Vehicle collision avoidance via control over a finite-time horizon

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Outline

Vehicle lateral collision avoidance

- The primary goal of collision avoidance system is to move the vehicle *as fast as possible* from an *unsafe* region to a *safe* one
- Once the vehicle is within the safe region a *nominal controller* can take over
- In our work we proposed a strategy based on control over a *finite-time horizon* for vehicle lateral collision avoidance
  - The finite-time strategy is used to bring the vehicle from the unsafe to the safe region
  - The proposed control approach is based only on steering (no braking)
  - The single vehicle case (subject to external disturbances) is considered in this work

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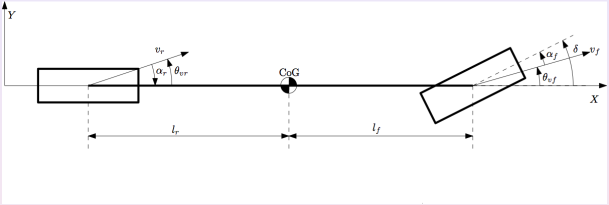
The bicycle model



- The behaviour of the vehicle is described by a 2-DoF model of a *bicycle* (a two-wheel model)
- The *bicycle model* is commonly used to describe the behaviour of a four-wheel vehicle, when the differences between the running condition of the left and the right side can be ignored
- The bicycle model describes both the lateral and yaw motions of the two-wheel vehicle (no roll)

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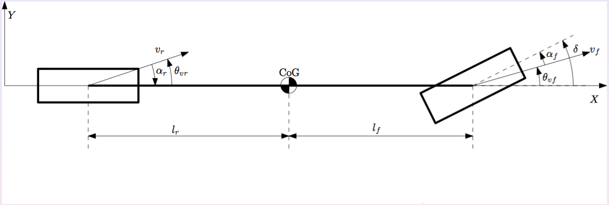

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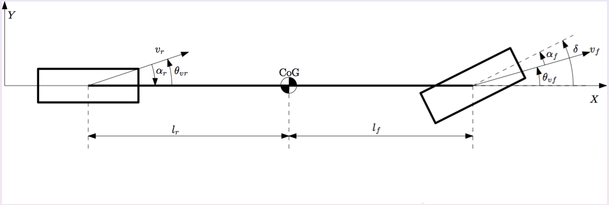

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Bicycle linear model

Letting

$$x = \begin{pmatrix} \dot{y} & \omega & \psi & Y \end{pmatrix}^T, \quad u = \delta$$

where

- $\dot{y}$ is the lateral velocity
- ψ and $\omega = \dot{\psi}$ are the yaw and yaw rate
- Y is the lateral displacement of the CoG (wrt the ground-fixed axes)
- δ is the steering angle of the front wheel (control input)

Bicycle linear model

$$\dot{x} = \begin{pmatrix} -\frac{C_{\alpha f}+C_{\alpha r}}{Mv_x} & -\frac{C_{\alpha f}l_f - C_{\alpha r}l_r}{Mv_x} - v_x & 0 & 0 \\ -\frac{C_{\alpha f}l_f - C_{\alpha r}l_r}{Iv_x} & -\frac{C_{\alpha f}l_f^2 + C_{\alpha r}l_r^2}{Iv_x} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & v_x & 0 \end{pmatrix} x + \begin{pmatrix} \frac{C_{\alpha f}}{M} \\ \frac{C_{\alpha f}l_f}{I} \\ 0 \\ 0 \end{pmatrix} u$$

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Model outputs and external disturbance

- The selected output vector z (corresponding monitored/constrained variables) is set equal to

$$z = \begin{pmatrix} \dot{y} & \omega & Y \end{pmatrix}^T ,$$

- The exogenous input w is included to model the effect of the *side wind* on the lateral position Y
- Denoting with η the efficiency of the side wind in moving the vehicle along the lateral direction, and

$$F = \begin{pmatrix} 0 & 0 & 0 & \eta \end{pmatrix}^T ,$$

the model becomes

$$\dot{x} = Ax + Fw + Bu \tag{1a}$$

$$z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} x \tag{1b}$$

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Structured Input-Output Finite-Time Stability - 1/2

Consider:

1. the system

$$\dot{x} = Ax + Fw + Bu\,, \quad x(0) = 0 \tag{2a}$$

$$z = Cx\,, \tag{2b}$$

2. a α-tuple of integer numbers $m_1, \dots, m_\alpha$, where $1 < \alpha < m$, with $\sum_{i=1}^{\alpha} m_i = m$, and the partition the output vector (2b)

$$z(t) = \left(z_1^T(t) \cdots z_\alpha^T(t) \right)^T , \quad t \in \Omega\,. \tag{3}$$

note: (3) induces the following partition on the C matrix

$$C = \left(C_1^T \cdots C_\alpha^T \right)^T .$$

3. α positive definite weighting matrices $Q_i(t) \in \mathbb{R}^{m_i \times m_i}$, and

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Structured Input-Output Finite-Time Stability - 2/2

Definition (Structured IO-FTS)

