Non-interference assessment in bounded Petri nets via Integer Linear Programming

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Outline

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Y.-C. Wu and S. Lafortune,
Comparative analysis of related notions of opacity in centralized and coordinated architectures,
Discrete Event Dyn. Syst., vol. 23, no. 3, pp. 307–339, 2013

N. Busi and R. Gorrieri,
A survey on non-interference with Petri nets,
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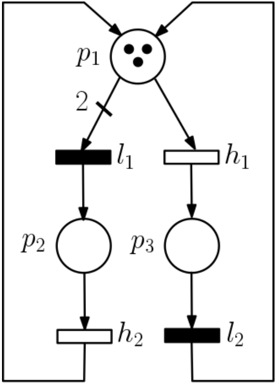
Non-interference in PN systems



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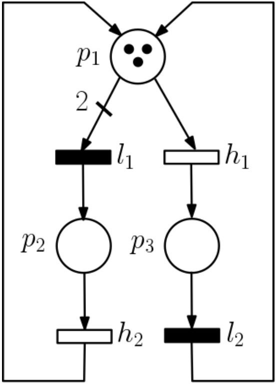

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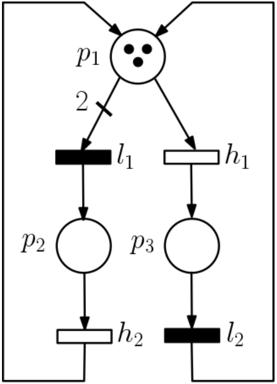

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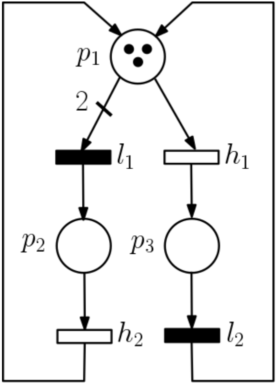

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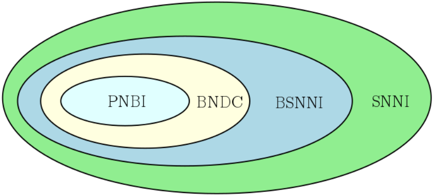
- Both high-level and low-level users know the system structure, but they interact with the system in two different ways (*views*)
- If the high-level view of the system *interferes* with the low-level one, information leaks may occur

Non-interference properties



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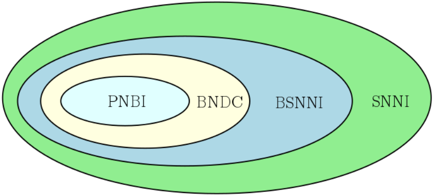

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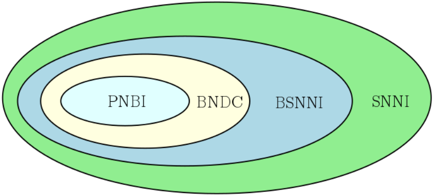

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- In a **Bisimulation SNNI (BSNNI)** the firing of a low-level transition cannot disable the firing of any high-level transition
- More restrictive non-interference properties exist
  - Bisimulation non-deducibility on composition (BNDC)
  - Place-based non-interference (PBNI)

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  - **to check SNNI** in bounded PNs
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- The proposed approach relies on the algebraic representation of the PN dynamic
- The proposed conditions are based on the solution of Integer Linear Programming (ILP) problems
  - *Off-the-shelf* commercial software can be used (e.g., CPLEX, FICO-Xpress)

Main assumptions

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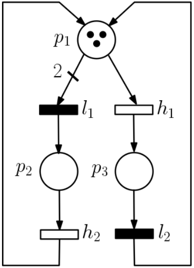
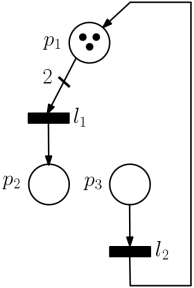
- The *low-level* subnet (subnet *induced* by the low-level transitions) is acyclic





