Input-output finite-time stabilization of LTV systems via dynamic output feedback

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Joint 50th IEEE Conference on Decision and Control & European Control Conference
December 12–15, 2011, Orlando, Florida

Outline

Input-output finite-time stability vs classic IO stability

IO stability

A system is said to be IO $\mathcal{L}_p$-stable if for any input of class $\mathcal{L}_p$, the system exhibits a corresponding output which belongs to the same class

IO-FTS

A system is defined to be IO-FTS if, given a class of norm bounded input signals over a specified time interval T, the outputs of the system do not exceed an assigned threshold during T

Main features of IO-FTS

IO-FTS:

- involves signals defined over a finite time interval
- does not necessarily require the inputs and outputs to belong to the same class
- specifies a *quantitative* bounds on both inputs and outputs

IO stability and IO-FTS are independent concepts

IO-FTS

The concept of IO-FTS has been first introduced in the framework of LTV systems

F. Amato, R. Ambrosino, G. De Tommasi, C. Cosentino
Input-output finite-time stabilization of linear systems
Automatica, 2010

and then it has been extended to impulsive dynamical linear systems

F. Amato, G. Carannante, G. De Tommasi
Input-output finite-time stabilisation of a class of hybrid systems via static output feedback
International Journal of Control, 2011

IO-FTS and state FTS

The definition of IO-FTS is fully consistent with the definition of (state) FTS, where the state of a zero-input system, rather than the input and the output, are involved.

P. Dorato
Short time stability in linear time-varying systems
Proc. IRE Int. Convention Record Pt. 4, 1961

F. Amato, M. Ariola, P. Dorato
Finite-time control of linear systems subject to parametric uncertanties and disturbances
Automatica, 2001

Contribution of the paper

In this paper we provide two conditions for the input-output finite-time stabilization of LTV systems via dynamic output feedback.

In particular:

- a (necessary and) sufficient condition is given when the input signals belong to $\mathcal{L}_2$
- a sufficient condition is provided when the inputs belong to $\mathcal{L}_\infty$

Notation

- $\mathcal{L}_p$ denotes the space of vector-valued signals whose p-th power is absolutely integrable over $[0, +\infty)$.
- The restriction of $\mathcal{L}_p$ to $\Omega := [t_0\,, t_0 + T]$ is denoted by $\mathcal{L}_p(\Omega)$.
- Given the time interval Ω, a symmetric positive definite matrix-valued function $R(\cdot)$, bounded on Ω, and a vector-valued signal $s(\cdot) \in \mathcal{L}_p(\Omega)$, the weighted signal norm

$$\left(\int_\Omega \left[s^T(\tau) R(\tau) s(\tau) \right]^{\frac{p}{2}} d\tau \right)^{\frac{1}{p}} ,$$

will be denoted by $\|s(\cdot)\|_{p,R}$. If $p = \infty$

$$\|s(\cdot)\|_{\infty,R} = \operatorname*{ess\,sup}_{t \in \Omega} \left[s^T(t) R(t) s(t) \right]^{\frac{1}{2}} .$$

IO-FTS of LTV systems

Given a positive scalar T, a class of input signals $\mathcal{W}$ defined over $\Omega = [t_0 \ , \ t_0 \ + \ T]$, a positive definite matrix-valued function $Q(\cdot)$ defined in Ω, system

$$\dot{x}(t) = A(t)x(t) + G(t)w(t)\,, \quad x(t_0) = 0 \tag{1a}$$

$$y(t) = C(t)x(t) \tag{1b}$$

is said to be IO-FTS with respect to $\big(\mathcal{W}\,, Q(\cdot)\,, \Omega\big)$ if

$$w(\cdot) \in \mathcal{W} \ \Rightarrow \ y^T(t)Q(t)y(t) < 1\,, \quad t \in \Omega\,.$$

IO finite-time stabilization via output feedback

Problem 1

Consider the LTV system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + G(t)w(t)\,, \quad x(t_0) = 0 \tag{2a}$$
$$y(t) = C(t)x(t) \tag{2b}$$

where $u(\cdot)$ is the control input and $w(\cdot)$ is the exogenous input. Given a class of disturbances $\mathcal{W}$ defined over Ω, and a positive definite matrix-valued function $Q(\cdot)$ defined over Ω, find a dynamic output feedback controller in the form

$$\dot{x}_c(t) = A_K(t)x_c(t) + B_K(t)y(t)\,, \tag{3a}$$
$$u(t) = C_K(t)x_c(t) + D_K(t)y(t) \tag{3b}$$

where $x_c(t)$ has the same dimension of $x(t)$, such that the closed loop system obtained by the connection of (2) and (3) is IO-FTS with respect to $\big(\mathcal{W}\,, Q(\cdot)\,, \Omega\big)$.

