a}$$

$$\begin{pmatrix} Q^{-1} & \widetilde{C}_{n0} Y + \widetilde{D}_{n0} W \\ (\widetilde{C}_{n0} Y + \widetilde{D}_{n0} W)^T & Y \end{pmatrix} > 0\,, \tag{11b}$$

$$Y > (\gamma_{n0} R)^{-1}\,, \tag{11c}$$

then system (10) *with* $K_{n0} = WY^{-1}$ *satisfies condition* (9)

$n = 0$ controller - comment

- In order to use the state feedback controller (8), an observer of the reduced plant (7) has been designed as a Kalman filter
- We want to minimize the output y_{n0} norm in the presence of a VDE, hence the weighting matrix R in (9) as been chosen as $R = \tilde{\xi}_{VDE}\tilde{\xi}_{VDE}^T + \varepsilon I$, where the term εI is needed to guarantee the full rank of R.

The $n = 1$ controller

- Similarly to what has been done in the $n = 0$ case, the design of the $n = 1$ controller has been carried out considering the reduced order model

$$\dot{\tilde{\zeta}} = \widetilde{A}_{n1}\tilde{\zeta} + \widetilde{B}_{n1}u_{n1} \tag{12a}$$

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- As for the $n = 0$ controller, a state observer has been designed as a Kalman filter

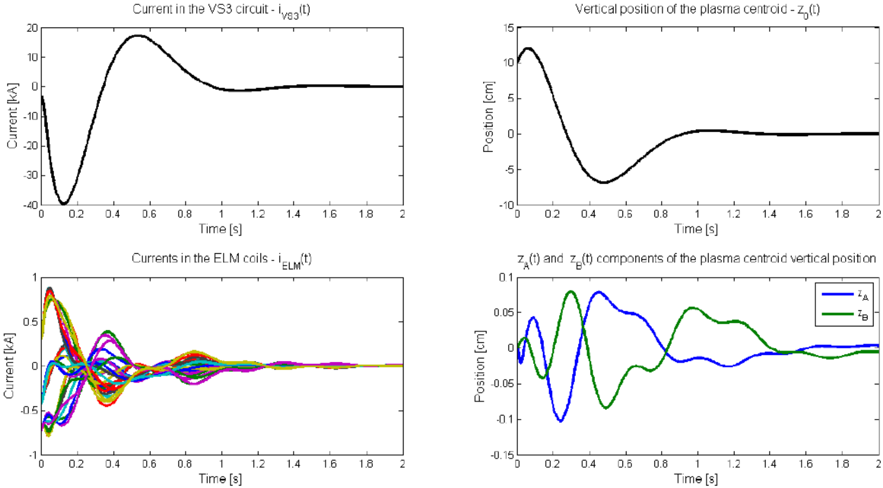
Controller design

- The linearized model around the equilibrium with $I_p = 9$ MA and normalized beta $\beta_N = 2.94$ has been considered
- The order of the model is 4135
- The order of the model has been reduced to 20 for the $n = 0$ controller and to 46 for the $n = 1$ controller.

VDE event - 1

- In the first simulation a VDE is considered
- It is shown that, resorting to a controller minimizing the index (9), the currents in the ELM coils remain well below 1 kA also during the transients

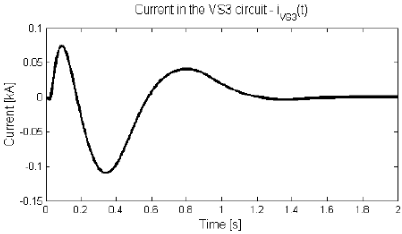
VDE event - 2

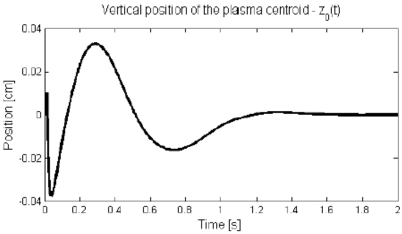

Current in the VS3 circuit - $i_{VS3}(t)$
Current [kA]
Time [s]
Vertical position of the plasma centroid - $z_0(t)$
Position [cm]
Time [s]
Currents in the ELM coils - $i_{ELM}(t)$
Current [kA]
Time [s]
$z_A(t)$ and $z_B(t)$ components of the plasma centroid vertical position
z_A
z_B
Position [cm]
Time [s]

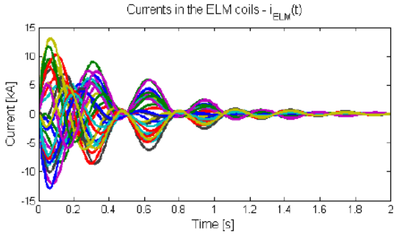
Kink instability - 1

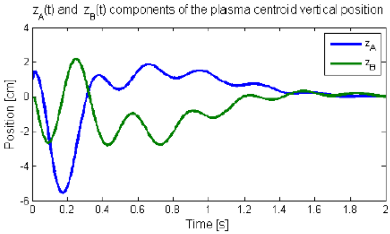
- In the second simulation, a disturbance along the $n = 1$ mode corresponding to a 1 cm displacement at the toroidal angle φ_1 is applied
- It is shown that the proposed architecture produces very little influence of the $n = 1$ loop on the $n = 0$ mode $\rightarrow$ the maximum variation of z_0 is less than one millimeter

Kink instability - 2









Conclusions

- Two separate control loops have been proposed for the simultaneous control of vertical and kink instabilities in ITER
- Scope of the proposed control architecture is to stabilize the plant, maximizing the operating region and minimizing the interaction between the two phenomena
- Simulation results, obtained for a suitable configuration of an ITER plasma, show the effectiveness of the proposed approach

Thank you!