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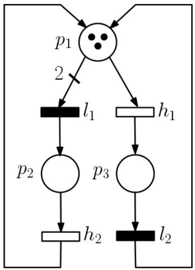

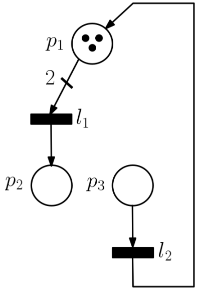

- *Unnecessary assumptions*
  - the net does not need to belong to any special class (ordinary or safe)

Notation

- The P/T net: $N = (P, L, H, \mathbf{Pre}, \mathbf{Post})$, with $L \cap H = \emptyset$
- The incidence matrix: $\boldsymbol{C} = \mathbf{Post} - \mathbf{Pre}$
- The net system $\mathcal{S} = \langle N, \boldsymbol{m}_0 \rangle$
- Projection of a string on the set of low-view transitions L
  - $Pr_L(\varepsilon) = \varepsilon$
  - $Pr_L(\sigma t) = \begin{cases} Pr_L(\sigma)t & \text{if } t \in L \\ Pr_L(\sigma) & \text{otherwise} \end{cases}$

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  The projection $Pr_L(\cdot)$ can be extended in the usual way to sets of sequences, i.e., if $\Sigma \subseteq (L \cup H)^*$ then

$$Pr_L(\Sigma) = \{Pr_L(\sigma) \mid \sigma \in \Sigma\} .$$

SNNI

Low-view trace equivalence

Two net systems $\mathcal{S}_1$ and $\mathcal{S}_2$ are said to be **low-view trace equivalent**, denoting it by

$$\mathcal{S}_1 \overset{Pr}{\approx}_{tr} \mathcal{S}_2 \,,$$

if and only if

$$Pr_{L_1}\left(\mathcal{L}(N_1\,, \boldsymbol{m}_{0_1})\right) = Pr_{L_2}\left(\mathcal{L}(N_2\,, \boldsymbol{m}_{0_2})\right) \,,$$

where $\mathcal{L}(N_i\,, \boldsymbol{m}_{0_i})$ is the language generate by the i-th net system.

SNNI

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Let $\mathcal{S} = \langle N \, , \boldsymbol{m}_0 \rangle$ be a net system and $\mathcal{S}_L = \langle N_L \, , \boldsymbol{m}_0 \rangle$ the system defined on the corresponding low-level subnet N_L. $\mathcal{S}$ is said to be **strong non-deterministic non-interference** if and only if

$$\mathcal{S} \overset{Pr}{\approx}_{tr} \mathcal{S}_L \, .$$

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In a SNNI system, the firings of a high-level transition cannot enable any *additional* firing of any low-level transition

Low-view bisimilarity

Let $\mathcal{S}_1$ and $\mathcal{S}_2$ be two net systems. A *low-view bisimulation* from $\mathcal{S}_1$ to $\mathcal{S}_2$ is a relation $\mathcal{R}$ on $R(N_1, \boldsymbol{m}_{0_1}) \times R(N_2, \boldsymbol{m}_{0_2})$ such that if $(\boldsymbol{m}_1, \boldsymbol{m}_2) \in \mathcal{R}$, then for all $t \in \bigcup_{i=1,2} L_i \cup H_i$ it is:

1. if $\boldsymbol{m}_1[t\rangle\boldsymbol{m}_1'$ then there exist τ and $\boldsymbol{m}_2'$ such that $\boldsymbol{m}_2[\tau\rangle\boldsymbol{m}_2'$, with $Pr_{L_1}(t) = Pr_{L_2}(\tau)$ and $(\boldsymbol{m}_1', \boldsymbol{m}_2') \in \mathcal{R}$;
2. if $\boldsymbol{m}_2[t\rangle\boldsymbol{m}_2'$ then there exist τ and $\boldsymbol{m}_1'$ such that $\boldsymbol{m}_1[\tau\rangle\boldsymbol{m}_1'$, with $Pr_{L_2}(t) = Pr_{L_1}(\tau)$ and $(\boldsymbol{m}_1', \boldsymbol{m}_2') \in \mathcal{R}$.