Given a time interval $\Omega = [0\,,\,T]$ with $T\ >\ 0$, a class of disturbances $\mathcal{W}$ defined over Ω, the output partition (3) and the corresponding positive definite weighting matrix $Q(\cdot)$ defined in (4), system (2) is said to be structured IO-FTS with respect to $(\Omega\,,\mathcal{W}\,,Q(\cdot))$ if

$$w(\cdot) \in \mathcal{W} \;\Rightarrow\; z_i^T(t)Q_i(t)z_i(t) < 1\,,\ t \in \Omega\,,\ i = 1,\ldots,\alpha\,.$$

▲

Constraining the control input using structured IO-FTS - 1/2

1. Given a β-tuple of integer numbers $p_1, \ldots, p_\beta$, where $\sum_{i=1}^{\beta} p_i = p$, let us partition the control input vector as

$$u(t) = \left(u_1^T(t) \cdots u_\beta^T(t) \right)^T , \quad t \in \Omega . \tag{5}$$

2. Consider β positive definite weighting matrices $T_i(t) \in \mathbb{R}^{q_i \times q_i}$, $i = 1, \ldots, \beta$, and

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Constraining the control input using structured IO-FTS - 2/2

Problem

Given a positive time interval $\Omega = [0\,, T]$ *with* $T > 0$, *a class of disturbances* $\mathcal{W}$ *defined over* Ω, *the output partition* (3), *the input partition* (5), *and the corresponding positive definite weighting matrices* $Q(\cdot)$, $T(\cdot)$ *defined in* (4) *and* (6), *find a time-varying state feedback control law*

$$u(t) = K(t)x(t)\,,$$

where $K(\cdot):\ \Omega \mapsto \mathbb{R}^{p\times n}$, *such that the closed-loop system*

$$\dot{x}(t) = (A + BK(t))\,x(t) + Fw(t)\,, \quad x(t_0) = 0 \tag{7a}$$

$$\begin{aligned} \begin{pmatrix} z(t) \\ u(t) \end{pmatrix} &= \begin{pmatrix} C \\ K(t) \end{pmatrix} x(t) \\ &= \begin{pmatrix} C_1 & \dots & C_\alpha & K_1(t) & \dots & K_\beta(t) \end{pmatrix}^T x(t)\,, \end{aligned} \tag{7b}$$

is structured IO-FTS wrt $(\Omega\,, \mathcal{W}\,, (Q(\cdot), T(\cdot)))$. ▲

Structured IO-FTS for norm bounded signals over Ω

Given the class of signals $\mathcal{W}_\infty$, Problem 2 is solvable *if* there exist a positive definite continuously differentiable matrix-valued function $\Pi(\cdot)$ and β matrix-valued functions $L_1(\cdot), \ldots, L_\beta(\cdot)$ such that

$$\begin{pmatrix} \Theta(t) & F \\ F^T & -R(t) \end{pmatrix} < 0\,, \tag{8a}$$

$$\begin{pmatrix} \Pi(t) & \Pi(t)C_i^T \\ C_i\Pi(t) & \widetilde{\Xi}_i(t) \end{pmatrix} \geq 0\,, \quad i = 1, \ldots, \alpha \tag{8b}$$

$$\begin{pmatrix} \Pi(t) & L_j^T(t) \\ L_j(t) & \widetilde{\Upsilon}_j(t) \end{pmatrix} \geq 0\,, \quad j = 1, \ldots, \beta\,, \tag{8c}$$

for all $t \in \Omega$, where

$$\begin{aligned} \Theta(t) &:= -\dot{\Pi}(t) + \Pi(t)A^T + A\Pi(t) \\ &\quad + B\left(L_1^T(t) \cdots L_\beta^T(t)\right)^T + \left(L_1^T(t) \cdots L_\beta^T(t)\right)B^T \\ \widetilde{\Xi}_i(t) &:= (tQ_i(t))^{-1} \\ \widetilde{\Upsilon}_j(t) &:= (tT_j(t))^{-1}\,. \end{aligned}$$

A controller which solves Problem 2 is given by $K_j(t) = L_j(t)\Pi^{-1}(t)\,, \quad j = 1, \ldots, \beta$.

Model parameters

Table : Two-wheel model parameters

Mass of the vehicle M (kg)	1200
Yaw inertia I (kg$\cdot$ m^2)	1500
Front cornering stiffness $C_{\alpha f}$ (N/rad)	125000
Rear cornering stiffness $C_{\alpha r}$ (N/rad)	80000
Distance from the CoG and the front axle l_f (m)	0.92
Distance from the CoG and the rear axle l_r (m)	1.38
Longitudinal velocity v_x (m/s)	22
Side wind efficiency η	0.15

Constrains

- In order to do not worse too much the passengers comfort when collision avoidance system is active, the lateral velocity $\dot{y}$ and the yaw rate ω are limited as follows

$$|\dot{y}|_{\text{max}} = 1.4 \text{ m/s } (\sim\ 5 \text{ km/h}),$$
$$|\omega|_{\text{max}} = 0.26 \text{ rad/s } (\sim\ 15 \text{ deg/s}),$$

  yielding the following choice for Q_1 and Q_2

$$Q_1 = 0.51, \quad Q_2 = 14.80.$$

- In order to limit the maximum steering angle as $|\delta|_{\text{max}} = 0.35$ rad ($\sim$ 20 deg), the corresponding weight is set equal to $T_1 = 8.16$.