Considered class of input signals

$\mathcal{W}_2$ signals

Norm bounded square integrable signals over Ω, defined as follows

$$\mathcal{W}_2\,(\Omega\,,R(\cdot)) := \{w(\cdot) \in \mathcal{L}_2(\Omega)\ :\ \|w\|_{2,R} \leq 1\}\ .$$

$\mathcal{W}_\infty$ signals

Uniformly bounded signals over Ω, defined as follows

$$\mathcal{W}_\infty\,(\Omega\,,R(\cdot)) := \{w(\cdot) \in \mathcal{L}_\infty(\Omega)\ :\ \|w\|_{\infty,R} \leq 1\}\,.$$

Preliminary results

- Two sufficient conditions to check IO-FTS of system (1) have been presented in Amato et al. Automatica, 2010.
- These two conditions have been used in this work to solve the problem of IO finite-time stabilization via dynamic output feedback

IO-FTS of LTV systems for $\mathcal{W}_2$ inputs

Theorem 1

If there exists a continuously differentiable positive definite solution $P(\cdot)$ such that

$$\dot{P}(t) + A(t)^T P(t) + P(t)A(t) + P(t)G(t)R^{-1}(t)G(t)^T P(t) < 0 \quad (4a)$$
$$P(t) \geq C(t)^T Q(t) C(t) \quad (4b)$$

are satisfied in the time interval Ω, then the LTV system (1) is IO-FTS with respect to $\left(\mathcal{W}_2\,, Q(\cdot)\,, \Omega\right)$.

IO-FTS of LTV systems for $\mathcal{W}_\infty$ inputs

Theorem 2

Let $\widetilde{Q}(t) = t\,Q(t)$. If there exists a continuously differentiable positive definite solution $P(\cdot)$ such that (4a) and

$$P(t) \geq C(t)^T \widetilde{Q}(t) C(t)\,, \quad \forall\ t \in \Omega \tag{5}$$

are satisfied in the time interval Ω, then LTV system (1) is IO-FTS with respect to $\big(\mathcal{W}_\infty\,, Q(\cdot)\,, \Omega\big)$.

$\mathcal{W}_2$ signals

Theorem 3

Given the exogenous input $w(t) \in \mathcal{W}_2$, Problem 1 is solvable if there exist two continuously differentiable symmetric matrix-valued functions $T(\cdot)$, $S(\cdot)$, a nonsingular matrix-valued function $N(\cdot)$ and matrix-valued functions $\hat{A}_K(\cdot)$, $\hat{B}_K(\cdot)$, $\hat{C}_K(\cdot)$ and $D_K(\cdot)$ such that the following DLMIs are satisfied (the time argument is omitted for brevity)

$$\begin{pmatrix} \Theta_{11} & \Theta_{12} & 0 \\ \Theta_{12}^T & \Theta_{22} & SG \\ 0 & G^T S & -R \end{pmatrix} < 0\,, \quad t \in \Omega \tag{6a}$$

$$\begin{pmatrix} \Psi_{11} & \Psi_{12} & 0 \\ \Psi_{12}^T & Q & QC^T \\ 0 & CQ & Q^{-1} \end{pmatrix} \geq 0\,, \quad t \in \Omega \tag{6b}$$

$\mathcal{W}_2$ signals (cont'd)

Theorem 3 (cont'd)

where

$$\Theta_{11} = -\dot{T} + AT + TA^T + B\hat{C}_K + \hat{C}_K^T B^T + GR^{-1}G^T$$
$$\Theta_{12} = A + \hat{A}_K^T + BD_K C + GR^{-1}G^T S$$
$$\Theta_{22} = \dot{S} + SA + A^T S + \hat{B}_K C + C^T \hat{B}_K^T$$
$$\Psi_{11} = S - C^T QC$$
$$\Psi_{12} = I - C^T QCT$$

Controller Design

If the hypotheses of Theorem 3 are satisfied, the following procedure has to be followed in order to design the controller:

1. Find $T(\cdot)$, $S(\cdot)$, $\hat{A}_K(\cdot)$, $\hat{B}_K(\cdot)$, $\hat{C}_K(\cdot)$ and $D_K(\cdot)$ such that (6) are satisfied.
2. Let $M(t) = (I - S(t)T(t))N^{-T}(t)$.
3. Obtain $A_K(\cdot)$, $B_K(\cdot)$ and $C_K(\cdot)$ by inverting the following equations

$$\begin{pmatrix} T & I \\ I & S \end{pmatrix} > 0 \tag{7a}$$

$$\hat{B}_K = MB_K + SBD_K \tag{7b}$$

$$\hat{C}_K = C_K N^T + D_K CT \tag{7c}$$

$$\begin{aligned} \hat{A}_K = {} & \dot{S}T + \dot{M}N^T + MA_K N^T + SBC_K N^T \\ & + MB_K CT + S\big(A + BD_K C\big)T \,. \end{aligned} \tag{7d}$$

$\mathcal{W}_\infty$ signals

A sufficient condition for IO finite-time stabilization via dynamic output feedback when $\mathcal{W}_\infty$ signals (Theorem 4 in the paper) can be obtained by letting $\widetilde{Q}(t) = t\,Q(t)$, and by solving the same DLMIs as in Theorem 3

Example - 1

Let us consider the second order unstable LTV system defined by

$$A = \begin{pmatrix} 0.5 + t & 0.1 \\ 0.4 & -0.3 + t \end{pmatrix}, B = \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

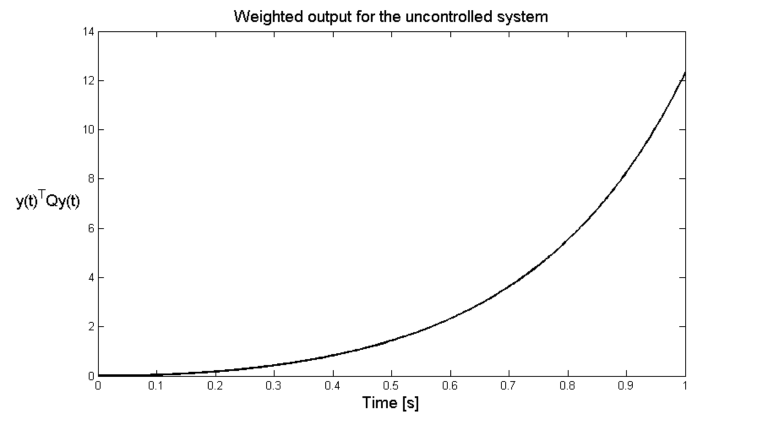
$$G = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 1 \end{pmatrix}.$$

and let

$$R = 1,\ Q = 1,\ \Omega = [0, 1].$$

Example - 2

By means of simulation it is easy to check that the considered LTV is IO finite-time unstable, when $\mathcal{W}_\infty$ disturbances are considered.

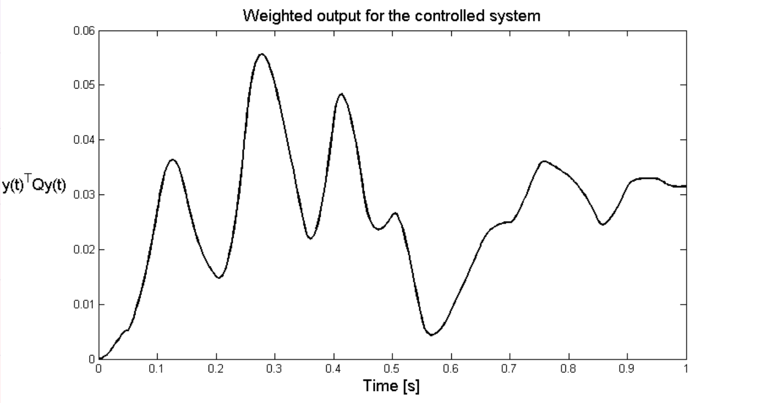


Example - 3

- In order to recast the DLMIs in terms of LMIs, the matrix-valued functions $T(\cdot)$ and $S(\cdot)$ have been assumed piecewise linear.
- The time interval Ω is divided in $n = T/T_s$ subintervals.
- Such a piecewise linear functions can approximate a generic continuous matrix-valued functions with adequate accuracy, provided that the length of T_s is sufficiently small.

Example - 4

Exploiting standard optimization tools such as the Matlab LMI Toolbox or TOMLAB, it is possible to find the matrix-valued functions $A_k(\cdot)\,, B_k(\cdot)\,, C_k(\cdot)\,, D_k(\cdot)$ that solve Problem 1



Conclusions

- Sufficient conditions for input-output finite-time stabilization of LTV systems via dynamic output feedback have been given
- The two classes of input signals $\mathcal{W}_2$ and $\mathcal{W}_\infty$ have been considered
- The effectiveness of the approach has been illustrated by means of numerical examples
- Further work is ongoing with the aim to add constraints on the control inputs

Thank you!