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$$\mathcal{S}_1 \overset{Pr}{\approx}_{bis} \mathcal{S}_2 ,$$

if and only if there exists a low-level bisimulation $\mathcal{R}$ from $\mathcal{S}_1$ and $\mathcal{S}_2$ such that $(\boldsymbol{m}_{0_1}, \boldsymbol{m}_{0_2}) \in \mathcal{R}$.

BSNNI

Let $\mathcal{S} = \langle N \, , \boldsymbol{m}_0 \rangle$ be a net system and $\mathcal{S}_L = \langle N_L \, , \boldsymbol{m}_0 \rangle$ the system defined on the corresponding low-level subnet N_L. $\mathcal{S}$ is said to be **bisimulation strong non-deterministic non-interference** if and only if

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- The class of SNNI systems includes the class of BSNNI systems, but the two classes are not equivalent
- In a BSNNI system the firing of a low-level transition cannot disable the firing of any high-level transition

SNNI for bounded net systems

A bounded net system $\mathcal{S} = \langle N, \boldsymbol{m}_0 \rangle$ is SNNI **if and only if** the set of constraints

$$\begin{cases} \boldsymbol{m}_0 + \boldsymbol{C}_L \cdot \hat{\boldsymbol{\sigma}} \geq \mathbf{Pre} \cdot \boldsymbol{s}_1 \\ \boldsymbol{m}_0 + \boldsymbol{C}_L \cdot \hat{\boldsymbol{\sigma}} + \boldsymbol{C} \cdot \boldsymbol{s}_1 \geq \mathbf{Pre} \cdot \boldsymbol{s}_2 \\ \dots \\ \boldsymbol{m}_0 + \boldsymbol{C}_L \cdot \hat{\boldsymbol{\sigma}} + \boldsymbol{C} \cdot \sum_{i=1}^{J-1} \boldsymbol{s}_i \geq \mathbf{Pre} \cdot \boldsymbol{s}_J \\ \boldsymbol{m}_0 + \boldsymbol{C}_L \cdot \hat{\boldsymbol{\sigma}} + \boldsymbol{C} \cdot \sum_{i=1}^{J} \boldsymbol{s}_i \geq \mathbf{0} \\ \sum_{i=1}^{J} \boldsymbol{s}_i(t) = 1 \end{cases} \tag{1}$$

does not admit any solution $\boldsymbol{s}_1, \dots, \boldsymbol{s}_J \in \mathbb{N}^{n_L+n_H}$ **for all** $t \in L$, with $J \geq \mathcal{J}_{\min}$ and $\hat{\boldsymbol{\sigma}}$ being equal to the solution of the ILP problem

$$\begin{aligned} & \max \boldsymbol{\sigma}(t) \\ & \text{s.t.} \\ & \begin{cases} \boldsymbol{m}_0 + \boldsymbol{C}_L \cdot \boldsymbol{\sigma} \geq \mathbf{0} \\ \boldsymbol{\sigma} \in \mathbb{N}^{n_L} \end{cases} \end{aligned} \tag{2}$$

BSNNI for bounded net systems

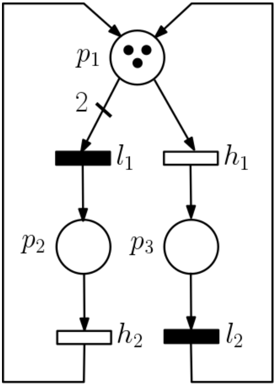
A SNNI bounded net system $\mathcal{S} = \langle N, \mathbf{m}_0 \rangle$ is BSNNI **if and only if** the set of constraints