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Controller synthesis

- The time-varying state feedback control law that guarantees lateral vehicle collision avoidance can computed from the solution of the following optimization problem with DLMI constraints

$$\max_{s.t.\ (8)} Q_3\,. \tag{9}$$

- This optimization problem can be efficiently solved by first recasting the DLMIs into standard LMIs

Wind gust

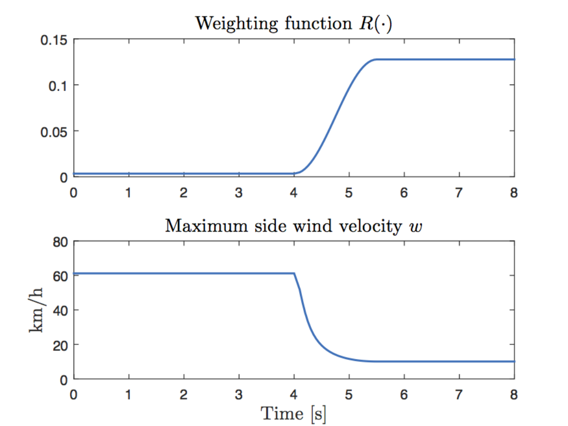

Figure : Input weighting function for the side wind and maximum considered wind velocity.

Simulation results - 1/2

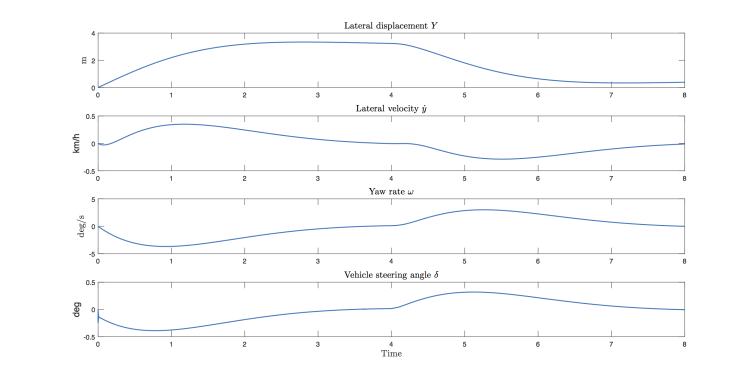

Figure : Simulation results obtained adopting the LTV state feedback as lateral collision avoidance system. for the bicycle model.

Simulation results - 2/2

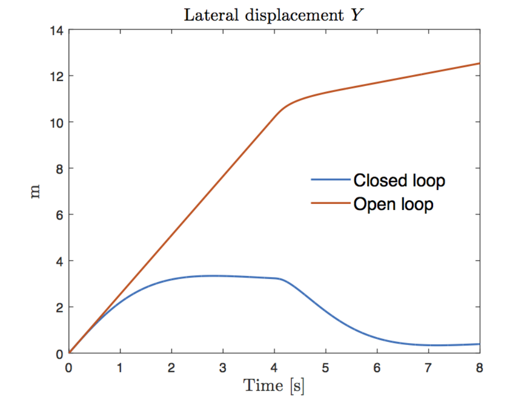

Figure : Comparison of the vehicle lateral displacement Y for both the open loop and the closed loop system.

Conclusion & future works

- A state feedback controller based on the concept of IO-FTS has been proposed as possible solution for vehicle lateral collision avoidance
- Possible future works:
  - implement a hybrid controller that switches from a standard (nominal) controller to the time-varying controller designed using IO-FTS
  - extend the approach to a platoon of vehicles with the aim of maintaining the minimum distance between the vehicles above a safety value even in the presence of disturbances.

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Solving DLMIs - 1/2

- If $\Pi(t)$ is the optimization matrix-valued function in (8), we sample $\Pi(t)$ in the time interval Ω with a sampling time equal to T_s
- In the k-th time interval we can approximate $\Pi(t)$ as follows

$$\Pi(t) = \Pi_k + \Xi_k \left(t - (k-1)\,T_s\right) + \Upsilon_k \left(t - (k-1)\,T_s\right)^2 , \quad t \in [(k-1)\,T_s\,, kTs) , \quad (10)$$

with $k = 1\,, 2\,, \ldots$.

Solving DLMIs - 2/2

When using the proposed approximation, the following additional constraints should be added in order to assure both the continuity of $\Pi(t)$ and of its derivative in $t_k = kT_s$

$$\Pi^{-}(kT_s) = \Pi_k + \Xi_k T_s + \Upsilon_k T_s^2 = \Pi_{k+1} = \Pi^{+}(kT_s)\,,$$

while, for the continuity of the derivative of $P(t)$, it should be

$$\dot{\Pi}^{-}(kT_s) = \Xi_k + 2\Upsilon_k T_s = \Xi_{k+1} = \dot{\Pi}^{+}(kT_s)\,,$$