$$\begin{cases} \mathbf{m}_0 \geq \mathbf{Pre} \cdot \mathbf{s}_1 \\ \mathbf{m}_0 + \mathbf{C} \cdot \mathbf{s}_1 \geq \mathbf{Pre} \cdot \mathbf{s}_2 \\ \dots \\ \mathbf{m}_0 + \mathbf{C} \cdot \sum_{i=1}^{J-1} \mathbf{s}_i \geq \mathbf{Pre} \cdot \mathbf{s}_J \\ \mathbf{m}_0 + \mathbf{C} \cdot \sum_{i=1}^{J} \mathbf{s}_i \geq \mathbf{0} \\ \sum_{i=1}^{J} \mathbf{s}_i(t_L) = \hat{\boldsymbol{\sigma}}(t_L) \\ \sum_{i=1}^{J} \mathbf{s}_i(t_H) = \sum_{i=1}^{J} \bar{\boldsymbol{\sigma}}_i(t_H) \end{cases} \tag{3}$$

admits a solution $\mathbf{s}_1, \dots, \mathbf{s}_J \in \mathbb{N}^{n_L+n_H}$ **for all** $t_L \in L$ **and** $t_H \in H$, with $J \geq \mathcal{J}_{\min}$, $\hat{\boldsymbol{\sigma}}$ being equal to the solution of (2), and $\bar{\boldsymbol{\sigma}}_1, \dots, \bar{\boldsymbol{\sigma}}_J$ equal to the solution of the ILP problem

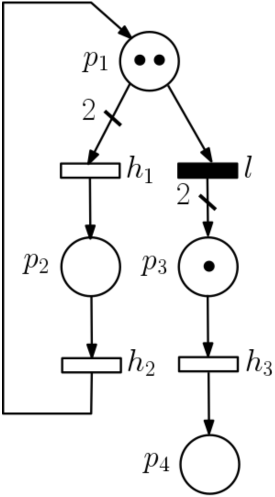
$$\begin{aligned} &\max \sum_{i=1}^{J} \boldsymbol{\sigma}_i(t_H) \\ &\text{s.t.} \\ &\begin{cases} \mathbf{m}_0 \geq \mathbf{Pre} \cdot \boldsymbol{\sigma}_1 \\ \mathbf{m}_0 + \mathbf{C} \cdot \boldsymbol{\sigma}_1 \geq \mathbf{Pre} \cdot \boldsymbol{\sigma}_2 \\ \dots \\ \mathbf{m}_0 + \mathbf{C} \cdot \sum_{i=1}^{J-1} \boldsymbol{\sigma}_i \geq \mathbf{Pre} \cdot \boldsymbol{\sigma}_J \\ \mathbf{m}_0 + \mathbf{C} \cdot \sum_{i=1}^{J} \boldsymbol{\sigma}_i \geq \mathbf{0} \\ \boldsymbol{\sigma}_i \in \mathbb{N}^{n_L+n_H}, \quad i = 1, 2, \dots, J \end{cases} \end{aligned} \tag{4}$$

Example (I)



- By setting $J = 5$, the solution of (2) for the transition $l_1 \in L$ returns $\hat{\sigma}(l_1) = 1$
- Given the solution $\hat{\sigma}$, also the feasibility problem (1) admits a solution
- The net system is NOT SNNI
- The time needed to solve a single instance of the ILP problem (2) and of the feasibility problem (1) is less the 500 μs using GLPK on a MacBook Pro equipped with an Intel® i5 at 3.1 GHz and with 16 GB of RAM

Example (II)



- By setting $J = 5$, in this case (1) does not admit any solution for any $t \in L$,
- The net system is SNNI
- The feasibility problem (3) does not admit a solution as well
- The firing of l prevents the firing of the two high level transitions
- The net system is NOT BSNNI
- About 2 ms are needed to check both SNNI and BSNNI on the considered hardware

Conclusions

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  - **non-interference enforcing** (submitted to the next CDC)

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  - **non-interference enforcing** (submitted to the next CDC)
  - **algebraic characterization of opacity in PNs** (WODES 2018)

F. Basile and G. De Tommasi,
An algebraic characterization of language-based opacity in labeled Petri nets,
WODES'18, Sorrento Coast, Italy, May 2018

Questions?